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Integral constraints based on stable pole-zero near cancellation in scalar feedback systems

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Abstract: This paper developed time domain integral constraints on error response for SISO feedback control systems caused by nominal plant's near cancellation of stable pole-zero near the $j\omega$-axis. These integral constraints should be satisfied by any feedback control systems. These integral constraints give new insight into the inherent trade-offs. It will result in the settling time longer or the infinite norm of the error response larger when there are near cancellations of stable pole-zero near the $j\omega$-axis. Hence, when feedback control systems are designed, it is necessary to avoid the compensator's poles and zeros nearly cancelling the nominal's zeros and poles (even if these poles and zeros are stable).

Key words: time domain integral constraints; feedback system; response; simulation

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纯量反馈系统稳定零极近似相消的积分约束

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摘要: 导出了单输入单输出反馈控制系统误差响应基于名义系统稳定的零、极点近似相消的时间域积分约束,此积分约束是任何反馈控制系统均应满足的.这一约束给出了单输入单输出反馈系统固有折中的新的观点.名义系统稳定的零、极点近似相消的存在导致反馈控制系统的调节时间延长或者误差响应的无穷范数变大.因此,在反馈控制系统设计中,尽量避免补偿器的零、极点与名义系统的极、零点近似相消(即使这些零、极点是稳定的).

关键词: 时间域积分约束; 反馈系统; 响应; 仿真

1 Introduction

There are always basic limitations on the achievable performance involved in the feedback control of any physical plant. These limitations arise from several sources. Bode developed the fundamental work on structural limitations in the control of linear time invariant systems. In [1,2] the waterbed effect for non-minimum-phase plant were given, which showed that if the system gain is pushed down on one frequency range, it pops up somewhere else. [2] also derived the area formula, which applies minimum and non-minimum phase plants. [3,4] extend the corresponding work to multivariable systems and to discrete time systems. [5] showed that performance and robust stability properties are limited by the presence of RHP poles and zeros for SISO system. [6] exploreed time-domain integral constraints to show that slow stable poles place constraints on the settling time of the closed-loop systems. [7] based on unit step response showed that fundamental limitations arise from the presence of stable zeros near or on the $j\omega$-axis. [8] treated multivariable systems by use singular values and the theory of subharmonic functions. Refinements of these results have also been presented in [9]. [2] converted the multivariable problem into a scalar one by pre-and post-multiplying the sensitivity function by vectors or by use of determinants[5]. A similar idea was advanced in the work of [10] which used directions associated with poles and zeros of the system resulting in a directional study of trade-offs. [3] developed integral constraints on sensitivity vectors for multivariable feedback systems due to either unstable poles or non-minimum-phase zeros of the plant; By use of these integral constraints[3] the inherent trade-offs in sensitivity reduction and the cost of decoup-

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ling were given. [11]and[12]extended the corresponding result in [7] to general tracking problem in SISO and MIMO feedback systems, showed that the fundamental limitations arise from the presence of stable zeros near or on the jω-axis. [2] showed that the near cancellation of unstable poles and zeros leads the feedback systems to loss of internal stability.

The aim of the present paper is to continue the research line of [11], and extend the corresponding result to near stable pole-zero cancellation. This paper shows the effect of near stable pole-zero cancellation near tie jω-axis for scalar feedback control systems tracking problem. Time domain integral constraints of the feedback control system tracking error are developed which shows near stable pole-zero cancellation near on the jω-axis imply a lower bound on the achievable settling time of the feedback control systems. One example explains the results of this paper.

2 Preliminaries

We consider the linear time-invariant feedback control systems shown in Fig. 1. The symbols in Fig. 1 have the following meaning. $p(s)$ is the proper rational plant transfer function; $c(s)$ is the proper rational controller transfer function; $u(t), e(t)$, and $y(t)$ are, respectively, the reference, error signal, and plant output.

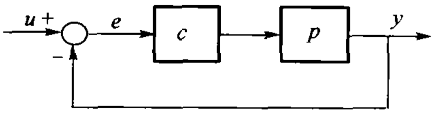

Fig. 1 Feedback control systems

Suppose the plant and the controller are described by coprime fractional representations (over the ring of proper stable transfer functions) [13]

$$p = \frac{n_p}{d_p},\ c = \frac{n_c}{d_c}. \tag{1}$$

Further, we assume that c is chosen so that the closed loop is internally stable (i.e. $n_p n_c + d_p d_c$ is analytic over the right-half plane). Then the sensitivity function and the complementary sensitivity function are defined, respectively, by

$$S(s) = (1 + pc)^{-1} = d_c d_p (d_c d_p + n_c n_p)^{-1} \tag{2}$$

and

$$T(s) = 1 - S(s) = (1 + pc)^{-1} pc = n_c n_p (d_c d_p + n_c n_p)^{-1}. \tag{3}$$

The L_∞ norm defined by

$$\| f \|_\infty = | f(t) |_{\text{ess sup} t \geqslant 0}. \tag{4}$$

When the plant $p(s)$ is stable, the set of all compensators that stabilize the plant $p(s)$ is given in [13] by

$$S(p) = \{c : c = q(1 - pq)^{-1}\}. \tag{5}$$

Where q is a proper, causal stable transfer function.

3 Time-domain constraints of the feedback systems for near stable pole-zero cancellation

Theorem 1 Consider the feedback control system shown in Fig. 1. Suppose the following two conditions hold: i) The plant $p(s)$ has at least one pair of near stable pole-zero cancellation near the jω-axis at $-(\sigma \pm |\delta_1|) \pm \mathrm{j}\omega_0 \pm |\delta_2|)$ and $-\sigma \pm \mathrm{j}\omega_0 (\sigma \geqslant 0, \omega_0 \geqslant 0, |\delta_1|, |\delta_2| \ll \sqrt{\sigma^2 + \omega_0^2})$; ii) All poles of the closed-loop system have real parts less than $-\alpha(\alpha > 0, \alpha > \sigma)$

Under these conditions the following integral constraints hold on the error signal $e(t)$ of the tracking problem shown in Fig. 1:

$$\int_0^\infty (\mathrm{e}^{(\sigma - \mathrm{j}\omega_0)t} - \mathrm{e}^{((\sigma \pm |\delta_1|) - \mathrm{j}(\omega_0 \pm |\delta_2|))t}) e(t) \mathrm{d}t = U(-\sigma + \mathrm{j}\omega_0), \tag{6}$$

$$\int_0^\infty (\mathrm{e}^{(\sigma + \mathrm{j}\omega_0)t} - \mathrm{e}^{((\sigma \pm |\delta_1|) + \mathrm{j}(\omega_0 \pm |\delta_2|))t}) e(t) \mathrm{d}t = U(-\sigma - \mathrm{j}\omega_0). \tag{7}$$

Proof The Laplace transform of the tracking error

$$e(t) = u(t) - y(t),$$

satisfying

$$E(s) = (1 - T(s)) U(s). \tag{8}$$

According to the internal theorem, in order to track the signal $u(t)$, the tracking error $e(t)$ satisfying

$$\lim_{t \to +\infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s(1 - T(s)) U(s) = 0.$$

So $s = 0$ is not a pole of (8). Hence, by assumption ii), $s = 0$ lies inside the region of convergence of the transform

$$\int_0^\infty \mathrm{e}^{-st} e(t) \mathrm{d}t = (1 - T(s)) U(s). \tag{9}$$

Because of $T(-\sigma \pm \mathrm{j}\omega_0) = 0$, set $s = -\sigma \pm \mathrm{j}\omega_0$ equation (9) gives

$$\int_0^\infty \mathrm{e}^{(\sigma \mp \mathrm{j}\omega_0)t} e(t) \mathrm{d}t = U(-\sigma \pm \mathrm{j}\omega_0). \tag{10}$$

Set $s = -(\sigma \pm |\delta_1|) \pm \mathrm{j}(\omega_0 \pm |\partial_2|)$ in equation (9), because of $T(-(\sigma \pm |\delta_1|) \pm \mathrm{j}(\omega_0 \pm |\delta_2|)) = 1$ equation (9) gives

$$\int_0^\infty \mathrm{e}^{(\sigma \pm |\delta_1|) \mp \mathrm{j}(\omega_0 \pm |\delta_2|)t} \mathrm{e}(t) \mathrm{d}t = 0. \tag{11}$$

Subtracting (11) from (10), we get the integral constraints (6) and (7).

From the integral constraints (6) and (7) we can infer that when the zeros approach the jω-axis, in order to satisfy the equations (6) and (7), $e(t)$ will change signs. That is, the feedback will appear overshoot.

Corollary 1 Consider the feedback control system shown in Fig.1, under the assumption of Theorem 1, suppose that $|\delta_1| = \frac{\sigma}{k}, \delta_2 = 0$, then the following integral constraints hold

$$\int_0^{\infty}(1 - e^{\pm\left|\frac{\sigma t}{k}\right|})e^{\sigma t}(\cos\omega_0 t)e(t)\mathrm{d}t = \frac{U(-\sigma + \mathrm{j}\omega_0) + U(-\sigma - \mathrm{j}\omega_0)}{2}, \tag{12}$$

$$\int_0^{\infty}(1 - e^{\pm\left|\frac{\sigma t}{k}\right|})e^{\sigma t}(\sin\omega_0 t)e(t)\mathrm{d}t = \frac{U(-\sigma + \mathrm{j}\omega_0) - U(-\sigma - \mathrm{j}\omega_0)}{2\mathrm{j}}. \tag{13}$$

Proof Using the relation that

$$\cos\omega_0 t = \frac{1}{2}(e^{-\mathrm{j}\omega_0 t} + e^{\mathrm{j}\omega_0 t}),$$

$$\sin\omega_0 t = \frac{1}{2\mathrm{j}}(e^{\mathrm{j}\omega_0 t} - e^{-\mathrm{j}\omega_0 t})$$

and set $|\delta_1| = \frac{\sigma}{k}, \delta_2 = 0$ in (6),(7), we get the time domain constraints (12) and (13).

4 Lower bounds on $\| e \|_\infty$

Definition 1 Define the exact settling time of the system to be

$$t_s = \inf\{\tau : |e(t)| = 0, \forall t > \tau\}. \tag{14}$$

Corollary 2 Consider the feedback control system shown in Fig.1. Under the assumption of Theorem 1, and assume that the exact settling time t_s satisfying $\omega_0 t_s \leqslant \frac{\pi}{2}$, $\delta_1 = -\frac{\sigma}{k}, \delta_2 = 0$, then the tracking error's infinite norm has the following lower bound:

$$\| e \|_\infty \geqslant ((1 - e^{-\frac{\sigma t_s}{k}})e^{\sigma t_s})^{-1} \cdot \max\left\{\frac{\omega_0 |U(-\sigma + \mathrm{j}\omega_0) + U(-\sigma - \mathrm{j}\omega_0)|}{2\sin\omega_0 t_s}, \frac{\omega_0 |U(-\sigma - \mathrm{j}\omega_0) - U(-\sigma + \mathrm{j}\omega_0)|}{2(1 - \cos\omega_0 t_s)}\right\}. \tag{15}$$

Proof From Definition 1 and the equations (12),(13), paying attention $1 - e^{-\frac{\sigma t}{k}} \leqslant 1 - e^{-\frac{\sigma t_s}{k}}$, $e^{\sigma t} \leqslant e^{\sigma t_s}$, and $\omega_0 t_s \leqslant \frac{\pi}{2}$, we get the following inequalities:

$$e^{\sigma t_s}(1 - e^{-\frac{\sigma t_s}{k}})\| e \|_\infty \frac{\sin\omega_0 t_s}{\omega_0} \geqslant \frac{|U(-\sigma + \mathrm{j}\omega_0) + U(-\sigma - \mathrm{j}\omega_0)|}{2},$$

$$e^{\sigma t_s}(1 - e^{-\frac{\sigma t_s}{k}})\| e \|_\infty \frac{1 - \cos\omega_0 t_s}{\omega_0} \geqslant \frac{|U(-\sigma - \mathrm{j}\omega_0) - U(-\sigma + \mathrm{j}\omega_0)|}{2}.$$

Hence, the inequality (15) holds.

Because the actual inputs are unit step, unit ramp, unit accelerate and their linear combinations, their corresponding Laplace transforms are $1/s, 1/s^2, 1/s^3$, and their linear combinations. [11] shows that for fixed t_s, as ω_0 becomes small, the lower bounds on $\| e \|_\infty$ become arbitrarily large. Compare the inequality (15) with the corresponding result in [11], it shows that because the existence of the factor $(1 - e^{-\frac{\sigma t_s}{k}})^{-1}$, the existence of near stable pole-zero cancellation near the $\mathrm{j}\omega$-axis deteriorate the feedback properties.

5 Example

Consider the plant $p(s) = \frac{100s^2 + 220s + 101}{100s^2 + 220s + 122}$, $p(s)$ has zeros at $-1 \pm 0.1\mathrm{j}$, poles at $-\frac{11}{10} \pm 0.1\mathrm{j}$. In order to find a compensator c that stabilize p and also track a step reference signal, let $q = a$, according to (2) and (5), we have

$$S(s) = (I + pc)^{-1} = I - pq = 1 - ap.$$

Let $S(0) = 0$, we have $a = \frac{122}{101}$, by (5), the compensator is given by $c(s) = q(1 - pq)^{-1}$.

The simulation response of $e(t)$ is shown in Fig.2. where

$$E(s) = S(s)U(s) = -\frac{2100s + 2180}{10100s^2 + 22220s + 12322}.$$

From the Fig.2, we can see, although the zeros do not near the $\mathrm{j}\omega$-axis, the feedback system shows long tracking time due to the near poles-zeros cancellation.

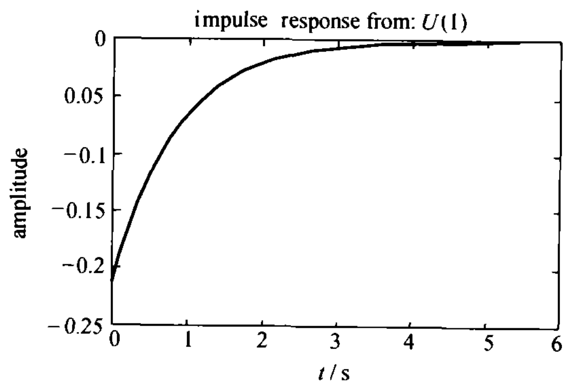

Fig. 2 Impulse response of $E(s)$

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