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Exponential stability of stochastic Hopfield neural networks with distributed parameters

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Abstract: Based on stochastic Fubini theorem, the Hopfield neural network system depicted by a stochastic partial differential equation is translated into a stochastic ordinary differential equation. By constructing a mean Lyapunov function with respect to the space variables and using Itô formula under the integral operators, the exponential stability of stochastic neutral systems with distributed parameters is investigated by deviating of the function along the trajectories of the systems. Also, the Lyapunov exponent estimate is obtained. Thus, the stability of stochastic systems with distributed parameters is studied by Lyapunov direct method.

Key words: Hopfield neural networks; distributed parameter; Lyapunov function

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具分布参数的随机 Hopfield 神经网络的指数稳定

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摘要: 基于随机 Fubini 定理,将随机偏微分方程描述的 Hopfield 神经网络系统转化为用相应的随机常微分方程来描述.利用关于空间变量平均的 Lyapunov 函数与 Itô 公式,通过对所构造的 Lyapunov 函数在 Itô 微分规则下对相应系统求导的方法,获得了系统指数稳定的代数判据及其 Lyapunov 指数估计.实现了运用 Lyapunov 直接法对分布参数系统稳定性的研究.

关键词: Hopfield 神经网络; 分布参数; Lyapunov 函数

1 Introduction

In recent years, there have been many results about the stability of deterministic Hopfield neural network[2~5]. In the aspect of stochastic Hopfield neural network including time delay and time-varying delay, many kinds of stability, such as exponential stability, mean-square exponential stability and almost sure asymptotic stability have been researched extensively and deeply in [6~12]. In fact, no one will ignore the diffusion phenomena when he studies the electrons moving in an asymmetric electron magnetic field. When he studies the stability or stabilization of Hopfield neural network, he should consider not only the stochastic disturbance but also the diffusion phenomena. Hence, the network model should be described by boundary value problems of partial differential equations. But so far, at the best knowledge of the authors, no achievement has been made about the stability of the stochastic Hopfield neural networks with distributed parameter. So it is of great value to study the stability of those systems.

In this paper, based on stochastic Fubini theorem, we will consider the integral of solution random field with regard to spatial variables as the solution of the corresponding stochastic ordinary differential equation and research its stability. By constructing an average Lyapunov functional and then using Itô formula, we will obtain some sufficient conditions of exponential stability criteria for stochastic system with distributed parameter.

2 Problem statements and preliminaries

In this paper, we will consider a more generalized Hopfield neural networks with distributed parameter:

$$C_i \mathrm{d}u_i(t,x) = \Big\{\sum_{k=1}^{m} \frac{\partial}{\partial x_k}\Big[D_{ik}(t,x,u)\frac{\partial u_i}{\partial x_k}\Big] - \frac{u_i}{R_i} + I_i + \sum_{j=1}^{n} T_{ij} g_j(u_j(t,x))\Big\}\mathrm{d}t + \sum_{l=1}^{k} \sigma_{il}(u_i(t,x)\mathrm{d}W_l(t) \tag{1}$$

with the initial conditions

$$u_i(0,x) = \varphi_i(x),\ i = N,\ x \in G \tag{2}$$

and the boundary conditions

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$$\frac{\partial u_i(t,x)}{\partial n} = 0,\ i \in N,\ (t,x) \in \mathbb{R}^+ \times \partial G, \quad (3)$$

where $G = \{x = (x_1,\cdots,x_r)^{\mathrm{T}},\ |x_k| < \omega\} \subset \mathbb{R}^r$, $N = \{1,\cdots,n\}$, $\mathbb{R}^+ = [0, +\infty)$ is a bounded convex domain with smooth boundary, ∂G is sufficiently smooth; $D_{ik} = D_{ik}(t,x,u) \geqslant 0$ is diffusion operator; C_i, R_i, I_i are capacitance, resistance and current respectively; $T = (T_{ij})_{n\times n}$ is weighting matrix; u_i, x_i are state variables and spatial variable; I_i is exoteric input and g_j is activation function, they are all global Lipshitz continuous; $W(t) = (W_1(t),\cdots,W_n(t))^{\mathrm{T}}$ is an m-dimensional Brownian motion defined on the complete probability space $(\Omega, F, (F_t)_{t\in I}, P)$ with natural filtration $\{F_t\}_{t\geqslant 0}$; $\sigma(u) = (\sigma_{il}(u_i(t,x)))_{n\times m}$ is local Lipshitz continuous and satisfies the linear increasing condition with $\sigma(0) = 0$; $\varphi_i(x), i \in N$ are measurable functions on G. Here we denote

$$\frac{\partial u_i}{\partial n} := \mathrm{col}\left(\frac{\partial u_i}{\partial x_1},\cdots,\frac{\partial u_i}{\partial x_r}\right),\ (t,x) \in \mathbb{R}^+ \times \partial G,$$

n is the outside unit normal vector of G.

Definition 1[1] We say the stochastic field $u(t,x) = \mathrm{col}(u_1(t,x),\cdots,u_n(t,x))$ is a stochastic field solution of problem (1) ~ (3), if the following conditions are satisfied

I) $u(t,x) = \mathrm{col}(u_1(t,x),\cdots,u_n(t,x))$ adapts $\{F_t\}_{t\geqslant 0}$;

II) For each $T \in \mathbb{R}^+$, $u(t,x) \in C([0,T] \times G, \mathbb{R}^n)$, $\mathrm{E}(\int_0^T |u(t,x)|^2 + |\nabla u(t,x)|^2) < +\infty$;

III) For each $T \in \mathbb{R}^+$, $t \in (0,T]$, it holds that

$$\begin{cases} \int_G u_i(t,x)\mathrm{d}x = \\ \int_G \varphi_i(x)\mathrm{d}x + \int_G\int_0^t \sum_{k=1}^r \frac{\partial}{\partial x_k}[D_{ik}(t,x,u)\frac{\partial v_i(\xi,x)}{\partial x_k}]\mathrm{d}\xi\mathrm{d}x + \\ \int_G\int_0^t[-\frac{u_i}{R_i} + I_i + \sum_{j=1}^n T_{ij}g_j(u_j(\xi,x))]\mathrm{d}\xi\mathrm{d}x + \\ \int_G\int_0^t\sum_{l=1}^m \sigma_{il}(u_i(\xi,x))\mathrm{d}W_l(t)\mathrm{d}x, \\ i = N,\ (t,x) \in (0,T] \times G, \qquad \text{a.s.} \end{cases} \quad (4)$$

Definition 2 We say the equilibrium of (1) ~ (3) about the given norm $\|\cdot\|_2$ is exponentially stable in mean square, if for every random field solution

$$u(t,x) = \mathrm{col}(u_1(t,x),\cdots,u_n(t,x))$$

of problems (1) ~ (3), there exist constants $M > 0, \lambda > 0$ such that

$$\mathrm{E}(\|u - u^*\|_G^2) \leqslant M\mathrm{e}^{-\lambda t}, \qquad \text{a.s.}$$

Let $L^2(G)$ be a space of real Lebesgue measurable functions on G with the L_2-norm defined as

$$\|u\|_2 = [\int_G |u(x)|^2\mathrm{d}x]^{1/2}.$$

Then $L^2(G)$ becomes a Banach space, and $|u|$ denotes the Euclid norm.

Note that stochastic Fubini theorem, from condition (5), if $g_i(i = 1,\cdots,n)$, $\sigma_{il}(i = 1,\cdots,n; l = 1,\cdots,m)$ are linear functions, then the stochastic process

$$\bar{u}(t) = \frac{1}{|G|}\int_G u(t,x)\mathrm{d}x,\ t \in \mathbb{R}^+ \quad (5)$$

is the solution process to the stochastic ordinary differential equation

$$\begin{aligned} &\mathrm{d}\bar{u}_i(t) = \\ &\{\int_G \sum_{k=1}^r \frac{\partial}{\partial x_k}[D_{ik}(t,x,u)\frac{\partial u_i(\xi,x)}{\partial x_k}]\mathrm{d}x + \\ &\int_G[-\frac{u_i}{R_i} + I_i + \sum_{j=1}^n T_{ij}g_j(u_j(\xi,x))]\mathrm{d}x\}\mathrm{d}t + \\ &\int_G\sum_{l=1}^m \sigma_{il}(u_i(\xi,x))\mathrm{d}x\mathrm{d}W_l(t),\ i = N,\ (t,x) \in \mathbb{R}^+ \times G \end{aligned} \quad (6)$$

with the initial condition

$$\bar{u}_i(0) = \bar{\varphi}_i,\ i \in N. \quad (7)$$

Where $|G|$ denotes the measure of bounded set G, $\bar{\varphi}_i = 1/G\int_G \varphi_i(x)\mathrm{d}x$. However, in a neural network, g_i and $\sigma_{il}(i = 1,\cdots,n; l = 1,\cdots,m)$ may not surely be linear functions. Therefore, we treat (5) as the solution process to (6) formally in the following discussion. By constructing an average Lyapunov functional and using the Itô differential formula, we reach our result via the computation of the differential of the Lyapunov functional along the system (1) ~ (3).

3 Main results

Consider the stochastic Hopfield neural network with distributed parameter

$$\begin{aligned} &C_i\mathrm{d}u_i(t,x) = \\ &\{\sum_{k=1}^m \frac{\partial}{\partial x_k}[D_{ik}(t,x,u)\frac{\partial u_i}{\partial x_k}) - \frac{u_i}{R_i} + I_i + \\ &\sum_{j=1}^n T_{ij}g_j(u_j(t,x))\}\mathrm{d}t + \sum_{l=1}^k \sigma_{il}(u_i(t,x)\mathrm{d}W_l(t) \end{aligned} \quad (8)$$

in the following discussion, we let u^* be the equilibrium of system (8), $u(t,x) = \mathrm{col}(u_1,\cdots,u_n)$ is random field solution of (8), then the system (8) can be rewritten as

$$C_i\mathrm{d}(u_i - u_i^*) =$$

$$\{\sum_{k=1}^{m}\frac{\partial}{\partial x_k}[D_{ik}(t,x,u)\frac{\partial(u_i-u_i^*)}{\partial x_k}]-$$
$$\frac{u_i-u_i^*}{R_i}+\sum_{j=1}^{n}T_{ij}(g_j(u_j)-g_j(u_j^*))\}dt+$$
$$\sum_{l=1}^{k}[\sigma_{il}(u_i)-\sigma_{il}(u_i^*)dW_l(t)]. \tag{9}$$

Theorem 1 Assume that

H1) $|g_j(u_1)-g_j(u_2)|\leqslant\frac{1}{2}l_j|u_1-u_2|, u_1,u_2\in\mathbb{R}^n, j=1,\cdots,n$;

H2) There exists a constant $\mu>0$, such that $\mathrm{tr}[\sigma(u)-\sigma(u^*)]^{\mathrm{T}}(\sigma(u)-\sigma(u^*))\leqslant\mu|u-u^*|^2$;

H3) $A=\mathrm{diag}(\frac{-2}{C_1R_1},\cdots,\frac{-2}{C_nR_n})+(b_{ij})_{n\times n}+\mu I_n$ is negative defined, here $b_{ii}=\frac{|T_{ii}|l_i}{C_i}, b_{ij}=b_{ji}=\frac{|T_{ij}|l_j}{2C_i}(i\neq j)$, and $(b_{ij})_{n\times n}$ is a symmetric matrix.

Suppose that $\lambda_{\max}(A)\varphi-\lambda$, then at the equilibrium $u=u^*$, the random field solution to the system of (9) has the following Lyapunov exponent estimate with respect to the spatial variables, namely

$$\sup\lim_{T\to\infty}(1/T)\lg(\mathrm{E}(\|u-u^*\|^2))\leqslant-\alpha.$$

Proof Let

$$\bar{V}(t,u(t,x))=\sum_{i=1}^{n}(u_i-u_i^*)^2. \tag{10}$$

Based on this, we construct a Lyapunov auxiliary functional

$$V(t,u(t,x))=$$
$$\int_G\bar{V}(t,u(t,x))dx=\int_G\sum_{i=1}^{n}(u_i-u_i^*)^2dx. \tag{11}$$

Obviously, it is positive definite. By using Itô differential formula, computing the differential of (11) along system (9), we obtain that

$$dV(t,u(t,x))|_{(9)}=$$
$$\int_G[L\bar{V}(t,u(t,x))dt+$$
$$\left(\frac{\partial\bar{V}}{\partial u}\right)^{\mathrm{T}}(\sigma(u)-\sigma(u^*))dW(t)]dx=$$
$$\int_G 2\sum_{i=1}^{n}(u_i-u_i^*)[-\frac{u_i-u_i^*}{C_iR_i}+$$
$$\sum_{j=1}^{n}\frac{T_{ij}}{C_i}(g_j(u_j)-g_j(u_j^*)]dxdt+$$
$$\int_G\mathrm{tr}(\sigma(u)-\sigma(u^*)^{\mathrm{T}}(\sigma(u)-\sigma(u^*))dxdt+$$
$$\int_G\sum_{k=1}^{r}\frac{\partial\bar{V}}{\partial u_i}\frac{\partial}{\partial x_k}[\frac{D_{ik}(t,x,u)}{C_i}\frac{\partial(u_i-u_i^*)}{\partial x_k}]dxdt+$$
$$2\int_G\sum_{i=1}^{n}(u_i-u_i^*)\sum_{l=1}^{m}(\sigma_{il}(u_i)-\sigma_{il}(u_i^*))dxdW_l(t). \tag{12}$$

Where

$$L\bar{V}(t,u(t,x))=$$
$$\frac{\partial\bar{V}}{\partial t}+\sum_{i=1}^{n}\left(\frac{\partial\bar{V}}{\partial u_i}\frac{\partial u_i}{\partial t}\right)_{(9)}+$$
$$\frac{1}{2}\mathrm{tr}[\sigma(u)-\sigma(u^*)]^{\mathrm{T}}(\sigma(u)-\sigma(u^*))\Delta\bar{V}(t,u(t,x)).$$

From the boundary condition and Green formula, we obtain that

$$\sum_{k=1}^{m}\int_G(u-u_i^*)\frac{D_{ik}}{C_i}\frac{\partial}{\partial x_k}\left(\frac{\partial(u-u_i^*)}{\partial x_k}\right)dx=$$
$$\int_G(u-u_i^*)\nabla\cdot\left(\frac{D_{ik}}{C_i}(\frac{\partial(u-u_i^*)}{\partial x_k})\right)_{k=1}^{m}dx=$$
$$\int_G\nabla\cdot(u-u_i^*)\left(\frac{D_{ik}}{C_i}(\frac{\partial(u-u_i^*)}{\partial x_k})\right)_{k=1}^{m}dx-$$
$$\int_G\left(\frac{D_{ik}}{C_i}(\frac{\partial(u-u_i^*)}{\partial x_k})\right)_{k=1}^{m}\Delta\cdot(u-u_i^*)dx=$$
$$\int_G(u-u_i^*)\left(\frac{D_{ik}}{C_i}(\frac{\partial(u-u_i^*)}{\partial x_k})\right)_{k=1}^{m}ds-$$
$$\sum_{k=1}^{m}\int_G\frac{D_{ik}}{C_i}\left(\frac{\partial(u-u_i^*)}{\partial x_k}\right)^2dx=$$
$$-\sum_{k=1}^{m}\int_G\frac{D_{ik}}{C_i}\left(\frac{\partial(u-u_i^*)}{\partial x_k}\right)^2dx\leqslant0. \tag{13}$$

Δ is Laplace operator,

$$\left(\frac{D_{ik}}{C_i}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)_{k=1}^{m}:=$$
$$\left(\frac{D_{i1}}{C_i}\frac{\partial(u_i-u_i^*)}{\partial x_1},\cdots,\frac{D_{im}}{C_i}\frac{\partial(u_i-u_i^*)}{\partial x_m}\right)$$

from the conditions II) and (13), the expression (12) can be rewritten as

$$dV(t,u(t,x))|_{(9)}\leqslant$$
$$\int_G 2\sum_{i=1}^{n}(u_i-u_i^*)[-\frac{u_i-u_i^*}{C_iR_i}+$$
$$\sum_{j=1}^{n}\frac{T_{ij}}{C_i}(g_j(u_j)-g_j(u_j^*)]dxdt+$$
$$\int_G\mathrm{tr}(\sigma(u)-\sigma(u^*))^{\mathrm{T}}(\sigma(u)-\sigma(u^*))dxdt+$$
$$\int_G\sum_{i=1}^{n}(u_i-u_i^*)\sum_{l=1}^{n}(\sigma_{il}(u_i)-\sigma_{il}(u_i^*))dxdW_l(t)\leqslant$$
$$-\int_G 2\sum_{i=1}^{n}\frac{(u_i-u_i^*)^2}{C_iR_i}dxdt+$$
$$\int_G 2\sum_{i=1}^{n}(u_i-u_i^*)\sum_{j=1}^{n}\frac{T_{ij}}{C_i}(g_j(u_j)-g_j(u_j^*))dxdt+$$
$$\int_G\mathrm{tr}(\sigma(u)-\sigma(u^*))^{\mathrm{T}}(\sigma(u)-\sigma(u^*))dxdt+$$

$$2\int_G \sum_{i=1}^n (u_i - u_i^*)\sum_{l=1}^m (\sigma_{il}(u_i) - \sigma_{il}(u_i^*))\mathrm{d}x\mathrm{d}W_l(t) \leqslant$$

$$-\int_G (u - u^*)^{\mathrm{T}}\mathrm{diag}(\frac{2}{C_1R_1}, \cdots, \frac{2}{C_nR_n})(u - u^*)\mathrm{d}x\mathrm{d}t +$$

$$\int_G (u - u^*)^{\mathrm{T}} \left| \frac{T_{ij}l_j}{C_i} \right|_{n\times n} (u - u^*)\mathrm{d}x\mathrm{d}t +$$

$$\frac{1}{2}\int_G \mathrm{tr}(\sigma(u) - \sigma(u^*))^{\mathrm{T}}(\sigma(u) - \sigma(u^*))\mathrm{d}x\mathrm{d}t +$$

$$\int_G \sum_{i=1}^n (u_i - u_i^*)\sum_{l=1}^m (\sigma_{il}(u_i) - \sigma_{il}(u_i^*))\mathrm{d}x\mathrm{d}W_l(t) \leqslant$$

$$-\lambda\int_G |u - u^*|^2\mathrm{d}x\mathrm{d}t +$$

$$\int_G \sum_{i=1}^n (u_i - u_i^*)\sum_{l=1}^m (\sigma_{il}(u_i) - \sigma_{il}(u_i^*))\mathrm{d}x\mathrm{d}W_l(t). \tag{14}$$

Taking $\alpha \in (0, \lambda)$, from the Itô formula, we have

$$\mathrm{d}[\mathrm{e}^{\alpha t}V(t,u)] =$$

$$\mathrm{e}^{\alpha t}[\alpha V(t,u)\mathrm{d}t + \mathrm{d}V(t,u)] \leqslant$$

$$\alpha\mathrm{e}^{\alpha t}\int_G \sum_{i=1}^n (u_i - u_i^*)^2\mathrm{d}x\mathrm{d}t - \lambda\mathrm{e}^{\alpha t}\int_G \| u - u^* \|^2\mathrm{d}x\mathrm{d}t +$$

$$2\mathrm{e}^{\alpha t}\int_G \sum_{i=1}^n (u_i - u_i^*)\sum_{l=1}^m (\sigma_{il}(u_i) - \sigma_{il}(u_i^*))\mathrm{d}x\mathrm{d}W_l(t).$$

Integrating the inequality (14) from 0 to $T > 0$ and take the mathematical expectation at its both sides, we have

$$\mathrm{E}[\mathrm{e}^{\alpha t}\bar{V}(T,u)] \leqslant$$

$$M + \mathrm{E}[\alpha\int_G\int_0^T \mathrm{e}^{\alpha t}\sum_{i=1}^n (u_i - u_i^*)^2\mathrm{d}t\mathrm{d}x -$$

$$\lambda\int_G\int_0^T \mathrm{e}^{\alpha t} | u - u^* |^2\mathrm{d}t\mathrm{d}x] \leqslant M, \tag{15}$$

where $\int_G (\sum_{i=1}^n \varphi_i^2(x))\mathrm{d}x$, then we have

$$\mathrm{e}^{\alpha t}\mathrm{E}(\int_G | u - u^* |^2\mathrm{d}x) \leqslant M$$

or

$$\mathrm{e}^{\alpha t}\mathrm{E}(\| u - u^* \|^2) \leqslant M.$$

Namely

$$\sup \lim_{T\to\infty}(1/T)\lg(\mathrm{E}(\| u - u^* \|^2)) \leqslant -\alpha.$$

The proof is completed.

Now we consider the following stochastic neural network with distributed parameter

$$C_i\mathrm{d}u_i(t,x) =$$

$$[d_i(t)\Delta u_i(t,x) - \frac{u_i}{R_i} + I_i +$$

$$\sum_{j=1}^n T_{ij}g_j(u_j(t,x))\mathrm{d}t + \sum_{l=1}^k \sigma_{il}(u_i(t,x)\mathrm{d}W_l(t), \tag{16}$$

the initial and boundary conditions are the same as (2), (3), $d_i(t) > 0 (i = 1, \cdots, n)$ are continuous functions with low boundary, Δ is Laplace operator.

Let $u^* = \mathrm{col}(u_1^*, u_2^*, \cdots, u_n^*)$ is equilibrium of system (17), similarly to the Theorem1, we have the following theorem.

Theorem 2 Assume that

H1) $| g_j(u_1) - g_j(u_2) | \leqslant \frac{1}{2}l_j | u_1 - u_2 |, u_1, u_2 \in \mathbb{R}^n, j = 1, \cdots, n;$

H2) There exists a constant $\mu > 0$, such that $\mathrm{tr}[(\sigma(u) - \sigma(u^*))]^{\mathrm{T}}(\sigma(u) - \sigma(u^*)) \leqslant \mu | u - u^* |^2$. We also let $\mu < 2\bar{d}/h^2, \bar{d} = \inf_{t\in\mathbb{R}^+}\max_{i\in\mathrm{N}}(d_i(t))$;

H3) $A = \mathrm{diag}(-\frac{2}{C_1R_1}, \cdots, -\frac{2}{C_nR_n}) + (b_{ij})_{n\times n}$ is negative defined, and suppose that $\lambda_{\max}(A) := -\lambda$.

Then, at the equilibrium $u = u^*$, random field solution to the system of (19) has a Lyapunov exponent estimate in mean square with respect to the spatial variables.

$$\sup \lim_{T\to\infty}(1/T)\lg(\mathrm{E}(\| u - u^* \|^2)) \leqslant -\alpha. \tag{17}$$

Proof Analogous to the proof of Theorem 1, let (11) be the corresponding Lyapunov function. We calculate the derivative of it along the system (16), we have

$$\mathrm{d}V(t, u(t,x)) |_{(19)} =$$

$$\int_G 2\sum_{i=1}^n (u_i - u_i^*)[d_i(t)\Delta(u_i - u_i^*) -$$

$$\frac{u_i - u_i^*}{C_iR_i} + \sum_{j=1}^n \frac{T_{ij}}{C_i}(g_j(u_j) - g_j(u_j^*)]\mathrm{d}x\mathrm{d}t +$$

$$\int_G \mathrm{tr}(\sigma(u) - \sigma(u^*))^{\mathrm{T}}(\sigma(u) - \sigma(u^*))\mathrm{d}x\mathrm{d}t +$$

$$2\int_G \sum_{i=1}^n (u_i - u_i^*)\sum_{l=1}^m (\sigma_{il}(u_i) - \sigma_{il}(u_i^*))\mathrm{d}x\mathrm{d}W_l(t). \tag{18}$$

From Gauss Divergence theorem and the boundary condition, we obtain that

$$\int_G (u_i - u_i^*)\Delta(u_i - u_i^*)\mathrm{d}x = -\int_G [\Delta(u_i - u_i^*)]^2\mathrm{d}x. \tag{19}$$

From Poincare inequality we obtain that

$$\int_G (u_i - u_i^*)\mathrm{d}x \leqslant h^2\int_G [\Delta(u_i - u_i^*)]^2\mathrm{d}x. \tag{20}$$

Substituting (19) and (20) into (18), analogous to the proof of Theorem 1, we can get the conclusion.

4 Conclusion

In this paper, we have considered a stochastic partial differential system, by constructing an average Lyapunov function with respect to the spatial variables and employing the Itô differential formula, we solve the problem of exponential stability of stochastic Hopfield neural networks with distributed parameter, we also obtain the corresponding rule for stochastic system with distributed parameter, the results obtained here are absolutely new.

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