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不确定双向联想记忆神经网络的稳定性分析

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摘要: 对双向联想记忆神经网络研究了平衡点的鲁棒稳定性. 该网络的参数不确定, 并且有时变时滞. 当神经网络的激励函数满足Lipschitz连续性条件时, 通过选取合适的Lyapunov-Krasovskii函数, 建立了两个全局鲁棒稳定判据. 由于这些判据考虑了神经元激励作用和抑制作用对网络的影响, 他们和时变时滞的数值无关, 并且易于使用内点算法进行检验. 在注释中和已有的结果进行了对比. 两个数值例子展示了所得结果的有效性.

关键词: 双向联想记忆神经网络; 时变时滞; 不确定性; 鲁棒稳定; 线性矩阵不等式; Lyapunov-Krasovskii函数

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Stability analysis of uncertain bi-directional associative memory neural networks with variable delays

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Abstract: The robust stability of equilibrium point is studied for bi-directional associative memory neural networks with parameter uncertainties and time-varying delays. When the activation function satisfies the condition of Lipschitz continuity, two sufficient conditions are established for the globally robust stability of the equilibrium point by suitably choosing Lyapunov-Krasovskii functional. The obtained results, which take account of the effects of neural inhibitory and excitatory on neural networks, are independent of the sizes of the time-varying delays and are easy to be checked by the interior-point algorithms in MATLAB toolbox. They are compared with prior results in a remark, and are demonstrated by two numerical examples for their effectiveness.

Key words: bi-directional associative memory neural networks; time varying delays; uncertainty; robust stability; linear matrix inequality (LMI); Lyapunov-Krasovskii functional

1 引言(Introduction)

双向联想记忆神经网络在求解优化问题及联想记忆等问题中已经被证明是很有用的数学模型, 因而在近几年得到了人们的广泛关注[1~8]. 基于一些代数不等式技术, 文献 [1~7]得到了关于定常时滞双向联想记忆神经网络全局渐近/指数稳定的若干充分判据. 虽然文献 [2~7]中的结果因包含了许多适量的未知参数而得到改进, 但由于没有系统的方法来调节这些参数, 这些结果通常不易验证. 此外, 文献 [1~7]中的结果没有考虑连接权系数的符号差, 进而没有考虑神经元激励和抑制对网络的影响. 目前, 线性矩阵不等式技术已用来研究神经网络的稳定性问题, 且所得到的结果具有既易于验证, 又考虑了神经元激励和抑制对网络的影响等特点[9,10]. 在神经网络的应用和设计中, 不可避免的存在如建模误差和参数摄动等不确定性, 这些不确定性将使网络产生复杂的动态行为, 因此, 所设计的神经网络必须对这些不确定性具有鲁棒性[11]. 到目前为止, 关于双向联想记忆神经网络鲁棒稳定性的结果尚不多见[12]. 文献 [12]仅针对单定常时滞的双向联想记忆神经网络建立了一个时滞依赖的鲁棒稳定判据. 一般来讲, 当时滞的幅值很大时, 时滞依赖的结果往往具有很大的保守性, 而时滞独立的结果却具有很小的保守性. 为此, 本文基于线性矩阵不等式技术, 针对具有参数不确定性的多时变时滞双向联想记忆神经网络, 给出其平衡点全局鲁棒稳定的两个充分判据.

2 问题描述与预备知识(Problem statement and preliminaries)

考虑如下多时变时滞双向联想记忆神经网络模型:

$$\dot{u}_i(t) = -[a_i + \delta a_i(t)]u_i(t)+$$

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$$\sum_{j=1}^{m}[b_{ij}+\delta b_{ij}(t)]\bar{g}_j(v_j(t-\tau_j(t)))+U_i,$$

$$\dot{v}_j(t)=-[c_j+\delta c_j(t)]v_j(t)+\sum_{i=1}^{n}[d_{ji}+\delta d_{ji}(t)]\bar{f}_i(u_i(t-z_i(t)))+J_i,$$

或写成矩阵–向量形式

$$\begin{aligned}\dot{u}(t)=&-(A+\Delta A)u(t)+\\&(B+\Delta B)\bar{g}(v(t-\tau(t)))+U,\\\dot{v}(t)=&-(C+\Delta C)v(t)+\\&(D+\Delta D)\bar{f}(u(t-z(t)))+J,\end{aligned}\tag{1}$$

其中u_i, v_j为神经元的状态,

$$\begin{aligned}&B=(b_{ij})_{n\times m},\\&D=(d_{ij})_{m\times n},\\&\tau(t)=(\tau_1(t),\cdots,\tau_m(t))^{\mathrm T},\\&z(t)=(z_1(t),\cdots,z_n(t))^{\mathrm T},\\&A=\mathrm{diag}\{a_1,\cdots,a_n\},\\&C=\mathrm{diag}\{c_1,\cdots,c_m\},\\&\Delta A=\mathrm{diag}\{\delta a_1(t),\cdots,\delta a_n(t)\},\\&\Delta C=\mathrm{diag}\{\delta c_1(t),\cdots,\delta c_m(t)\},\\&\Delta B=(\delta b_{ij}(t))_{n\times m},\\&\Delta D=(\delta d_{ij}(t))_{m\times n},\end{aligned}$$

$\delta a_i(t)$, $\delta c_j(t)$, $\delta b_{ij}(t)$ 和$\delta d_{ij}(t)$ 为不确定时变参数, $a_i>0$ 和$c_j>0$, b_{ij}, d_{ij} 为突触连接强度, $\bar{g}_j(\cdot)$ 和$\bar{f}_i(\cdot)$ 为神经元激励函数; $\bar{g}(v(t))=(\bar{g}_1(v_1(t)),\cdots,\bar{g}_m(v_m(t)))^{\mathrm T}$, $\bar{f}(u(t))=(\bar{f}_1(u_1(t)),\cdots,\bar{f}_n(u_n(t)))^{\mathrm T}$, U_i, J_j 为外部常值偏置, $\tau_j(t)>0$, $z_i(t)>0$, $\dot{\tau}_j(t)$ 和$\dot{z}_i(t)$ 分别表示时变时滞的变化率, $i=1,\cdots,n,\ j=1,\cdots,m$.

假设 1 激励函数$\bar{g}_j(\cdot)$, $\bar{f}_k(\cdot)$ 是有界的, 且满足$0\leqslant(\bar{g}_j(\varsigma)-\bar{g}_j(\xi))/(\varsigma-\xi)\leqslant\delta_j^g$, $0\leqslant(\bar{f}_k(\varsigma)-\bar{f}_k(\xi))/(\varsigma-\xi)\leqslant\delta_k^f$, 其中, $\varsigma,\ \xi\in\mathbb{R}$, $\varsigma\neq\xi$, $\delta_j^g>0$, $\delta_k^f>0$, $k=1,\cdots,n$, $j=1,\cdots,m$. 令$\Delta_g=\mathrm{diag}\{\delta_1^g,\cdots,\delta_m^g\}$, $\Delta_f=\mathrm{diag}\{\delta_1^f,\cdots,\delta_n^f\}$.

假设 2 $\Delta A=M_aF(t)N_a$, $\Delta C=M_cF(t)N_c$, $\Delta B=M_bF(t)N_b$和$\Delta D=M_dF(t)N_d$, 其中, M_a, N_a, M_c, N_c, M_b, N_b 和M_d, N_d 为已知矩阵, $F(t)$表示未知时变函数, 且$F^{\mathrm T}(t)F(t)\leqslant I$, I为适维单位矩阵.

引理 1[12] 对于两个适维向量X 和Y, 两个适维矩阵P, $Q=Q^{\mathrm T}>0$, 则不等式$-X^{\mathrm T}QX+2X^{\mathrm T}PY\leqslant Y^{\mathrm T}P^{\mathrm T}Q^{-1}PY$ 成立.

引理 2[13] 对于适维矩阵Y, $F(t)$, Z 及对称矩阵$\Lambda=\Lambda^{\mathrm T}$, 当且仅当存在正常数$\alpha>0$使得$\Lambda+\alpha^{-1}YY^{\mathrm T}+\alpha Z^{\mathrm T}Z<0$时, 有$\Lambda+YF(t)Z+Z^{\mathrm T}F^{\mathrm T}(t)Y^{\mathrm T}<0$, 其中$F^{\mathrm T}(t)F(t)\leqslant I$.

令u^* 和v^* 表示模型(1)的平衡点, 采用坐标变换$x(t)=u(t)-u^*$和$y(t)=v(t)-v^*$, 则模型(1)转变成如下形式:

$$\begin{cases}\dot{x}(t)=-(A+\Delta A)x(t)+(B+\Delta B)g(y(t-\tau(t))),\\\dot{y}(t)=-(C+\Delta C)y(t)+(D+\Delta D)f(x(t-z(t))),\end{cases}\tag{2}$$

其中:

$$\begin{aligned}&g(y(t-\tau(t)))=\\&(g_1(y_1(t-\tau_1(t))),\cdots,g_m(y_m(t-\tau_m(t))))^{\mathrm T},\\&f(x(t-z(t)))=\\&(f_1(x_1(t-z_1(t))),\cdots,f_n(y_n(t-z_n(t))))^{\mathrm T},\\&g_i(y_i(t-\tau_i(t)))=\bar{g}_i(y_i(t-\tau_i(t))+v_i^*)-\bar{g}_i(v_i^*),\\&f_k(x_k(t-z_k(t)))=\bar{f}_k(x_k(t-z_k(t))+u_k^*)-\bar{f}_k(u_k^*),\\&\forall\varsigma\neq0,0\leqslant g_i(\varsigma)/\varsigma\leqslant\delta_i^g,\ 0\leqslant f_k(\varsigma)/\varsigma\leqslant\delta_k^f,\\&k=1,\cdots,n,\ i=1,\cdots,m.\end{aligned}$$

显然, 模型(1)的稳定性问题等价于模型(2)的稳定性问题.

3 鲁棒稳定结果(Robust stability results)

定理 1 假定$\dot{\tau}_j(t)<1$ 和$\dot{z}_i(t)<1$. 如果存在正定对称矩阵P, Q, 正定对角矩阵 E, F, 正常数ε_a, ε_b, ε_c, ε_d, 使得下面LMIs成立:

$$\begin{bmatrix}\varXi_1&\varXi_2\\\varXi_2^{\mathrm T}&\varXi_3\end{bmatrix}<0,\tag{3}$$

$$\begin{bmatrix}\varXi_4&\varXi_5\\\varXi_5^{\mathrm T}&\varXi_6\end{bmatrix}<0,\tag{4}$$

则系统(2)的原点是全局鲁棒稳定的, 其中:

$$\begin{aligned}&\varXi_1=-PA-A^{\mathrm T}P+\Delta_fF\Delta_f+\varepsilon_aN_a^{\mathrm T}N_a,\\&\varXi_2=[PB\ PM_a\ PM_b],\\&\varXi_3=\mathrm{diag}\{-\eta_1E+\varepsilon_bN_b^{\mathrm T}N_b,-\varepsilon_aI,-\varepsilon_bI\},\\&\varXi_4=-QC-C^{\mathrm T}Q+\Delta_gE\Delta_g+\varepsilon_cN_c^{\mathrm T}N_c,\\&\varXi_5=[QD\ QM_c\ QM_d],\\&\varXi_6=\mathrm{diag}\{-\eta_2F+\varepsilon_dN_d^{\mathrm T}N_d,-\varepsilon_cI,\ -\varepsilon_dI\},\\&\eta_1=\min(1-\dot{\tau}_j(t)),\\&\eta_2=\min(1-\dot{z}_i(t)),\\&i=1,\cdots,n,\ j=1,\cdots,m.\end{aligned}$$

证 考虑如下Lyapunov-Krasovskii泛函:

$$\begin{aligned}V(t)=&x^{\mathrm T}(t)Px(t)+\sum_{j=1}^{m}\int_{t-\tau_j(t)}^{t}e_jg_j^2(y_j(s))\mathrm{d}s+\\&y^{\mathrm T}(t)Qy(t)+\sum_{j=1}^{n}\int_{t-z_j(t)}^{t}F_jf_j^2(x_j(s))\mathrm{d}s,\end{aligned}\tag{5}$$

沿着系统(2)的轨迹对(5)求导数得

$$\begin{aligned}\dot{V}(t)\leqslant&-2x^{\mathrm{T}}(t)P(A+\Delta A)x(t)+\\&2x^{\mathrm{T}}(t)P(B+\Delta B)g(y(t-\tau(t)))]-\\&2y^{\mathrm{T}}(t)Q(C+\Delta C)y(t)+\\&2y^{\mathrm{T}}(t)Q(D+\Delta D)f(x(t-z(t)))-\\&\eta_1 g^{\mathrm{T}}(y(t-\tau(t)))Eg(y(t-\tau(t)))-\\&\eta_2 f^{\mathrm{T}}(x(t-z(t)))Ff(x(t-z(t)))+\\&g^{\mathrm{T}}(y(t))Eg(y(t))+f^{\mathrm{T}}(x(t))Ff(x(t)),\end{aligned}\tag{6}$$

其中: $E=\mathrm{diag}\{e_1,\cdots,e_m\}$, $F=\mathrm{diag}\{F_1,\cdots,F_n\}$. 根据引理1和假设1,下面不等式成立:

$$\begin{aligned}&2x^{\mathrm{T}}(t)P(B+\Delta B)g(y(t-\tau(t)))-\\&\eta_1 g^{\mathrm{T}}(y(t-\tau(t)))Eg(y(t-\tau(t)))\leqslant\\&\frac{1}{\eta_1}x^{\mathrm{T}}(t)P(B+\Delta B)E^{-1}(B+\Delta B)^{\mathrm{T}}Px(t),\end{aligned}\tag{7}$$

$$\begin{aligned}&2y^{\mathrm{T}}(t)Q(D+\Delta D)f(x(t-z(t)))-\\&\eta_2 f^{\mathrm{T}}(x(t-z(t)))Ff(x(t-z(t)))\leqslant\\&\frac{1}{\eta_2}y^{\mathrm{T}}(t)Q(D+\Delta D)F^{-1}(D+\Delta D)^{\mathrm{T}}Qy(t),\end{aligned}\tag{8}$$

$$g^{\mathrm{T}}(y(t))Eg(y(t))\leqslant y^{\mathrm{T}}(t)\Delta_g E\Delta_g y(t),\tag{9}$$

$$f^{\mathrm{T}}(x(t))Ff(x(t))\leqslant x^{\mathrm{T}}(t)\Delta_f F\Delta_f x(t).\tag{10}$$

将式(7)～(10)代入式(6), 得

$$\begin{aligned}\dot{V}(t)\leqslant&x^{\mathrm{T}}(t)[-P(A+\Delta A)-(A+\Delta A)^{\mathrm{T}}P+\\&\frac{1}{\eta_1}P(B+\Delta B)E^{-1}(B+\Delta B)^{\mathrm{T}}P+\\&\Delta_f F\Delta_f]x(t)+y^{\mathrm{T}}(t)[-Q(C+\Delta C)-\\&(C+\Delta C)^{\mathrm{T}}Q+\Delta_g E\Delta_g+\\&\frac{1}{\eta_2}Q(D+\Delta D)F^{-1}(D+\Delta D)^{\mathrm{T}}Q]y(t).\end{aligned}\tag{11}$$

显然, 如果下面两个条件成立:

$$\begin{aligned}&-P(A+\Delta A)-(A+\Delta A)^{\mathrm{T}}P+\Delta_f F\Delta_f+\\&\frac{1}{\eta_1}P(B+\Delta B)E^{-1}(B+\Delta B)^{\mathrm{T}}P<0,\end{aligned}\tag{12}$$

$$\begin{aligned}&-Q(C+\Delta C)-(C+\Delta C)^{\mathrm{T}}Q+\Delta_g E\Delta_g+\\&\frac{1}{\eta_2}Q(D+\Delta D)F^{-1}(D+\Delta D)^{\mathrm{T}}Q<0,\end{aligned}\tag{13}$$

则对于任意的$x(t)\neq 0$和$y(t)\neq 0$, $\dot{V}(t)<0$. 当且仅当$x(t)=y(t)=0$时, 才有$\dot{V}(t)=0$. 根据Lyapunov 稳定理论, 系统(2)的原点是全局稳定的.

根据Schur补引理[14], 假设2和引理2, 式(12)成立当且仅当存在正常数ε_a和ε_b使得下式成立:

$$\begin{aligned}&\begin{bmatrix}-PA-A^{\mathrm{T}}P+\Delta_f F\Delta_f & PB\\ B^{\mathrm{T}}P & -\eta_1 E\end{bmatrix}+\\&\varepsilon_a^{-1}\begin{bmatrix}-PM_a\\0\end{bmatrix}\begin{bmatrix}-M_a^{\mathrm{T}}P & 0\end{bmatrix}+\varepsilon_a\begin{bmatrix}N_a^{\mathrm{T}}\\0\end{bmatrix}\begin{bmatrix}N_a & 0\end{bmatrix}+\\&\varepsilon_b^{-1}\begin{bmatrix}PM_b\\0\end{bmatrix}\begin{bmatrix}M_b^{\mathrm{T}}P & 0\end{bmatrix}+\varepsilon_b\begin{bmatrix}0\\N_b^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}0 & N_b\end{bmatrix}<0.\end{aligned}\tag{14}$$

再次应用Schur补引理[14], 式(14)等价于式(3). 同理, 当且仅当存在正常数ε_c和ε_d使得下式成立

$$\begin{aligned}&\begin{bmatrix}-QC-C^{\mathrm{T}}Q+\Delta_g E\Delta_g & QD\\ D^{\mathrm{T}}Q & -\eta_2 F\end{bmatrix}+\\&\varepsilon_c^{-1}\begin{bmatrix}-QM_c\\0\end{bmatrix}\begin{bmatrix}-M_c^{\mathrm{T}}Q & 0\end{bmatrix}+\varepsilon_c\begin{bmatrix}N_c^{\mathrm{T}}\\0\end{bmatrix}\begin{bmatrix}N_c & 0\end{bmatrix}+\\&\varepsilon_d^{-1}\begin{bmatrix}QM_d\\0\end{bmatrix}\begin{bmatrix}M_d^{\mathrm{T}}Q & 0\end{bmatrix}+\varepsilon_d\begin{bmatrix}0\\N_d^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}0 & N_d\end{bmatrix}<0,\end{aligned}\tag{15}$$

才有(13)成立, 再应用Schur补引理[14]即得式(4). 这样, 系统(2)的原点是全局鲁棒稳定的. 证毕.

推论 1 假定$\dot{\tau}_j(t)<1$和$\dot{z}_i(t)<1$. 如果存在正定对称矩阵P, Q, 正定对角矩阵 E, F, 使得下面的LMIs成立:

$$-2PA+\frac{1}{\eta_1}PBE^{-1}B^{\mathrm{T}}P+\Delta_f F\Delta_f<0,$$

$$-2QC+\frac{1}{\eta_2}QDF^{-1}D^{\mathrm{T}}Q+\Delta_g E\Delta_g<0,\tag{16}$$

则无参数摄动的系统(2)是全局渐近稳定的, 其中: $\eta_1=\min(1-\dot{\tau}_j(t))$, $\eta_2=\min(1-\dot{z}_i(t))$, $i=1,\cdots,n$, $j=1,\cdots,m$.

在模型(1)中, 如果$\tau_j(t)=\bar{\tau}(t)>0$和$z_j(t)=\bar{z}(t)>0$, 即单时滞情况, 则模型(2)可表示成如下形式:

$$\begin{aligned}&\dot{x}(t)=-(A+\Delta A)x(t)+(B+\Delta B)g(y(t-\bar{\tau}(t))),\\&\dot{y}(t)=-(C+\Delta C)y(t)+(D+\Delta D)f(x(t-\bar{z}(t))),\end{aligned}\tag{17}$$

其中的符号同(2)中的定义. 此时定理1仍适用于系统(17), 但结果有些保守. 下面将采用另一种Lyapunov-Krasovskii泛函形式, 给出保证系统(17)全局鲁棒稳定的新判据.

定理 2 假定$\dot{\bar{\tau}}(t)<1$和$\dot{\bar{z}}(t)<1$. 如果存在正定对称矩阵P, Q, E, F, 正定对角矩阵H, L, M, N, 正常数ε_x, ε_y, α_i, $i=1,\cdots,10$, 使得下面LMI成立:

$$\begin{bmatrix} \varOmega_1 & 0 & 0 & 0 & 0 & 0 & 0 & \varTheta_1 & 0 & 0 & \varTheta_2 \\ * & \varOmega_2 & \varTheta_3 & \varTheta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \varTheta_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varOmega_3 & \bar{\varOmega}_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \bar{\varOmega}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \varOmega_5 & \bar{\varOmega}_5 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \varTheta_6 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \varOmega_6 & \bar{\varOmega}_6 & 0 & 0 \\ * & * & * & * & * & * & * & * & \bar{\varOmega}_7 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \varOmega_8 & \bar{\varOmega}_8 \\ * & * & * & * & * & * & * & * & * & * & \varTheta_7 \end{bmatrix} < 0, \tag{18}$$

则系统(17)的原点是全局鲁棒稳定的, 其中:

$\varOmega_1 = -PA/2 - A^{\mathrm{T}}P/2 + \varepsilon_x I + \alpha_1 N_a^{\mathrm{T}} N_a + \alpha_2 N_a^{\mathrm{T}} N_a$,

$\varOmega_2 = -QC/2 - C^{\mathrm{T}}Q/2 + \varepsilon_y I + \alpha_3 N_c^{\mathrm{T}} N_c + \alpha_4 N_c^{\mathrm{T}} N_c$,

$\varOmega_3 = E - M + \alpha_6 N_b^{\mathrm{T}} N_b$,

$\bar{\varOmega}_3 = [HM_c \ \ HM_d \ \ HD]$,

$\bar{\varOmega}_4 = \mathrm{diag}\{-\alpha_4 I, \ -\alpha_5 I, \ \varOmega_4\}$,

$\varOmega_4 = -\eta_2 F + \alpha_5 N_d^{\mathrm{T}} N_d$,

$\varOmega_5 = -\eta_1(PA + \varepsilon_x I) + \alpha_7 N_a^{\mathrm{T}} N_a$,

$\bar{\varOmega}_5 = [PM_b \ , \eta_1 PM_a]$,

$\varOmega_6 = F - N + \alpha_9 N_d^{\mathrm{T}} N_d$,

$\bar{\varOmega}_6 = [LM_a, \ \ LM_b, \ \ 0]$,

$\bar{\varOmega}_7 = \mathrm{diag}\{-\alpha_2 I, \ -\alpha_8 I, \ \varOmega_7\}$,

$\varOmega_7 = -\eta_1 E + \alpha_8 N_b^{\mathrm{T}} N_b$,

$\varOmega_8 = -\eta_2(QC + \varepsilon_y I) + \alpha_{10} N_c^{\mathrm{T}} N_c$,

$\bar{\varOmega}_8 = [QM_d, \ \eta_2 QM_c, \ 0]$,

$\varTheta_1 = -A^{\mathrm{T}}L + N\Delta_f/2$,

$\varTheta_2 = [0, \ 0, \ PM_a/2], \varTheta_3 = QM_c/2$,

$\varTheta_4 = -C^{\mathrm{T}}H + M\Delta_g/2$,

$\varTheta_5 = -\alpha_3 I$,

$\varTheta_6 = \mathrm{diag}\{-\alpha_6 I, \ -\alpha_7 I\}$,

$\varTheta_7 = \mathrm{diag}\{-\alpha_9 I, \ -\alpha_{10} I, \ -\alpha_1 I\}$,

$*$表示相应元素的对称部分, $\eta_1 = 1 - \dot{\bar{\tau}}(t)$, $\eta_2 = 1 - \dot{\bar{z}}(t)$.

证 考虑如下Lyapunov-Krasovskii泛函:

$$\begin{aligned} V(t) =& x^{\mathrm{T}}(t)Px(t) + y^{\mathrm{T}}(t)Qy(t) + \\ & 2\sum_{j=1}^{m}\int_0^{y_j(t)} H_j g_j(s)\mathrm{d}s + \\ & 2\sum_{j=1}^{n}\int_0^{x_j(t)} L_j f_j(s)\mathrm{d}s + \\ & \int_{t-\bar{\tau}(t)}^{t} g^{\mathrm{T}}(y(s))(E_0 + E)g(y(s))\mathrm{d}s + \\ & \int_{t-\bar{z}(t)}^{t} f^{\mathrm{T}}(x(s))(F_0 + F)f(x(s))\mathrm{d}s, \end{aligned} \tag{19}$$

其中E_0, F_0将在后面定义. 沿着系统(17)的轨迹对(19)求导得,

$$\begin{aligned} \dot{V}(t) =& 2x^{\mathrm{T}}(t)P\Big[-(A + \Delta A)x(t) + \\ & (B + \Delta B)g(y(t - \bar{\tau}(t)))\Big] + \\ & 2y^{\mathrm{T}}(t)Q\Big[-(C + \Delta C)y(t) + \\ & (D + \Delta D)f(x(t - \bar{z}(t)))\Big] + \\ & 2g^{\mathrm{T}}(y(t))H\Big[-(C + \Delta C)y(t) + \\ & (D + \Delta D)f(x(t - \bar{z}(t)))\Big] + \\ & 2f^{\mathrm{T}}(x(t))L\Big[-(A + \Delta A)x(t) + \\ & (B + \Delta B)g(y(t - \bar{\tau}(t)))\Big] + \\ & g^{\mathrm{T}}(y(t))(E + E_0)g(y(t)) - \\ & \eta_1 g^{\mathrm{T}}(y(t - \bar{\tau}(t)))(E + E_0)g(y(t - \bar{\tau}(t))) + \\ & f^{\mathrm{T}}(x(t))(F + F_0)f(x(t)) - \\ & \eta_2 f^{\mathrm{T}}(x(t - \bar{z}(t)))(F + F_0)f(x(t - \bar{z}(t))) + \\ & \varepsilon_x x^{\mathrm{T}}(t)x(t) - \varepsilon_x x^{\mathrm{T}}(t)x(t) + \\ & \varepsilon_y y^{\mathrm{T}}(t)y(t) - \varepsilon_y y^{\mathrm{T}}(t)y(t), \end{aligned} \tag{20}$$

其中: $H = \mathrm{diag}\{H_1, \cdots, H_m\}$, $L = \mathrm{diag}\{L_1, \cdots, L_n\}$. 为方便考虑, 下面令$\bar{A} = A + \Delta A$, $\bar{B} = B + \Delta B$, $\bar{C} = C + \Delta C$, $\bar{D} = D + \Delta D$. 根据引理1和假设1, 下面不等式成立:

$$\begin{aligned} & -x^{\mathrm{T}}(t)(P\bar{A} + \varepsilon_x I)x(t) + \\ & 2x^{\mathrm{T}}(t)P\bar{B}g(y(t - \bar{\tau}(t))) \leqslant \\ & g^{\mathrm{T}}(y(t - \bar{\tau}(t)))\bar{B}^{\mathrm{T}}P(P\bar{A} + \varepsilon_x I)^{-1}P\bar{B} \times \\ & g(y(t - \bar{\tau}(t))), \end{aligned} \tag{21}$$

$$\begin{aligned} & -y^{\mathrm{T}}(t)(Q\bar{C} + \varepsilon_y I)y(t) + \\ & 2y^{\mathrm{T}}(t)Q\bar{D}f(x(t - \bar{z}(t))) \leqslant \\ & f^{\mathrm{T}}(x(t - \bar{z}(t)))\bar{D}^{\mathrm{T}}Q(Q\bar{C} + \varepsilon_y I)^{-1}Q\bar{D} \times \\ & f(x(t - \bar{z}(t))), \end{aligned} \tag{22}$$

$$\begin{aligned} & 2g^{\mathrm{T}}(y(t))H\bar{D}f(x(t - \bar{z}(t))) - \\ & \eta_2 f^{\mathrm{T}}(x(t - \bar{z}(t)))Ff(x(t - \bar{z}(t))) \leqslant \\ & \frac{1}{\eta_2}g^{\mathrm{T}}(y(t))H\bar{D}F^{-1}\bar{D}^{\mathrm{T}}Hg(y(t)), \end{aligned} \tag{23}$$

$$2f^{\mathrm{T}}(x(t))L\bar{B}g(y(t - \bar{\tau}(t))) -$$

$$\eta_1 g^{\mathrm T}(y(t-\bar\tau(t)))Eg(y(t-\bar\tau(t))) \leqslant \frac{1}{\eta_1}f^{\mathrm T}(x(t))L\bar B E^{-1}\bar B^{\mathrm T}Lf(x(t)), \tag{24}$$

$$0 \leqslant f^{\mathrm T}(x(t))\Delta_f Nx(t) - f^{\mathrm T}(x(t))Nf(x(t)), \tag{25}$$

$$0 \leqslant g^{\mathrm T}(y(t))\Delta_g My(t) - g^{\mathrm T}(y(t))Mg(y(t)). \tag{26}$$

令

$$E_0 = \frac{1}{\eta_1}\bar B^{\mathrm T}P(P\bar A+\varepsilon_x I)^{-1}P\bar B,$$

$$F_0 = \frac{1}{\eta_2}\bar D^{\mathrm T}H(H\bar C+\varepsilon_y I)^{-1}H\bar D,$$

则将式(21)~(26)代入式(20),得

$$\dot V(t) \leqslant \begin{bmatrix} x^{\mathrm T}(t) & y^{\mathrm T}(t) & g^{\mathrm T}(y(t)) & f^{\mathrm T}(x(t))\end{bmatrix} \times \begin{bmatrix} \bar\Theta_1 & 0 & 0 & \bar\Theta_2 \\ 0 & \bar\Theta_3 & \bar\Theta_4 & 0 \\ 0 & \bar\Theta_4^{\mathrm T} & \bar\Theta_5 & 0 \\ \bar\Theta_2^{\mathrm T} & 0 & 0 & \bar\Theta_6 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ g(y(t)) \\ f(x(t)) \end{bmatrix}. \tag{27}$$

其中:

$$\bar\Theta_1 = -P\bar A/2 - \bar A^{\mathrm T}P/2 + \varepsilon_x I,$$

$$\bar\Theta_2 = -\bar A^{\mathrm T}L + N\Delta_f/2,$$

$$\bar\Theta_3 = -Q\bar C/2 - \bar C^{\mathrm T}Q/2 + \varepsilon_y I,$$

$$\bar\Theta_4 = -\bar C^{\mathrm T}H + M\Delta_g/2,$$

$$\bar\Theta_5 = E + H\bar D F^{-1}\bar D^{\mathrm T}H/\eta_2 - M + \bar B^{\mathrm T}P(P\bar A+\varepsilon_x I)^{-1}P\bar B/\eta_1,$$

$$\bar\Theta_6 = F + L\bar B E^{-1}\bar B^{\mathrm T}L/\eta_1 - N + \bar D^{\mathrm T}Q(Q\bar C+\varepsilon_y I)^{-1}Q\bar D/\eta_2.$$

根据Schur补引理、假设2和引理2,

$$\begin{bmatrix} \bar\Theta_1 & 0 & 0 & \bar\Theta_2 \\ 0 & \bar\Theta_3 & \bar\Theta_4 & 0 \\ 0 & \bar\Theta_4^{\mathrm T} & \bar\Theta_5 & 0 \\ \bar\Theta_2^{\mathrm T} & 0 & 0 & \bar\Theta_6 \end{bmatrix} < 0 \tag{28}$$

等价于

$$\begin{bmatrix} \varPsi_{11} & 0 & 0 & 0 & 0 & \hat\Theta_2 & 0 & 0 \\ 0 & \varPsi_{22} & \hat\Theta_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \hat\Theta_4^{\mathrm T} & \varPsi_{33} & HD & B^{\mathrm T}P & 0 & 0 & 0 \\ 0 & 0 & D^{\mathrm T}H & \varPsi_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & PB & 0 & \varPsi_{55} & 0 & 0 & 0 \\ \hat\Theta_2^{\mathrm T} & 0 & 0 & 0 & 0 & \varPsi_{66} & LB & D^{\mathrm T}Q \\ 0 & 0 & 0 & 0 & 0 & B^{\mathrm T}L & \varPsi_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & QD & 0 & \varPsi_{88} \end{bmatrix} < 0, \tag{29}$$

其中:

$$\hat\Theta_1 = -PA/2 - A^{\mathrm T}P/2 + \varepsilon_x I,$$

$$\hat\Theta_2 = -A^{\mathrm T}L + N\Delta_f/2,$$

$$\hat\Theta_3 = -QC/2 - C^{\mathrm T}Q/2 + \varepsilon_y I,$$

$$\hat\Theta_4 = -C^{\mathrm T}H + M\Delta_g/2,$$

$$\varPsi_{11} = \hat\Theta_1 + \alpha_1 N_a^{\mathrm T}N_a + \alpha_2 N_a^{\mathrm T}N_a + \alpha_1^{-1}PM_aM_a^{\mathrm T}P/4,$$

$$\varPsi_{22} = \hat\Theta_3 + \alpha_3 N_c^{\mathrm T}N_c + \alpha_4 N_c^{\mathrm T}N_c + \alpha_3^{-1}QM_cM_c^{\mathrm T}Q/4,$$

$$\varPsi_{33} = E - M + \alpha_6 N_b^{\mathrm T}N_b + \alpha_4^{-1}HM_cM_c^{\mathrm T}H + \alpha_5^{-1}HM_dM_d^{\mathrm T}H,$$

$$\varPsi_{44} = -\eta_2 F + \alpha_5 N_d^{\mathrm T}N_d,$$

$$\varPsi_{55} = -\eta_1(PA+\varepsilon_x I) + \alpha_6^{-1}PM_bM_b^{\mathrm T}P + \alpha_7^{-1}\eta_1^2 PM_aM_a^{\mathrm T}P + \alpha_7 N_a^{\mathrm T}N_a,$$

$$\varPsi_{66} = F - N + \alpha_2^{-1}LM_aM_a^{\mathrm T}L + \alpha_8^{-1}LM_bM_b^{\mathrm T}L + \alpha_9 N_d^{\mathrm T}N_d,$$

$$\varPsi_{77} = -\eta_1 E + \alpha_8 N_b^{\mathrm T}N_b,$$

$$\varPsi_{88} = -\eta_2(QC+\varepsilon_y I) + \alpha_9^{-1}QM_dM_d^{\mathrm T}Q + \alpha_{10}^{-1}\eta_2^2 QM_cM_c^{\mathrm T}Q + \alpha_{10}N_c^{\mathrm T}N_c.$$

再一次应用Schur补引理, 即得式(18).　　证毕.

注 1　因为定理1 针对多时滞情况而定理2仅针对单时滞情况, 进而定理1的适用范围更宽些, 如对于网络(2), 定理2就不适用. 然而对于网络(17), 虽然定理1和定理2均适用, 但定理1却通常具有很大的保守性, 这一点体现了一般性和特殊性的辩证关系.

注 2　具有定常时滞的无摄动模型(1) 在文献[1,5]中得到研究. 文献 [1]中的定理1给出了保证模型(1)全局渐近稳定的简单判据, 但没能考虑连接权矩阵的符号差, 进而未能考虑神经元的激励和抑制对网络的影响. 文献 [5]的定理2利用一个复杂的代数不等式, 得到了模型(1)全局稳定的充分判据. 虽然文献 [5]的定理2因包含了大量的可调参数使得结果得到改进, 但没有系统的方法来调节这些未知参数, 进而所得结果不易验证. 而本文的推论1考虑的是时变时滞的模型(1), 同时, 基于线性矩阵不等式技术得到的结果既易于验证, 又消除了神经元激励和抑制对网络的影响. 因此, 本文定理克服了上述两篇文献中的不足. 文献 [3,4,6,7]基于不同的代数不等式技术, 针对定常时滞的无摄动模型(17)给出了若干全局渐近稳定充分判据. 但无论从稳定判据是否容易验证及是否考虑了神经元的激励和抑制对网络的影响等方面来说, 本文的结果都明显扩展和改进了文献 [3,4,6,7]中的结果.

4　数值例子(Numerical examples)

4.1　例 1(Example 1)

考虑如下定常时滞双向联想记忆神经网络

$$\begin{cases} \dot{x}(t) = -Ax(t) + Bg(y(t-\tau)) + U, \\ \dot{y}(t) = -Cy(t) + Df(x(t-z)) + J, \end{cases} \tag{30}$$

其中

$$A = C = \mathrm{diag}\{2,\ 2\},$$

$$B = D = \begin{bmatrix} c & c \\ c & -c \end{bmatrix},$$

$$g_j(u_j) = f_j(u_j) = \tanh\ u_j,$$

$U,\ J$为2维实常向量, $\tau > 0,\ z > 0$为任意有界时滞. 如果在本文的推论1中令$P = Q = I, E = F = I$, 则为保证推论1成立, 则c必须满足$|c| < 1$. 对于本例, 文献 [1]的定理1也要求$|c| < 1$才成立. 同样, 文献 [3,4]的结果也要求$|c| < 1$时才成立. 显然, 对于本例(30), 本文的定理1与文献 [1]的定理1, 文献 [3,4]的结果分别提供了不同的全局渐近稳定充分条件.

4.2 例 2(Example 2)

考虑双向联想记忆网络(17), 其中参数如下:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 1.2 & 1.0 \\ -0.3 & 0.2 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.9 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, C = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix},$$

$$M_a = \begin{bmatrix} -0.2 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}, M_b = \begin{bmatrix} -0.4 & 0.4 \\ 0.3 & 0.5 \end{bmatrix},$$

$$M_c = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & -0.3 \end{bmatrix}, F(t) = \begin{bmatrix} \sin\ t & 0 \\ 0 & \cos\ t \end{bmatrix},$$

$$M_d = \begin{bmatrix} -0.2 & 0.4 \\ 0.4 & 0.3 \end{bmatrix}, M_a = N_a, M_b = N_b,$$

$$M_c = N_c, M_d = N_d, \Delta_g = \Delta_f = I,$$

$$\eta_1 = 0.5, \eta_2 = 0.3.$$

应用定理1可得

$$P = \begin{bmatrix} 0.4908 & -0.0004 \\ -0.0004 & 0.6148 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.5474 & 0.0070 \\ 0.0070 & 0.8879 \end{bmatrix},$$

$$E = \begin{bmatrix} 1.8654 & 0 \\ 0 & 2.0881 \end{bmatrix},$$

$$F = \begin{bmatrix} 1.4710 & 0 \\ 0 & 2.2279 \end{bmatrix},$$

$$\varepsilon_a = 1.1062, \varepsilon_b = 0.9140,$$

$$\varepsilon_c = 1.1266, \varepsilon_d = 0.8709.$$

显然, 所考虑的网络是全局鲁棒稳定的.

5 结论(Conclusion)

本文研究了变时滞不确定双向联想记忆神经网络的全局稳定性问题, 建立了两个鲁棒稳定充分判据. 所得到的结果建立了网络参数之间的关系, 且能够表示成线性矩阵不等式形式, 进而具有易于验证的特点. 通过两个数值例子验证了所得结果的有效性.

参考文献(References):

[1] ARIK S. Global asymptotic stability analysis of bi-directional associative memory neural networks with time delays[J]. *IEEE Transactions on Neural Networks*, 2005, 16(3): 580 – 586.

[2] ARIK S, TAVSANOGLU V. Global asymptotic stability analysis of bi-directional associative memory neural networks with constant time delays[J]. *Neurocomputing*, 2005, 68(1): 161 – 176.

[3] LIAO X, YU J, CHEN G. Novel stability criteria for bi-directional associative memory neural networks with time delays[J]. *International Journal of Circuits Theory and Applications*, 2002, 30(3): 519 – 546.

[4] ZHANG J, YANG Y. Global stability analysis of bidirectional associative memory neural networks with time delay[J]. *International Journal of Circuit Theory and Applications*, 2001, 29(1): 185 – 196.

[5] ZHAO H. Exponential stability and periodic oscillatory of bi-directional associative memory neural network involving delays[J]. *Neurocomputing*, 2006, 69(2): 424 – 448.

[6] LIAO X, YU J. Qualitative analysis of bi-directional associative memory with time delay[J]. *International Journal of Circuits Theory and Applications*, 1998, 26(1): 219 – 229.

[7] CAO J. Global asymptotic stability of delayed bi-directional associative memory networks[J]. *Applied Mathematics and Computations*, 2003, 142(2): 333 – 339.

[8] 张化光, 季策, 王占山. 递归人工神经网络的定性分析和综合[M]. 北京: 科学出版社, 2004.

[9] LIAO X, CHEN G, SANCHEZ E N. LMI-based approach for asymptotic stability analysis of delayed neural networks[J]. *IEEE Transactions on Circuits and Systems-I*, 2002, 49(7): 1033 – 1039.

[10] WANG Z, ZHANG H. Exponential stability analysis of neural networks with multiple time varying delays[J]. *Chinese Journal of Electronics*, 2006, 6(4): 649 – 653.

[11] 季策, 张化光. 多时滞Hopfield 神经网络的鲁棒稳定性及吸引域的估计[J]. 控制理论与应用, 2005, 22(4): 538 – 542.
(JI Ce, ZHANG Huaguang. Robust stability of Hopfield neural networks with multiple time-delay and estimation of the domain of attraction[J]. *Control Theory & Applications*, 2005, 22(4): 538 – 542.)

[12] PARK J. Robust stability of bi-directional associative memory neural networks with delays[J]. *Physics Letters A*, 2006, 349(3): 494 – 499.

[13] XIE L, FU M, DE SOUZA C E. Control and quadratic stabilization of system with parameter uncertainty via output feedback[J]. *IEEE Transactions on Automatic Control*, 1992, 37(8): 1253 – 1256.

[14] BOYD S, EI GHAOUI L, FERON E, et al. *Linear Matrix Inequalities in System and Control Theory*[M]. Philadelphia: Society for Industrial and Applied Mathematics, 1994.

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