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一类不确定非线性系统的鲁棒自适应控制

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摘要: 针对一类MIMO不确定非线性系统的输出跟踪问题, 基于自适应反步法和滑模控制为其设计了鲁棒自适应控制器. 模型包含3种不确定性: 1) 参数不确定性; 2) 输入增益的不确定性; 3) 代表系统未建模动态和干扰的不确定函数, 该函数有界. 以非完整移动机械臂的输出跟踪控制为目标, 对其进行仿真实验, 实验结果表明所提出的控制算法是正确有效的.

关键词: 自适应反步; 鲁棒控制; 滑模控制; 不确定非线性系统
中图分类号: TP13　　**文献标识码**: A

Robust adaptive control of a class of uncertain nonlinear systems

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Abstract: Combining the adaptive Backstepping with the sliding mode control, we present a robust adaptive controller for a class of MIMO uncertain nonlinear systems. The model contains three forms of model uncertainty: 1) parametric uncertainty; 2) uncertainty in the input gains; 3) bounded uncertain functions which represent unmodelled dynamics and disturbances. The results of numerical simulation for the output tracking control of nonholonomic mobile manipulators are presented to demonstrate the effectiveness of the proposed approach.

Key words: adaptive Backstepping; robust control; sliding mode control; uncertain nonlinear systems

1 引言(Introduction)

对于不确定非线性系统, 滑模控制是一种较为有效的鲁棒控制方法, 出现了许多研究成果[1~3]. 虽然滑模控制具有良好的跟踪性能, 但它要求系统的不确定性满足所谓匹配条件[1], 且不能处理具有未知参数的系统. 近10多年来, 反步设计法引起了国内外有关学者的高度重视. 反步法不仅可以处理非匹配不确定性, 而且可以处理带有未知参数的非线性系统, 故反步法在非线性系统的控制设计中取得了巨大成功[4~6], 并被推广到自适应控制、鲁棒控制、滑模控制等多个领域. 这主要是因为反步法可通过反向设计使控制V函数和控制器的设计过程系统化、结构化, 且可以控制相对阶为n的非线性系统, 消除了经典无源性设计中相对阶为1的限制. 文献[7, 8]将自适应反步法与滑模控制相结合很好地实现了系统的控制性能.

本文研究的是具有3种不确定性的MIMO非线性系统的输出跟踪问题. 将自适应反步法与滑模控制结合起来为其设计了鲁棒自适应控制器. 然后以非完整移动机械臂的输出跟踪控制为目标, 对其进行仿真实验, 仿真结果证明了它的正确有效性.

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2 问题的提出(Problem statement)

考虑如下形式的MIMO非线性不确定系统:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x_1, \cdots, x_i) + \varphi_i(x_1, \cdots, x_i)\theta + \\ \qquad d_i(x, w, t), 1 \leqslant i \leqslant n-1, \\ \dot{x}_n = f_n(x) + \eta g(x)u + \varphi_n(x)\theta + d_n(x, w, t), \\ y = x_1, \end{cases} \tag{1}$$

其中: $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$为系统状态, y为输出, $x_i, f_i, d_i, u \in \mathbb{R}^m$, $\varphi_i \in \mathbb{R}^{m\times p}$, $i = 1, 2, \cdots, n$, $g(x) \in \mathbb{R}^{m\times m}$, $\theta \in \mathbb{R}^p$为未知的常参数向量; $d_i(x, w, t)$, $i = 1, 2, \cdots, n$为包含系统未建模动态和干扰的有界未知非线性函数, w为不确定时变参数, $\eta \in \mathbb{R}$为未知的控制输入增益常系数.

控制目标: 设计鲁棒自适应控制器使系统输出能跟踪期望输出. 设计控制器之前, 先做如下假设:

假设 1 给定的有界参考信号$y_r(t)$连续可导且n阶导数有界.

假设 2 $g(x)$可逆.

假设 3 存在已知正值函数$\delta_i(x_1, \cdots, x_i)$使得

$$|d_i(x, w, t)| \leqslant \delta_i(x_1, \cdots, x_i),\ i = 1, 2, \cdots, n. \tag{2}$$

3 鲁棒自适应控制器设计(Robust adaptive controller design)

第1步 定义误差变量:

$$z_1 = x_1 - y_r, z_2 = x_2 - \dot{y}_r - \alpha_1.$$

对z_1求导有

$$\dot{z}_1 = z_2 + \alpha_1 + f_1(x_1) + \omega_1^{\mathrm{T}}(x_1)\hat{\theta} + d_1 + \omega_1^{\mathrm{T}}(x_1)\tilde{\theta}, \tag{3}$$

其中: $\omega_1(x_1) = \varphi_1^{\mathrm{T}}(x_1)$, $\hat{\theta}$为θ的估计值, $\tilde{\theta} = \theta - \hat{\theta}$. 取Lyapunov函数

$$V_1 = \frac{1}{2}z_1^{\mathrm{T}}z_1 + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\varGamma^{-1}\tilde{\theta}.$$

其中$\varGamma$为正定的自适应增益矩阵, 则

$$\dot{V}_1 = z_1^{\mathrm{T}}(z_2 + \alpha_1 + f_1 + \omega_1^{\mathrm{T}}\hat{\theta} + d_1) + \tilde{\theta}^{\mathrm{T}}\varGamma^{-1}(\varGamma\omega_1 z_1 - \dot{\hat{\theta}}).$$

取虚拟控制$\alpha_1 = -c_1 z_1 - f_1 - \omega_1^{\mathrm{T}}\hat{\theta} - \frac{\gamma}{4}\delta_1^2 z_1 \mathrm{e}^{at}$, ($a$和$\gamma$都是正数, $c_1 = \mathrm{diag}\{c_1\}, c_1 > 0$, 下同), 则有

$$\dot{z}_1 = -c_1 z_1 + z_2 + \omega_1^{\mathrm{T}}(x_1)\tilde{\theta} + d_1 - \frac{\gamma}{4}\delta_1^2 z_1 \mathrm{e}^{at}, \tag{4}$$

$$\dot{V}_1 \leqslant -c_1 z_1^{\mathrm{T}} z_1 + z_1^{\mathrm{T}} z_2 + \frac{1}{\gamma}\mathrm{e}^{-at} + \tilde{\theta}^{\mathrm{T}}\varGamma^{-1}(\tau_1 - \dot{\hat{\theta}}).$$

其中$\tau_1 = \varGamma\omega_1 z_1$.

第*k*步 $k = 2, 3, \cdots, n-1$, 依此类推, 有

$$z_{k+1} = x_{k+1} - y_r^{(k)} - \alpha_k,$$

$$\dot{z}_k = z_{k+1} + \alpha_k + f_k + \omega_k^{\mathrm{T}}\hat{\theta} - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}x_{i+1} - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}f_i - \frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_{k-1}}{\partial t} + \omega_k^{\mathrm{T}}\tilde{\theta} + \zeta_k. \tag{5}$$

取虚拟控制为

$$\begin{aligned}&\alpha_k(x_1, \cdots, x_k, \hat{\theta}, t) = \\ &-z_{k-1} - c_k z_k - \omega_k^{\mathrm{T}}\hat{\theta} + \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}x_{i+1} + \\ &\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}f_i + \frac{\partial\alpha_{k-1}}{\partial t} + \frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\tau_k + \\ &\omega_k^{\mathrm{T}}\varGamma^{\mathrm{T}}(\sum_{i=1}^{k-2}(\frac{\partial\alpha_i}{\partial\hat{\theta}})^{\mathrm{T}}z_{i+1}) - \varDelta_k,\end{aligned} \tag{6}$$

则有

$$\begin{aligned}&\dot{z}_k = \\ &-z_{k-1} - c_k z_k + z_{k+1} + \omega_k^{\mathrm{T}}\tilde{\theta} + \frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\tau_k - \dot{\hat{\theta}}) + \\ &(\sum_{i=1}^{k-2}z_{i+1}^{\mathrm{T}}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}})\varGamma\omega_k + \zeta_k - \varDelta_k.\end{aligned} \tag{7}$$

取Lyapunov函数

$$V_k = \frac{1}{2}\sum_{i=1}^{k}z_i^{\mathrm{T}}z_i + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\varGamma^{-1}\tilde{\theta},\ k = 2, 3, \cdots, n-1,$$

则有

$$\dot{V}_k \leqslant -\sum_{i=1}^{k}z_i^{\mathrm{T}}c_i z_i + z_k^{\mathrm{T}}z_{k+1} + \frac{k(k+1)}{2\gamma}\mathrm{e}^{-at} + \tilde{\theta}^{\mathrm{T}}\varGamma^{-1}(\tau_k - \dot{\hat{\theta}}) + (\sum_{i=1}^{k-1}z_{i+1}^{\mathrm{T}}\frac{\partial\alpha_i}{\partial\hat{\theta}})(\tau_k - \dot{\hat{\theta}}).$$

其中:

$$c_k > 0,\ \omega_k = \varphi_k^{\mathrm{T}} - \sum_{i=1}^{k-1}(\frac{\partial\alpha_{k-1}}{\partial x_i}\varphi_i)^{\mathrm{T}},\ k \geqslant 2,$$

$$\tau_k = \tau_{k-1} + \varGamma\omega_k z_k = \varGamma\sum_{i=1}^{k}\omega_i z_i,$$

$$\zeta_k = d_k - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}d_i,$$

$$\varDelta_k = \frac{\gamma}{4}\mathrm{e}^{at}z_k(\delta_k^2 + \sum_{i=1}^{k-1}(\frac{\partial\alpha_{k-1}}{\partial x_i})^{\mathrm{T}}(\frac{\partial\alpha_{k-1}}{\partial x_i})\delta_i^2).$$

第*n*步 与滑模控制的结合.

在算法的最后一步加入下面定义的滑动流形:

$$S = \sum_{i=1}^{n-1}m_i z_i + z_n. \tag{8}$$

其中$m_i = \mathrm{diag}\{m_i\}(i = 1, 2, \cdots, n-1)$为使下列多项式为Hurwitz的正常数:

$$\xi^{n-1} + m_{n-1}\xi^{n-2} + \cdots + m_2\xi + m_1 = 0.$$

定义$z_n = x_n - y_r^{(n-1)} - \alpha_{n-1}$, 则

$$\begin{aligned}&\dot{z}_n = \dot{x}_n - y_r^{(n)} - \dot{\alpha}_{n-1} = \\ &f(x) + \eta g(x)u + \omega_n^{\mathrm{T}}(x)\hat{\theta} - y_r^{(n)} + \xi_n - \\ &\sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}x_{i+1} - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}f_i - \\ &\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_{n-1}}{\partial t} + \omega_n^{\mathrm{T}}(x)\tilde{\theta}.\end{aligned}$$

令

$$\alpha_n = -\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\tau_n - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}x_{i+1} - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}f_i - \frac{\partial\alpha_{n-1}}{\partial t},$$

$$\zeta_n = d_n - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}d_i,$$

$$\omega_n = \varphi_n^{\mathrm{T}} - \sum_{i=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_i}\varphi_i)^{\mathrm{T}},$$

则有

$$\dot{z}_n = f(x) + \eta g(x)u + \omega_n^{\mathrm{T}}\hat{\theta} - y_r^{(n)} + \zeta_n + \alpha_n + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\tau_n - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + \omega_n^{\mathrm{T}}\tilde{\theta}. \tag{9}$$

取Lyapunov函数

$$V_n = V_{n-1} + \frac{1}{2}S^{\mathrm{T}}S + \frac{1}{2\lambda}\tilde{\eta}^2,$$

其中: $\tilde{\eta} = \eta - \hat{\eta}$, $\hat{\eta}$为η的估计值, $\lambda > 0$为设计常数.

则有

$$\begin{aligned}\dot V_n \leqslant & -\sum_{i=1}^{n-1} z_i^{\mathrm T}c_iz_i + z_{n-1}^{\mathrm T}z_n + \frac{(n-1)n}{2\gamma}\mathrm e^{-at} + \\ & \tilde\theta^{\mathrm T}\varGamma^{-1}(\tau_{n-1}-\dot{\hat\theta})+(\sum_{i=1}^{n-2} z_{i+1}^{\mathrm T}\frac{\partial\alpha_i}{\partial\hat\theta})(\tau_{n-1}-\dot{\hat\theta}) + \\ & \frac{1}{\sigma}\tilde\eta(-\dot{\hat\eta}) + S^{\mathrm T}[f(x)+\eta g(x)u+\omega_n^{\mathrm T}\hat\theta - y_r^{(n)} + \\ & \zeta_n+\alpha_n - \frac{\partial\alpha_{n-1}}{\partial\hat\theta}\dot{\hat\theta} + \frac{\partial\alpha_{n-1}}{\partial\hat\theta}\tau_n + \omega_n^{\mathrm T}\tilde\theta + \\ & \sum_{i=1}^{n-1} m_i(-z_{i-1}-c_iz_i+z_{i+1}) + \sum_{i=1}^{n-1} m_i\omega_i^{\mathrm T}\tilde\theta + \\ & \sum_{i=1}^{n-1} m_i(\xi_i-\varDelta_i) + \sum_{i=1}^{n-1} m_i\frac{\partial\alpha_i}{\partial\hat\theta}(\tau_i-\dot{\hat\theta}) + \\ & \sum_{i=1}^{n-1} m_i\omega_i^{\mathrm T}\varGamma^{\mathrm T}(\sum_{j=1}^{i-1}(\frac{\partial\alpha_j}{\partial\hat\theta})^{\mathrm T}z_{j+1})].\end{aligned}$$

令

$$\tau_n = \tau_{n-1} + \varGamma(\omega_n + \sum_{i=1}^{n-1} m_i\omega_i)S, \tag{10}$$

则有

$$\begin{aligned}\dot V_n \leqslant & -\sum_{i=1}^{n-1} z_i^{\mathrm T}c_iz_i + z_{n-1}^{\mathrm T}z_n + \frac{(n-1)n}{2\gamma}\mathrm e^{-at} + \\ & \tilde\theta^{\mathrm T}\varGamma^{-1}(\tau_n-\dot{\hat\theta}) + (\sum_{i=1}^{n-2} z_{i+1}^{\mathrm T}\frac{\partial\alpha_i}{\partial\hat\theta})(\tau_n-\dot{\hat\theta}) + \\ & S^{\mathrm T}[f(x)+\hat\eta g(x)u+\omega_n^{\mathrm T}\hat\theta - y_r^{(n)} + \zeta_n + \alpha_n + \\ & \frac{\partial\alpha_{n-1}}{\partial\hat\theta}(\tau_n-\dot{\hat\theta}) + \sum_{i=1}^{n-1} m_i(-z_{i-1}-c_iz_i+z_{i+1}) + \\ & \sum_{i=1}^{n-1} m_i(\xi_i-\varDelta_i) + \sum_{i=1}^{n-1} m_i\frac{\partial\alpha_i}{\partial\hat\theta}(\tau_i-\dot{\hat\theta}) + \\ & \sum_{i=1}^{n-1} m_i\omega_i^{\mathrm T}\varGamma^{\mathrm T}(\sum_{j=1}^{i-1}(\frac{\partial\alpha_j}{\partial\hat\theta})^{\mathrm T}z_{j+1})] - \\ & S^{\mathrm T}(\omega_n + \sum_{i=1}^{n-1} m_i\omega_i)^{\mathrm T}\varGamma^{\mathrm T}(\sum_{i=1}^{n-2} z_{i+1}^{\mathrm T}\frac{\partial\alpha_i}{\partial\hat\theta})^{\mathrm T} + \\ & \frac{1}{\lambda}\tilde\eta(-\dot{\hat\eta} + S^{\mathrm T}g(x)u).\end{aligned}$$

取参数更新律为: $\dot{\hat\theta} = \tau_n$, $\dot{\hat\eta} = S^{\mathrm T}g(x)u$; 并考虑下面的自适应滑模控制器:

$$\begin{aligned}u = & -\frac{1}{\hat\eta}g^{-1}(x)[f(x)+\omega_n^{\mathrm T}\hat\theta - y_r^{(n)} + \alpha_n + \\ & \sum_{i=1}^{n-1} m_i(-z_{i-1}-c_iz_i+z_{i+1}) - \\ & \sum_{i=1}^{n-1} m_i\varDelta_i + \sum_{i=1}^{n-1} m_i\frac{\partial\alpha_i}{\partial\hat\theta}(\tau_i-\tau_n) + \\ & \sum_{i=1}^{n-1} m_i\omega_i^{\mathrm T}\varGamma^{\mathrm T}(\sum_{j=1}^{i-1}(\frac{\partial\alpha_j}{\partial\hat\theta})^{\mathrm T}z_{j+1}) - \\ & (\omega_n + \sum_{i=1}^{n-1} m_i\omega_n)^{\mathrm T}\varGamma^{\mathrm T}(\sum_{i=1}^{n-2} z_{i+1}^{\mathrm T}\frac{\partial\alpha_i}{\partial\hat\theta})^{\mathrm T} + KS + \\ & W\mathrm{sgn}(S) + v_n\mathrm{sgn}(S) + \sum_{i=1}^{n-1} m_iv_i\mathrm{sgn}(S)].\end{aligned} \tag{11}$$

其中:

$$K = \mathrm{diag}\{k_i\},\ W = \mathrm{diag}\{w_i\},\ k_i, w_i > 0,$$

$$v_i = \delta_i + \sum_{j=1}^{i-1}\left\|\frac{\partial\alpha_{i-1}}{\partial x_j}\right\|\delta_j,\ i = 1,2,\cdots,n,$$

$\mathrm{sgn}(\cdot)$为符号函数. 则有

$$\begin{aligned}\dot V_n \leqslant & -\sum_{i=1}^{n-1} c_iz_i^{\mathrm T}z_i + z_{n-1}^{\mathrm T}z_n + \frac{(n-1)n}{2\gamma}\mathrm e^{-at} - \\ & k_iS^{\mathrm T}S - w_i\|S\| + S^{\mathrm T}[\zeta_n - v_n\mathrm{sgn}(S) + \\ & \sum_{i=1}^{n-1} m_i(\zeta_i - v_i\mathrm{sgn}(S))] \leqslant \\ & -[z_1^{\mathrm T}\ \ z_2^{\mathrm T}\ \ \cdots\ \ z_n^{\mathrm T}]Q[z_1\ \ z_2\ \ \cdots\ \ z_n]^{\mathrm T} + \\ & \frac{(n-1)n}{2\gamma}\mathrm e^{-at} - w_{i\min}\|S\|.\end{aligned}$$

其中

$$Q = \begin{bmatrix} c_1+Km_1^2 & Km_1m_2 & \cdots & Km_1m_{n-1} & Km_1 \\ Km_1m_2 & c_2+Km_2^2 & \cdots & Km_2m_{n-1} & Km_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Km_1m_{n-1} & Km_2m_{n-1} & \cdots & c_{n-1}+Km_{n-1}^2 & -\dfrac{I}{2}+Km_{n-1} \\ Km_1 & Km_2 & \cdots & -\dfrac{I}{2}+Km_{n-1} & K \end{bmatrix}.$$

只要适当选择参数$K, m_i, c_i (i = 1,2,\cdots,n-1)$就可保证矩阵$Q$正定. 令$\tilde W_n = [z_1\ \ z_2\ \ \cdots\ \ z_n]Q[z_1\ \ z_2\ \ \cdots\ \ z_n]^{\mathrm T} + w_{i\min}\|S\|$, $\varepsilon = n/\gamma$, 只要γ取足够大, ε就可以足够小. 则有

$$\dot V_n \leqslant -\tilde W_n + \varepsilon(n-1)\mathrm e^{-at}/2.$$

故有

$$0 \leqslant \int_0^t \tilde W_n\mathrm dt \leqslant V_n(0) + \varepsilon(n-1)(1-\mathrm e^{-at})/(2a),$$

且

$$\lim_{t\to\infty}\int_0^t \tilde W_n\mathrm dt \leqslant V_n(0) + \varepsilon(n-1)/(2a) < \infty.$$

由 Barbalat 定理[9]可知, $\lim\limits_{t\to\infty}\tilde W_n = 0$; 则有$\lim\limits_{t\to\infty} z_i = 0 (i = 1,\cdots,n)$, $\lim\limits_{t\to\infty} S = 0$, $\lim\limits_{t\to\infty}(x_1 - y_r) = 0$, 从而保证了系统在滑动面$S = 0$上的稳定性, 达到轨迹跟踪的目的.

如果ε为足够小的正数, a适当大, 则有$\dot{V}_n < 0$. 另外, 也可以适当选取设计参数K, W, c_i, m_i, 使得$\dot{V}_n < 0$, 进而达到控制目的.

4 仿真实验(Simulation experiment)

4.1 包含驱动电机动态特性的移动机械臂简化动态模型(Reduced dynamic model including motor dynamics of mobile manipulators)

考虑如图1所示的移动机械臂系统, 其包含驱动电机动态特性的简化动态方程可表示为[10]

$$\begin{cases} \dot{q} = S(q)v, \\ \bar{M}(q)\dot{v} + \bar{V}(q,\dot{q})v + \bar{G}(q) = \tau, \\ \dot{\tau} = H_1 u - H_2\tau - H_3 v. \end{cases} \tag{12}$$

其中: $q = [x_c\ \ y_c\ \ \varphi\ \ \theta_1\ \ \theta_2]^{\mathrm{T}}$, $\bar{M} = S^{\mathrm{T}}MS$, $\bar{V} = S^{\mathrm{T}}(M\dot{S} + VS)$, $\bar{G} = S^{\mathrm{T}}G$, $\bar{B} = S^{\mathrm{T}}B = I$. $\tau = [\tau_r\ \ \tau_l\ \ \tau_1\ \ \tau_2]^{\mathrm{T}}$, $M(q)$为对称正定惯性矩阵, $V(q,\dot{q})$为哥氏力和向心力矩阵, $G(q)$为万有引力矢量, $B(q)$为输入转换矩阵, τ为转矩输入矢量, $v = [v_1\ \ v_2\ \ v_3\ \ v_4]^{\mathrm{T}} = [\dot{\theta}_r\ \ \dot{\theta}_l\ \ \dot{\theta}_1\ \ \dot{\theta}_2]^{\mathrm{T}}$,

$$S(q) = \begin{bmatrix} \frac{l}{2}\cos\varphi - \frac{l}{R}d\sin\varphi & \frac{l}{2}\cos\varphi + \frac{l}{R}d\sin\varphi & 0 & 0 \\ \frac{l}{2}\sin\varphi + \frac{l}{R}d\cos\varphi & \frac{l}{2}\sin\varphi - \frac{l}{R}d\cos\varphi & 0 & 0 \\ \frac{l}{R} & -\frac{l}{R} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

l为驱动轮半径, R为两驱动后轮之间的距离, d为移动平台的质心C到两驱动后轮的中心P之间的距离, φ为移动平台的方向角; $H_1 = \mathrm{diag}\{\beta_i k_{Ti}/L_i\}$, $H_2 = \mathrm{diag}\{r_i/L_i\}$, $H_3 = \mathrm{diag}\{\beta_i^2 k_{Ti} k_{ei}/L_i\}$, $i = 1,2,3,4$, $u = [u_1, u_2, u_3, u_4]^{\mathrm{T}}$, L, r, k_e, k_T为驱动电机的电枢电感、电阻、反电势常数和转矩常数, β为齿轮减速比.

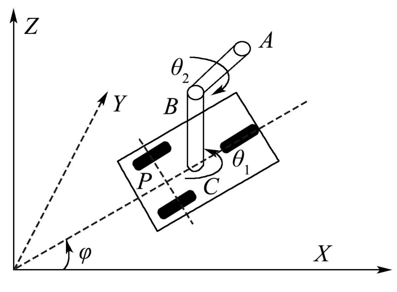


图 1 两连杆移动机械臂

Fig. 1 Mobile manipulators with two link

4.2 鲁棒自适应控制器设计(Robust adaptive controller design)

基于第3部分的设计原理为系统(12)设计鲁棒自适应控制器. 不失一般性, 假设4台驱动电机都采用相同的DC电机, 参数都未知, 且系统存在有界扰动, 则系统简化动态方程(12)可表示为

$$\begin{cases} \dot{q} = S(q)v, \\ \dot{v} = \bar{M}^{-1}(q)(\tau - \bar{V}(q,\dot{q})v - \bar{G}(q)), \\ \dot{\tau} = \phi_1\bar{H}_1 u - \phi_2\bar{H}_2\tau - \phi_3\bar{H}_3 v + d. \end{cases} \tag{13}$$

其中: $\phi_1 = k_T/L$, $\phi_2 = r/L$, $\phi_3 = k_T k_e/L$, $H_1 = \phi_1\bar{H}_1, H_2 = \phi_2\bar{H}_2, H_3 = \phi_3\bar{H}_3$, $\|d\| \leqslant \delta$.

定义移动机械臂的输出

$$Y = [y_1\ \ y_2\ \ y_3\ \ y_4]^{\mathrm{T}} = [x_c\ \ y_c\ \ \theta_1\ \ \theta_2]^{\mathrm{T}}. \tag{14}$$

控制目标: 设计控制u以使$\lim\limits_{t\to\infty}(Y - Y_d) = 0$, $\lim\limits_{t\to\infty}(\dot{Y} - \dot{Y}_d) = 0$, 其中$Y_d$为期望的连续可微的输出轨迹.

微分方程(14)得

$$\dot{Y} = B_1(\varphi)v,\ \ddot{Y} = B_1(\varphi)\dot{v} + \dot{B}_1(\varphi)v,$$
$$\dddot{Y} = B_1(\varphi)\ddot{v} + 2\dot{B}_1(\varphi)\dot{v} + \ddot{B}_1(\varphi)v =$$
$$P + D(\phi_1\bar{H}_1 u - \phi_2\bar{H}_2\tau - \phi_3\bar{H}_3 v + d).$$

其中: $P = 2\dot{B}_1\dot{v} + \ddot{B}_1 v - D(\tilde{M}\dot{v} + \dot{\bar{V}}v + \dot{\bar{G}})$, $D = B_1\bar{M}^{-1}$, 因B_1非奇异且其元素仅为正弦、余弦函数的组合, 故D为非奇异且有界, 即$\|D\| \leqslant f_d$.

$$B_1(\varphi) = \begin{bmatrix} \frac{l}{2}\cos\varphi - \frac{l}{R}d\sin\varphi & \frac{l}{2}\cos\varphi + \frac{l}{R}d\sin\varphi & 0 & 0 \\ \frac{l}{2}\sin\varphi + \frac{l}{R}d\cos\varphi & \frac{l}{2}\sin\varphi - \frac{l}{R}d\cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\dot{\varphi} = \frac{l}{R}(v_1 - v_2),\ \tilde{M} = \dot{\bar{M}} + \bar{V}.$$

取状态变换和非线性输入变换

$$Z_1 = Y, Z_2 = \dot{Y}, Z_3 = \ddot{Y}, \tag{15}$$

$$\nu = D\bar{H}_1 u. \tag{16}$$

令$w_1 = D\bar{H}_2\tau$, $w_2 = D\bar{H}_3 v$为中间变量, 则输出式(14)满足如下的动态方程:

$$\begin{cases} \dot{Z}_1 = Z_2, \\ \dot{Z}_2 = Z_3, \\ \dot{Z}_3 = P + \phi_1\nu - \phi_2 w_1 - \phi_3 w_2 + Dd. \end{cases} \tag{17}$$

可见式(17)具有式(1)的形式, 其中: $f_1 = f_2 = 0$,

$\varphi_1 = \varphi_2 = 0$, $d_1 = d_2 = 0$, $f_3 = P$, $\eta = \phi_1$, $\varphi_3 = [-w_1 \ \ -w_2]$, $\theta = [\phi_2 \ \ \phi_3]^{\mathrm{T}}$, $d_3 = Dd$, 且$\|d_3\| \leqslant f_d\delta$.

假设给定轨迹为$Z_d = Y_d = [z_{1d} \ \ z_{2d} \ \ z_{3d} \ \ z_{4d}]^{\mathrm{T}}$, 为使系统(13)的输出跟踪上给定的输出, 按着第3部分的设计方法得参数更新律为

$$\dot{\hat{\phi}}_1 = \lambda S^{\mathrm{T}}\nu,\ \dot{\hat{\phi}}_2 = -\gamma_2 S^{\mathrm{T}} w_1,\ \dot{\hat{\phi}}_3 = -\gamma_3 S^{\mathrm{T}} w_2, \tag{18}$$

控制器ν满足如下的方程:

$$\begin{aligned}&P + \hat{\phi}_1\nu - \hat{\phi}_2 w_1 - \hat{\phi}_3 w_2 - Z_d^{(3)} - \\ &\dot{\alpha}_2 + m_2(E_3 + \alpha_2 - \dot{\alpha}_1) + m_1(E_2 + \alpha_1) = \\ &-k_1 S - (k_2 + v_3)\mathrm{sgn}(S).\end{aligned} \tag{19}$$

其中: $v_3 = f_d\delta_2$, $E_1 = Z_1 - Z_d$, $E_2 = Z_2 - \dot{Z}_d - \alpha_1$, $E_3 = Z_3 - \ddot{Z}_d - \alpha_2$为输出误差变量; $\alpha_1 = -c_1E_1$, $\alpha_2 = -E_1 - c_2E_2 + \dot{\alpha}_1$为虚拟控制; $S = m_1E_1 + m_2E_2 + E_3$ 为切换函数; $c_i = \mathrm{diag}\{c_i\}(i = 1, 2)$为正定矩阵, λ 为大于0的设计常数, $\varGamma = \mathrm{diag}\{\gamma_2, \gamma_3\}$为正定设计矩阵, $\hat{\phi}_i(i = 1, 2, 3)$为$\phi_i(i = 1, 2, 3)$的估计值.

将得到的控制ν代入式(16), 即得系统所需的控制输入u.

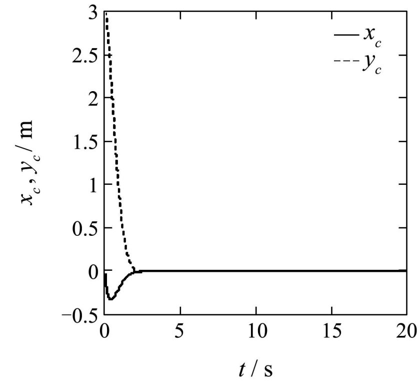
4.3　仿真研究(Simulation study)

为了验证上面所设计的鲁棒自适应控制器的有效性, 进行了仿真实验. 在仿真中, 假定移动机械臂系统的期望轨迹为

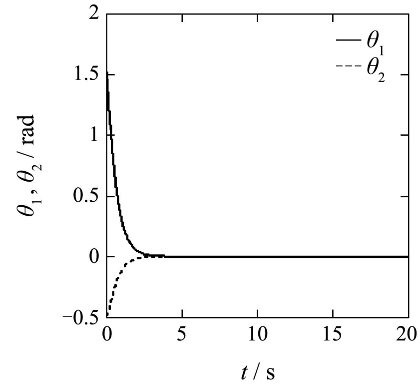
$$\begin{aligned}&Z_d = [z_{1d} \ \ z_{2d} \ \ z_{3d} \ \ z_{4d}]^{\mathrm{T}} = \\ &[2\sin(t) \ \ -3\cos(t) \ \ \sin(2t) \ \ \cos(2t)]^{\mathrm{T}}.\end{aligned} \tag{20}$$

使移动平台的中心位置跟踪椭圆运动, 二连杆角位移跟踪正弦运动.

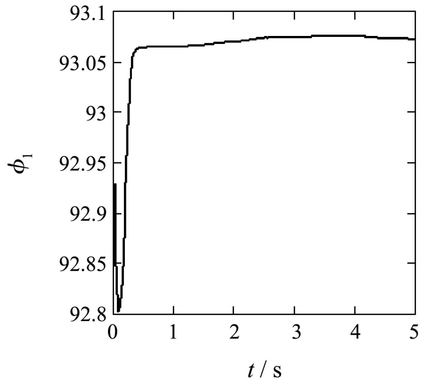
在仿真过程中, 移动机械臂相关参数为: $m_0 = 50\,\mathrm{kg}, m_1 = 4\,\mathrm{kg}, m_2 = 3.5\,\mathrm{kg}$, $R = 0.3\,\mathrm{m}$, $d = 0.3\,\mathrm{m}$, $J_0 = 1.417\,\mathrm{kgm^2}$, $J_1 = 0.03\,\mathrm{kgm^2}$, $J_2 = 0.036\,\mathrm{kgm^2}$, $l = 0.1\,\mathrm{m}$, $L_{BC} = 0.5\,\mathrm{m}, L_{AB} = 0.35\,\mathrm{m}$; 初始条件为$q_0 = [0 \ \ 0 \ \ 0 \ \ \pi/2 \ \ \pi/6]$. 为简化仿真, 4台驱动电机均采用80 W的直流伺服电机, 有关的电机参数为: $L = 2.03\,\mathrm{mH}, r_1 = 5.41\,\Omega, k_e = 0.02\,\mathrm{V/(rad \cdot s)}, k_T = 0.191\,\mathrm{N \cdot m/A}$, 齿轮减速比分别为$\beta_1 = \beta_2 = 71, \beta_3 = \beta_4 = 51$. 加随机扰动$d = [\delta_1 \ \ \delta_2 \ \ \delta_3 \ \ \delta_4]^{\mathrm{T}}$, $\delta_1, \delta_2 \in [-5 \ \ 5]$, $\delta_3, \delta_4 \in [-1 \ \ 1]$. 设计参数的选取: $c_1 = c_2 = I$, $m_1 = \mathrm{diag}\{1, 1, 3, 3\}$, $m_2 = \mathrm{diag}\{2, 2, 6, 6\}$, $k_1 = \mathrm{diag}\{50, 50, 150, 150\}$, $k_2 = \mathrm{diag}\{10, 10, 30, 30\}$, $\lambda = \gamma_2 = \gamma_3 = 1$. 仿真结果如图2所示.



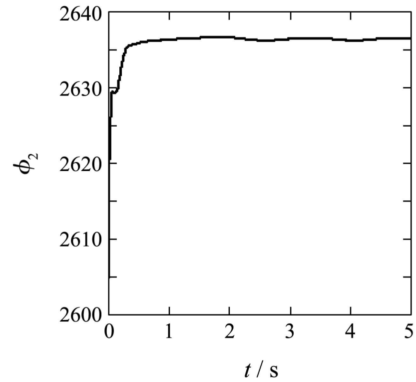
(a) 移动平台的跟踪误差曲线



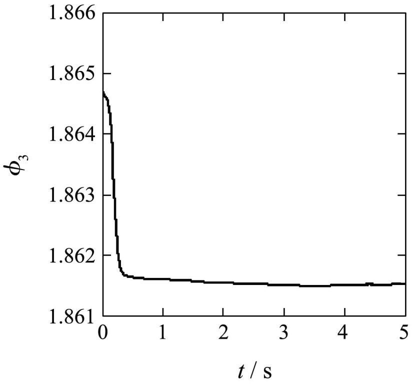
(b) 连杆的跟踪误差曲线



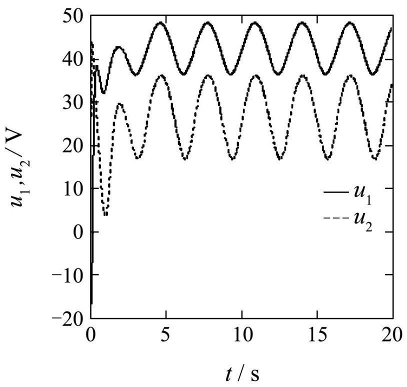
(c) 参数ϕ_1的估计



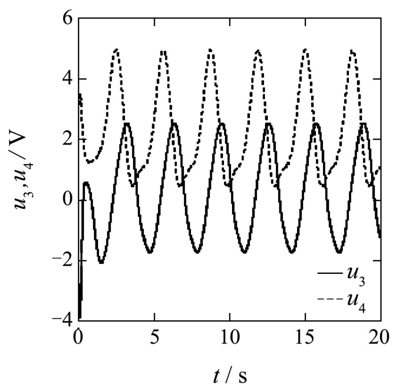
(d) 参数ϕ_2的估计



(e) 参数ϕ_3的估计



(f) 移动平台驱动电机控制电压



(g) 连杆驱动电机控制电压

图 2 仿真曲线

Fig. 2 Simulation curve

从仿真结果可以看出, 该控制器成功地实现了非完整移动机械臂对给定轨迹的跟踪. 即使在初始误差很大的情况下, 跟踪效果也很好；该控制器对模型中的未知参数具有良好的自适应性, 对外界随机干扰也具有很强的鲁棒性.

5 结论(Conclusion)

本文针对一类包含了3种不确定性的MIMO非线性系统的输出跟踪问题, 基于自适应反步法和滑模控制为其设计了鲁棒自适应控制器, 并给出了详细的推导过程. 3种不确定性表现为: 1) 参数不确定性; 2) 输入增益的不确定性; 3) 代表系统未建模动态和干扰的不确定函数, 该函数有界. 最后, 以非完整移动机械臂的输出跟踪控制为目标, 对其进行仿真实验, 实验结果表明所提出的控制算法是正确有效的.

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