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Fuzzy indirect adaptive controller for manipulator trajectory tracking applications

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Abstract: In order to apply the existing fuzzy control algorithm to real time control of multi-DOF robot manipulator, the original indirect adaptive fuzzy control algorithm was extended from the SISO case to the MIMO case, a strict mathematical description for each step was gave and its convergence was proved. The constructed controller was further shown to have the property of uniform ultimate boundedness (u.u.b.) and adopted for the control application of an n -link robot manipulator trajectory tracking. Simulated results with a 2-link revolute joint arm with remotely driven link were presented to demonstrate its feasibility.

Key words: fuzzy control; adaptive control; intelligent control; uniform ultimate boundedness

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机器人轨迹跟踪的间接自适应模糊控制

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摘要: 为把已有的模糊控制算法应用于多自由度机器人的实时控制中,将间接自适应模糊控制算法从单输入单输出(SISO)系统推广至多输入多输出(MIMO)系统,并给出了系统收敛的严格数学证明.另外对于 n 关节的机器人轨迹跟踪问题,设计了一种新型控制器能够保证系统的最终一致有界性(u.u.b.).通过对具有远程独立电机驱动的双连杆机械臂的仿真试验证明了该方案的可行性.

关键词: 模糊控制;自适应控制;智能控制;最终一致有界

1 Introduction

Control engineers have been pursuing a reliable, applicable and effective nonlinear controller. A variety of plants possess in nature parametric uncertainty or/and inaccuracy to some extent. As a result, a control scheme based on a "perfectly developed" mathematical model may be excellent in theory but still may be far away from practical applications[1~3]. Recent years witness exciting achievements of fuzzy control in a number of complicated nonlinear control areas[4,5]. Unfortunately this control methodology, which describe the plant as one controlled by its performance behavior with human languages, seems a little bit clumsy when such a complex dynamic system as robot manipulator is to be handled. To bridge over the prominent gap between the traditional mathematical model based on control strategy and human heuristic experiences based on control scheme, the authors intend to develop a new fuzzy control algorithm, which will be capable of making use of the experts' linguistic experience to determine the controller initial values and, at the same time, extracting some eigen-information (e.g. data, function) from the mathematical models[6]. Accordingly, a fuzzy controller with superior performance can be formed by means of a fuzzy inference engine. Possible controllers are discussed but most of them remain in the field of SISO case, not suitable for robotic systems[1]. In section 2, we break this barrier by extending the algorithm in[1] into the MIMO case. Its convergence is then justified in section 3.

As far as robotic systems are concerned, there are sufficient systematic theories to tackle the control problems. Most of these theories are based on "exact" mathematical modeling schemes[7,8]. At present, it remains open to search for intelligent robotic control methodologies based on robot behavior models such as fuzzy control, neural network control[9,10]. In section 4, a two-link robot manipulator is used to testify the feasibility of the suggested MIMO adaptive fuzzy control algorithm.

Currently, the development of an applied intelligent control strategy is far from perfect or completed. This is also true for robot fuzzy control, which deals with either the off-line modeling phase for the fuzzy logic system or a

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"pure" fuzzy control strategy or some slowly moving robot up to now[11~14]. Further improved possibilities are suggested in section 5.

2 MIMO adaptive fuzzy controller

Consider the compound system by inter-connecting p nonlinear coupling subsystems:

$$\begin{cases} \dot{x}_{11} = x_{12}, \\ \quad\vdots \\ \dot{x}_{1n_1} = f_1(x_{11},\cdots,x_{1n_1};x_{21},\cdots,x_{2n_2};\cdots;x_{p1},\cdots,x_{pn_p}) + \\ \qquad g_1(x_{11},\cdots,x_{1n_1};x_{21},\cdots,x_{2n_2};\cdots;x_{p1},\cdots,x_{pn_p})u_1, \\ y_1 = x_{11}; \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}_{21} = x_{22}, \\ \quad\vdots \\ \dot{x}_{2n_2} = f_2(x_{11},\cdots,x_{1n_1};x_{21},\cdots,x_{2n_2};\cdots;x_{p1},\cdots,x_{pn_p}) + \\ \qquad g_2(x_{11},\cdots,x_{1n_1};x_{21},\cdots,x_{2n_2};\cdots;x_{p1},\cdots,x_{pn_p})u_2, \\ y_2 = x_{21}; \end{cases} \tag{2}$$

$$\vdots$$

$$\begin{cases} \dot{x}_{p1} = x_{p2}, \\ \quad\vdots \\ \dot{x}_{pn_p} = f_p(x_{11},\cdots,x_{1n_1};x_{21},\cdots,x_{2n_2};\cdots;x_{p1},\cdots,x_{pn_p}) + \\ \qquad g_p(x_{11},\cdots,x_{1n_1};x_{21},\cdots,x_{2n_2};\cdots;x_{p1},\cdots,x_{pn_p})u_p, \\ y_p = x_{p1}; \end{cases} \tag{3}$$

or equivalently in compact form:

$$\begin{cases} x_j^{(n_j)} = f_j(x_1,\dot{x}_1,\cdots,x_1^{(n_1-1)};x_2,\dot{x}_2,\cdots,x_2^{(n_2-1)};\cdots; \\ \qquad x_p,\dot{x}_p,\cdots,x_p^{(n_p-1)}) + g_j(x_1,\dot{x}_1,\cdots,x_1^{(n_1-1)}; \\ \qquad x_2,\dot{x}_2,\cdots,x_2^{(n_2-1)};\cdots,x_p,\dot{x}_p,\cdots,x_p^{(n_p-1)})u_j \\ y_j = x_j (j = 1,\cdots,p), \end{cases} \tag{4}$$

where f_j, g_j are unknown continuous functions for $j = 1,2,\cdots,p$. $u_j \in \Re$ and $y_j \in \Re$ are the input and output of the corresponding subsystem respectively. $\underline{x}_j = (x_{j1},x_{j2},\cdots,x_{jn_j})^{\mathrm{T}} = (x_{j1},\dot{x}_{j1},\cdots,x_{j1}^{(n_j-1)}) \in \Re^{n_j}$ is the observable subsystem state vector. The state vector of the compound system is denoted as $\underline{x} = (\underline{x}_1^{\mathrm{T}},\underline{x}_2^{\mathrm{T}},\cdots,\underline{x}_p^{\mathrm{T}})^{\mathrm{T}} \in \Re^{\sum_{j=1}^{p} n_j}$; the output vector, $\underline{y} = (y_1,y_2,\cdots,y_p)^{\mathrm{T}} \in \Re^p$. For the system to be controllable, it is necessary to assume that: when $\underline{x} \in U_c \subset \Re^{\sum_{j=1}^{p} n_j}$, i.e., the state vector lies in some controllable region, $g_j(\underline{x}) \neq 0$, for $j = 1,2,\cdots,p$. Without loss of generality, let $g_j(\underline{x}) > 0$, for $j = 1,2,\cdots,p$ in the following discussion.

The first type of adaptive fuzzy controller[1], which is linear in its parameters, will be used in the subsequent estimation of the bounds for the unknown continuous functions for the compound system as

$$\begin{cases} \hat{f}_i(\underline{x}) = \sum_{l=1}^{M} \theta_{fjl}\,\xi_l(\underline{x}) = \underline{\theta}_{fj}^{\mathrm{T}}\xi(\underline{x}), \\ \hat{g}_j(\underline{x}) = \sum_{l=1}^{M} \theta_{gjl}\xi_l(\underline{x}) = \underline{\theta}_{gj}^{\mathrm{T}}\xi(\underline{x}), \end{cases} \quad j = 1,2,\cdots,p. \tag{5}$$

where $\underline{\theta}_{*j} = (\theta_{*j1},\theta_{*j2},\cdots,\theta_{*jM})^{\mathrm{T}}$, $\underline{\xi} = (\underline{x}) = (\xi_1(\underline{x}),\xi_2(\underline{x}),\cdots,\xi_M(\underline{x}))^{\mathrm{T}}$. The footnote star "$*$" standing for functions $f_j(\underline{x}), g_j(\underline{x})$ will be discarded in the following discussion if no ambiguous meaning results. In addition, M is the total number of available rules in fuzzy controller's rule base; $\underline{\theta}_{*j}$ is the adjustable parameters; $\xi_l(\underline{x})$ is the fuzzy basis function (abbreviated as FBF) for the l-th rule, which in MIMO case can be defined as

$$\xi_l(\underline{x}) = \frac{\prod_{j=1}^{p}\prod_{i=1}^{n_i} \mu_{F_{ji}^l}(x_{ji})}{\sum_{l=1}^{M}\prod_{j=1}^{p}\prod_{i=1}^{n_i} \mu_{F_{ji}^l}(x_{ji})}, \tag{6}$$

where $\mu_{F_{ji}^l}$ is the given membership function.

The control objective is: when the signals are bounded, the system output should track the pre-defined reference 1 trajectory, i.e., $\underline{y} \to \underline{y}_m(t) := (y_{1m},y_{2m},\cdots,y_{pm})^{\mathrm{T}} \in \Re^p$. Denote the control as $\underline{u} = (u_1,u_2,\cdots,u_p)^{\mathrm{T}}$, then the control objective can be rephrased as : obtain a feedback control law $u_j = u_j(\underline{x}\,|\,\underline{\theta}_{*j})$, and parametric adaptation law for $\underline{\theta}_{*j}, j = 1,2,\cdots,p$, such that

1) The resultant system should be stable, i.e., for all variables $\underline{x}_j(t), \underline{\theta}_{*j}$ and $u_j = u_j(\underline{x}\,|\,\underline{\theta}_{*j}) (j = 1,2,\cdots,p$ for all $t \geqslant 0)$, there should exist: $|\underline{x}_j(t)| \leqslant M_{jx} < \infty$, $|\underline{\theta}_j(t)| \leqslant M_{j\theta}$ and $|u_j(\underline{x}\,|\,\underline{\theta}_{*j})| \leqslant M_{ju} < \infty$, where $M_{jx}, M_{j\theta}, M_{ju}$ are pre-defined parameters by control system designer.

2) The tracking error $e_j = y_{jm} - y_j$ should be as small as possible under the precondition that 1) is satisfied.

We will now construct an adaptive fuzzy controller step by step for the MIMO case.

Define the individual subsystem error vector as $\underline{e}_j = (e_j, \dot{e}_j,\cdots,e_j^{(n_j-1)})^{\mathrm{T}}$; parameter vector, $\underline{k}_j = (k_{j1},k_{j2},\cdots,k_{jn_j})^{\mathrm{T}} \in \Re^{n_j}$ so that the polynomial $h_j(s) = k_{j1} + k_{j2}s + \cdots + k_{jn_j}s^{n_j-1} + s^{n_j}$ is Hurwitz (with all its roots in the left half plane).

According to the pole assignment philosophy, the equivalent control component can be conveniently constructed as follows:

$$u_{jc} = \frac{1}{\hat{g}_j(\underline{x}|\theta_{gj})}[-\hat{f}_j(\underline{x}|\theta_{fj}) + y_{jm}^{(n)} + \underline{k}_j^{\mathrm{T}}\underline{e}_j]. \quad (7)$$

Therefore the error equation for the individual subsystem is

$$e_j^{(n_j)} = -\underline{k}_j^{\mathrm{T}}\underline{e}_j + [\hat{f}_j(\underline{x}|\theta_{fj}) - f_j(\underline{x})] + [\hat{g}_j(\underline{x}|\theta_{gj}) - g_j(\underline{x})]u_{jc}, \quad (8)$$

which can be rewritten in matrix form as

$$\dot{\underline{e}}_j = \Lambda_{jc}\underline{e}_j + \underline{b}_{jc}[\hat{f}_j(\underline{x}|\theta_{fj}) - f_j(\underline{x}) + (\hat{g}_j(\underline{x}|\theta_{gj}) - g_j(\underline{x}))u_{jc}], \quad (9)$$

where system and control matrices for the error system are

$$\begin{cases} \Lambda_{jc} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ -k_{j1} & -k_{j2} & \cdots & \cdots & \cdots & \cdots & -k_{jn_j} \end{bmatrix}_{n_j\times n_j}, \\ \underline{b}_{jc} = [0 \ \cdots \ 0 \ 1]^{\mathrm{T}}_{1\times n_j}. \end{cases} \quad (10)$$

To apply the independent decoupled control and variable structure control methodology[6], synthesize the control as a combination of certainty equivalent and supervisory control as follows:

$$u_j = u_{jc} + u_{js}. \quad (11)$$

This leads to the following error equation:

$$\dot{\underline{e}}_j = \Lambda_{jc}\underline{e}_j + \underline{b}_{jc}[\hat{f}_j(\underline{x}|\theta_{fj}) - f_j(\underline{x}) + (\hat{g}_j(\underline{x}|\theta_{gj}) - g_j(\underline{x}))u_{jc} - g_j(\underline{x})u_{js}]. \quad (12)$$

Depending on the required error convergence speed for each subsystem, the designer can determine individual symmetric positive definite matrices $Q_j \in \Re^{n_j\times n_j}$, with which the solution $P_j \in \Re^{n_j\times n_j}$ of the following Lyapunov algebraic equation can be obtained:

$$\Lambda_{jc}^{\mathrm{T}}P_j + P_j\Lambda_{jc} = -Q_j. \quad (13)$$

Selecting the corresponding Lyapunov function for the individual subsystem yields

$$V_{je} = \frac{1}{2}\underline{e}_j^{\mathrm{T}}P_j\underline{e}_j. \quad (14)$$

By substituting the reformulated new error equation and Lyapunov algebraic equation, we have

$$\dot{V}_{je} = -\frac{1}{2}\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j + \underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}[\hat{f}_j(\underline{x}|\theta_{fj}) - f_j(\underline{x}) + (\hat{g}_j(\underline{x}|\theta_{gj}) - g_j(\underline{x}))u_{jc} - g_j(\underline{x})u_{js}]. \quad (15)$$

Denoting

$$P = \mathrm{diag}(P_1, P_2, \cdots, P_j) \in \Re^{(\sum_{j=1}^{p} n_j)\cdot(\sum_{j=1}^{p} n_j)},$$

$$Q = \mathrm{diag}(Q_1, Q_2, \cdots, Q_j) \in \Re^{(\sum_{j=1}^{p} n_j)\cdot(\sum_{j=1}^{p} n_j)},$$

$$\Lambda_c = \mathrm{diag}(\Lambda_{c1}, \Lambda_{c2}, \cdots, Q_{cj}) \in \Re^{(\sum_{j=1}^{p} n_j)\cdot(\sum_{j=1}^{p} n_j)},$$

and by the operation rules for block-wise matrices we can formulate the Lyapunov equation for the compound system as

$$\Lambda_c^{\mathrm{T}}P + P\Lambda_c = -Q. \quad (16)$$

Choose the Lyapunov function for the compound system as

$$V_e = \frac{1}{2}\underline{e}^{\mathrm{T}}P\underline{e}, \quad (17)$$

i.e.,

$$V_e = V_{1e} + V_{2e} + \cdots + V_{pe} = \sum_{j=1}^{p} V_{je} \quad (18)$$

Therefore it follows

$$\dot{V}_e = \sum_{j=1}^{p}\dot{V}_{je} = -\frac{1}{2}\sum_{j=1}^{p}\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j + \sum_{j=1}^{p}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}[\hat{f}_j(\underline{x}|\theta_{fj}) - f_j(\underline{x})] + \sum_{j=1}^{p}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}[(\hat{g}_j(\underline{x}|\theta_{gj}) - g_j(\underline{x}))u_{jc} - g_j(\underline{x})u_{js}], \quad (19)$$

$$\dot{V}_e = \sum_{j=1}^{p}\dot{V}_{je}, \quad (20)$$

$$\dot{V}_e < \sum_{j=1}^{p}\{-\frac{1}{2}\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j + |\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}|[|\hat{f}_j(\underline{x}|\theta_{fj})| + |f_j(\underline{x})| + |\hat{g}_j(\underline{x}|\theta_{gj})u_{jc}| + |g_j(\underline{x})u_{jc}| - g_j(\underline{x})u_{js}]\}. \quad (21)$$

For the above Lyapunov function's derivative to be nonpositive, the upper and / or lower bound(s) for f_j, g_j should be made available. Thus we have the following assumption.

Assumption 1 The bounds for $f_j^U(\underline{x})$, $g_j^U(\underline{x})$, $g_j^L(\underline{x})$, $j = 1,2,\cdots,p$ can be determined such that when $\underline{x} \in U_c$, we have $|f_j(\underline{x})| \leqslant f_j^U(\underline{x})$, $|g_j^L(\underline{x})| \leqslant g_j(\underline{x}) \leqslant g_j^U(\underline{x})$, where $f_j^U(\underline{x}) < \infty$, $g_j^U(\underline{x}) < \infty$, $g_j^L(\underline{x}) > 0$, $\forall\ \underline{x} \in U_c$.

Thus we can reach the following sufficient (but not necessarily unique) condition to make the above equalities hold

$$u_{js} = I_j\mathrm{sgn}(\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc})\frac{1}{g_j^L(\underline{x})}[|\hat{f}_j(\underline{x}|\theta_{fj})| + f_j^U(\underline{x}) + |\hat{g}_j(\underline{x}|\theta_{gj})u_{jc}| + |g_j^U(\underline{x})u_{jc}|], \quad (22)$$

where $I_j = 1$ when $V_{je} > \bar{V}_j$ (constant designated by the designer); $I_j = 0$ when $V_{je} \leqslant \bar{V}_j$. The function sgn(*) is sign function. It takes on a value of "1" when the input variable is "0".

$$\dot{V}_e \leqslant -\sum_{j=1}^{p}\frac{1}{2}\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j + \sum_{j=1}^{p}|\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}|[|\hat{f}_j| + |f_j| + |\hat{g}_ju_{jc}| + |g_ju_{jc}| - \frac{g_j}{g_j^L}(|\hat{f}_j| + f_j^U + |\hat{g}_ju_{jc}| + |g_j^Uu_{jc}|)] \leqslant$$

$$-\sum_{j=1}^{p}\frac{1}{2}\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j=-\frac{1}{2}\underline{e}^{\mathrm{T}}Q\underline{e}. \tag{23}$$

From what has already been shown above, it can be concluded that the inequality $V_e \leqslant \bar{V} < \infty$ is guaranteed by means of the given control, which consists of certainty control component and a supervisory control component represented respectively by equations (7) and (22). This also justifies the synthesis of u_{js}.

The adaptation laws for the parameters $\underline{\theta}_{fj}$ and $\underline{\theta}_{gj}$ will be discussed in the next section.

Firstly let us define the optimized parametric values as follows:

$$\underline{\theta}_{fj}^{*}=\arg\min_{\underline{\theta}_{fj}\in\Omega_{fj}}\left[\sup_{\underline{x}\in U_c}\left|\hat{f}_j(\underline{x}\mid\underline{\theta}_{fj})-f_j(\underline{x})\right|\right], \tag{24}$$

$$\underline{\theta}_{gj}^{*}=\arg\min_{\underline{\theta}_{gj}\in\Omega_{gj}}\left[\sup_{\underline{x}\in U_c}\left|\hat{g}_j(\underline{x}\mid\underline{\theta}_{gj})-g_j(\underline{x})\right|\right], \tag{25}$$

where Ω_{fj} and Ω_{gj} are the constraint sets for $\underline{\theta}_{fj}$ and $\underline{\theta}_{gj}$ which can be pre-defined by the designer. In case of the first type of adaptive system, we have

$$\Omega_{fj}=\{\underline{\theta}_{fj}:|\underline{\theta}_{fj}|\leqslant M_{fj},\sigma_{ji}^{l}\geqslant\sigma_j\}, \tag{26}$$

$$\Omega_{gj}=\{\underline{\theta}_{gj}:|\underline{\theta}_{gj}|\leqslant M_{gj},\bar{y}_j^{l}\geqslant\varepsilon,\sigma_{ji}^{l}\geqslant\sigma_j\}. \tag{27}$$

Define the minimum approximation error as

$$w_j(\underline{x})=\hat{f}_j(\underline{x}\mid\underline{\theta}_{fj}^{*})-f_j(\underline{x})+(\hat{g}_j(\underline{x}\mid\underline{\theta}_{gj}^{*})-g_j(\underline{x}))u_{jc} \tag{28}$$

This value deserves the best possible estimation that we can achieve within the constraint for the corresponding parameter.

By rewriting the error equation in terms of the minimum approximation error as

$$\begin{aligned}\dot{\underline{e}}_j=\Lambda_{jc}\underline{e}_j-\underline{b}_c g_j(\underline{x})u_{js}+\underline{b}_{jc}[\hat{f}_j(\underline{x}\mid\underline{\theta}_{fj}^{*})-\hat{f}_j(\underline{x}\mid\underline{\theta}_{fj})+\\ \hat{g}_j(\underline{x}\mid\underline{\theta}_{gj}^{*})-\hat{g}_j(\underline{x}\mid\underline{\theta}_{gj}))u_{jc}+w_j(\underline{x})],\end{aligned} \tag{29}$$

and choosing the first type of fuzzy control system to estimate $\hat{f}_j,\hat{g}_j$, the error equation becomes

$$\begin{aligned}\dot{\underline{e}}_j&=\Lambda_{jc}\underline{e}_j-\underline{b}_{jc}g_j(\underline{x})u_{js}+\underline{b}_{jc}w_j+\\ &\underline{b}_{jc}[\underline{\phi}_{fj}^{\mathrm{T}}\xi_f(\underline{x})+\underline{\phi}_{gj}^{\mathrm{T}}\xi_g(\underline{x})u_{jc}],\end{aligned} \tag{30}$$

where $\underline{\phi}_{fj}=\underline{\theta}_{fj}-\underline{\theta}_{fj}^{*}$, $\underline{\phi}_{gj}=\underline{\theta}_{gj}-\underline{\theta}_{gi}^{*}$, and $\xi_f(\underline{x})$, $\xi_g(\underline{x})$ are FBFs. Choose Lyapunov function as

$$V=\frac{1}{2}\underline{e}^{\mathrm{T}}P\underline{e}+\sum_{j=1}^{p}\frac{1}{2\gamma_{j1}}\underline{\phi}_{fj}^{\mathrm{T}}\underline{\phi}_{fj}+\sum_{j=1}^{p}\frac{1}{2\gamma_{j2}}\underline{\phi}_{gj}^{\mathrm{T}}\underline{\phi}_{gj}, \tag{31}$$

where γ_{j1},γ_{j2} are positive constants. The derivative of the Lyapunov function above is

$$\begin{aligned}\dot{V}=&-\sum_{j=1}^{p}\left[\frac{1}{2}\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j+g_j(\underline{x})\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}u_{js}-\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}w_j\right]+\\ &\sum_{j=1}^{p}\frac{1}{\gamma_{j1}}\underline{\phi}_{fi}^{\mathrm{T}}[\dot{\underline{\theta}}_{fj}+\gamma_{j1}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_f(\underline{x})]+\\ &\sum_{j=1}^{p}\frac{1}{\gamma_{j2}}\underline{\phi}_{gi}^{\mathrm{T}}[\dot{\underline{\theta}}_{gj}+\gamma_{j2}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_g(\underline{x})u_{jc}],\end{aligned} \tag{32}$$

where we employed the Lyapunov equation and the formulae $\dot{\underline{\phi}}_{fj}=\dot{\underline{\theta}}_{fj}$, $\dot{\underline{\phi}}_{gj}=\dot{\underline{\theta}}_{gj}$. Thus the parametric adaptation law can be determined as

$$\dot{\underline{\theta}}_{fj}=-\gamma_{j1}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_f(\underline{x}), \tag{33}$$

$$\dot{\underline{\theta}}_{gj}=-\gamma_{j2}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_g(\underline{x})u_{jc}. \tag{34}$$

It should be noted at this point that the above parameter adaptation law guarantee the parameters' remaining within the constraint set, which means that further modification consideration should be made. One effective way to achieve this end is by employing the projection algorithm suggested by L.X. Wang[1], i.e. the values for $\underline{\theta}_{fj}$ and $\underline{\theta}_{gj}$ should be determined according to the following rules:

a) For $i=1,2,\cdots,M$, suppose $|\underline{\theta}_{fji}|<M_{fj}$ and $|\underline{\theta}_{fj}|<m_j\times M_{fj}$ hold; or, $|\underline{\theta}_{fj}|\geqslant m_j\times M_{fj}$ but $\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\cdot\underline{\theta}_{fj}^{\mathrm{T}}\xi_f(\underline{x})\geqslant 0$ holds; then

$$\dot{\underline{\theta}}_{fj}=-\gamma_{j1}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_f(\underline{x}), \tag{35}$$

otherwise

$$\dot{\underline{\theta}}_{fj}=-\gamma_{j1}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_f(\underline{x})+\gamma_{j1}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\frac{\underline{\theta}_{fj}\underline{\theta}_{fj}^{\mathrm{T}}\xi_f(\underline{x})}{|\underline{\theta}_{fj}|^2}, \tag{36}$$

where m_j is the dimensionality of $\underline{\theta}_{fj}$.

b) Suppose a component of $\underline{\theta}_{gj}$'s: $\underline{\theta}_{gji}\leqslant\varepsilon_j$ and $\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\cdot\xi_{gi}(\underline{x})u_{jc}<0$, then

$$\dot{\underline{\theta}}_{gji}=-\gamma_{j2}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_{gi}(\underline{x})u_{jc}, \tag{37}$$

otherwise (i.e. when $\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_{gi}(\underline{x})u_{jc}\geqslant 0$)

$$\dot{\underline{\theta}}_{gji}=0. \tag{38}$$

c) Suppose a component of $\underline{\theta}_{gj}$'s $\underline{\theta}_{gji}>\varepsilon_j$. In addition, $|\underline{\theta}_{gji}|<M_{gj}$ and $|\underline{\theta}_{gj}|<m_j\times M_{gj}$, or $|\underline{\theta}_{gj}|\geqslant m_j\times M_{gj}$ but $\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\underline{\theta}_{gj}^{\mathrm{T}}\xi_g(\underline{x})u_{jc}\geqslant 0$, then

$$\dot{\underline{\theta}}_{gj}=-\gamma_{j2}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_g(\underline{x})u_{jc}, \tag{39}$$

otherwise

$$\begin{aligned}\dot{\underline{\theta}}_{gj}=&-\gamma_{j2}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\xi_g(\underline{x})u_{jc}+\\ &\gamma_{j2}\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\frac{\underline{\theta}_{gj}\underline{\theta}_{gj}^{\mathrm{T}}\xi_g(\underline{x})u_{jc}}{|\underline{\theta}_{gj}|^2}.\end{aligned} \tag{40}$$

We have so far constructed the adaptive fuzzy control algorithm in MIMO case.

3 Convergence of MIMO adaptive fuzzy control algorithm

To facilitate the proof of convergence of the suggested adaptive MIMO fuzzy algorithm, we cite the following Barbalat Lemma and its corollary[15].

Barbalat lemma If a differentiable function $f(t)$ is bounded and uniformly continuous as $t\to\infty$, then $\dot{f}(t)\to 0$ as $t\to\infty$.

Lyapunov-like corollary If a scalar function

$V(x,t)$ satisfies the following conditions:

- $V(x,t)$ has lower bound;
- $\dot{V}(x,t)$ is nonpositive definite;
- $\dot{V}(x,t)$ is uniformly continuous w.r.t time t;

then $\dot{V}(x,t) \to 0$ as $t \to \infty$.

We now state a theorem for the MIMO adaptive fuzzy control algorithm.

Theorem 1 (Adaptive fuzzy control for MIMO case)

Consider the plant represented by (4), and apply control (11), where u_{jc} is given by (7), u_{js} by (22), and $\hat{f}_j$, $\hat{g}_j$ by (5). The parameter vectors $\underline{\theta}_{fj}$, $\underline{\theta}_{gj}$ are adjusted by the adaptation law (33), (34). Suppose Assumption 1 is true. Then the resulting system possesses the following properties:

a) $|\underline{\theta}_{fj}| \leqslant m_j \times M_{fj}$, $|\underline{\theta}_{gj}| \leqslant m_j \times M_{gj}$, each component of $\underline{\theta}_{gj}$'s $\geqslant \varepsilon_j \geqslant \varepsilon := \min(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_p)$, $|\underline{x}| \leqslant$

$$|\underline{y}_m| + \left(\frac{2\bar{V}}{\lambda_{\min 0}}\right)^{1/2},$$

$$\lambda_{\min 0} = \min(\lambda_{\min 1}, \lambda_{\min 2}, \cdots, \lambda_{\min p}), \tag{41}$$

$$\begin{aligned}|u_j(t)| \leqslant{}& \frac{1}{\varepsilon_j}\left(m_j \times M_{fj} + |y_{jm}^{(n_j)}| + |\underline{k}_j|\left(\frac{2\bar{V}_j}{\lambda_{\min j}}\right)^{1/2}\right) + \\ & \frac{1}{|g_j^L(\underline{x})|}\Big[m_j \times M_{fj} + |f_j^U(\underline{x})| + \\ & \frac{1}{\varepsilon_j}(m_j \times M_{gj} + g_j^U)(m_j \times M_{fj} + \\ & |y_{jm}^{(n_j)}| + |\underline{k}_j|\left(\frac{2\bar{V}_j}{\lambda_{\min j}}\right)^{1/2})\Big],\end{aligned} \tag{42}$$

where λ_j is the minimum eigenvalue of matrix P_j and $\underline{y}_{jm} = (y_{jm}, \dot{y}_{jm}, \cdots, y_{jm}^{(n_j-1)})^{\mathrm{T}}$.

b) For all $t \geqslant 0$, there exists

$$\int_0^t |\underline{e}(\tau)|^2 \mathrm{d}\tau \leqslant a + \sum_{j=1}^{p} b_j \int_0^t |w_j(\tau)|^2 \mathrm{d}\tau, \tag{43}$$

where a, b_j are constants; w_j is the minimum approximation error defined by (28).

c) If w_j is absolutely integrable, i.e., $\int_0^\infty |w_j(t)|^2 \mathrm{d}t < \infty$, then $\lim_{t\to\infty} |\underline{e}(t)| = 0$.

Proof

a) Choose $V_{fj} = \frac{1}{2}\underline{\theta}_{fj}^{\mathrm{T}}\underline{\theta}_{fj}$. If equation (35) holds, it corresponds to the first case in the projection algorithm in a) of Section 2, then one of the following two conditions must be true:

$$|\underline{\theta}_{fji}| < M_{fj} \text{ and } |\underline{\theta}_{fj}| < m_j \times M_{fj}, \tag{44}$$

or

$$|\underline{\theta}_{fj}| = m_j \times M_{fj}.$$

When $|\underline{\theta}_{fj}| = m_j \times M_{fj}$ holds, substituting (35) into V_{fj} yields

$$\dot{V}_{fj} = -\gamma_{j1}\,\underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc}\,\underline{\theta}_{fj}^{\mathrm{T}}\,\xi_f(\underline{x}) \leqslant 0. \tag{45}$$

Hence we always have

$$|\underline{\theta}_{fj}| \leqslant m_j \times M_{fj}. \tag{46}$$

Alternatively, if equation (36) holds, it corresponds to the second case in the projection algorithm in (a) of Section 2, then

$$\dot{V}_{fj} = -\gamma_{j1}\,\underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc}\,\underline{\theta}_{fj}^{\mathrm{T}}\,\xi_f(\underline{x}) + \gamma_{j1}\,\underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} \times \frac{|\underline{\theta}_{fj}|^2\,\underline{\theta}_{fj}^{\mathrm{T}}\,\xi_f(x)}{|\underline{\theta}_{fj}|^2} = 0. \tag{47}$$

So we have proved that

$$|\underline{\theta}_{fj}| \leqslant m_j \times M_{fj},\ \forall t \geqslant 0. \tag{48}$$

Similarly we can show

$$|\underline{\theta}_{gj}| \leqslant m_j \times M_{gj},\ \forall t \geqslant 0. \tag{49}$$

Summarizing equations (37) and (38) results in the conclusion:

$$\underline{\theta}_{gji} > \varepsilon_j > \min(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_p). \tag{50}$$

As a result of the fact that $V_e \leqslant \bar{V} = \sum_{j=1}^{p} V_{je}$, which is ensured in Section 2:

$$\begin{aligned}&\frac{1}{2}\min(\lambda_{\min 1}, \lambda_{\min 2}, \cdots, \lambda_{\min p})|\underline{e}|^2 \leqslant \\ &\frac{1}{2}\sum_{j=1}^{p}\lambda_{\min j}|e_j|^2 \leqslant \frac{1}{2}\underline{e}^{\mathrm{T}}P\underline{e} = \\ &\frac{1}{2}\sum_{j=1}^{p}\underline{e}_j^{\mathrm{T}}P_j\underline{e}_j \leqslant \sum_{j=1}^{p} V_{je}\end{aligned} \tag{51}$$

Further we have

$$|\underline{e}| \leqslant \left(\frac{2\bar{V}}{\lambda_{\min 0}}\right)^{1/2}. \tag{52}$$

Because $\underline{e} = \underline{y}_m - \underline{x}$, we conclude that

$$|\underline{x}| \leqslant |\underline{y}_m| + |\underline{e}| \leqslant |\underline{y}_m| + \left(\frac{2\bar{V}}{\lambda_{\min 0}}\right)^{1/2},$$

which is (41).

Because the estimates $\hat{f}_j(\underline{x}\,|\,\underline{\theta}_{fj})$ and $g_j(\underline{x}\,|\,\underline{\theta}_{gj})$ are the weighted averages of each component for the parameter vectors $\underline{\theta}_{fj}$ and $\underline{\theta}_{gj}$, therefore it follows

$$\hat{f}_j(\underline{x}\,|\,\underline{\theta}_{fj} \leqslant |\underline{\theta}_{fj}|) \leqslant m_j \times M_{fj},\ \hat{g}_j(\underline{x}\,|\,\underline{\theta}_{gj}) \geqslant \varepsilon.$$

From equation (22), we have

$$|u_{js}| \leqslant \frac{1}{\varepsilon_j}\left[m_j \times M_{fj} + |y_{jm}^{(n_j)}| + |\underline{k}_j|\left(\frac{2\bar{V}_j}{\lambda_{\min j}}\right)^{1/2}\right]. \tag{53}$$

From equation (22), we have

$$|u_{js}| \leqslant \left|\frac{1}{g_j^L(\underline{x})}\right|\Big[m_j \times M_{fj} + |f_j^U(\underline{x})| + m_j \times M_{gj}|u_{jc}| + |g_j^U(\underline{x})u_{jc}|\Big]. \tag{54}$$

Combining equations (53) and (54), we get (42).

b) From (32) and (33) ~ (40), we obtain

$$\dot{V} = -\sum_{j=1}^{p}\left[\frac{1}{2}\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j + g_j(\underline{x})\,\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}u_{js} - \underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}w_j\right] +$$

$$\sum_{j=1}^{p} I_{j1}\, \underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} \frac{\underline{\phi}_{fj}^{\mathrm{T}} \underline{\theta}_{fj} \underline{\theta}_{fj}^{\mathrm{T}}}{|\underline{\theta}_{fj}|^2} \underline{\xi}_f(\underline{x}) +$$

$$\sum_{j=1}^{p} I_{j2}\, \underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} \frac{\underline{\phi}_{gj+}^{\mathrm{T}} \underline{\theta}_{gj+} \underline{\theta}_{gj+}^{\mathrm{T}}}{|\underline{\theta}_{gj+}|^2} \underline{\xi}_{j+}(\underline{x}) u_{jc} +$$

$$\sum_{j=1}^{p} I_{j3} \underline{\phi}_{gj\varepsilon}^{\mathrm{T}}\, \underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc}\, \underline{\xi}_{j\varepsilon}(\underline{x}) u_{jc}, \tag{55}$$

where $I_{j1} = 0(1)$ when (36) applies (when (37) applies); $I_{j2} = 0(1)$ when (39) applies (when (40) applies); $I_{j3} = 0(1)$ when (37) applies (when (38) applies); and where $\underline{\theta}_{gj+}$ is the vector formed by those components of $\underline{\theta}_{gj}$ whose values are larger than ε_j; $\underline{\theta}_{gj\varepsilon}$ is the vector formed by those components whose values are equal to ε_j. In addition, $\underline{\phi}_{gj+} = \underline{\theta}_{gj+} - \underline{\theta}_{gj+}^*$, $\underline{\phi}_{gj\varepsilon} = \underline{\theta}_{gj\varepsilon} - \underline{\theta}_{gj\varepsilon}^*$. For $\underline{\xi}_j(\underline{x})$, the formed vectors $\underline{\xi}_{j+}(\underline{x})$ and $\underline{\xi}_{j\varepsilon}(\underline{x})$ resemble $\underline{\theta}_{gj+}$ and $\underline{\theta}_{gj\varepsilon}$ respectively.

We shall now demonstrate that each of the last three terms in (55) is nonpositive.

The second term. The result is obvious with $I_{j1} = 0$. Now suppose $I_{j1} = 1$, i.e., $|\underline{\theta}_{fj}| = m_j \times M_{fj}$ and $\underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} \underline{\theta}_{fj}^{\mathrm{T}} \underline{\xi}_f(\underline{x}) < 0$. By utilizing the associated definitions in linear algebra, e.g.[16], we can conveniently form the "cosine theorem" in the multidimensional case:

$$(\underline{a} - \underline{b})^{\mathrm{T}} \underline{a} = \frac{1}{2}(|\underline{a}|^2 - |\underline{b}|^2 + |\underline{a} - \underline{b}|^2), \tag{56}$$

where $\underline{a}, \underline{b} \in \Xi^n$.

Based on this relationship, we proceed as follows:

$$\underline{\phi}_{fj}^{\mathrm{T}} \underline{\theta}_{fj} = (\underline{\theta}_{fj} - \underline{\theta}_{fj}^*)\, \underline{\theta}_{fj} =$$

$$\frac{1}{2}[\,|\underline{\theta}_{fj}|^2 - |\underline{\theta}_{fj}^*|^2 + |\underline{\theta}_{fj} - \underline{\theta}_{fj}^*|^2] \geqslant 0$$

$$(|\underline{\theta}_{fj}| = m_j \times M_{fj} \geqslant |\underline{\theta}_{fj}^*|).$$

Up to now, we have proved that the term containing I_{j1} in (55) is nonpositive.

The third term. The nonpositiveness for the term containing I_{j2} can similarly be justified.

The fourth term. Based on (37), (38) and considering the fact that $\phi_{gji} = \theta_{gji} - \theta_{gji}^* = \varepsilon_j - \theta_{gji}^* \leqslant 0$, we conclude that the term containing I_{j3} is nonpositive too. Thus equation (55) reduces to

$$\dot{V} \leqslant$$

$$-\sum_{j=1}^{p} \frac{1}{2} \underline{e}_j^{\mathrm{T}} Q_j \underline{e}_j - \sum_{j=1}^{p} g_j(\underline{x})\, \underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} u_{js} + \sum_{j=1}^{p} \underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} w_j. \tag{57}$$

Reconsidering equation (22) and $g_j(\underline{x}) > 0$ leads to the conclusion: $g_j(\underline{x})\, \underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} u_{js} \geqslant 0$, making (57) further reduced to

$$\dot{V} \leqslant -\sum_{j=1}^{p} \frac{1}{2} \underline{e}_j^{\mathrm{T}} Q_j \underline{e}_j + \sum_{j=1}^{p} \underline{e}_j^{\mathrm{T}} P_j \underline{b}_{jc} w_j \leqslant$$

$$-\frac{\lambda_{Q\min 0} - 1}{2} |\underline{e}|^2 + \sum_{j=1}^{p} \frac{1}{2} |P_j \underline{b}_{jc} w_j|^2. \tag{58}$$

Integrating both end sides of formula (58) yields

$$\int_0^t |\underline{e}(\tau)|^2 \mathrm{d}\tau \leqslant$$

$$\frac{2}{\lambda_{Q\min 0} - 1}[\,|V(0)| - |V(t)|\,] +$$

$$\frac{1}{\lambda_{Q\min 0} - 1} \sum_{j=1}^{p} |P_j \underline{b}_{jc}| \int_0^t |w_j(\tau)|^2 \mathrm{d}\tau. \tag{59}$$

Defining

$$a = \frac{2}{\lambda_{Q\min 0} - 1}[\,|V(0)| - \sup_{t \geqslant 0} |V(t)|\,],$$

$$b_j = \frac{|P_j \underline{b}_{jc}|}{\lambda_{Q\min 0} - 1},$$

(59) then becomes (43).

c) If $w_j \in L_2$, and from (43) we know $\underline{e} \in L_2$. Since we have already proved that each of the variables in the right hand side of equation (29) is bounded, we conclude that $\dot{\underline{e}}_j \in L_\infty$. Applying the Barbalat Lemma and its corollary[15] (if $\underline{e} \in L_2 \cap L_\infty$ and $\dot{\underline{e}} \in L_\infty$, then $\lim_{t\to\infty} |\underline{e}(t)| = 0$) results in the conclusion: of $\lim_{t\to\infty} |\underline{e}(t)| = 0$.

4 Robot manipulator applications

The robot manipulator control problem, owing to its strong effect between the joints and high degree of nonlinearity in its dynamics, may serve as an excellent plant to test the suggested MIMO indirect adaptive fuzzy control scheme. We are now ready to apply the MIMO fuzzy system developed in the previous sections to try its ability in application to a multi-DOF robot manipulator trajectory tracking problem.

A) Indirect adaptive fuzzy control strategy for multi-DOF manipulator.

The multi-DOF rigid robot manipulator dynamics can be characterized by the following equation:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + \phi(q) = \tau, \tag{60}$$

where q is the link's generalized coordinate; $D(q)$ is the inertia matrix; $g(q)$ is the gravity term; and τ is the generalized force applied at corresponding joints.

Choosing the state vector as $x_k = q_k; x_{n+k} = \dot{q}_k, k = 1, 2, \cdots, n$ and denoting $x_{\mathrm{I}} = (x_1, \cdots, x_n)^{\mathrm{T}}, x_{\mathrm{II}} = (x_{n+1}, \cdots, x_{n+n})^{\mathrm{T}}$, due to the positive definiteness of the inertia matrix $D(q)$[7], equation (60) becomes

$$\begin{cases}\dot{x}_{\mathrm{I}} = \dot{x}_{\mathrm{II}},\\ \dot{x}_{\mathrm{II}} = -D^{-1}(x_{\mathrm{I}})[C(x_{\mathrm{I}},x_{\mathrm{II}})+\phi(x_{\mathrm{I}})]+D^{-1}(x_{\mathrm{I}})\tau.\end{cases} \tag{61}$$

Making the transformation: $u(x_{\mathrm{I}}) = D^{-1}(x_{\mathrm{I}})\tau$, denoting $f = -D^{-1}(x_{\mathrm{I}})[C(x_{\mathrm{I}},x_{\mathrm{II}}) + \phi(x_{\mathrm{I}})]$, we obtain a model, which is suitable for applying the indirect adaptive fuzzy control for the multi-DOF robot manipulator. In fact, it is the standard decoupled form, or the standard form for the computed torque control[7,8].

Note that the above equation takes exactly the same form as that obtained in Section 3 and consequently leads to the following theorem.

Theorem 2 (Robot indirect adaptive fuzzy controller) Consider the robot system (61). Suppose we can determine the bounds for functions $f_j^U(\underline{x}), j = 1,\cdots,n$, i.e. $|f_j(\underline{x})| \leqslant f_j^U(\underline{x})$, the right-hand side of which is the polynomial of state variable, when $\underline{x}: = [x_{\mathrm{I}}^{\mathrm{T}}, x_{\mathrm{II}}^{\mathrm{T}}]^{\mathrm{T}} \in U_c$, where $f_j^U(\underline{x}) < \infty, \forall\, \underline{x} \in U_c$. Apply control (62), embracing the certainty equivalent control u_{jc} in (63) and supervisory control u_{js} in (64). The estimate for the fuzzy system $\hat{f}_j$ is given by (65). Suppose the parameter vector $\underline{\theta}_{fj}$ is adjusted by the adaptation law (66), i.e.,

$$u_j = u_{jc} + u_{js}, \tag{62}$$

$$u_{jc} = -\hat{f}_j(\underline{x}\,|\,\underline{\theta}_{fj}) + \ddot{q}_j^d + \underline{k}_j^{\mathrm{T}}\underline{e}_j, \tag{63}$$

$$u_{js} = I_j\mathrm{sgn}(\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc})[\,|\hat{f}_j(\underline{x}\,|\,\underline{\theta}_{fj})| + f_j^U(\underline{x})], \tag{64}$$

$$\hat{f}_j(\underline{x}) = \sum_{l=1}^{M}\theta_{fjl}\xi_l(\underline{x}) = \underline{\theta}_{fj}^{\mathrm{T}}\,\xi(\underline{x}), \tag{65}$$

$$\dot{\underline{\theta}}_{fj} = -\gamma_{j1}\,\underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\,\xi_f(\underline{x}), \tag{66}$$

then the resulted robot manipulator control system has the following properties:

Property 1

$$|\,\underline{\theta}_{fj}\,| \leqslant m_j\cdot M_{fj},\ |\,\underline{x}\,| \leqslant |\,\underline{x}^d\,| + \sqrt{\frac{2\bar{V}}{\lambda_{\min 0}}},$$

$$\lambda_{\min 0} = \min(\lambda_{\min 1},\lambda_{\min 2},\cdots,\lambda_{\min n})$$

$$|\,u_j(t)\,| \leqslant$$

$$2m_j\cdot M_{fj} + |\,\ddot{q}_j^d\,| + |\,\underline{k}_j\,|\sqrt{\frac{2\bar{V}_j}{\lambda_{\min j}}} + |\,f_j^U(\underline{x})\,|,$$

where m_j is the dimensionality of vector θ_{fj}, λ_j is the minimum eigenvalue of matrix P_j, and $\underline{x}^d$ is the required tracking trajectory.

Property 2

$$\int_0^t|\,\underline{e}(t)\,|^2\mathrm{d}t \leqslant a + \sum_{j=1}^{n}b_j\int_0^t|\,w_j(t)\,|^2\mathrm{d}t,\ \forall\, t \geqslant 0,$$

where a, b_j are constants; and w_j is the minimum approximation error.

Property 3 If w_j is absolutely integrable, i.e., $\int_0^\infty|\,w_j(t)\,|^2\mathrm{d}t < \infty$, then $\lim\limits_{t\to\infty}|\,\underline{e}(t)\,| = 0$.

Property 4 For arbitrary initial error state $\underline{e}(t_0) = \underline{e}_0, (\underline{e}_0, t_0) \in \Xi^{2n}\times\Xi^1$, the system (61) is u.u.b..

Proof

Properties 1,2,3 The proof procedure is similar. Refer to the proof of Theorem 1 in Section 3.

Property 4 Examine the j-th subsystem (located at the j-th joint). Suppose that the initial error state be $(\underline{e}_{j0}, t_0) \in \Xi^2\times\Xi^1$. Define the ellipsoids as

$$E_j(k) = \left\{\underline{e}_j \in \Xi^2\,\middle|\,\frac{1}{2}\underline{e}_j^{\mathrm{T}}P_j\underline{e}_j \leqslant k \equiv \text{const.} > 0\right\}.$$

According to section 2, we should exert supervisory control (meaning $I_j = 1$) when $V_{je} > \bar{V}_j$, or $\underline{e}_j \in E_j(k), k > \bar{V}_j$. Meanwhile the Lyapunov function $\frac{1}{2}\,\underline{e}_j^{\mathrm{T}}P_j\underline{e}_j$ is monotonically decreasing. By the following relationship for matrix eigenvalues, we have

$$0 < \lambda_{\min j}\,\|\,\underline{e}_j(t)\,\|^2 \leqslant \underline{e}_j^{\mathrm{T}}(t)P_j\underline{e}_j(t) \leqslant \underline{e}_{j0}^{\mathrm{T}}P_j\underline{e}_{j0} \leqslant \lambda_{\max j}\,\|\,\underline{e}_{j0}\,\|^2. \tag{67}$$

On the other hand, supervisory control should be removed, i.e., $I_j = 0$, when $V_{je} \leqslant \bar{V}_j$, i.e., $\underline{e}_j \in E_j(\bar{V}_j)$. In the same way, we have

$$0 < \lambda_{\min j}\,\|\,\underline{e}_j(t)\,\|^2 \leqslant \underline{e}_j^{\mathrm{T}}(t)P_j\underline{e}_j(t) \leqslant 2\bar{V}_j. \tag{68}$$

Combining (67) and (68) leads to a bound of $\underline{e}_j(t)$, $t \in [t_0, +\infty)$:

$$d_j(\underline{e}_{j0}) = \begin{cases}\|\,\underline{e}_{j0}\,\|\sqrt{\dfrac{\lambda_{\max j}}{\lambda_{\min j}}}, & \underline{e}_{j0} \in E_j(k), k > \bar{V}_j,\\ \sqrt{\dfrac{2\bar{V}_j}{\lambda_{\min j}}}, & \underline{e}_{j0} \in E_j(\bar{V}_j).\end{cases} \tag{69}$$

According to section 3, when $V_{je} > \bar{V}_j$, i.e., $\underline{e}_j \in E_j(k), k > \bar{V}_j$, the derivative of the Lyapunov function is

$$\dot{V}_{je} = -\frac{1}{2}\,\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j + \underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}\times[\hat{f}_j(\underline{x}\,|\,\theta_{fj}) - f_j(\underline{x}) - u_{js}]. \tag{70}$$

Denoting

$$c_0 = \min\{\frac{1}{2}\,\underline{e}_j^{\mathrm{T}}Q_j\underline{e}_j - \underline{e}_j^{\mathrm{T}}P_j\underline{b}_{jc}[\hat{f}(\underline{x}\,|\,\underline{\theta}_{fj}) - f_j(\underline{x}) - u_{js}]\,|\,\underline{e}_j \in E_j(k_0)\setminus\mathrm{int}[E_j(\bar{V}_j)]\},$$

where $k_0 = \frac{1}{2}\,\underline{e}_{j0}^{\mathrm{T}}P_j\underline{e}_{j0}$, $\mathrm{int}[E_j(\bar{V}_j)]$ represents the interior of $E_j(\bar{V}_j)$. Because the choice of the supervisory control guarantees that V_{je} decreases until $\underline{e}_j$ approaches the boundary of $E_j(\bar{V}_j)$, i.e., $\underline{e}_j \in Bd[E_j(\bar{V}_j)]$. Therefore the time interval for this error state transfer is calculated as

$$\bar{t} - t_0 \leqslant \frac{k_0 - \bar{V}_j}{c_0}. \tag{71}$$

Considering the fact that the time interval is zero when $V_{je} \leqslant \bar{V}_j$, i.e., when $\underline{e}_j \in E_j(\bar{V}_j)$, we can generalize the transfer time interval as follows:

$$T_j(\underline{e}_{j0}, E_j(k_0)) = \begin{cases} \dfrac{k_0 - \bar{V}_j}{c_0}, & \underline{e}_{j0} \in E_j(\bar{k}), \bar{k} > \bar{V}_j, \\ 0, & \underline{e}_{j0} \in E_j(\bar{V}_j). \end{cases} \tag{72}$$

Considering the error states of all the subsystem error $\underline{e}_j$, we can summarize as: the error state for plant (61) will converge within the time interval not longer than $T(\underline{e}_0, E(\sum_{j=1,\cdots,n} k_{j0}) = \max_{j=1,\cdots,n} \{T_j(\cdot)\}$ to the set bounded by $d(\underline{e}_0) = \sqrt{\sum_{j=1,\cdots,n} d_j^2(\underline{e}_{j0})}$.

B) Simulation trial

To verify the feasibility of the suggested control scheme, we take a two-link manipulator with remote drive[7] for simulation trial. The mechanism is characterized by an independent remote driving motor for the second link, which simplifies the model of the whole system without loss of inter-coupling effect between its two revolute joints. The joint variables are selected as in Fig. 1. The corresponding dynamic equations are

$$d_{11}\ddot{p}_1 + d_{12}\ddot{p}_2 + c_{221}\dot{p}_2^2 + \phi_1 = \tau_1,$$
$$d_{12}\ddot{p}_1 + d_{22}\ddot{p}_2 + c_{112}\dot{p}_1^2 + \phi_2 = \tau_2,$$
$$d_{11} = m_1 l_{c1}^2 + m_2 l_1^2 + I_1,$$
$$d_{12} = m_2 l_1 l_{c2}\cos(p_2 - p_1),$$
$$d_{21} = d_{12} d_{22} = m_2 l_{c2}^2 + I_2,$$
$$c_{221} = -m_2 l_1 l_{c2}\sin(p_2 - p_1),$$
$$c_{112} = m_2 l_1 l_{c2}\sin(p_2 - p_1),$$
$$\phi_1 = (m_1 l_{c1} + m_2 l_1) g\cos p_1,$$
$$\phi_2 = m_2 l_{c2} g\cos p_2.$$

where the angles in work space of links 1 and 2 are respectively p_1, p_2; the exerted torques are τ_1, τ_2; gravity terms are ϕ_1, ϕ_2; d_{ijk} is Christoffel symbol, where $i = 1, 2; j = 1,2,; k = 1,2$[7].

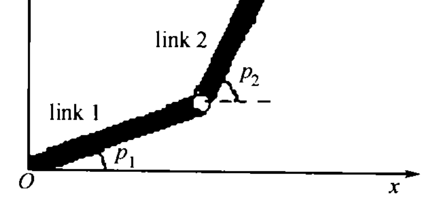

Fig. 1 Joint variable selection scheme

The physical parameters used in the simulation are given in Table 1.

Table 1 Gravity acceleration $g = 9.8$ m/s^2

link	mass m_i/kg	length l_i/m	center of mass l_{ci}/m	moment of inertia I_i/kg·m^2
1	32	0.5	$l_1/2$	19.48
2	6	0.4	$l_2/2$	18.55

Suppose the required tracking trajectory is $q_1^d = q_2^d = \frac{\pi}{30}\sin t$ (rad), $t \in [0, \pi]$. We divide the variable universe of discourse $q_i \in [-\frac{\pi}{6}, \frac{\pi}{6}]$, $(i = 1,2)$ into 5 equally distributed fuzzy values and take Gaussian form FBF (Fuzzy Basis Function). In order to apply parameter adaptation a number of (625 + 625) auxiliary state variables should be selected.

The simulation is conducted by using Runge-Kutta 5 integration method with minimum step size as 0.001 second, maximum step size as 10 seconds and accuracy as 10^{-10} radians. The actual simulation lasted for approximately 28 hours on a 586 (Pentium 100 M) personal computer with 16 megabyte work memory. The target simulated time interval is $t \in [0, \pi]$, with initial state selected as $\underline{x}_0 = [-\frac{\pi}{60}, 0, -\frac{\pi}{60}, 0]^T$.

The simulation results are plotted in Figs. 2 ~ 7, which show the gradual convergence tendency for the tracking errors (the required trajectories are plotted as dotted lines).

Special attention should be given to Figs. 6 and 7, which cause the chattering effect appearing in the resultant torques to be exerted at individual joint. The control chattering amplitude is mainly due to the discontinuity of the supervisory part, which, in the robot manipulator case, is obviously harmful because it may excite the unwanted unmodelled dynamics. This has to be alleviated if not to say removed.

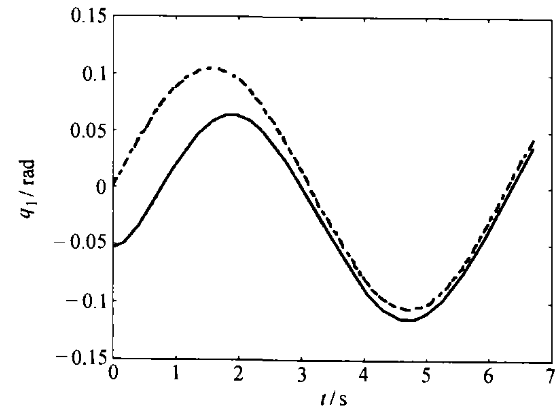

Fig. 2 Angle trajectory at link 1

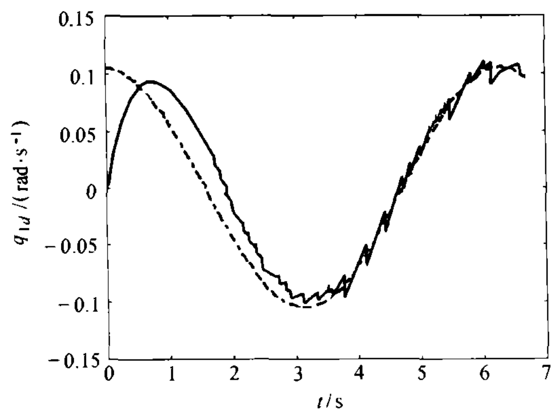

Fig.3 Velocity trajectory at link 1

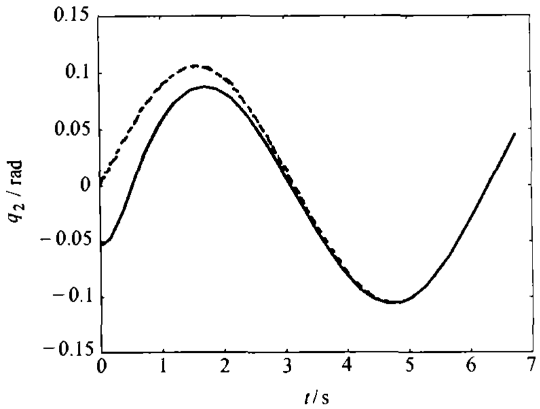

Fig. 4 Angle trajectory at link 2

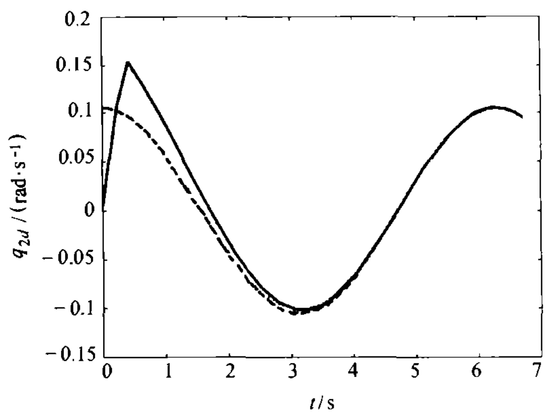

Fig.5 Velocity trajectory at link 2

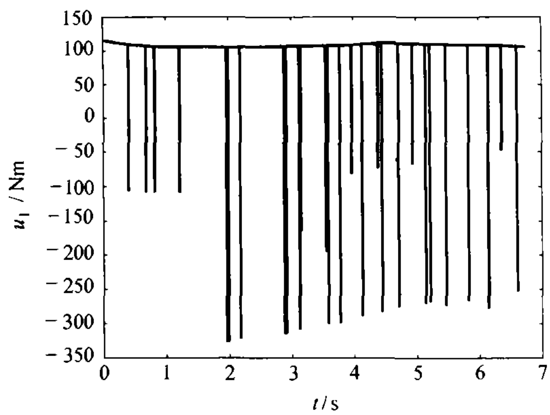

Fig. 6 Torque at link 1

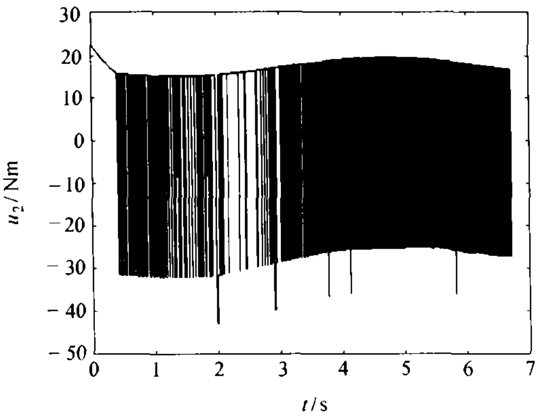

Fig. 7 Torque at link 2

5 Conclusion

In this paper we discuss the expansion of indirect adaptive fuzzy control algorithm from its original SISO case into MIMO case, the construction of the controller and its control algorithm convergence, with simulations on digital computer for a multi-DOF robot manipulator trajectory tracking application.

From the simulated results we may take heed of the potential harmfulness arising from the chattering control torques (see for example Figs.6 and 7). This can possibly be alleviated by employing the achievement made in the relevant literature [6].

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附录(Appendix):

个别符号说明.

q_r 表示机器人其中心点坐标, q_d 表示其目标点(如加下标 i 用于区别不同机器人); O 或 O_i 表示障碍物,用下标 i 或 j 来区别不同障碍物; ∂D 表示点集 D 的边界;黑体表示一个矢量或矩阵,否则为一标量或函数(文中的 p,q 和 q_r 一般均表示一点,虽是二维向量,但为方便没用黑体表示); $\boldsymbol{F}_s(q)_i$ 表示二维力矢量场函数(或简记为 $\boldsymbol{F}_{si}$),而 K_{si} 和 k_{si} 表示其场参数,其中下标 $s = r$ 和 n 分别表示排斥力和协调力, i 表示障碍物 O_i 或 i 所形成的场, $\boldsymbol{F}_a(q)_i$ 表示第 i 个机器人所受的目标点 q_{d_i} 处引力场的吸引力矢量, K_{ai} 表示其场参数; $d(O_1, O_2)$ 表示两封闭区域 O_1 和 O_2 边界的最小距离; $\boldsymbol{g}_O(q)$ 表示封闭区域 O 边界上距点 q 距离最近的点.

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