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Swing-up controller design for cart-type double inverted pendulum

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Abstract: This paper presents experimental results of swing-up controller designs for a parallel cart-type double inverted pendulum and a serial cart-type double inverted pendulum. The proposed control scheme for the two kinds of cart-type double inverted pendulum involves two steps: 1) to swing up the two pendulums, 2) to stabilize the two pendulums around the unstable equilibrium state. Since the pendulums are cart-type, this paper gives an analysis of traveling distance of the cart and the proposed controller also includes a controller to restrict the traveling distance of the cart. Experiments presented include the above two steps for each cart-type double inverted pendulum.

Key words: parallel (serial) cart-type double inverted pendulum system; swing-up control; distance control of cart
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小车二级摆摆起控制器设计

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摘要: 研究了小车二级并行摆系统及小车二级串行摆系统的摆起控制器设计问题,并给出了这两种系统的实验结果.首先,针对上述两种系统,设计了两步控制器,即 1)摆起双摆达到倒立稳摆位置的控制器,2)进行稳摆控制的控制器.其次,由于小车二级摆位移受轨道长度限制,又考虑了小车位移的控制问题.上述两种实际系统的摆起及稳摆成功,验证了所提出设计方法的有效性.

关键词: 小车二级并行(串行)摆系统; 摆起控制; 小车位移控制

1 Introduction

Inverted pendulum systems are nonlinear and well known for evaluating the validation of various kinds of control theories. Among inverted pendulum systems, much attention has been paid to the control of a (parallel or serial) double pendulum, because of their strong underactuatedness and nonlinearity. From the control engineering point of view, there are two control problems for the inverted pendulum: 1) swing-up control of pendulum from the downward position to the upward unstable equilibrium position, and 2) stabilizing control of pendulum at the upward unstable equilibrium position. In the swing-up control, the angle of pendulum changes dramatically from 180 degree at downward position to 0 degree at upward position, and the effect of non-linearity appears so large that the nonlinear control scheme has to be adopted. The swing-up control has attracted the attention from many researchers. First, an energy method is proposed and it implemented the swing-up of a single inverted pendulum experimentally[1]. Then the swing-up control of a rotationally driven serial double inverted pendulum is implemented experimentally by using the energy method plus an optimal control[2]. The basic idea of the control scheme is to consider the joint position of the two pendulums as the position of the virtual cart supporting the second pendulum and to generate the input to the virtual cart by controlling the first pendulum. As for the stabilizing control, first, a single inverted pendulum is controlled successfully by linearizing around the upward unstable equilibrium position and using a linear controller[3]. Then, the swing up controls of a serial double inverted pendulum[4] which consists of two pendulums connected serially and a parallel inverted pendulum are tried[5]. These ideas can not be applied to cart-type double inverted pendulum where the pendulums are placed on a linearly moving single cart. Hence in this paper, swing-up control schemes of the parallel cart-type double inverted pendulum and the serial cart-type double inverted pendulum are considered by using the experimental systems in our laboratory.

For control of cart-type inverted pendulum system, the control problem can be addressed by two approaches, namely, control of swing-up the pendulum and control of the traveling position of the cart. So far some control schemes for the above control problems are proposed. With regard to the swing-up control, a method is presented and is to swing up pendulums one by one sequentially[6]. That is, the design scheme given by[6] utilizes, after

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the first pendulum has been swung up, the robustness of the sliding mode controller to stabilize the first pendulum while the second pendulum is in swinging up. In the case of the energy based control for the parallel cart-type double inverted pendulum, simulation study is presented for the convergence of the energies of the two pendulums[7]. There are also several papers[8,9] to restrict cart position, but all of them are for a single inverted pendulum. Hence, this paper considers swinging up control of two kinds of cart-type double inverted pendulum and restricting moving position of the cart. The paper addresses the experimental study of swing-up control of cart-type double inverted pendulum. Namely, swing-up control of the two pendulums of a parallel cart-type double inverted pendulum simultaneously, and swing-up control of the serial cart-type double inverted pendulum one by one are considered.

The remainder of this paper is organized as follows: Section 2 presents a brief description of the two kinds of double cart-type inverted pendulum experimental systems. Meanwhile, dynamics equations are derived for the two experimental systems. Swing-up control algorithm for the two systems is presented in Section 3. The section also provides the details on a scheme to restrict the traveling position of the cart for the cart-type double inverted pendulum. Finally, Section 4 presents the results from the swing-up experiments with the two kinds of inverted pendulums.

2 The two kinds of cart-type double inverted pendulum systems

In this paper, the parallel cart-type double inverted pendulum and the serial cart-type double inverted pendulum are considered. Fig. 1 is an illustration of the parallel cart-type double inverted pendulum system, and Fig. 2 is an illustration of the serial cart-type double inverted pendulum system. A picture of a cart for the serial double inverted! pendulum experiment is shown in Fig. 3.

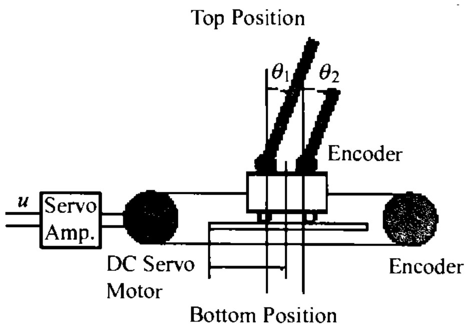

Fig. 1 Parallel cart-type double inverted pendulum system

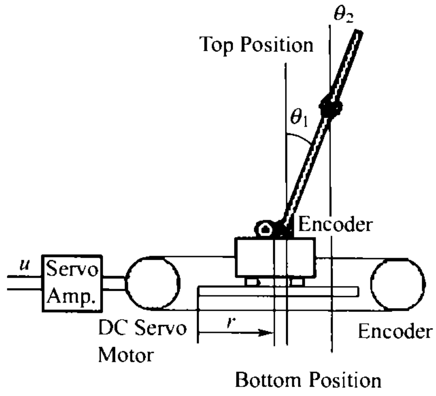

Fig. 2 Serial cart-type double inverted pendulum system

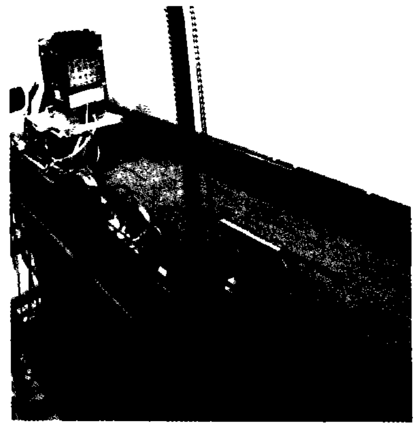

Fig. 3 Cart on the rail

2.1 Dynamics of the parallel cart-type double inverted pendulum

The dynamical equation for the parallel cart-type double inverted pendulum in Fig. 1 is given

$$\ddot{\theta}_i = \frac{m_i g l_i}{I_i}\sin\theta_i - \frac{m_i l_i}{I_i}\ddot{r}\cos\theta_i - \frac{c_i}{I_i}\dot{\theta}_i, \qquad (1)$$

where $i = 1,2$ and the following notations are used through the paper:

r – the displacement of the cart [m];

L – the length of the first pendulum [m];

θ_i – the angle from the upright position of the pendulums [rad];

l_i – the length from the pivot to the center of mass of the pendulums [m];

m_i – the mass of the pendulums [kg];

c_i – the friction of the pendulums around the pivot;

n_i – the mass of the pendulum's joint [kg];

I_i – the moment of inertia of the pendulums at the pivot [kgm^2];

g – the acceleration of gravity [m/s^2];

J_i – the moment of inertia of the pendulums at the pivot [kgm^2];

J_{n_i} – the moment of inertia of the pendulums arourd its center of mass $[\mathrm{kgm}^2]$.

where, the mass of the pendulum's joint is negligible. The control problem is to swing up the both pendulums from downward position ($\theta_1, \theta_2 = \pi$) to upward position ($\theta_1, \theta_2 = 0$) simultaneously and to stabilize the both pendulums at the upward unstable position (see Fig. 4), under the consideration of restricting the cart traveling position.

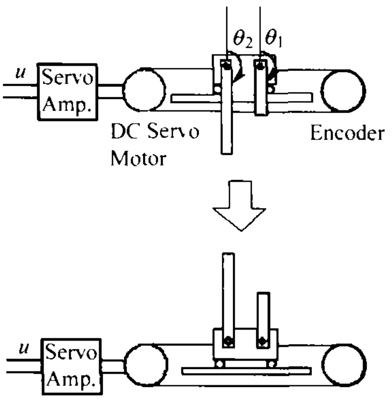

Fig. 4 Illustration of the control problem for the parallel cart-type double inverted pendulum

2.2 Dynamics of the serial cart-type double inverted pendulum

From Fig. 2, the dynamical equation for the pendulums can be obtained by using Lagrange equation as

$$\ddot{\theta}_1 = -\frac{b_2}{b_1}\ddot{r}\cos\theta_1 + \frac{b_2}{b_1}g\sin\theta_1 - \frac{c_1 + c_2}{b_1}\dot{\theta}_1 + \frac{c_2}{b_1}\dot{\theta}_2 - \frac{m_2 l_2 L}{b_1}\sin(\theta_1 - \theta_2)\dot{\theta}_2^2 - \frac{m_2 l_2 L}{b_1}\cos(\theta_1 - \theta_2)\ddot{\theta}_2, \quad (2)$$

$$\ddot{\theta}_2 = -\frac{m_2 l_2}{b_3}\ddot{r}\cos\theta_2 + \frac{b_2 l_2}{b_3}g\sin\theta_2 + \frac{c_2}{b_3}(\dot{\theta}_1 - \dot{\theta}_2) + \frac{m_2 l_2 L}{b_3}\sin(\theta_1 - \theta_2)\dot{\theta}_1^2 - \frac{m_2 l_2 L}{b_3}\cos(\theta_1 - \theta_2)\ddot{\theta}_1, \quad (3)$$

$$b_1 = J_1 + m_1 l_1^2 + m_2 L^2 + n_2 L^2 + J_{n_1}, \quad (4)$$

$$b_2 = m_1 l_1 + m_2 L + n_2 L, \quad (5)$$

$$b_3 = J_2 + J_{n_2} + m_2 l_2^2. \quad (6)$$

The control problem for the serial double pendulum is the swing up and stabilization in the same way as for the parallel double pendulum (see Fig. 5).

3 Swing-up controller design for the two kinds of cart-type double inverted pendulum systems

In this section, we discuss the swing-up control of the two systems. A step response in preliminary experiment

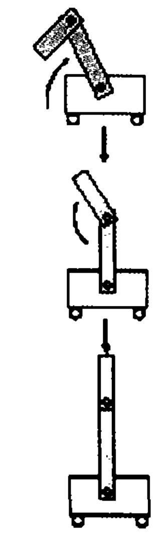

Fig. 5 Illustration of the control problem for the serial cart-type double inverted pendulum

shows that the response of the cart to the input is faster than that of pendulums, so that the equation for the displacement of the carts r [m] in the two systems is approximated as

$$\ddot{r} = aU. \quad (7)$$

where, U is the acceleration command input to the amplifier of the servo motor, the coeffcient a is the ratio of the attained acceleration $[\mathrm{m/s^2}]$ to the acceleration command input. This approximation is justified by the fact that in the experiment a suffciently powerful DC servo motor is used and the cart velocity follows the velocity command input instantaneously. By our preliminary experiments, $a = 0.32$ for the parallel cart-type double inverted pendulum system and $a = 0.232$ for the serial cart-type double inverted pendulum system are used. In the following, energy-based swing-up controller design for the parallel cart-type double inverted pendulum is considered. The design method is based on the energy control by using a Lyapunov function. An additional controller is also proposed to restrict the cart travel while swinging up the two pendulums. A bound of the traveling distance of the cart is given mathematically.

3.1 Swing-up controller design for the parallel cart-type double inverted pendulum system

To derive the control law to swing up the two inverted pendulums, a Lyapunov function is defined:

$$V = \frac{1}{2}w_1(E_{10} - E_1)^2 + \frac{1}{2}w_2(E_{20} - E_2)^2, \quad (8)$$

where $w_i > 0$ is a weighting factor and E_i is the total of kinetic energy of ith pendulum,

$$E_i = m_i g l_i \cos\theta_i + \frac{1}{2}I_i\dot{\theta}_i^2 \quad (9)$$

and $E_{i0} = m_i g l_i$, the energy at the upright unstable equilibrium position ($\theta_i = 0, \dot{\theta}_i = 0$). Thus, the function (8) is positive except at the upright position.

The control input is given by

$$U = -u_a \operatorname{sgn}(D) \tag{10}$$

where $u_a > 0$ is a weighting factor and sgn($\cdot$) is the sign function,

$$\operatorname{sgn}(D) = \begin{cases} 1, & D > 0, \\ 0, & D = 0, \\ -1, & D < 0 \end{cases} \tag{11}$$

and D is designed so that $\dot{V} < 0$, that is,

$$D = w_1(E_{10} - E_1) m_1 l_1 \dot{\theta}_1 \cos\theta_1 + w_2(E_{20} - E_2) m_2 l_2 \dot{\theta}_2 \cos\theta_2. \tag{12}$$

Using formulas (1),(7),(9) and (10), we obtain

$$\dot{V} = -a u_a \operatorname{sgn}(D) D \leqslant 0. \tag{13}$$

When D is not equal to 0, the Lyapunov function (8) is decreasing with respect to time and converges on zero. In other words, E_1 and E_2 are increasing gradually, and the energy of pendulums converges to E_{10} and E_{20}, respectively. Hence, by using this control input, two pendulums swing up and approach a homoclinic orbit[10], on which the energy of the pendulums is kept at E_{10} and E_{20}.

For some thresholds θ_{10} and θ_{20}, there exists a moment when two pendulums reach within the regions ($|\theta_1| < \theta_{10}$) and ($|\theta_2| < \theta_{20}$) simultaneously. At the moment, the swing up control law is switched to a linear feedback control in order to stabilize two pendulums to the upright position and the cart to the initial position. The linear feedback control law is obtained by pole placement state feedback control law[11,12].

The existence of the moment is proven as follows: First, by the swing-up control, two pendulums approach the homoclinic orbit. Then, on the homoclinic orbit, the energies are $E_I = E_{i0}$, that is, $D = 0$, this implies $U = 0$ and $\ddot{r} = 0$. With assuming $c_i \approx 0$, the dynamic equations of the pendulums (1) are approximated by

$$\ddot{\theta}_i = \frac{m_i g l_i}{I_i} \sin\theta_i. \tag{14}$$

This equation is of harmonic oscillations[13], the movements of the pendulums are close to harmonic oscillations and the oscillating periods[13] are

$$T_i = 4\sqrt{\frac{I_i}{m_i g l_i}} \int_0^{\pi/2} \frac{\mathrm{d}\tau}{\sqrt{1 - a_i^2 \sin^2\tau}}, \tag{15}$$

$$a_i = \sin\frac{\pi - \theta_{i0}}{2}. \tag{16}$$

If the period when the pendulum is in the region ($|\theta_i| < \theta_{i0}$) in T_i is denoted by δT_i and the period of the first oscillation which depends on the initial condition by T_{i0}, ($0 < T_{i0} < T_i$), then at the n_ith oscillation, the pendulums are in the region respectively, during

$$n_i T_i + T_{i0} - \delta T_i < t < n_i T_i + T_{i0}, i = 1,2. \tag{17}$$

If there exist some integers n_1 and n_2 such that the next relations hold, then two pendulums are in the region ($|\theta_1| < \theta_{10}$) and ($|\theta_2| < \theta_{20}$) simultaneously, that is, they are close to the upright position simultaneously in a finite time. The time is the maximum of the left hand side of the above equations.

$$-\delta T_2 < n_1 T_1 - n_2 T_2 + T_{10} - T_{20} < \delta T_1. \tag{18}$$

This holds when the length of the both pendulums is designed such as the ratio T_1/T_2 is close to an irrational number.

In the following, a control law to restrict cart traveling position is considered. In order to restrict the cart travel, a controller denoted by K is added to the swing up control input U of equation (10):

$$u = U + K. \tag{19}$$

The control input K is given by

$$K = \begin{cases} u_a, & \bar{K} > u_a, \\ \bar{K}, & -u_a \leqslant \bar{K} \leqslant u_a, \\ 0, & D = 0, \\ -u_a, & \bar{K} < -u_a, \end{cases} \tag{20}$$

where $\bar{K}$ is a pole assignment control input and derived by the following procedure. First, rewrite equation (7) to a state equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} r \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 \\ a \end{bmatrix} u. \tag{21}$$

The control $u = \bar{K}$ is derived such that the closed loop system of (21) have assigned poles of α_1 and α_2, where α_1 and α_2 are negative real numbers and $\alpha_2 < \alpha_1 < 0$, that is, $k_1 = \alpha_1\alpha_2, k_2 = -(\alpha_1 + \alpha_2)/a$ and $\bar{K}$ is given by

$$\bar{K} = -\begin{bmatrix} k_1 & k_2 \end{bmatrix}\begin{bmatrix} r \\ \dot{r} \end{bmatrix}. \tag{22}$$

Then, from Eq.(20), the next relations hold.

$$\begin{cases} u_a \geqslant |K|, & \text{if } D \neq 0, \\ K = 0, & \text{if } D = 0. \end{cases} \tag{23}$$

This makes

$$\operatorname{sgn}(U) = \operatorname{sgn}(U + K). \tag{24}$$

From (10), Eq.(13) and the Lyapunov stability hold. By using Eq.(19), the maximum displacement of the cart is restricted to L_1[11,12]:

$$L_1 = \frac{a u_a}{\alpha_1 \alpha_2}. \tag{25}$$

3.2 Swing-up controller design for the serial cart-type double inverted pendulum

In this section, a swing-up controller for the serial double pendulum is considered. The swing-up control algorithm is based on energy method and sliding mode control. The scheme consists of three steps. In the first step, a control method to swing up the first pendulum proposed by using Lyapunov method[14] is applied.

From Eqs. (2), (3) and (7), by neglecting the second pendulum and viscosity friction, the dynamics of θ_1 is

$$\ddot{\theta}_1 = \frac{m_1 l_1}{J_1}(g\sin\theta_1 - U\cos\theta_1). \qquad (26)$$

Define a Lyapunov function as

$$V_1 = \frac{1}{2}E_1^2. \qquad (27)$$

where E_1 is the energy of the first pendulum given as

$$E_1 = \frac{1}{2}J_1\dot{\theta}_1^2 + m_1 g l_1(\cos\theta_1 - 1). \qquad (28)$$

When the pendulum is at the upright position, i. e., $\theta_2 = 0$ and $\dot{\theta}_2 = 0$, then $E_1 = 0$ and when the pendulum is at the pendant position, i.e., $\theta_2 = \pi$ and $\dot{\theta}_2 = 0$, then $E_1 = -2m_1 l_1 g$.

Calculating the derivative of V_1 along the trajectory of Eq. (26) yields

$$\dot{V}_1 = -m_1 l_1(E_1\dot{\theta}_1\cos\theta_1)U. \qquad (29)$$

From Eq. (29), to make $\dot{V} < 0$, the control input is given as

$$U = u_{a1}\mathrm{sgn}(E_1\dot{\theta}_1\cos\theta_1),\ u_{a1} > 0. \qquad (30)$$

From Eqs. (29) and (30), the derivative of V_1 is

$$\dot{V}_1 = -m_1 l_1(E_1\dot{\theta}_1\cos\theta_1)(u_{a1}\mathrm{sgn}(E_1\dot{\theta}\cos\theta_1)) \leqslant 0. \qquad (31)$$

It follows that $V_1(t)$ is a non-increasing function and $V_1 \to 0$ as $t \to \infty$. When $V_1 = 0$, there are three cases, 1) $\cos\theta_1 = 0$, 2) $\dot{\theta}_1 = 0$), and 3) $E_1 = 0$. Since the horizontal position of the pendulum is not an equilibrium point, case 1) can not be maintained. Also, if case 2) can be kept without control input, then $\theta_1 = 0$ must hold simultaneously, which follows $E_1 = 0$. Otherwise, the pendulum will fall again. Therefore, the necessary and suffcient condition for $\dot{V}_1 \equiv 0$ is $E_1 = 0$. Thus,

$$\lim_{t\to\infty} V_1(t) = \frac{1}{2}E_1^2(t) = 0 \qquad (32)$$

is obtained. Using the input of Eq. (30), the energy of the first pendulum increases to zero, and the first pendulum is swung up.

Here, in designing the control law which swings up the first pendulum, the influences of the second pendulum and viscosity friction are ignored. However, these influences exist in the real system, and it is predicted that the energy of the first pendulum decreases by these influences. The influence of viscosity friction makes always loss of the energy of the first pendulum and avoided by setting up a large input gain and giving energy to a system larger than the energy loss of the system by the influence. However, as for the influence of the second pendulum there are two cases that it may lose the energy of the first pendulum, and it may give energy to the first pendulum. Therefore, we assume that we can also ignore the influence of second pendulum by making the input gain u_{a1} large. Further, considering the traveling position of the cart, using the same design scheme as previous section, we can obtain a controller K. Then, a combined controller is given as $u = U + K$. When the first pendulum swung up in the neighborhood of $\theta_1 = 0$ by the controller u, the controller will be switched to the controller of Step 2. Namely, swing up the second pendulum while stabilizing the first pendulum. In Step 2, the proposed method consists of two control laws, to stabilize the first pendulum at the unstable equilibrium point, and to swing up the second pendulum. For designing the stabilizing controller of the first pendulum at the unstable equilibrium point, a state feedback stabilization controller is derived by using sliding mode controller[15].

First, the control law which stabilizes the first pendulum is considered. By neglecting θ_2 in (2), the dynamics of θ_1 becomes

$$b_1\ddot{\theta}_1 + (c_1 + c_2)\dot{\theta}_1 - b_2 g\sin\theta_1 + b_2\cos\theta_1\ddot{r} = 0. \qquad (33)$$

The following state variables are chosen as

$$x_1 = [r, \dot{r}, \theta_1, \dot{\theta}_2]^{\mathrm{T}}. \qquad (34)$$

With Eqs. (7) and (34), the linearized state-space equation of (33) around the unstable equilibrium point of the first pendulum $((\theta_1, \dot{\theta}_1) = (0,0))$ is

$$\dot{x}_1 = A_1 x_1 + B_1 u = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{b_2 g}{b_1} & -\frac{c_1 + c_2}{b_1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -\frac{b_2}{b_1} \\ 0 \\ 1 \end{bmatrix} u. \qquad (35)$$

Meanwhile, a sliding mode controller is used to keep robustness for stabilization of the first pendulum. The control input U which stabilizes Eq. (35) is

$$u_1 = -(S_1 B_1)^{-1}(S_1 A_1 x_1 + R_1\mathrm{sgn}(\sigma_1) + K_1\sigma_1), \qquad (36)$$

$$\sigma_1 = S_1 x_1 \qquad (37)$$

where $R_1 > 0$, $K_1 > 0$ and S_1 is the solution of the following Riccati equation with $\varepsilon_1 > 0$,

$$P_1(A_1 + \varepsilon_1 I) + (A_1 + \varepsilon_1 I)^{\mathrm{T}}P_1 - P_1 B_1 B_1^{\mathrm{T}} P_1 + Q_1 = 0,$$

$$S_1 = B_1^{\mathrm{T}} P_1.$$

Then, the control law which swings up the second pendulum will be designed. The energy control method is used as the controller which swings up the second pendulum as the same way to Step 1. In Step 2, the pendulum to be swung up is the second one and the control input to swing up is the displacement of the joint of the two pendulums, whereas, in Step 1, the pendulum is the first one and the control input is the cart position. Hence, in Step 2, the joint position is controlled in the same way as the cart position in Step 1. Therefore, the control input which for the swing up of the second pendulum is

$$u_2 = u_{a2}\mathrm{sgn}(\dot{\theta}_2\cos\theta_2),\ u_{a2} > 0. \tag{38}$$

The description of swinging up the second pendulum is shown in Fig. 6.

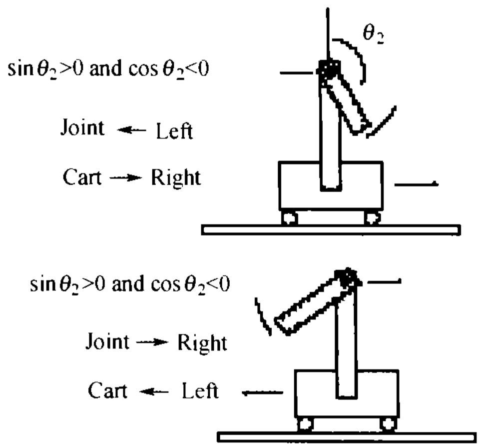

Fig. 6 Swinging up the second pendulum

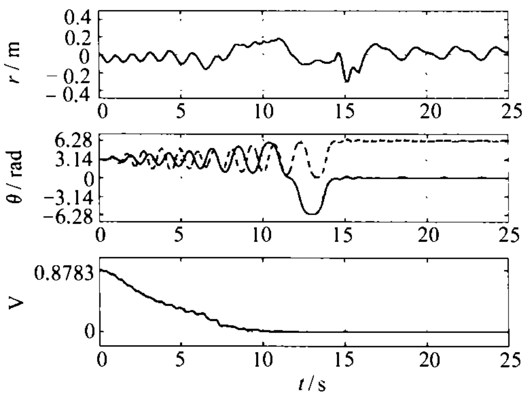

Fig. 7 Experimental results by the proposed method

By combining u_1 of (36) and u_2 of (38), the following controller is obtained, which swings up the second pendulum while stabilizing the first pendulum:

$$u = u_1 + u_2. \tag{39}$$

The sliding mode control input u_1 which stabilizes the first pendulum has strong robustness to disturbances and the swing-up input u_2 can be treated as disturbance. Therefore, it is not influential to stabilization under the following condition

$$|u_{a2}| \leqslant (S_1B_1)^{-1}(R_1 + K_1|\sigma|). \tag{40}$$

When the second pendulum is swung up in the neighborhood of $\theta_2 = 0$ by controller (39), the controller will be switched to the controller of Step 3. That is, stabilize two pendulums. The state $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2) = (0,0,0,0)$ is an unstable equilibrium point of the both pendulums. Choose x_2 as the following state variables

$$x_2 = [x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}]^{\mathrm{T}} = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2 r, \dot{r}]^{\mathrm{T}}. \tag{41}$$

With Eqs. (19) and (41), the linearized state-space equations of (2) and (3) around $x_2 = 0$ is

$$\dot{x}_2 = A_2x_2 + B_2u =$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \\ 0 \\ 1 \end{bmatrix} u, \tag{42}$$

where

$$a_{21} = \frac{b_2b_3g}{b_3b_1 - (m_2l_2L)^2},$$
$$a_{22} = -\frac{m_2l_2Lc_2 + b_3(c_1 + c_2)}{b_3b_1 - (m_2l_2L)^2},$$
$$a_{23} = -\frac{N_1N_2g}{b_3b_1 - (m_2l_2L)^2},$$
$$a_{24} = \frac{b_3c_2 + N_1c_2}{b_3b_1 - (m_2l_2L)^2},$$
$$a_{41} = -\frac{N_1b_2g}{b_3b_1 - (m_2l_2L)^2},$$
$$a_{42} = \frac{b_1c_2 + N_1(c_1 + c_2)}{b_3b_1 - (m_2l_2L)^2},$$
$$a_{43} = \frac{N_2b_1g}{b_3b_1 - (m_2l_2L)^2},$$
$$a_{44} = -\frac{b_1c_2 + N_1c_2}{b_3b_1 - (m_2l_2L)^2},$$
$$b_2 = \frac{N_1N_2 - b_2b_3}{b_3b_1 - (m_2l_2L)^2},$$
$$b_4 = \frac{N_1b_2 - N_2b_1}{b_3b_1 - (m_2l_2L)^2},$$
$$N_1 = m_2l_2L,\ N_2 = m_2l_2.$$

By the result of the control in Step 2, the state variables (x_{21}, x_{22}, x_{23}) are in the neighborhood of $(0,0,0)$. However, the state variable x_{24} is not in the neighborhood of 0, because the second pendulum is swinging. Therefore, a sliding mode controller keeping robustness is used as stabilizing controller for the two pendulums. The input control u which stabilizes (42) is

$$u = -(S_2B_2)^{-1}(S_2A_2x_2 + R_2\mathrm{sgn}(\sigma_2) + K_2\sigma_2, \tag{43}$$

$$\sigma_2 = S_2x_2 \tag{44}$$

where $R_2 > 0, K_2 > 0$ and S_2 is the solution of the following Riccati equation with $\varepsilon_2 > 0$:

$$P_2(A_2 + \varepsilon_2 I) + (A_2 + \varepsilon_2 I)^{\mathrm{T}} P_2 - P_2 B_2 B_2^{\mathrm{T}} P_2 + Q_2 = 0,$$
$$S_2 = B_2^{\mathrm{T}} P_2.$$

4 Experimental results

4.1 Swing-up the parallel cart-type double inverted pendulum

In the first experiment swing-up control for the parallel cart-type double inverted pendulum is considered. The parameters of the system and the control algorithm are given in Table 1.

Table 1 Parameters of the system and control law

l_1	m_1	c_1	I_1	w_1	θ_{10}	T_1
0.25	0.225	0.00833	0.0182	1.0	0.1	3.1905
l_2	m_2	c_2	I_2	w_2	θ_{20}	T_2
0.15	0.135	0.00109	0.004	2.8	0.1	2.5016
α_1	α_2	k_1	k_2	u_a	L	T_s
-0.5	-1.0	2.1552	6.4655	2.4	1.1136	7.9814
closed loop ploes of stabilizing control						
-3.0, -3.5, -4.0, -4.5, -5.0, -5.5						

The experimental results using the proposed method are given in Fig. 7[12]. The first graph of the figure of r shows that the displacement of the cart is restricted between -0.3 m and 0.2 m, that is within $-L_1 < r < L_1$. The second graph of θ_1 (solid line) and θ_2 (dotted line) shows that the parallel double inverted pendulums are swung up simultaneously at 15 s. Especially, the period between the moment that both pendulums go in oscillating movement and the moment that both pendulums reach the close positions to the upright simultaneously is 4 s.

Fig. 7 shows that the Lyapunov function V decreases monotonically and converges to zero. Meanwhile, animation of the swing-up control is also shown in Fig. 8.

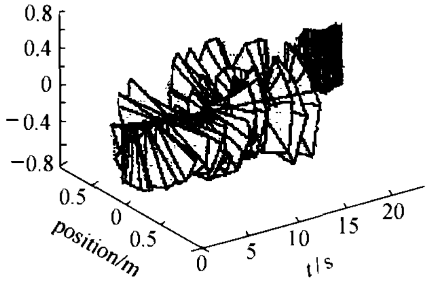

Fig. 8 Animation of the swing-up control

4.2 Swing-up the serial cart-type double inverted pendulum

The next experimental example involves swing-up the serial cart-type double inverted pendulum. In this case, the parameters of the system and the control algorithm are given in Table 2. Applying the proposed control algorithm for the experimental system the results shown in Fig. 9, which shows cart traveling position, angle of the first pendulum and the second pendulum, and step number of the controller. It is shown that at 3 s, the first pendulum is swung up, from 3 ~ 8 s, the second pendulum is swinging while the first one is kept at the upright position, finally at 8 s, both pendulums are swung up. Animation of the swing-up control is also given in Fig. 10 to show the experimental result. From Fig. 9, the displacement of the cart satisfies the maximum length [- 1, 1] (unit: m) of the rail.

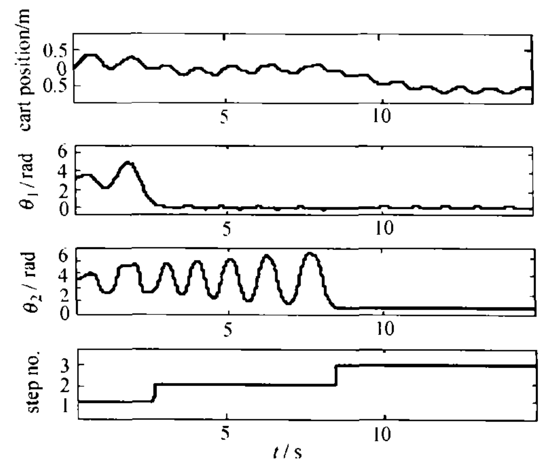

Fig. 9 Experimental results by the proposed method

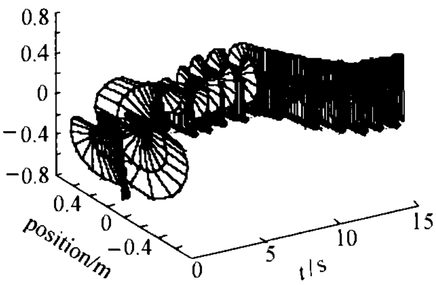

Fig. 10 Animation of the swing-up control

Table 2 Parameters of the system and control law

l_1	m_1	c_1	L	n_1	J_1	J_{n1}	u_{a1}	θ_{10}	ε	R
0.19	0.18	0.0001	0.38	0.078	0.0024	2.8×10^{-5}	16.0	0.6	1	4
l_2	m_2	c_2	a	n_2	J_2	J_{n2}	u_{a2}	θ_{20}	Q	K
0.115	0.1	0.002	0.232	0.05	0.00052	2.0×10^{-6}	1.0	0.2	I	4

5 Conclusion

In this paper, experimental results of controller design for a parallel cart-type double inverted pendulum and a serial cart-type double inverted pendulum are given. Our control algorithm for the two experimental systems involves two steps. First, swing up the two pendulums controller designs using energy method and sliding mode control. Next, stabilize the two pendulums around the unstable equilibrium state using pole placements state feedback control for the parallel pendulum system and sliding mode control for the serial pendulum. The experimental results of the application of the proposed control schemes to the two systems have been presented. In each experiment, the proposed scheme has been shown to obtain the desired swing-up control result. The further work will be on the swing-up controller design for the cart-type double inverted pendulum systems with uncertainties. Meanwhile, the influences of uncertain acceleration ratio a will also be discussed.

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