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# 基于级联观测器的多输入多输出非线性系统的故障诊断

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**摘要**：将块观测方法应用于非线性系统的故障检测和分离．首先给出了非线性系统的块观测形式，针对传感器故障和执行器故障对非线性系统进行分块，得到了带有故障系统的块观测器形式．利用滑模观测器实现系统状态观测，得到观测器误差；利用所设计的观测器对非线性系统进行故障的诊断和分离；采用等效输出注入概念重构了故障信号，使得多变量输入输出非线性系统的故障诊断问题得到了解耦；针对异步电动机系统实现了传感器故障的分离．仿真结果证明了算法的有效性．

**关键词**：故障诊断；滑模；块观测器；等效输出注入

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# Fault detection and isolation of MIMO nonlinear systems based on the cascade observer

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**Abstract**: This paper considers the application of a block-observable method to the problem of fault detection and isolation. First, a block-observable form of nonlinear systems is introduced. The cascade observer of the system can be obtained by decoupling the system into blocks according to the faults of the sensor and the performer. The observer error can be got through the application of the sliding mode state observer. The observer designed in this paper is applied to the problem of the fault detection and isolation of the nonlinear system. The equivalent output injection concept can explicitly reconstruct fault signals, in which the problem of the fault diagnosis for multi-input and output nonlinear system will be decoupled. The isolation of the sensor fault is obtained in the system of an asynchronous motor, and the result of the simulation validates the algorithm.

**Key words**: fault detection and isolation; sliding mode control; block observer; equivalent output injection

## 1 引言(Introduction)

在控制系统中，基于模型的故障诊断的核心问题是产生残差[1,2]，将残差作为故障的指示信号．目前有许多不同的方法用来设计残差发生器[2~6]．基于观测器的故障诊断方法也引起了广泛的注意[6,7]．非线性系统的故障诊断与分离一直备受关注，有许多综述研究对其进行总结[1~3,6,8,9]．基于线性化方法的非线性系统故障诊断分两步：1）将非线性系统在其平衡点处线性化；2）对线性化后的系统利用现有的鲁棒观测器进行故障诊断与分离[8]，但是由于非线性系统的大范围工作和参数变化，人们不得不采用多个观测器叠加来使其与平衡点符合，这样就有很多的故障诊断与分离的观测器，增加了系统的复杂性．文献[8]首先采用了非线性观测器进行故障诊断的方法，但这种方法未能保证所设计的观测器增益使观测器稳定．随着非线性系统理论的发展，许多非线性系统观测器被应用于故障诊断，如未知输入观测器[2]，滑模观测器[8,10]，自适应观测器[9]，非线性自适应滤波器等[5]．但上述方法大部分需要进行坐标变换或系统为某种特殊形式．由于神经网络和模糊的万能逼近特性和学习能力，目前也被应用于系统的故障诊断和分离，但相对来说需要大量样本和先进的硬件设备[10]．

非线性系统理论目前被普遍重视，应用最广泛的是反向递推法[11]，这种方法虽然被广泛地应用于一类非线性系统，但一般要求系统满足强反馈条件．

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文献[12,13]提出的块控制原理是非线性系统到非线性系统的变换,保持系统的非线性特性并达到系统的解耦;并且许多种其他控制方法可直接应用于分块系统的控制和观测器设计,如滑模控制,高增益控制等[13~16].

本文针对得到的块观测器,采用滑模观测器[16~18]重构故障而不是通过残差检测是否存在故障.考虑当系统状态不完全已知的情况,观测器设计保证在故障出现时系统仍然具有滑模运动,通过分析"等效输出注入"信号来检测故障[10].首先利用非线性系统的块观测形式,对故障系统进行坐标变换,使之为块观测形式,以构造滑模观测器.然后利用等效注入的方法重构各块的故障,使非线性故障得到诊断和分离.

## 2 级联观测器设计(Design of cascades observer)

研究如下非线性多变量系统

$$\begin{cases}\dot{x} = f(x,u),\\ y = h(x,u).\end{cases} \tag{1}$$

其中: $x\in\mathbb{R}^n$ 是状态矢量, $y\in\mathbb{R}^m$ 是输入矢量, $u\in\mathbb{R}^r$ 是控制矢量,矢量函数 $f(x,u)$ 和 $h(x,u)$ 在开集状态空间中是连续可微的,原点是系统稳定的平衡点: $f(0,0)=0$, $h(0,0)=0$. 系统(1)是局部可观测的并简化为如下块观测形式[13]:

$$\begin{cases}\dot{y}_i = y_{i+1}^*,\\ \dot{\bar{y}}_i = \bar{h}_i(y^*,u_i^*),\ i=1,\cdots,\gamma-1,\\ \dot{y}_\gamma^* = \bar{h}_\gamma(y^*,u_\gamma^*),\\ y_i^*=(y_i^{\mathrm{T}},\bar{y}_i^{\mathrm{T}})^{\mathrm{T}},\ y_i^*\in\mathbb{R}^{m_i},\ y_i\in\mathbb{R}^{m_{i+1}},\ \bar{y}_i\in\mathbb{R}^{m_i-m_{i+1}}.\end{cases} \tag{2}$$

与其他的方法不同,块观测形式揭示了控制目标的执行机构的非线性特性,其变换保持了系统的非线性特性.根据得到的块观测形式(2),状态观测问题在每个块中可以独立解决,状态观测器的形式为

$$\begin{cases}\dot{z}_i = z_{i+1}^* + v_i,\\ \dot{\bar{z}}_i = \bar{h}(z^*,u_i^*)+\bar{v}_i,\ i=1,\cdots,\gamma-1,\\ \dot{z}_\gamma^* = \bar{h}_\gamma(z^*,u_\gamma^*)+v_\gamma^*,\\ z_i^{*\mathrm{T}}=(z_i^{\mathrm{T}},\bar{z}_i^{\mathrm{T}}),\ \dim z_i^* = \dim y_i^*,\ i=1,\cdots,\gamma.\end{cases} \tag{3}$$

其中 $v_i^{*\mathrm{T}}=(v_i^{\mathrm{T}},\bar{v}_i^{\mathrm{T}},v_\gamma^{*\mathrm{T}})$ 是观测器控制,选择其以解决系统方程的不匹配误差 $\varepsilon_i^*=y_i^*-z_i^*$ 的稳定性问题,有

$$\begin{cases}\dot{\varepsilon}_i = \varepsilon_{i+1}^* - v_i,\\ \dot{\bar{\varepsilon}} = \bar{h}_i((z^*+\varepsilon^*),u_i^*)-\bar{h}_i(z^*,u_i^*)-\bar{v}_i,\\ \quad i=1,\cdots,\gamma-1,\\ \dot{\varepsilon}_\gamma^* = \bar{h}_\gamma((z^*+\varepsilon^*),u_\gamma^*)-\bar{h}_n(z^*,u_\gamma^*)-v_v^*.\end{cases} \tag{4}$$

其中:系统(4)的状态矢量 $\varepsilon^{*\mathrm{T}}=(\varepsilon_i^{\mathrm{T}},\bar{\varepsilon}_i^{\mathrm{T}},\varepsilon_\gamma^{*\mathrm{T}})\in\mathbb{R}^n$,包括矢量 $\varepsilon_i\in\mathbb{R}^{m_{i+1}}$, $\bar{\varepsilon}_i\in\mathbb{R}^{m_i-m_{i+1}}$, $\varepsilon_\gamma^*\in\mathbb{R}^{m_v}$. $\|\varepsilon^*\|\leqslant F^*$ - const, 并且

$$\begin{aligned}&\|\Delta\bar{h}_i(\varepsilon^*,u_i^*)\| =\\ &\|\bar{h}_i((z^*+\varepsilon^*),u_i^*)-\bar{h}_i(z^*,u_i^*)\|\leqslant\\ &\bar{F}_i - \text{const},\ i=1,\cdots,\gamma.\end{aligned} \tag{5}$$

对于观测器(4)可以采用不同的观测器形式,如滑模观测器[13]、高增益观测器及自适应观测器等,本文利用滑模观测器实现故障诊断.

## 3 非线性系统的故障诊断(Fault isolation of the nonlinear system)

考虑系统(1)在存在故障情况下为

$$\begin{cases}\dot{x} = f(x,u)+D_if_{ia}\ 或\ \dot{x}=f(x,u)+Df_a;\\ y = h(x,u)+G_j\bar{f}_{jo}\ 或\ y=h(x,u)+G\bar{f}_o.\end{cases} \tag{6}$$

其中: $D_i$ 是 $n\times1$ 维故障方向, $f_{ia}$ 表示第 $i$ 个执行器或元器件的故障, $i=1,2,\cdots,r$. $r$ 是故障方向的个数, $D\in\mathbb{R}^{n\times r}$, $G_j$ 是 $m\times1$ 维故障方向, $\bar{f}_{jo}$ 表示第 $j$ 个传感器故障, $G\in\mathbb{R}^{m\times r}$, $D_j$, $G_j$ 的列向量是线性无关的,且故障向量都有界.

假定 $r\leqslant m<n$,将多变量输入输出系统转化为 $\gamma$ 个可观测的低阶子系统,针对每个子系统设计级联观测器,其形式为

$$\dot{z}_i = h_i(z^*,u_i^*)+v_i.$$

$v_i$ 使得误差向量 $\varepsilon_i^*=y_i^*-z_i^*$ 在观测器产生滑模动态后在有限时间内趋于0.因此,当系统到达滑动模态就可以采用等效输出注入的方法对 $f$ 进行逼近,获得对故障的估计.

### 3.1 输入故障 $f_{ia}(t)$ 的检测和分离(Detection and isolation of input fault $f_{ia}(t)$)

下面详细阐述将带有故障的非线性系统模型划分为块观测形式.

第1步 在系统(6)中,假定 $\mathrm{rank}\{\frac{\partial h}{\partial x}\}=m_1\neq0$, $m_1<n$,

a) 系统的状态矢量可以表示为 $x^{\mathrm{T}}=(x_1^{\mathrm{T}},\bar{x}_2^{\mathrm{T}})$,

$x_1 \in \mathbb{R}^{m_1}, \bar{x}_2 = \in \mathbb{R}^{n-m_1}$,如果 $\operatorname{rank}\{\frac{\partial h(x_1,\bar{x}_2,u)}{\partial x_1}\} = m_1$,则对 $m_1 < m$,通过变更输出矢量组成部分的顺序,有 $y^{\mathrm{T}} = ((y_1^*)^{\mathrm{T}}, (\bar{y}_1^*)^{\mathrm{T}}), y_1^* \in \mathbb{R}^{m_1}, \bar{y}_1^* \in \mathbb{R}^{n-m_1}, D = ((D_1^*)^{\mathrm{T}}, (\bar{D}_2)^{\mathrm{T}})^{\mathrm{T}}, D_1^* \in \mathbb{R}^{m_1}, \bar{D}_2 \in \mathbb{R}^{n-m_1}$,这样 $\det(\frac{\partial h_1^*}{\partial x_1}) \neq 0$.

b) 由变量 $y_1^* \in \mathbb{R}^{m_1}$ 和 $\bar{x}_2 \in \mathbb{R}^{n-m_1}$ 表示系统(6)有

$$\begin{cases} \dot{y}_1^* = h_1^*(y_1^*, \bar{x}_2, u, \dot{u}) + \left(D_1^* \dfrac{\partial h(x,u)}{\partial x_1}\right) f_{ia}, \\ \dot{\bar{x}}_2 = \bar{f}_2(\tilde{y}_1^*, \bar{x}_2, u) + \bar{D}_2 f_{ia}, \\ \tilde{y}_1^* = y_1^* - \left(D_1^* \dfrac{\partial h(x,u)}{\partial x_1}\right) f_{ia}, \\ \operatorname{rank}\left(D_1^* \dfrac{\partial h(x,u)}{\partial x_1}\right) = m_1. \end{cases} \tag{7}$$

由于 $\dim x_1 = \dim \tilde{y}_1^* = m_1$,坐标转化 $x^{\mathrm{T}} = ((\tilde{y}_1^*)^{\mathrm{T}}, \bar{x}_2^{\mathrm{T}})$ 是非奇异的.

c) 在系统(7)中, $\operatorname{rank}\{\frac{\partial h_1^*}{\partial \bar{x}_2}\} = m_2 \neq 0, m_2 < \dim \bar{x}_2$. 如果 $m_2 < m_1$,对系统(7)进行状态变换有 $(y_1^*)^{\mathrm{T}} = (y_1^{\mathrm{T}}, \bar{y}_1^{\mathrm{T}}), y_1 \in \mathbb{R}^{m_2}, \bar{y}_1 \in \mathbb{R}^{m_1-m_2}$. 当满足 $\operatorname{rank}\{\frac{\partial h_1}{\partial \bar{x}_2}\} = m_2$ 时,有下列方程:

$$\begin{cases} \dot{y}_1 = h_1(\tilde{y}_1^*, \bar{x}_2, u, \dot{u}) + D_1 f_{ia}, \\ \dot{\bar{y}} = \bar{h}_1(\tilde{y}_1^*, \bar{x}_2, u, \dot{u}) + \bar{D}_1 f_{ia}, \\ \dot{\bar{x}}_2 = \bar{f}_2(\tilde{y}_1^*, \bar{x}_2, u) + \bar{D}_2 f_{ia}. \end{cases} \tag{8}$$

其中: $\left(D_1^* \dfrac{\partial h(x,u)}{\partial x_1}\right)^{\mathrm{T}} = (D_1^{\mathrm{T}}, \bar{D}_1^{\mathrm{T}}), D_1 \in \mathbb{R}^{m_2}, \bar{D}_1 \in \mathbb{R}^{m_1-m_2}$.

如果 $m_1 + m_2 < n$ 或 $m_2 \neq 0$,则继续第 2 步.

第 2 步　如果方程(8)中的最后一个方程的输出变量由第一个方程的右边定义,则有

$$\begin{cases} \dot{\bar{x}}_2 = \bar{f}_2(\tilde{y}_1^*, \bar{x}_2, u) + \bar{D}_2 f_{ia}, \\ y_2^* = h_1(\tilde{y}_1^*, \bar{x}_2, u, \dot{u}), \\ \bar{x}_2 \in \mathbb{R}^{n-m_1},\ y_2^* \in \mathbb{R}^{m_2}. \end{cases} \tag{9}$$

a) 由于 $m_2 < n - m_1$,如果

$$\operatorname{rank}\left\{\frac{\partial h_1(\tilde{y}_1^*, x_2, \bar{x}_3, u, \dot{u})}{\partial x_2}\right\} = m_2,$$

则系统(9)的状态矢量可表示为 $\bar{x}_2^{\mathrm{T}} = (x_2^{\mathrm{T}}, \bar{x}_3^{\mathrm{T}}), x_2 \in \mathbb{R}^{m_2}, \bar{x}_3 \in \mathbb{R}^{n-m_1-m_2}, \bar{D}_2^{\mathrm{T}} = ((D_2^*)^{\mathrm{T}}, \bar{D}_3^{\mathrm{T}}), D_2^* \in \mathbb{R}^{m_1}, \bar{D}_3 \in \mathbb{R}^{n-m_1-m_2}$.

b) 由变量 $y_2^*$ 和 $\bar{x}_3$ 表示系统(9)有

$$\begin{cases} \dot{y}_2^* = h_2^*(y_1^*, y_2^*, \bar{x}_3, u, \dot{u}, \ddot{u}) + \left(D_2^* \dfrac{\partial h_1(\cdot)}{\partial x_2}\right) f_{ia}, \\ \dot{\bar{x}}_3 = \bar{f}_3(\tilde{y}_1^*, \tilde{y}_2^*, \bar{x}_3, u) + \bar{D}_3 f_{ia}, \\ \tilde{y}_2^* = y_2^* - \left(D_2^* \dfrac{\partial h_1(\cdot)}{\partial x_2}\right) f_{ia}, \\ \operatorname{rank}\left(D_2^* \dfrac{\partial h_1(\cdot)}{\partial x_2}\right) = m_2. \end{cases} \tag{10}$$

由 $\dim x_2 = \dim \tilde{y}_2^* = m_2$,坐标变换 $\bar{x}_2^{\mathrm{T}} = ((\tilde{y}_2^*)^{\mathrm{T}}, \bar{x}_3^{\mathrm{T}})$ 是非奇异的.

c) 在系统(10)中,如果 $\operatorname{rank}\{\frac{\partial h_2^*}{\partial \bar{x}_3}\} = m_3 \neq 0$, $m_3 < \dim \bar{x}_3, m_3 < m_2$,则改变系统(10)的状态矢量组成部分 $(y_2^*)^{\mathrm{T}} = (y_2^{\mathrm{T}}, \bar{y}_2^{\mathrm{T}}), y_2 \in \mathbb{R}^{m_3}, \bar{y}_2 \in \mathbb{R}^{m_2-m_3}$,方程满足 $\operatorname{rank}\{\frac{\partial h_2}{\partial \bar{x}_3}\} = m_3$:

$$\begin{cases} \dot{y}_2 = h_2(y_1^*, y_2^*, \bar{x}_3, u, \dot{u}, \ddot{u}) + D_2 f_{ia}, \\ \dot{\bar{y}}_2 = \bar{h}_2(\tilde{y}_1^*, \tilde{y}_2^*, \bar{x}_3, u, \dot{u}, \ddot{u}) + \bar{D}_2 f_{ia}, \\ \dot{\bar{x}} = \bar{f}_3(\tilde{y}_1^*, \tilde{y}_2^*, \bar{x}_3, u) + \bar{D}_3 f_{ia}. \end{cases} \tag{11}$$

其中: $\left(D_2^* \dfrac{\partial h(\cdot)}{\partial x_2}\right)^{\mathrm{T}} = (D_2^{\mathrm{T}}, \bar{D}_2^{\mathrm{T}}), D_2 \in \mathbb{R}^{m_3}, \bar{D}_2 \in \mathbb{R}^{m_2-m_3}$.

第 $\mu$ 步　此时系统状态方程为

$$\begin{cases} \dot{\bar{x}}_\mu = \bar{f}_\mu(\tilde{y}_1^*, \cdots, \tilde{y}_{\mu-1}^*, \bar{x}_\mu, u) + \bar{D}_\mu f_{ia}, \\ y_\mu^* = h_{\mu-1}(\tilde{y}_1^*, \cdots, \tilde{y}_{\mu-1}^*, \bar{x}_\mu, u, \dot{u}, \ddot{u}, \cdots, u^{(\mu-1)}), \\ \qquad \bar{x}_\mu \in \mathbb{R}^{n-m_1-\cdots-m_{\mu-1}},\ y_\mu^* \in \mathbb{R}^{m_\mu}, \\ \operatorname{rank}\left\{\dfrac{\partial h_{\mu-1}}{\partial \bar{x}_\mu}\right\} = m_\mu \neq 0,\ m_\mu < \dim \bar{x}_\mu. \end{cases} \tag{12}$$

a) 系统(12)的状态矢量是 $\bar{x}_\mu^{\mathrm{T}} = (x_\mu^{\mathrm{T}}, \bar{x}_{\mu+1}^{\mathrm{T}}), x_\mu \in \mathbb{R}^{m_\mu}, \bar{x}_{\mu+1} \in \mathbb{R}^{n-m_1-\cdots-m_\mu}, \bar{D}_\mu^{\mathrm{T}} = ((D_\mu^*)^{\mathrm{T}}, \bar{D}_{\mu+1}^{\mathrm{T}}), D_\mu^* \in \mathbb{R}^{m_\mu}, \bar{D}_{\mu+1} \in \mathbb{R}^{n-m_1-\cdots m_\mu}$,则

$$\operatorname{rank}\left\{\frac{\partial h_{\mu-1}(\tilde{y}_1^*, \cdots, \tilde{y}_{\mu-1}^*, x_\mu, \bar{x}_{\mu+1}, u, \dot{u}, \cdots, u^{(\mu+1)})}{\partial x_\mu}\right\} = m_\mu.$$

b) 系统(12)由变量 $y_\mu^*$ 和 $\bar{x}_{\mu+1}$ 表达为

$$\begin{cases} \dot{y}_\mu^* = h_\mu^*(\tilde{y}_1^*, \cdots, \tilde{y}_{\mu-1}^*, y_\mu^*, \bar{x}_{\mu+1}, u, \dot{u}, \cdots, u^{(\mu)}) + \\ \qquad \left(D_\mu^* \dfrac{\partial h_{\mu-1}(\cdot)}{\partial x_\mu}\right) f_{ia}, \\ \dot{\bar{x}}_{\mu+1} = \bar{f}_{\mu+1}(\tilde{y}_1^*, \cdots, \tilde{y}_{\mu-1}^*, x_\mu, \bar{x}_{\mu+1}, u) + \bar{D}_{(\mu+1)} f_{ia}, \\ \tilde{y}_\mu^* = y_\mu^* - \left(D_\mu^* \dfrac{\partial h_{\mu-1}(\cdot)}{\partial x_\mu}\right) f_{ia}, \\ \operatorname{rank}\left(D_\mu^* \dfrac{\partial h_{\mu-1}(\cdot)}{\partial x_\mu}\right) = m_\mu. \end{cases} \tag{13}$$

由于 $\dim x_\mu = \dim \tilde{y}_\mu^*$,坐标变换 $\bar{x}_\mu^{\mathrm{T}} = [(\tilde{y}_\mu^*)^{\mathrm{T}}, \bar{x}_{\mu+1}^{\mathrm{T}}]$

是非奇异的.

c) 在系统(13)中, $\mathrm{rank}\{\frac{\partial h_\mu^*(\cdot)}{\partial \bar{x}_{\mu+1}}\} = m_{\mu+1} \neq 0$, $m_{\mu+1} < \dim \bar{x}_{\mu+1}$. 如果 $m_{\mu+1} < m_\mu$, 对系统(13)的状态矢量进行坐标转换 $y_\mu \in \mathbb{R}^{m_{\mu+1}}, \bar{y}_\mu \in \mathbb{R}^{m_\mu - m_{\mu+1}}$, 并且 $\mathrm{rank}\{\frac{\partial h_\mu(\cdot)}{\partial \bar{x}_{\mu+1}}\} = m_{\mu+1}$, 则有

$$\begin{cases} \dot{y}_\mu = h_\mu(\tilde{y}_1^*, \cdots, \tilde{y}_\mu^*, \bar{x}_{\mu+1}, u, \cdots, u^{(\mu)}) + D_\mu f_{ia}, \\ \dot{\bar{y}}_\mu = \bar{h}_\mu(\tilde{y}_1^*, \cdots, \tilde{y}_\mu^*, \bar{x}_{\mu+1}, u, \dot{u} \cdots, u^{(\mu)}) + \bar{D}_\mu f_{ia}, \\ \dot{\bar{x}}_{\mu+1} = \bar{f}_{\mu+1}(\tilde{y}_1^*, \cdots, \tilde{y}_\mu^*, \bar{x}_{\mu+1}, u) + \bar{D}_{\mu+1} f_{ia}. \end{cases} \tag{14}$$

其中: $\left(D_\mu^* \frac{\partial h_{\mu-1}(\cdot)}{\partial x_\mu}\right)^{\mathrm{T}} = (D_\mu^{\mathrm{T}}, \bar{D}_\mu^{\mathrm{T}}), D_\mu \in \mathbb{R}^{m_\mu}$, $\bar{D}_\mu \in \mathbb{R}^{m_\mu - m_{\mu-1}}$.

系统(5)分为 $\gamma$ 块,如果第 $\gamma$ 步中有 $m_\gamma = 0$, 系统(5)在新变量中仅仅是部分采取块观测形式,状态矢量 $\bar{x}_{\gamma+1}$ 形成不可观测子空间. 当 $\sum_{i=1}^{\gamma} m_i = n$ 或 $\dim(\bar{x}_{\gamma+1}) = 0$ 时,系统(3)可以简化为

$$\begin{cases} \dot{y}_i = y_{i+1}^* - D_i f_{ia}, \\ \dot{\bar{y}} = \bar{h}_i(y^*, u_i^*) + \bar{D}_i f_{ia}, \ i = 1, \cdots, \gamma - 1, \\ \dot{y}_\gamma^* = \bar{h}_\gamma(\tilde{y}^*, u_\gamma^*) + \left(D_\gamma^* \frac{\partial h_{\gamma-1}(\cdot)}{\partial x_\gamma}\right) f_{ia}, \\ y_i^* = (y_i^{\mathrm{T}}, \bar{y}_i^{\mathrm{T}})^{\mathrm{T}}, \ y_i^* \in \mathbb{R}^{m_i}, \ y_i \in \mathbb{R}^{m_{i+1}}, \ \bar{y}_i \in \mathbb{R}^{m_j - m_{i+1}}, \\ \dot{y}_i^* = \tilde{y}_i^* - \left(D_i \frac{\partial h_{i-1}(\cdot)}{\partial x_i}\right) f_{ia}, \\ \left(D_i^* \frac{\partial h_{i-1}(\cdot)}{\partial x_i}\right)^{\mathrm{T}} = (D_i^{\mathrm{T}}, \bar{D}_i^{\mathrm{T}}), \ \mathrm{rank}\left(D_i^* \frac{\partial h_{i-1}(\cdot)}{\partial x_i}\right) = m_i, \\ \dot{y}_i^* = h_i^*(\tilde{y}_1^*, \cdots, \tilde{y}_i^*, \bar{x}_{i+1}, u, \dot{u}, \cdots, u^{(i)}) + \\ \qquad \left(D_i^* \frac{\partial h_{i-1}(\cdot)}{\partial x_i}\right) f_{ia}. \end{cases} \tag{15}$$

设计滑模观测器如式(2)所示,则得到如下误差系统:

$$\begin{cases} \dot{\varepsilon}_i = (E_i + E_i^s)\varepsilon_{i+1}^* + D_i f_{ia}(t) - v_i, \\ \dot{\bar{\varepsilon}} = \bar{h}_i((z^* + \varepsilon^*), u_i^*) - \bar{h}_i(z^*, u_i^*) + \\ \qquad \bar{D}_i f_{ia}(t) - \bar{v}_i, \ i = 1, \cdots, \gamma - 1, \\ \dot{\varepsilon}_\gamma^* = \bar{h}_\gamma((z^* + \varepsilon^*), u_\gamma^*) - \bar{h}_\gamma(z^*, u_\gamma^*) + \\ \qquad \left(D_\gamma \frac{\partial h_{\gamma-1}(\cdot)}{\partial x_\gamma}\right) f_{ia}(t) - v_\gamma^*. \end{cases} \tag{16}$$

其中: $E_i$ 是相应维数的单位矩阵, $E_i^s \in \mathbb{R}^{m_i \times m_i}$ 是修正矩阵以保证误差系统的有效性,不连续项 $v_i$ 定义为

$$v_i = \begin{cases} -\rho \| D_i \| \frac{P_2 \varepsilon_i}{\| P_2 \varepsilon_i \|}, & \varepsilon_i^* \neq 0, \\ 0, & \varepsilon_i^* = 0. \end{cases} \tag{17}$$

其中: $P_2 \in \mathbb{R}^{m_i \times m_i}$ 为矩阵 $E_i^s$ 的Lyapunov矩阵,而标量 $\rho$ 的选择只要求满足

$$\| f_{ia}(t) \| < \rho. \tag{18}$$

**定理 1** 误差系统(16)在不连续控制 $v_i$ 作用下,将到达如下式定义的理想滑动模态:

$$S_i = \{\varepsilon_i^* \in \mathbb{R}^n : \varepsilon_i^* = 0\}. \tag{19}$$

**证** 利用滑模存在条件和式(17),(18)可直接得出.

根据定理1,误差系统将到达由 $S_i$ 定义的滑动模态,这样,当误差处于滑动模态运动时,有 $\varepsilon_i^* = 0$ 和 $\dot{\varepsilon}_i^* = 0$, 从式(16)可得到

$$\varepsilon_i^*(t) \to 0, \ 0 = \varepsilon_{i+1}^* + D_i f_{ia} - v_{\mathrm{eq}}. \tag{20}$$

其中: $v_{\mathrm{eq}}$ 是所谓的等效输出注入信号(Equivalent Output Injection EOI)[10],它类似于Utkin[17]所提出的等效控制概念,表示了不连续项 $v$ 的平均作用,这种平均作用是维持滑模运动所必须的[17].这样进一步可得到

$$v_{i\mathrm{eq}} \to D_i f_{ia}(t). \tag{21}$$

从上式可看出,通过引入EOI信号 $v_{\mathrm{eq}}$ 的概念获得了对输入故障 $f_{ia}(t)$ 的重构.只要获得EOI信号,就可以获得输入故障信号.可以采用Utkin[17]的低通滤波器(Low-pass Filter)方法获得EOI信号,也可采用下面的替代方法:

$$v_{\mathrm{eq}} \approx v_\delta = -\rho \| D_i \| \frac{P_2 \varepsilon_i}{\| P_2 \varepsilon_i \| + \delta}. \tag{22}$$

其中 $\delta$ 为一个很小的正数,从而当 $\mathrm{rank} D_i = m_i$ 时可得到输入误差的估算表达式

$$f_{ia}(t) \approx -\rho \| D_i \| (D_i^{\mathrm{T}} D_i)^{-1} D_i^{\mathrm{T}} \frac{P_2 \varepsilon_i}{\| P_2 \varepsilon_i \| + \delta}. \tag{23}$$

### 3.2 输出故障 $f_o(t)$ 的检测和分离(Detection and isolation of output fault $f_o(t)$)

系统简化为块观测形式:

$$\begin{cases} \dot{y}_i = y_{i+1}^* = h_i(y^*, u_i^*) + G_{i+1}^* f_o, \\ \dot{\bar{y}} = \bar{h}_i(y^*, u_i^*) + \bar{G}_{i+1}^* f_o, \ i = 1, \cdots, r - 1, \\ \dot{y}_\gamma^* = \bar{h}_\gamma(y^*, u_\gamma^*) + \dot{G}_\gamma^* f_o + G_\gamma^* \dot{f}_o, \\ \dim G_{i+1}^* = \dim y_i^* = \dim x_i = m_i, \\ \dot{G}_i^* f_o + G_i^* \dot{f}_o = G_{i+1}^* f_o. \end{cases} \tag{24}$$

此时误差系统为

$$\begin{cases}\varepsilon_i^* = y_i^* - z_i^*, \\ \dot{\varepsilon}_i = (E_i + E_i^s)\varepsilon_{i+1}^* + G_{i+1}^* f_o - v_i, \\ \dot{\bar{\varepsilon}}_i = \bar{h}_i((z^* + \varepsilon^*), u_i^*) - \bar{h}_i(z^*, u_i^*) + \bar{G}_i f_o - \bar{v}_i, \\ \qquad i = 1,\cdots,\gamma - 1, \\ \varepsilon_\gamma^* = \bar{h}_\gamma((z^* + \varepsilon^*), u_\gamma^*) - \bar{h}_\gamma(z^*, u_\gamma^*) + \\ \qquad G_\gamma^* f_o + G_\gamma^* \dot{f}_o - v_\gamma^*.\end{cases} \tag{25}$$

从方程(25)可以看到, $f_o$ 和 $\dot{f}_o$ 在误差系统中是以输出干扰形式出现的,因而必须选择足够大的系数 $\rho$ 来维持干扰下的滑模运动.根据 3.1 的讨论,有

$$0 = E_i \varepsilon_{i+1}^* + E_i^s \varepsilon_i^* + G_i^* f_o + G_i^* \dot{f}_o - v_{eq}^*, \tag{26}$$

这样,对于缓变故障,有 $\dot{f}_o(t) = 0$ 和 $\varepsilon_{i+1}^*(t) = 0$, 从而由式(26)可得

$$v_{eq} = \dot{G}_i^* f_o + G_i^* \dot{f}_o = G_{i+1}^* f_o. \tag{27}$$

从上面式子可以看出,同样采用 EOI 信号 $v_{eq}$ 实现了对输出故障的重构.

**注**　从上面的论述中发现,当采用 EOI 信号重构系统的故障时,观测器的增益修正矩阵 $E_i^s$ 没有在重构表达式中出现.也就是说该系数的大小不影响对故障的估算.这是因为在观测器达到滑动模态的情况下, $\varepsilon_i^* \equiv 0$, 从而 $E_i^s \varepsilon_i^*$ 没有在等效输出注入信号中表现出来.但实际上,该系数矩阵对达到滑模平面前的运动有影响,因而在设计时要在达到速度和抗抖振性能之间进行折中考虑.

## 4　仿真(Simulation)

根据上面讨论的结果,本文研究异步电动机的状态观测和故障诊断.异步电动机是非线性多变量耦合系统,其中的磁通是不可测量的状态变量.对异步电动机的故障诊断有很多种方法,本文采用上述方法实现故障诊断.异步电动机在固定坐标 $(\alpha,\beta)$ 下的系统模型为[13]

$$\begin{cases}\dot{x}_1 = \alpha_1[\alpha_2 P(x_3)x_2 - \alpha_5 x_1 + U], \\ \dot{x}_2 = -P(x_3)x_2 + \alpha_4 x_1, \\ \dot{x}_3 = \dfrac{1}{J}(\alpha_2 x_1^{\mathrm{T}} S x_2 - x_4), \\ \dot{x}_4 = 0, \\ P(x_3) = \begin{vmatrix} \alpha_3 & x_3 \\ -x_3 & \alpha_3 \end{vmatrix} > 0,\ S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\ \alpha_1 = \dfrac{L_r}{L_s L_r - L_h^2}, \\ \alpha_2 = \dfrac{L_h}{L_r},\ \alpha_3 = \dfrac{R_r}{L_r},\ \alpha_4 = R_r\alpha_2,\ \alpha_5 = \alpha_2\alpha_4 + R_s.\end{cases} \tag{28}$$

式中: $x_1^{\mathrm{T}} = (x_{1\alpha}, x_{1\beta})$ 是定子电流矢量, $x_2^{\mathrm{T}} = (x_{2\alpha}, x_{2\beta})$ 是转子磁通矢量, $x_3 \in \mathbb{R}^1$ 是转子速度, $x_4 \in \mathbb{R}^1$ 为负载转矩,并且

$$P(x_3) = \begin{vmatrix} \alpha_3 & x_3 \\ -x_3 & \alpha_3 \end{vmatrix} > 0,\ S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$\alpha_1 = \frac{L_r}{L_s L_r - L_h^2},\ \alpha_2 = \frac{L_h}{L_r},$$

$$\alpha_3 = \frac{R_r}{L_r},\ \alpha_4 = R_r\alpha_2,\ \alpha_5 = \alpha_2\alpha_4 + R_s.$$

$R_s, R_r, L_s, L_r, L_h$ 是定转子的电阻、电感和互感, $U^{\mathrm{T}} = (U_\alpha, U_\beta)$ 是控制电压.

第 1 步　假设只有电子电流矢量可测,设 $y_1^* = x_1 \in \mathbb{R}^2$, 能表示式(28)的第一个方程为

$$\dot{y}_1^* = -\alpha_1\alpha_5 y_1^* + \alpha_1\alpha_2 h_1. \tag{29}$$

式中: $h_1 = p(x_3)x_2 = p(0)x_2 + x_3 S x_2$; $\mathrm{rank}\{\frac{\partial h_1}{\partial x_2}\} = 2$.

第 2 步　设 $y_2^* \in \mathbb{R}^2$, $y_2^* = h_1$, 作为(28)中第二个方程的伪输出,则有

$$\begin{aligned}\dot{y}_2^* &= p(0)\dot{x}_2 + \dot{x}_3 S x_2 + x_3 S \dot{x}_2 = \\ &\alpha_3\alpha_4 y_1^* - \alpha_3 y_2^* + h_2,\end{aligned} \tag{30}$$

式中: $h_2 = x_3 S x_2 + x_3 S \dot{x}_2$, $\mathrm{rank}\{\frac{\partial h_2}{\partial(x_3, x_4)}\} = 2$.

第 3 步　设 $y_3^* \in \mathbb{R}^2$, $y_3^* = h_2$ 作为第二个方程的伪输出,有

$$\dot{y}_3^* = \ddot{x}_3 S x_2 + 2\dot{x}_3 S \dot{x}_2 + x_3 S \ddot{x}_2 = h_3. \tag{31}$$

这里

$$\ddot{x}_3 = \frac{\alpha_2}{J}(\dot{y}_1^{*\mathrm{T}} S P^{-1}(x_3) y_2^* + y_1^{*\mathrm{T}} S(-y_2^* + \alpha_4 y_1^*)),$$

$$\ddot{x}_2 = (-\alpha_3\alpha_4 - \alpha_5\alpha_1\alpha_4) y_1^* + (\alpha_3 + \alpha_1\alpha_2\alpha_4) y_2^* - h_2.$$

由变换的非奇异性,有

$$\begin{cases} x_2 = h_1^{-1}(x_3, y_2^*), \\ x_3 = h_2^{-1}(x_4, y_1^*, y_2^*, y_3^*), \\ x_4 = h_2^{-1}(x_2, y_1^*, y_2^*, y_3^*). \end{cases} \tag{32}$$

式(29)~(32)是异步电动机的分块系统,作者考虑传感器的故障,利用本文所提的观测器方法构造故障观测器.从仿真结果看出,利用等效注入的方法可以分离故障.我们考虑如下故障:电流传感器快跳变和电流传感器的缓慢跳变,在本系统中即为 $x_1$ 故障.

仿真参数:三相感应电动机功率为 2.2 kW;定子电感 $L_s = 0.0008$ H;定子电阻 $R_s = 0.8\,\Omega$;定子电感 $L_r = 0.0008$ H;定子电阻 $R_r = 0.8\,\Omega$;互感 $L_m = 0.0008$ H;转动惯量 $J = 1.662\,\mathrm{kg\cdot m^2}$. 图 1 和图 2 为

传感器发生快跳变的观测输出和等效注入输出.

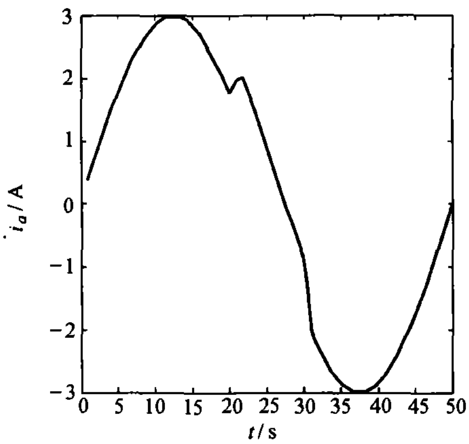


图1 传感器发生跳变时($t=20$ ms)观测器的输出
Fig. 1 Observer output of sensor jump ($t=20$ ms)

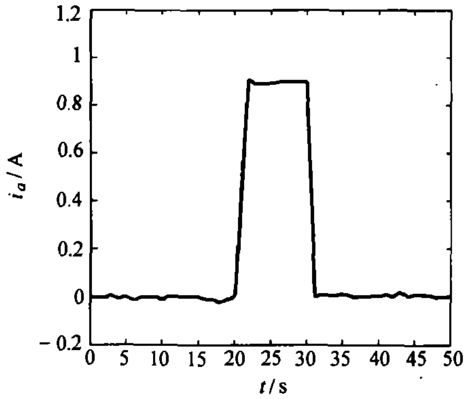


图2 在传感器跳变故障($t=20$ ms),第一块等效故障输出
Fig. 2 Output of observer 1 at sensor jump ($t=20$ ms)

## 5 总结(Conclusion)

研究了非线性系统基于块观测器的故障诊断与分离.所进行的变换是针对系统的观测器的设计,其非线性没有损失.对于传感器故障和状态故障分别给出其模型变换.利用滑模特性和等效输入原理设计故障诊断观测器.采用本文所提出的滑模观测器的故障诊断策略,系统的执行器故障和传感器故障能够根据等效输出注入项直接估算得到,而且该方法具有较强鲁棒性.仿真的结果证明了系统的有效性.由于许多系统都能满足本文中的变换条件,如电机驱动系统,机器人控制系统和电力系统,它们本身直接满足级联和分块条件,所以本方法具有很好的适应性.

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