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相关观测融合Kalman估值器及其全局最优性

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摘要: 对于带相关观测噪声和带不同观测阵的多传感器线性离散时变随机控制系统, 用加权最小二乘法(WLS)提出了两种加权观测融合Kalman估值器, 它们包括状态滤波、状态预报和状态平滑. 基于信息滤波器形式下的Kalman滤波器, 证明了在相同初值下, 它们在数值上恒等于相应的集中式观测融合Kalman估值器, 因而具有全局最优性. 但是它们可明显减轻计算负担. 数值仿真例子验证了它们在功能上等价于集中式观测融合Kalman估值器.

关键词: 多传感器信息融合; 加权观测融合; 相关观测噪声; Kalman滤波器; 全局最优性

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Correlated measurement fusion Kalman estimators and their global optimality

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Abstract: For the multi-sensor linear discrete time-varying stochastic control systems with correlated measurement noises and different measurement matrices, two weighted measurement fusion Kalman estimators are developed by using the weighted least squares (WLS) method. They include the state filtering, state prediction and state smoothing. Based on the Kalman filter in the information filter form, it is proved that under the same initial values, they are numerically identical to the corresponding centralized measurement fusion Kalman estimators, so that they have the global optimality. However, they can obviously reduce the computational burden. A numerical simulation example verifies their functional equivalence to the centralized measurement fusion Kalman estimator.

Key words: multi-sensor information fusion; weighted measurement fusion; correlated measurement noises; Kalman filter; global optimality

1 引言(Introduction)

由于多传感器信息融合技术在军事、国防、目标跟踪、GPS定位、机器人、信号处理、通信、控制等领域的广泛应用, 目前它已成为备受人们关注的热门领域. 观测融合Kalman滤波是一类重要的融合估计方法[1~3]. 它包括集中式和分布式观测融合, 集中式观测融合方法通过简单地合并所有观测方程为一个增广的观测方程, 然后与状态方程联立, 应用一个单个Kalman滤波器得到全局最优状态估计. 分布式观测融合方法直接加权局部观测得到一个加权融合观测方程, 然后与状态方程联立应用一个单一Kalman滤波器得到最终融合状态估计. 在一定的条件下, 用加权观测融合方法可得到全局最优状态估计[1~3], 文献[3]在不相关观测噪声假设下, 证明了两种加权观测融合Kalman估值器在功能上完全等价于集中式观测融合Kalman估值器, 因而具有全局最优性. 然而在许多应用中观测噪声是相关的, 例如各传感器有公共的背景噪声(干扰噪声)[4]. 文献[5]在各传感器具有相同观测阵的假设下用Lagrange乘数法提出了带相关观测噪声的一种加权观测融合Kalman滤波器和预报器, 并证明了它们的全局最优性. 通常, 用集中观测融合方法得到的增广观测方程的观测向量维数远大于用加权观测融合方法得到的融合观测方程的观测向量维数, 因而采用加权观测融合方法可明显减少计算负担.

本文对带相关观测噪声和带不同观测阵的多传感器系统, 用加权最小二乘(WLS)法[6]提出了两种加权观测融合Kalman估值器, 并用信息滤波器证明了

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它们完全功能等价于集中式观测融合Kalman估值器, 因而具有全局最优性.

2 两种加权观测融合方法(Two weighted measurement fusion methods)

考虑带相关观测噪声和不同观测阵的多传感器线性离散时变随机控制系统

$$x(t+1)=\varPhi(t)x(t)+B(t)u(t)+\varGamma(t)w(t), \quad (1)$$

$$y_i(t) = H_i(t)x(t) + v_i(t), i = 1, \cdots, L, \quad (2)$$

$$s(t) = Cx(t), \quad (3)$$

其中: t为离散时间, $x(t) \in \mathbb{R}^n$为状态, $u(t) \in \mathbb{R}^p$为已知控制, $y_i(t) \in \mathbb{R}^{m_i}$为第$i$个传感器的观测, $v_i(t) \in \mathbb{R}^{m_i}$为观测噪声, $w(t) \in \mathbb{R}^r$为输入白噪声, $s(t) \in \mathbb{R}^q$为有关信号, $\varPhi(t), B(t), \varGamma(t)$和$H_i(t)$是已知的适当维数时变矩阵, C为常阵.

假设 1　$w(t)$和$v_i(t)(i = 1, \cdots, L)$为零均值、方差阵各为$Q(t)$和$R_{ii}(t)$的不相关白噪声, 而$v_i(t)$和$v_j(t)$是相关阵为$R_{ij}(t)$的相关观测噪声,

$$\mathrm{E}\left\{\begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(k), v_j^{\mathrm{T}}(k) \end{bmatrix}\right\}=\begin{bmatrix} Q(t) & 0 \\ 0 & R_{ij}(t) \end{bmatrix}\delta_{tk}, \quad (4)$$

其中: E为数学期望, T为转置号, $\delta_{tt} = 1, \delta_{tk} = 0(t \neq k)$.

假设 2　各传感器带不同观测阵$H_i(t)$, 它们有公共的右因子$m \times n$ 矩阵$H^{(\mathrm{I})}(t)$, 即

$$H_i(t) = M_i(t)H^{(\mathrm{I})}(t), \quad (5)$$

且矩阵$M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)M^{(0)}(t)$是可逆的, 或矩阵$H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t)$是可逆的, 其中定义

$$M^{(0)}(t) = [M_1^{\mathrm{T}}(t), \cdots, M_L^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$H^{(0)}(t) = [H_1^{\mathrm{T}}(t), \cdots, H_L^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$R^{(0)}(t) = \begin{bmatrix} R_{11}(t) & \cdots & R_{1L}(t) \\ \vdots & & \vdots \\ R_{L1}(t) & \cdots & R_{LL}(t) \end{bmatrix}, \quad (6)$$

且定义$R^{(0)-1}(t) = (R^{(0)}(t))^{-1}$.

2.1 集中式观测融合方法(Centralized measurement fusion method)

用增广观测向量方法有集中式观测融合方程

$$y^{(0)}(t) = H^{(0)}(t)x(t) + v^{(0)}(t), \quad (7)$$

$$y^{(0)}(t) = [y_1^{\mathrm{T}}(t), \cdots, y_L^{\mathrm{T}}(t)]^{\mathrm{T}}, \quad (8)$$

$$v^{(0)}(t) = [v_1^{\mathrm{T}}(t), \cdots, v_L^{\mathrm{T}}(t)]^{\mathrm{T}}, \quad (9)$$

其中$H^{(0)}(t)$和白噪声$v^{(0)}(t)$的方差阵$R^{(0)}(t)$由式(6)给出. 对系统式(1)和式(7)应用标准Kalman滤波算法[6]可得全局最优Kalman滤波器$\hat{x}^{(0)}(t|t)$和预报器$\hat{x}^{(0)}(t+1|t)$及相应误差方差阵$P^{(0)}(t|t)$ 和$P^{(0)}(t+1|t)$.

2.2 加权观测融合方法(I)(Weighted measurement fusion method(I))

由式(5)有集中式观测方程

$$y^{(0)}(t) = M^{(0)}(t)H^{(\mathrm{I})}(t)x(t) + v^{(0)}(t). \quad (10)$$

可视式(10)为对$H^{(\mathrm{I})}(t)x(t)$的观测模型, 于是应用加权最小二乘(WLS)法可得$H^{(\mathrm{I})}(t)x(t)$ 的WLS估值(即Gauss-Markow估值)为[6]

$$y^{(\mathrm{I})}(t) = (M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)M^{(0)}(t))^{-1} \times M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)y^{(0)}(t). \quad (11)$$

可视估值$y^{(\mathrm{I})}(t)$为对$H^{(\mathrm{I})}(t)x(t)$的观测, 由式(11)可知$y^{(\mathrm{I})}(t)$是 $y_1(t), \cdots, y_L(t)$的加权和, 将式(10)代入式(11), 则有加权融合观测方程

$$y^{(\mathrm{I})}(t) = H^{(\mathrm{I})}(t)x(t) + v^{(\mathrm{I})}(t), \quad (12)$$

其中观测白噪声$v^{(\mathrm{I})}(t)$及其方差阵$R^{(\mathrm{I})}(t)$各为

$$v^{(\mathrm{I})}(t) = (M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)M^{(0)}(t))^{-1} \times M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)v^{(0)}(t), \quad (13)$$

$$R^{(\mathrm{I})}(t) = (M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)M^{(0)}(t))^{-1}. \quad (14)$$

对系统式(1)和式(12)应用标准Kalman滤波算法可得到加权观测融合Kalman滤波器 $\hat{x}^{(\mathrm{I})}(t|t)$和预报器 $\hat{x}^{(\mathrm{I})}(t+1|t)$ 及相应的误差方差阵 $P^{(\mathrm{I})}(t|t)$ 和$P^{(\mathrm{I})}(t+1|t)$.

2.3 加权观测融合方法(II)(Weighted measurement fusion method(II))

式(7)可看成是对$x(t)$的观测模型, 于是可得$x(t)$的WLS估值

$$y^{(\mathrm{II})}(t) = (H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t))^{-1} \times H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)y^{(0)}(t). \quad (15)$$

将式(7)代入式(15)引出加权融合观测方程

$$\begin{cases} y^{(\mathrm{II})}(t) = H^{(\mathrm{II})}(t)x(t) + v^{(\mathrm{II})}(t), \\ H^{(\mathrm{II})}(t) = I_n, \end{cases} \quad (16)$$

其中观测白噪声$v^{(\mathrm{II})}(t)$及其方差阵$R^{(\mathrm{II})}(t)$各为

$$v^{(\mathrm{II})}(t) = (H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t))^{-1} \times H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)v^{(0)}(t), \quad (17)$$

$$R^{(\mathrm{II})}(t) = (H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t))^{-1}. \quad (18)$$

对系统式(1)和式(16)应用标准Kalman滤波算法可得加权观测融合Kalman滤波器$\hat{x}^{(\mathrm{II})}(t|t)$和

预报器$\widehat{x}^{(\mathrm{II})}(t+1|t)$及相应的误差方差阵分别为$P^{(\mathrm{II})}(t|t)$和$P^{(\mathrm{II})}(t+1|t)$.

3 两种加权观测融合方法的部分功能等价性(Partially functional equivalence of the measurement fusion methods)

融合观测方程式(7)(12)和式(16)有统一的形式

$$y^{(i)}(t)=H^{(i)}(t)x(t)+v^{(i)}(t),\ i=0,\mathrm{I},\mathrm{II}. \tag{19}$$

定义方差阵$P^{(i)}(t_1|t_2)$的逆矩阵$P^{(i)-1}(t_1|t_2)$为信息矩阵, 定义信息滤波器和预报器为$\widehat{z}^{(i)}(t_1|t_2)=P^{(i)-1}(t_1|t_2)\widehat{x}^{(i)}(t_1|t_2)$, 则有在信息滤波器形式下的Kalman滤波器和预报器[1,6]:

$$\begin{aligned}&\widehat{z}^{(i)}(t|t-1)=\\&P^{(i)-1}(t|t-1)\Phi(t-1)\times\\&P^{(i)}(t-1|t-1)\widehat{z}^{(i)}(t-1|t-1)+\\&P^{(i)-1}(t|t-1)B(t-1)u(t-1),\end{aligned}\tag{20}$$

$$\begin{aligned}&\widehat{z}^{(i)}(t|t)=\\&\widehat{z}^{(i)}(t|t-1)+H^{(i)\mathrm{T}}(t)R^{(i)-1}(t)y^{(i)}(t),\end{aligned}\tag{21}$$

$$\begin{aligned}&P^{(i)}(t|t-1)=\\&\Phi(t-1)P^{(i)}(t-1|t-1)\Phi^{\mathrm{T}}(t-1)+\\&\Gamma(t-1)Q(t-1)\Gamma^{\mathrm{T}}(t-1),\end{aligned}\tag{22}$$

$$\begin{aligned}&P^{(i)-1}(t|t)=\\&P^{(i)-1}(t|t-1)+H^{(i)\mathrm{T}}(t)R^{(i)-1}(t)H^{(i)}(t)\end{aligned}\tag{23}$$

带初值$\widehat{x}^{(i)}(0|0)$和$P^{(i)}(0|0)$.

定理 1 (观测融合方法的部分功能等价性) 对带相关观测噪声和不同观测阵的多传感器系统式(1)~式(3)在假设1和假设2下, 两种加权观测融合Kalman滤波器和预报器是全局最优的, 它们分别在数值上是恒同于相应的集中式观测融合Kalman滤波器和预报器, 即

$$\widehat{x}^{(0)}(t|t)=\widehat{x}^{(\mathrm{I})}(t|t)=\widehat{x}^{(\mathrm{II})}(t|t),\forall t, \tag{24}$$

$$\widehat{x}^{(0)}(t+1|t)=\widehat{x}^{(\mathrm{I})}(t+1|t)=\widehat{x}^{(\mathrm{II})}(t+1|t),\ \forall t, \tag{25}$$

$$P^{(0)}(t|t)=P^{(\mathrm{I})}(t|t)=P^{(\mathrm{II})}(t|t), \tag{26}$$

$$P^{(0)}(t+1|t)=P^{(\mathrm{I})}(t+1|t)=P^{(\mathrm{II})}(t+1|t), \tag{27}$$

只要它们有相同的初值

$$\begin{aligned}&\widehat{x}^{(0)}(0|0)=\widehat{x}^{(\mathrm{I})}(0|0)=\widehat{x}^{(\mathrm{II})}(0|0),\\&P^{(0)}(0|0)=P^{(\mathrm{I})}(0|0)=P^{(\mathrm{II})}(0|0).\end{aligned}\tag{28}$$

证 由式(20)~(23)看到, 在相同初值式(28)下, 只要对$i=0,\mathrm{I},\mathrm{II}$, $H^{(i)\mathrm{T}}(t)R^{(i)-1}(t)H^{(i)}(t)$三者恒同, 且$H^{(i)\mathrm{T}}(t)R^{(i)-1}(t)y^{(i)}(t)$三者恒同, 就有式(24)~式(27)成立. 事实上由式(5)和式(6)有$H^{(0)}(t)=M^{(0)}(t)H^{(\mathrm{I})}(t)$, 因而由式(14)(16)和式(18)有

$$\begin{aligned}&H^{(\mathrm{I})\mathrm{T}}(t)R^{(\mathrm{I})-1}(t)H^{(\mathrm{I})}(t)=\\&H^{(\mathrm{I})\mathrm{T}}(t)M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)M^{(0)}(t)H^{(\mathrm{I})}(t)=\\&H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t),\end{aligned}\tag{29}$$

$$\begin{aligned}&H^{(\mathrm{II})\mathrm{T}}(t)R^{(\mathrm{II})-1}(t)H^{(\mathrm{II})}(t)=\\&I_nH^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t)I_n=\\&H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t).\end{aligned}\tag{30}$$

又应用式(11)(14)(15)和式(18)有

$$\begin{aligned}&H^{(\mathrm{I})\mathrm{T}}(t)R^{(\mathrm{I})-1}(t)y^{(\mathrm{I})}(t)=\\&H^{(\mathrm{I})\mathrm{T}}(t)M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)M^{(0)}(t)\times\\&(M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)M^{(0)}(t))^{-1}\times\\&M^{(0)\mathrm{T}}(t)R^{(0)-1}(t)y^{(0)}(t)=\\&H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)y^{(0)}(t),\end{aligned}\tag{31}$$

$$\begin{aligned}&H^{(\mathrm{II})\mathrm{T}}(t)R^{(\mathrm{II})-1}(t)y^{(\mathrm{II})}(t)=\\&I_nH^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t)\times\\&(H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)H^{(0)}(t))^{-1}\times\\&H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)y^{(0)}(t)=\\&H^{(0)\mathrm{T}}(t)R^{(0)-1}(t)y^{(0)}(t).\end{aligned}\tag{32}$$

故由式(20)~(23)有式(24)~(27)成立. 因为集中式观测融合器是全局最优的, 故上述两种加权观测融合器也是全局最优的. 证毕.

4 观测融合方法的完全功能等价性(Completely functional equivalence of the measurement fusion methods)

式(1)和式(19)的Kalman平滑器[7]为

$$\begin{aligned}&\widehat{x}^{(i)}(t|t+N)=\\&\widehat{x}^{(i)}(t|t)+\sum_{k=1}^{N}K_x^{(i)}(t|t+k)\varepsilon^{(i)}(t+k),\end{aligned}\tag{33}$$

其中$i=0,\mathrm{I},\mathrm{II}$, $N>0$, 且新息过程$\varepsilon^{(i)}(t)$为

$$\varepsilon^{(i)}(t)=y^{(i)}(t)-H^{(i)}(t)\widehat{x}^{(i)}(t|t-1), \tag{34}$$

平滑增益为

$$\begin{aligned}&K_x^{(i)}(t|t+k)=\\&P^{(i)}(t|t-1)\{\prod_{j=0}^{k-1}\Psi_p^{(i)\mathrm{T}}(t+j)\}\times\\&H^{(i)\mathrm{T}}(t+k)Q_\varepsilon^{(i)-1}(t+k),\end{aligned}\tag{35}$$

$$\Psi_p^{(i)}(t)=\Phi(t)[I_n-K_f^{(i)}(t)H^{(i)}(t)], \tag{36}$$

$$K_f^{(i)}(t)=P^{(i)}(t|t)H^{(i)\mathrm{T}}(t)R^{(i)-1}(t), \tag{37}$$

$$Q_\varepsilon^{(i)}(t)=H^{(i)}(t)P^{(i)}(t|t-1)H^{(i)\mathrm{T}}(t)+R^{(i)}(t). \tag{38}$$

平滑误差方差阵为

$$P^{(i)}(t|t+N)= P^{(i)}(t|t)-\sum_{k=1}^{N}K_x^{(i)}(t|t+k)Q_\varepsilon^{(i)}(t+k)K_x^{(i)\mathrm{T}}(t|t+k). \tag{39}$$

输入白噪声估值器为[7]

$$\widehat{w}^{(i)}(t|t+N)=\sum_{k=0}^{N}K_w^{(i)}(t|t+k)\varepsilon^{(i)}(t+k), \tag{40}$$

$$\begin{aligned}&K_w^{(i)}(t|t)=0,\\&K_w^{(i)}(t|t+1)=\\&Q(t)\varGamma^{\mathrm{T}}(t)H^{(i)\mathrm{T}}(t+1)\times Q_\varepsilon^{(i)-1}(t+1),\\&K_w^{(i)}(t|t+k)=\\&Q(t)\varGamma^{\mathrm{T}}(t)\{\prod_{j=0}^{k-1}\varPsi_p^{(i)\mathrm{T}}(t+j)\}\times\\&H^{(i)\mathrm{T}}(t+k)Q_\varepsilon^{(i)-1}(t+k).\end{aligned} \tag{41}$$

相应的误差方差阵为

$$P_w^{(i)}(t|t+N)=\begin{cases}Q(t)-\sum_{k=0}^{N}K_w^{(i)}(t|t+k)Q_\varepsilon^{(i)}(t+k)K_w^{(i)\mathrm{T}}(t|t+k),\ N\geqslant 0,\\ P_w^{(i)}(t|t+N)=Q(t),\qquad N<0.\end{cases} \tag{42}$$

由式(3)有信号估值器及其误差方差阵为

$$\widehat{s}^{(i)}(t|t+N)=C\widehat{x}^{(i)}(t|t+N), \tag{43}$$

$$P_s^{(i)}(t|t+N)=CP^{(i)}(t|t+N)C^{\mathrm{T}}. \tag{44}$$

定理 2(观测融合方法的完全功能等价性)　对带相关观测噪声和带不同观测阵的多传感器系统在假设1和假设2下, 两种加权观测融合Kalman估值器完全功能等价于集中式观测融合Kalman估值器. 即在相同初值条件下, 它们分别在数值上恒同于相应的集中式观测融合Kalman估值器(状态、白噪声、信号估值器和滤波、预报和平滑估值器), 因而它们具有全局最优性.

证　部分功能等价性见定理1.

由式(26)(29)(30)(36)和式(37)引出

$$\varPsi_p^{(\mathrm{I})}(t)=\varPsi_p^{(\mathrm{II})}(t)=\varPsi_p^{(0)}(t). \tag{45}$$

应用标准Kalman滤波[6]有

$$K_f^{(i)}(t)=P^{(i)}(t|t-1)H^{(i)\mathrm{T}}(t)Q_\varepsilon^{(i)-1}(t). \tag{46}$$

于是有

$$H^{(i)\mathrm{T}}(t)Q_\varepsilon^{(i)-1}(t)\varepsilon^{(i)}(t)= P^{(i)-1}(t|t-1)K_f^{(i)}(t)\varepsilon^{(i)}(t). \tag{47}$$

应用(29)～(32)(34)和式(37)及定理1易知

$$K_f^{(\mathrm{I})}(t)\varepsilon^{(\mathrm{I})}(t)=K_f^{(\mathrm{II})}(t)\varepsilon^{(\mathrm{II})}(t)=K_f^{(0)}(t)\varepsilon^{(0)}(t). \tag{48}$$

进而由式(47)和式(48)有

$$\begin{aligned}&H^{(\mathrm{I})\mathrm{T}}(t)Q_\varepsilon^{(\mathrm{I})-1}(t)\varepsilon^{(\mathrm{I})}(t)=\\&H^{(\mathrm{II})\mathrm{T}}(t)Q_\varepsilon^{(\mathrm{II})-1}(t)\varepsilon^{(\mathrm{II})}(t)=\\&H^{(0)\mathrm{T}}(t)Q_\varepsilon^{(0)-1}(t)\varepsilon^{(0)}(t).\end{aligned} \tag{49}$$

于是应用定理1, 式(45)和式(49)引出$\widehat{x}^{(i)}(t|t+N)$, $\widehat{w}^{(i)}(t|t+N)$和$\widehat{s}^{(i)}(t|t+N)(i=0,\mathrm{I},\mathrm{II})$彼此相等, 且$P^{(i)}(t|t+N)$, $P_w^{(i)}(t|t+N)$和$P_s^{(i)}(t|t+N)(i=0,\mathrm{I},\mathrm{II})$彼此相等.　　证毕.

定理 3　对带相关观测噪声和不同观测阵的定常系统式(1)～式(3), 在假设1和假设2下, 其中$\varPhi(t)=\varPhi$, $B(t)=B$, $\varGamma(t)=\varGamma$, $H_i(t)=H_i$, $C(t)=C$, $R_{ij}(t)=R_{ij}$, $Q(t)=Q$, $H_i(t)=M_iH^{(\mathrm{I})}$, 假设观测融合系统是完全可观、完全可控的, 则用两种加权观测融合方法在相同初值下所得到的稳态Kalman估值器在数值上分别恒同于相应的集中式观测融合稳态Kalman估值器, 因而具有完全功能等价性和渐近全局最优性.

证　用稳态信息滤波器平行于定理1和定理2的推导可得定理3. 因集中式观测融合稳态Kalman估值器同集中式观测融合时变Kalman估值器相比是次优的, 且是渐近全局最优的[8], 故加权观测融合稳态Kalman估值器也是具有渐近全局最优性.

证毕.

5　数值仿真例子(Numerical simulation examples)

考虑带相关观测噪声的3传感器跟踪系统式(1)和式(2), 其中:

$$v_i(t)=\xi(t)+e_i(t),\ i=1,2,3 \tag{50}$$

$$\varPhi=\begin{bmatrix}1 & T_0\\0 & 1\end{bmatrix},\ \varGamma=\begin{bmatrix}0.5T_0^2\\T_0\end{bmatrix},\ H_1=H_2=\begin{bmatrix}1 & 0\end{bmatrix},\ H_3=\begin{bmatrix}0 & 1\end{bmatrix}. \tag{51}$$

其中: $T_0=0.5$为采样周期, $x(t)=[x_1(t),x_2(t)]^{\mathrm{T}}$, $\xi(t)$和$e_i(t)$是零均值, 方差分别为$Q_w=0.81$和$\sigma_\xi^2=0.64,\sigma_{e_1}^2=1,\sigma_{e_2}^2=2.25,\sigma_{e_3}^2=4$的独立的高斯白噪声, 目的是用本文中提到的3种观测融合方法求局部和融合Kalman预报器, 其MATLAB真结果如表1所示.

表 1 3种观测融合方法的功能等价性

Table 1 Functional equivalence between three measurement fusion methods

t	$\widehat{x}^{(0)}(t+1\|t)$	$\widehat{x}^{(\mathrm{I})}(t+1\|t)$	$\widehat{x}^{(\mathrm{II})}(t+1\|t)$
10	$\begin{bmatrix}-1.5028\\-1.0366\end{bmatrix}$	$\begin{bmatrix}-1.5028\\-1.0366\end{bmatrix}$	$\begin{bmatrix}-1.5028\\-1.0366\end{bmatrix}$
15	$\begin{bmatrix}-3.6390\\-1.2996\end{bmatrix}$	$\begin{bmatrix}-3.6390\\-1.2996\end{bmatrix}$	$\begin{bmatrix}-3.6390\\-1.2996\end{bmatrix}$
20	$\begin{bmatrix}-7.2722\\-1.0972\end{bmatrix}$	$\begin{bmatrix}-7.2722\\-1.0972\end{bmatrix}$	$\begin{bmatrix}-7.2722\\-1.0972\end{bmatrix}$
t	$P^{(0)}(t+1\|t)$	$P^{(\mathrm{I})}(t+1\|t)$	$P^{(\mathrm{II})}(t+1\|t)$
10	$\begin{bmatrix}1.0877 & 0.6504\\0.6504 & 0.6967\end{bmatrix}$	$\begin{bmatrix}1.0877 & 0.6504\\0.6504 & 0.6967\end{bmatrix}$	$\begin{bmatrix}1.0877 & 0.6504\\0.6504 & 0.6967\end{bmatrix}$
15	$\begin{bmatrix}1.0887 & 0.6529\\0.6529 & 0.6993\end{bmatrix}$	$\begin{bmatrix}1.0887 & 0.6529\\0.6529 & 0.6993\end{bmatrix}$	$\begin{bmatrix}1.0887 & 0.6529\\0.6529 & 0.6993\end{bmatrix}$
20	$\begin{bmatrix}1.0888 & 0.6529\\0.6529 & 0.6992\end{bmatrix}$	$\begin{bmatrix}1.0888 & 0.6529\\0.6529 & 0.6992\end{bmatrix}$	$\begin{bmatrix}1.0888 & 0.6529\\0.6529 & 0.6992\end{bmatrix}$

6 结论(Conclusion)

本文对带相关观测噪声和带不同观测阵的多传感器系统, 用加权最小二乘法(WLS)提出两种观测融合Kalman估值器, 用信息滤波器证明了它们是完全功能等价于集中式观测融合Kalman估值器, 因而它们是全局最优的, 且可减少计算负担. 它们包括文献[1~3,5]的结果作为特殊情形.

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