

一类非线性时滞输出反馈系统的自适应控制

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摘要: 针对一类参数化非线性时滞输出反馈系统, 提出了一种无记忆自适应跟踪控制器的设计方案. 采用时滞滤波器估计系统状态, 用 Domination 处理非线性时滞项, 应用 Backstepping 技术设计控制器和参数自适应律. 放宽了对时滞项的要求. 通过构建一个 Lyapunov-Krasovskii 泛函, 证明了闭环系统的稳定性, 实现了对目标轨线的渐近跟踪, 保证了所有信号一致有界. 实例仿真说明了该方案的可行性.

关键词: 非线性时滞系统; 输出反馈; 自适应控制; backstepping

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Adaptive control for a class of nonlinear time-delay output-feedback systems

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Abstract: A design approach of memoryless adaptive tracking controller is proposed for a class of parametric nonlinear time-delay output-feedback systems. Time-delay filters are introduced to estimate the states of the system. Domination method is employed to deal with the nonlinear time-delay terms. Backstepping technique is used to design controller and parameters adaptive laws. The requirement on the time-delay terms is relaxed. The stability of the closed-loop system is proved by constructing a Lyapunov-Krasovskii functional. The asymptotical tracking of given trajectories is achieved and the boundedness of all signals is also guaranteed. The feasibility of the approach is illustrated by a simulation example.

Key words: nonlinear time-delay systems; output feedback; adaptive control; backstepping

1 引言 (Introduction)

时滞(或时间延迟)存在于各类工程系统之中, 它的存在会对系统的性能产生重要影响, 甚至破坏系统的稳定性^[1], 因此, 时滞系统的控制问题一直是一个比较活跃的研究领域, 并取得了一些重要的成果^[1-4]. 文献[2]研究了一类带有不确定界的非线性时滞系统的自适应控制问题, 但要求系统时滞项满足 Lipschitz 条件并且拟有界. 文献[3]解决了一类不确定非线性时滞系统的输出反馈镇定问题, 接着将这一结果推广到随机非线性时滞系统^[4], 但未考虑非线性时滞系统的自适应跟踪问题.

作者针对一类参数化非线性时滞输出反馈系统, 采用时滞滤波器, 结合 Backstepping 的设计方法, 提出了一种无记忆自适应控制器的设计方案. 采用 Domination 方法处理非线性时滞项. 它只要求时滞项光滑, 不仅可以解决调节问题, 而且可以解决自适

应跟踪问题. 实例仿真说明了该方案的可行性.

2 问题描述 (Problem description)

考虑以下非线性时滞输出反馈系统

$$\begin{cases} \dot{x} = Ax + \phi(y) + \Phi(y)a + \begin{bmatrix} 0 \\ b \end{bmatrix} \sigma(y)u + \\ \quad H(y(t-\tau)) + G(y(t-\tau))\theta_2, \\ y = e_1^T x \end{cases} \quad (1)$$

其中 $u \in \mathbb{R}$, $y \in \mathbb{R}$, $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ 分别表示系统的输入、输出和状态;

$$A = \begin{bmatrix} 0 & & & \\ \vdots & I_{n-1} & & \\ 0 & \dots & 0 & \end{bmatrix},$$
$$\Phi(\cdot) = \begin{bmatrix} \phi_{1,1}(\cdot) & \dots & \phi_{q,1}(\cdot) \\ \vdots & & \vdots \\ \phi_{1,n}(\cdot) & \dots & \phi_{q,n}(\cdot) \end{bmatrix},$$

$$G(\cdot) = \begin{bmatrix} 0 & \cdots & 0 \\ g_{1,2}(\cdot) & \cdots & g_{p,2}(\cdot) \\ \vdots & & \vdots \\ g_{1,n}(\cdot) & \cdots & g_{p,n}(\cdot) \end{bmatrix},$$

$\phi(\cdot) = [\phi_{0,1}(\cdot), \dots, \phi_{0,n}(\cdot)]^T, H(\cdot) = [h_1(\cdot), \dots, h_n(\cdot)]^T; \phi_{j,i}(\cdot) (0 \leq j \leq q, 1 \leq i \leq n), g_{j,i}(\cdot) (1 \leq j \leq p, 2 \leq i \leq n), h_i(\cdot) (1 \leq i \leq n)$ 及 $\sigma(\cdot) \neq 0$ 是已知光滑函数; $a \in \mathbb{R}^q, \theta_2 \in \mathbb{R}^p, b = [b_m, \dots, b_0]^T \in \mathbb{R}^{m+1}$ 是未知的常值参数向量; $e_1 = [1, 0, \dots, 0]^T \in \mathbb{R}^n$, 时滞 τ 已知, 仅输出 y 可测.

本文的目标是设计一个无记忆自适应跟踪控制器, 使得系统输出渐近跟踪给定的目标轨线 $y_r(t)$.

假设 2.1^[5] b_m 符号已知, $B(s) = b_m s^m + \dots + b_1 s + b_0$ 是 Hurwitz 多项式.

假设 2.2 参考信号满足 $y_r^{(i)}(t), 0 \leq i \leq \rho = n - m$ 已知且有界, $y_r^{(\rho)}$ 分段连续.

由微分中值定理, 存在已知的光滑非负函数 $s_{h_i}(t) = s_{h_i}(y, y_r), s_{g_{j,i}}(t) = s_{g_{j,i}}(y, y_r)$, 满足

$$\begin{cases} |h_i(y) - h_i(y_r)| \leq |y - y_r| s_{h_i}(t), \\ |g_{j,i}(y) - g_{j,i}(y_r)| \leq |y - y_r| s_{g_{j,i}}(t). \end{cases} \quad (2)$$

3 自适应控制器设计 (Adaptive controller design)

由于系统状态不可测, 首先设计以下时滞滤波器:

$$\begin{cases} \dot{\xi} = A_0 \xi + ky + \phi(y) + H(y(t - \tau)), \\ \dot{\Psi}^T = A_0 \Psi^T + G(y(t - \tau)), \\ \dot{\Xi} = A_0 \Xi + \Phi(y), \\ \dot{\lambda} = A_0 \lambda + e_n \sigma(y) u, \\ v_j = A_0^j \lambda, 0 \leq j \leq m, \\ \Omega^T = [v_m, \dots, v_1, v_0, \Xi]. \end{cases} \quad (3)$$

其中 $e_n = [0, \dots, 0, 1]^T \in \mathbb{R}^n$, 选择 $k = [k_1, \dots, k_n]^T$ 使得矩阵 $A_0 = A - ke_1^T$ 为稳定矩阵. 因此, 存在正定矩阵 P 满足

$$PA_0 + A_0^T P = -I. \quad (4)$$

定义状态估计 $\hat{x} = \xi + \Omega^T \theta_1 + \Psi^T \theta_2$, 其中 $\theta_1 = [b^T, a^T]^T$, 则误差 $\varepsilon = x - \hat{x}$ 满足

$$\dot{\varepsilon} = A_0 \varepsilon. \quad (5)$$

类似于无时滞的情况^[5], 采用滤波器(3), 系统(1)可转化为以下形式:

$$\begin{cases} \dot{y} = b_m v_{m,2} + w_0 + \bar{w}^T \theta_1 + \Psi_{(2)}^T \theta_2 + \varepsilon_2 + h_1(y(t - \tau)), \\ \dot{v}_{m,i} = v_{m,i+1} - k_i v_{m,1}, 2 \leq i \leq \rho - 1, \\ \dot{v}_{m,\rho} = \sigma(y) u + v_{m,\rho+1} - k_\rho v_{m,1}. \end{cases} \quad (6)$$

其中 $w_0 = \phi_{0,1} + \xi_2, \bar{w} = [0, v_{m-1,2}, \dots, v_{0,2}, \Phi_{(1)} + \Xi_{(2)}]^T, X_{(i)}$ 表示矩阵 X 的第 i 行.

采用 Backstepping 技术设计控制律和参数自适应律. 定义坐标变换

$$\begin{cases} z_1 = y - y_r, \\ z_i = v_{m,i} - \hat{\beta} y_r^{(i-1)} - \alpha_{i-1}, 2 \leq i \leq \rho. \end{cases} \quad (7)$$

其中 $\hat{\beta}$ 是 $\beta = 1/b_m$ 的估计值. 为了描述方便, 采用 $y_\tau, y_{r\tau}$ 分别表示 $y(t - \tau), y_r(t - \tau)$.

选择稳定化函数

$$\begin{cases} \alpha_1 = \hat{\beta} \alpha_1, \\ \bar{\alpha}_1 = -c_1 z_1 - (d_1 + \frac{1}{2}) z_1 - W(t) z_1 - w_0 - \bar{w}^T \hat{\theta}_1 - \Psi_{(2)}^T \hat{\theta}_2 - h_1(y_{r\tau}), \\ \alpha_2 = -\hat{b}_m z_1 - \Lambda_2 z_2 - \Delta_2 - \frac{\partial \alpha_1}{\partial y} h_1(y_{r\tau}) - \frac{\partial \alpha_1}{\partial \xi} H(y_{r\tau}) - \frac{\partial \alpha_1}{\partial \Psi^T} G(y_{r\tau}), \\ \alpha_i = -z_{i-1} - \Lambda_i z_i - \Delta_i - \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \theta_1} \Gamma_1 \frac{\partial \alpha_{i-1}}{\partial y} w z_j - \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \theta_2} \Gamma_2 \frac{\partial \alpha_{i-1}}{\partial y} [\Psi_{(2)}^T]^T z_j - \frac{\partial \alpha_{i-1}}{\partial y} h_1(y_{r\tau}) - \frac{\partial \alpha_{i-1}}{\partial \xi} H(y_{r\tau}) - \frac{\partial \alpha_{i-1}}{\partial \Psi^T} G(y_{r\tau}), 3 \leq i \leq \rho. \end{cases} \quad (8)$$

其中

$$c_i, d_i > 0,$$

$$\alpha_\rho := \sigma(y) u + v_{m,\rho+1} - \hat{\beta} y_r^{(\rho)}, z_{\rho+1} := 0.$$

$$W(t) = W(y, y_r) =$$

$$\frac{\rho}{2} s_{h_1}^2(t) + \frac{\rho-1}{2} \sum_{j=1}^n s_{h_j}^2(t) + \frac{\rho-1}{2} \sum_{j=1}^p \sum_{k=2}^n s_{g_{j,k}}^2(t),$$

$$\Lambda_i = c_i + d_i \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + \frac{1}{2} \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + \frac{1}{2} \sum_{j=1}^n \left(\frac{\partial \alpha_{i-1}}{\partial \xi_j} \right)^2 +$$

$$\frac{1}{2} \sum_{j=1}^p \sum_{k=2}^n \left(\frac{\partial \alpha_{i-1}}{\partial \Psi_{j,k}^T} \right)^2, 2 \leq i \leq \rho,$$

$$\Delta_i = -k_i v_{m,1} - \frac{\partial \alpha_{i-1}}{\partial \Psi^T} A_0 \Psi^T - \frac{\partial \alpha_{i-1}}{\partial y} (w_0 + \bar{w}^T \theta_1 + \Psi_{(2)}^T \theta_2) -$$

$$\frac{\partial \alpha_{i-1}}{\partial \xi} (A_0 \xi + ky + \phi(y)) - \frac{\partial \alpha_{i-1}}{\partial \Xi} (A_0 \Xi + \Phi(y)) -$$

$$\sum_{j=1}^{m+i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j} (-k_j \lambda_1 + \lambda_{j+1}) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r} y_r^{(j)} -$$

$$\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_{rr}^{(i-1)} y_r^{(j)}} - \left(\frac{\partial \alpha_{i-1}}{\partial \hat{\beta}} + y_r^{(i-1)} \right) \dot{\hat{\beta}} - \frac{\partial \alpha_{i-1}}{\partial \theta_1} \Gamma_1 \tau_{\theta_1, i} - \frac{\partial \alpha_{i-1}}{\partial \theta_2} \Gamma_2 \tau_{\theta_2, i}, 2 \leq i \leq \rho,$$

$$w = [v_{m,2}, v_{m-1,2}, \dots, v_{0,2}, \Phi_{(1)} + \Xi_{(2)}]^T.$$

选择调节函数为

$$\begin{cases} \tau_{\theta_1,1} = (w - \hat{\beta}(\dot{y}_r + \bar{\alpha}_1) e_1) z_1, \\ \tau_{\theta_1,i} = \tau_{\theta_1,i-1} - \frac{\partial \alpha_{i-1}}{\partial y} w z_i, 2 \leq i \leq \rho, \\ \tau_{\theta_2,1} = [\Psi_{(2)}^T] z_1, \\ \tau_{\theta_2,i} = \tau_{\theta_2,i-1} - \frac{\partial \alpha_{i-1}}{\partial y} [\Psi_{(2)}^T]^T z_i, 2 \leq i \leq \rho. \end{cases} \quad (9)$$

控制律为

$$u = \frac{1}{\sigma(y)} (\alpha_\rho - v_{m,\rho+1} + \hat{\beta} y_r^{(\rho)}); \quad (10)$$

参数自适应律为

$$\begin{cases} \dot{\hat{\beta}} = -\gamma \text{sgn}(b_m) (\dot{y}_r + \bar{\alpha}_1) z_1, \gamma > 0, \\ \dot{\theta}_1 = \Gamma_1 \tau_{\theta_1, \rho}, \Gamma_1 > 0, \\ \dot{\theta}_2 = \Gamma_2 \tau_{\theta_2, \rho}, \Gamma_2 > 0. \end{cases} \quad (11)$$

$$W_d(t - \tau) = \begin{bmatrix} h_1(y_\tau) - h_1(y_{r\tau}) \\ -\frac{\partial \alpha_1}{\partial y} [h_1(y_\tau) - h_1(y_{r\tau})] - \frac{\partial \alpha_1}{\partial \xi} [H(y_\tau) - H(y_{r\tau})] - \frac{\partial \alpha_1}{\partial \Psi^T} [G(y_\tau) - G(y_{r\tau})] \\ \vdots \\ -\frac{\partial \alpha_{\rho-1}}{\partial y} [h_1(y_\tau) - h_1(y_{r\tau})] - \frac{\partial \alpha_{\rho-1}}{\partial \xi} [H(y_\tau) - H(y_{r\tau})] - \frac{\partial \alpha_{\rho-1}}{\partial \Psi^T} [G(y_\tau) - G(y_{r\tau})] \end{bmatrix}. \quad (13)$$

选择 Lyapunov-Krasoviskii 泛函

$$V = \frac{1}{2} z^T z + \frac{1}{2} \bar{\theta}_1^T \Gamma_1^{-1} \bar{\theta}_1 + \frac{1}{2} \bar{\theta}_2^T \Gamma_2^{-1} \bar{\theta}_2 + \frac{|b_m|}{2\gamma} \bar{\beta}^2 + \sum_{i=1}^{\rho} \frac{1}{4d_i} \epsilon^T P \epsilon + \int_{t-\tau}^t S(\delta) d\delta \quad (14)$$

其中 $S(\delta)$ 为待定的非负函数.

沿式(5),(11),(12),对式(14)求导得

$$\begin{aligned} \dot{V} = & \frac{1}{2} z^T (A_z + A_z^T) z - \sum_{i=1}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y} \epsilon_2 - \\ & \sum_{i=1}^{\rho} \frac{1}{4d_i} |\epsilon|^2 - \sum_{i=2}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial \xi} [H(y_\tau) - H(y_{r\tau})] - \\ & \sum_{i=2}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial \Psi^T} [G(y_\tau) - G(y_{r\tau})] + (z_1(t) - \\ & \sum_{i=2}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y}) [h_1(y_\tau) - h_1(y_{r\tau})] + S(t) - S(t-\tau) \leq \end{aligned}$$

定理 在假设 2.1,2.2 下,由系统(1),控制律(10)自适应律(11)和滤波器(3)组成的闭环系统,所有信号全局一致有界,并且 $\lim_{t \rightarrow \infty} [y(t) - y_r(t)] = 0$.

证 由以上设计过程,误差系统可最终表示为

$$\begin{aligned} \dot{z} = & A_z(z, t) z + W_\epsilon(z, t) \epsilon_2 + \\ & W_{\theta_1}(z, t)^T \bar{\theta}_1 + W_{\theta_2}(z, t)^T \bar{\theta}_2 - \\ & b_m (\dot{y}_r + \bar{\alpha}_1) e_1 \bar{\beta} + W_d(t - \tau). \end{aligned} \quad (12)$$

其中 $A_z(z, t)$ 为斜反对称矩阵,对角线元素为

$$a_{11} = -c_1 - d_1 - \frac{1}{2} - W(t),$$

$$a_{ii} = -\Lambda_i, 2 \leq i \leq \rho,$$

$$z = [z_1, \dots, z_\rho]^T,$$

$$W_\epsilon(z, t) = \left[1, -\frac{\partial \alpha_1}{\partial y}, \dots, -\frac{\partial \alpha_{\rho-1}}{\partial y} \right]^T,$$

$$W_{\theta_1}(z, t)^T = W_\epsilon(z, t) w^T - \hat{\beta} (\dot{y}_r + \bar{\alpha}_1) e_1 e_1^T,$$

$$W_{\theta_2}(z, t)^T = W_\epsilon(z, t) \Psi_{(2)}^T,$$

$$\begin{aligned} & \frac{1}{2} z^T (A_z + A_z^T) z - \sum_{i=1}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y} \epsilon_2 - \sum_{i=1}^{\rho} \frac{1}{4d_i} |\epsilon|^2 + \\ & \frac{1}{2} \sum_{i=2}^{\rho} z_i^2 \left[\sum_{j=1}^n \left(\frac{\partial \alpha_{i-1}}{\partial \xi_j} \right)^2 + \sum_{j=1}^n \sum_{k=2}^n \left(\frac{\partial \alpha_{i-1}}{\partial \Psi_{j,k}^T} \right)^2 + \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^2 \right] + \\ & \frac{1}{2} z_1^2(t) + z_1^2(t-\tau) \left[\frac{\rho}{2} s_{h_1}^2(t-\tau) + \frac{\rho-1}{2} \sum_{j=1}^n s_{h_j}^2(t-\tau) + \right. \\ & \left. \frac{\rho-1}{2} \sum_{j=1}^n \sum_{k=2}^n s_{g_{j,k}}^2(t-\tau) \right] + S(t) - S(t-\tau). \end{aligned} \quad (15)$$

其中定义 $\frac{\partial \alpha_0}{\partial y} := -1$, 选择非负函数 $S(\delta) = z_1^2(\delta) W(\delta)$, 则有

$$\begin{aligned} \dot{V} \leq & - \sum_{i=1}^{\rho} c_i z_i^2 - \sum_{i=1}^{\rho} d_i \left(z_i \frac{\partial \alpha_{i-1}}{\partial y} + \frac{1}{2d_i} \epsilon_2 \right)^2 - \\ & \sum_{i=1}^{\rho} \frac{1}{4d_i} (\epsilon_1^2 + \epsilon_3^2 + \dots + \epsilon_n^2) \leq \end{aligned}$$

$$- \sum_{i=1}^p c_i z_i^2. \quad (16)$$

由时滞系统稳定性定理^[6], $\lim_{t \rightarrow \infty} z(t) = 0$, 其余部分的证明类似于无时滞的情况^[5].

4 仿真 (Simulation)

考虑以下非线性时滞系统:

$$\begin{cases} \dot{x}_1 = x_2 + \theta_1 y^2 + \sin(y(t - \tau)), \\ \dot{x}_2 = u + \theta_2 \cos(y(t - \tau)), \\ y = x_1. \end{cases}$$

在仿真中,参考信号选为 $y_r(t) = \sin t \sin 2t$, 初始条件为: $x_1(t) = 0.1, t \in [-\tau, 0]; x_2(0) = 1, y_r(t) = 0, t \in [-\tau, 0]; \tau = 2$, 其余全取为 $0, t \in [0, 20]; c_1 = c_2 = d_1 = d_2 = 0.1; \gamma_1 = 0.65, \gamma_2 = 0.5; k_1 = k_2 = 1; \theta_1 = 0.5, \theta_2 = -1$. 仿真结果如图 1, 2 所示.

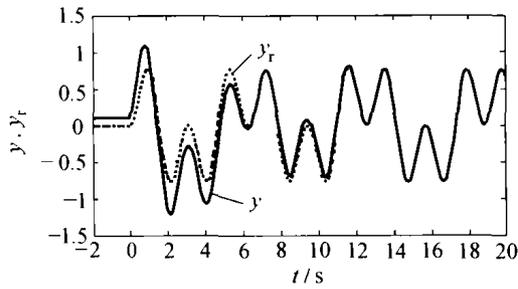


图 1 系统输出仿真结果
Fig. 1 Simulation result of system output

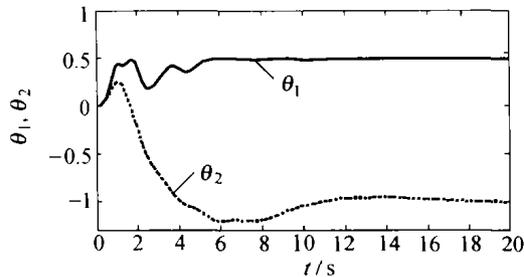


图 2 估计参数仿真结果
Fig. 2 Simulation result of estimated parameters

5 结语 (Conclusion)

针对一类具有参数化输出反馈形式的非线性时滞系统,采用时滞滤波器估计系统状态,运用占优化方法处理非线性时滞项,结合 Backstepping 的设计技巧,提出了一种无记忆自适应输出跟踪控制方法.但仍然要求时滞矩阵 G 的第一行为零,如何取消这假设,仍然是一个有待于进一步研究的问题.

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