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基于观测器的切换模糊时滞系统的反馈控制

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摘要: 针对状态不可测的切换模糊时滞系统, 根据平行分布补偿算法(PDC), 设计了切换模糊观测器和反馈控制器, 应用共同Lyapunov函数方法使观测误差系统在任意切换下渐近稳定, 应用多Lyapunov函数方法, 使系统状态在设计切换律下渐近稳定, 并给出了时滞相关的切换模糊系统渐近稳定的矩阵不等式条件. 仿真结果表明结论的有效性.

关键词: 切换系统; 模糊控制; 时滞相关; 观测器; Lyapunov函数

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Observer-based feedback control for switching fuzzy time-delay systems

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Abstract: The stabilization problem is addressed for a class of switching fuzzy time-delay systems. For the non-measurable states, a switching fuzzy observer and a feedback controller are designed based on the parallel distributed compensation(PDC) method. Asymptotic stability of the observed error system is then obtained by the common-Lyapunov-function method. A switching law is also designed by the multiple-Lyapunov-function method to guarantee the asymptotic stability of the systems. Delay-dependent conditions for asymptotic stability of the switching fuzzy systems are obtained in terms of matrix inequalities. Finally, simulation results show effectiveness of the design method.

Key words: switching system; fuzzy control; delay dependent; observer; Lyapunov functions

1 引言(Introduction)

近年来, 基于T-S模型的模糊系统[1,2]和切换系统[3~6]的研究受到了广泛重视. 如果一个系统用模糊方法建模, 即描述为一个模糊系统, 同时系统中又含有离散动态, 如离散切换信号, 则这类系统就成为切换模糊系统. 关于切换模糊系统的研究成果, 文献[7]提出了切换Lyapunov函数方法, 并给出控制器设计和稳定条件, 文献[8]利用单Lyapunov函数和多Lyapunov函数方法, 设计切换模糊状态反馈控制器和切换律使系统镇定, 但并未考虑系统存在时滞情况下的稳定控制问题.

和一般切换系统的研究情形一样, 各个模糊时滞子系统都不稳定时, 仍有可能通过设计适当的切换律而获得稳定性, 这是本文研究的一个出发点. 本文考虑一类状态不可观测的切换模糊时滞系统, 利用共同Lyapunov函数和多Lyapunov函数方法, 给出系统的稳定条件和切换律设计. 最后通过仿真例子验证了结论的有效性.

2 问题描述(Problem statement)

考虑子系统全是模糊时滞系统的切换系统:

$$\begin{cases} \dot{x}(t)=\sum\limits_{\sigma=1}^{l}\sum\limits_{i=1}^{N_\sigma}\nu_\sigma(\hat{x}(t))\mu_{\sigma i}(z(t))[A_{\sigma i}x(t)+ \\ \qquad A_{h\sigma i}x(t-d_\sigma(t))+B_{\sigma i}u_\sigma(t)], \\ y(t)=\sum\limits_{\sigma=1}^{l}\sum\limits_{i=1}^{N_\sigma}\nu_\sigma(\hat{x}(t))\mu_{\sigma i}(z(t))C_{\sigma i}x(t), \\ x(t)=\Psi(t),\ t\in[-\tau,0],\ 0\leqslant d_\sigma(t)\leqslant\tau, \\ i=1,2,\cdots,N_\sigma. \end{cases} \tag{1}$$

其中: $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^\vartheta$, $u_\sigma(t)\in\mathbb{R}^m$; $A_{\sigma i}$, $A_{h\sigma i}$, $B_{\sigma i}$, $C_{\sigma i}$是适当维数的常数矩阵; $d_\sigma(t)$为时滞时间, 且$|\dot{d}_\sigma(t)|\leqslant d<1$; $\sigma\in M=\{1,2,\cdots,l\}$为切换信

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号, 描述为

$$\nu_\sigma(\hat{x}(t))=\begin{cases}1,\ \hat{x}(t)\in\widetilde{\Omega}_\sigma,\\0,\ \hat{x}(t)\notin\widetilde{\Omega}_\sigma,\end{cases}$$

即当$\hat{x}(t)\in\widetilde{\Omega}_\sigma$时, $\nu_\sigma(\hat{x}(t))=1$; $\mu_{\sigma i}(z(t))=\dfrac{\prod\limits_{j=1}^{p}M_{\sigma j}^{i}(z_j(t))}{\sum\limits_{i=1}^{N_\sigma}\prod\limits_{j=1}^{p}M_{\sigma j}^{i}(z_j(t))}$, $0\leqslant\mu_{\sigma i}(z(t))\leqslant 1$, $\sum\limits_{i=1}^{N_\sigma}\mu_{\sigma i}(z(t))=1$, $z(t)\in\mathbb{R}^p$是模糊前件变量, $M_{\sigma j}^{i}(z_j(t))$表示$z_j(t)$属于模糊集$M_{\sigma j}^{i}$的隶属度, $M_{\sigma j}^{i}$是模糊集合.

对于系统(1), 设计观测器和反馈控制器:

$$\begin{aligned}\dot{\hat{x}}(t)=&\sum_{\sigma=1}^{l}\sum_{i=1}^{N_\sigma}\sum_{j=1}^{N_\sigma}\nu_\sigma(\hat{x}(t))\mu_{\sigma i}(z(t))\mu_{\sigma j}(z(t))\times\\&\{A_{\sigma i}\hat{x}(t)+A_{h\sigma i}\hat{x}(t-d_\sigma(t))+\\&B_{\sigma i}u_\sigma(t)+L_{\sigma i}[C_{\sigma j}x(t)-C_{\sigma j}\hat{x}(t)]\},\end{aligned}\tag{2}$$

$$u_\sigma(t)=-\sum_{i=1}^{N_\sigma}\mu_{\sigma i}(z(t))K_{\sigma i}\hat{x}(t),\tag{3}$$

其中$L_{\sigma i}$和$K_{\sigma i}$为观测器和控制器增益矩阵.

定义观测误差为$e(t)=x(t)-\hat{x}(t)$.

3　主要结果(Main results)

本节讨论切换模糊时滞系统的稳定控制和切换律设计.

定理 1　假设存在同时非负或同时非正实数$\beta_{\sigma\lambda}(\sigma,\lambda\in M,\sigma\neq\lambda)$、对称正定矩阵$P_\sigma,R_\sigma,Q_\sigma,P_e,W,S$和适当维数矩阵$Y_{\sigma j},F_{\sigma i}(i,j=1,2,\cdots,N_\sigma)$, 使得矩阵不等式

$$\begin{bmatrix}\theta_{11}+\sum\limits_{\lambda=1,\lambda\neq\sigma}^{l}\beta_{\sigma\lambda}(P_\lambda-P_\sigma) & * & * & *\\ \theta_{21} & \theta_{22} & * & *\\ \theta_{31} & \theta_{32} & \theta_{33} & *\\ 0 & 0 & \theta_{43} & \theta_{44}\end{bmatrix}<0\tag{4}$$

成立, 其中:

$$\begin{aligned}\theta_{11}=&A_{\sigma i}^{\mathrm T}P_\sigma+P_\sigma A_{\sigma i}-P_\sigma Y_{\sigma j}^{\mathrm T}B_{\sigma i}^{\mathrm T}P_\sigma-\\&P_\sigma B_{\sigma i}Y_{\sigma j}P_\sigma+(A_{\sigma i}-\\&B_{\sigma i}Y_{\sigma j}P_\sigma)^{\mathrm T}\tau R_\sigma(A_{\sigma i}-B_{\sigma i}Y_{\sigma j}P_\sigma)+Q_\sigma,\\\theta_{21}=&A_{h\sigma i}^{\mathrm T}P_\sigma+A_{h\sigma i}^{\mathrm T}\tau R_\sigma(A_{\sigma i}-B_{\sigma i}Y_{\sigma j}P_\sigma),\\\theta_{22}=&A_{h\sigma i}^{\mathrm T}\tau R_\sigma A_{h\sigma i}-(1-d)Q_\sigma,\\\theta_{31}=&(P_e^{-1}F_{\sigma i}C_{\sigma j})^{\mathrm T}P_\sigma+(P_e^{-1}F_{\sigma i}C_{\sigma j})^{\mathrm T}\tau R_\sigma\times\\&(A_{\sigma i}-B_{\sigma i}Y_{\sigma j}P_\sigma),\\\theta_{32}=&(P_e^{-1}F_{\sigma i}C_{\sigma j})^{\mathrm T}\tau R_\sigma A_{h\sigma i},\\\theta_{33}=&A_{\sigma i}^{\mathrm T}P_e+P_eA_{\sigma i}-C_{\sigma j}^{\mathrm T}F_{\sigma i}^{\mathrm T}-F_{\sigma i}C_{\sigma j}+\\&\tau(A_{\sigma i}-P_e^{-1}F_{\sigma i}C_{\sigma j})^{\mathrm T}W(A_{\sigma i}-\\&P_e^{-1}F_{\sigma i}C_{\sigma j})^{\mathrm T}+S+\\&(P_e^{-1}F_{\sigma i}C_{\sigma j})^{\mathrm T}\tau R_\sigma(P_e^{-1}F_{\sigma i}C_{\sigma j}),\\\theta_{43}=&A_{h\sigma i}^{\mathrm T}P_e+\tau A_{h\sigma i}^{\mathrm T}W(A_{\sigma i}-P_e^{-1}F_{\sigma i}C_{\sigma j}),\\\theta_{44}=&\tau A_{h\sigma i}^{\mathrm T}WA_{h\sigma i}-(1-d)S.\end{aligned}$$

则存在观测器(2)和反馈控制器(3), 其中:

$$L_{\sigma i}=P_e^{-1}F_{\sigma i},\tag{5}$$

$$K_{\sigma i}=Y_{\sigma i}P_\sigma,\tag{6}$$

及切换律$\sigma=\sigma(\hat{x}(t))$, 使得系统(1)的闭环系统在原点是渐近稳定的. $*$表示对称位置矩阵块的转置.

证　不失一般性, 假设$\beta_{\sigma\lambda}\geqslant 0$, 显然对任意$\hat{x}(t)\in\mathbb{R}^n\backslash\{0\}$, 至少存在一个$\sigma\in M$使得$\hat{x}^{\mathrm T}(t)(P_\lambda-P_\sigma)\hat{x}(t)\geqslant 0,\forall\lambda\in M$, 则由矩阵不等式(4)可知

$$\begin{bmatrix}\hat{x}(t)\\\hat{x}(t-d_\sigma(t))\\e(t)\\e(t-d_\sigma(t))\end{bmatrix}^{\mathrm T}\begin{bmatrix}\theta_{11} & * & * & *\\\theta_{21} & \theta_{22} & * & *\\\theta_{31} & \theta_{32} & \theta_{33} & *\\0 & 0 & \theta_{43} & \theta_{44}\end{bmatrix}\begin{bmatrix}\hat{x}(t)\\\hat{x}(t-d_\sigma(t))\\e(t)\\e(t-d_\sigma(t))\end{bmatrix}<0.\tag{7}$$

令$\Omega_\sigma=\{\hat{x}(t)\in\mathbb{R}^n\backslash\{0\}|\hat{x}^{\mathrm T}(t)(P_\lambda-P_\sigma)\hat{x}(t)\geqslant 0,\ \lambda\in M,\ \lambda\neq\sigma\}$, 则$\bigcup\limits_{\sigma}\Omega_\sigma=\mathbb{R}^n\setminus\{0\}$. 构造集合$\widetilde{\Omega}_1=\Omega_1,\cdots,\widetilde{\Omega}_i=\Omega_i-\bigcup\limits_{j=1}^{i-1}\widetilde{\Omega}_j,\cdots$, 显然有$\bigcup\limits_{i=1}^{l}\widetilde{\Omega}_i=\mathbb{R}^n\setminus\{0\}$, 且$\widetilde{\Omega}_i\bigcap\widetilde{\Omega}_j=\varPhi,i\neq j$. 设计切换律

$$\sigma=\sigma(\hat{x}(t)),\ \hat{x}(t)\in\widetilde{\Omega}_\sigma.\tag{8}$$

取Lyapunov函数

$$\begin{aligned}&V(t)=\\&\hat{x}^{\mathrm T}(t)P_\sigma\hat{x}(t)+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\hat{x}}^{\mathrm T}(s)R_\sigma\dot{\hat{x}}(s)\mathrm{d}s\mathrm{d}\theta+\\&\int_{t-d_\sigma(t)}^{t}\hat{x}^{\mathrm T}(s)Q_\sigma\hat{x}(s)\mathrm{d}s+\\&e^{\mathrm T}(t)P_ee(t)+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}^{\mathrm T}(s)W\dot{e}(s)\mathrm{d}s\mathrm{d}\theta+\\&\int_{t-d_\sigma(t)}^{t}e^{\mathrm T}(s)Se(s)\mathrm{d}s.\end{aligned}\tag{9}$$

由切换律设计和式(7)可以证明, 对任意的$\hat{x}(t)\neq 0$, $e(t)\neq 0$, 即$x(t)\neq 0$, 有$\dot{V}(t)<0$, 所以闭环系统在原点是渐近稳定的, 且观测误差趋于零. 由于篇幅有限, 证明过程从略.　　证毕.

4　仿真算例(Simulation example)

本节给出的仿真例子说明了系统在引入切换机制以后, 对系统的保守性带来改进. 考虑切换模糊时滞系统(1), 其中:

$$\sigma=1,2,\ i=1,2,$$

$$A_{11}=\begin{bmatrix}1.6 & -1\\ -1 & 0.1\end{bmatrix},\ A_{12}=\begin{bmatrix}1.3 & -1\\ -1 & 0.1\end{bmatrix},$$

$$A_{21}=\begin{bmatrix}0.8 & -1\\ -1 & 0.7\end{bmatrix},\ A_{22}=\begin{bmatrix}0.2 & -1\\ -1 & -0.8\end{bmatrix},$$

$$A_{h11}=\begin{bmatrix}0 & 0\\ 1.9 & 0.4\end{bmatrix},\ A_{h12}=\begin{bmatrix}0 & 0\\ 2.8 & 0.3\end{bmatrix},$$

$$A_{h21}=\begin{bmatrix}0 & 0\\ 2.6 & 0.4\end{bmatrix},\ A_{h22}=\begin{bmatrix}0 & 0\\ 1.5 & 0.1\end{bmatrix},$$

$$B_{\sigma i}=I,\ C_{\sigma i}=[1\ \ 0].$$

隶属度函数为

$$\mu_{i1}(x_1(t))=1-1/(1+\mathrm{e}^{-4x_1(t)}),$$
$$\mu_{i2}(x_1(t))=1/(1+\mathrm{e}^{-4x_1(t)}),\ i=1,2.$$

假设系统的控制器增益:

$$K_{11}=\begin{bmatrix}1.6 & -1\\ -1 & 11\end{bmatrix},\ K_{12}=\begin{bmatrix}1.3 & -1\\ -1 & 10.5\end{bmatrix},$$

$$K_{21}=\begin{bmatrix}10.2 & -1\\ -1 & 0.7\end{bmatrix}\ K_{22}=\begin{bmatrix}12.5 & -1\\ -1 & -0.8\end{bmatrix}.$$

容易看出, 各模糊子系统单独使用时均不稳定. 解矩阵不等式(4)得

$$P_1=\begin{bmatrix}0.4622 & 0.0012\\ 0.0012 & 0.5835\end{bmatrix},\ P_2=\begin{bmatrix}0.7224 & 0.0008\\ 0.0008 & 0.7387\end{bmatrix}.$$

令

$$\varOmega_1=$$
$$\{\hat{x}(t)\in\mathbb{R}^2|\hat{x}^{\mathrm{T}}(t)(P_2-P_1)\hat{x}(t)\geqslant 0,\hat{x}(t)\neq 0\},$$
$$\varOmega_2=$$
$$\{\hat{x}(t)\in\mathbb{R}^2|\hat{x}^{\mathrm{T}}(t)(P_1-P_2)\hat{x}(t)\geqslant 0,\hat{x}(t)\neq 0\},$$

则$\varOmega_1\bigcup\varOmega_2=\mathbb{R}^2\setminus\{0\}$. 给出切换律

$$\sigma(\hat{x}(t))=\begin{cases}1,\ \hat{x}(t)\in\varOmega_1,\\ 2,\ \hat{x}(t)\in\varOmega_1\backslash\varOmega_2.\end{cases}$$

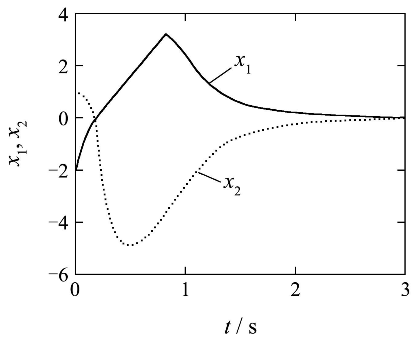

图 1 切换模糊时滞系统的状态曲线
Fig. 1 The state trajectories of the switching fuzzy time-delay system

图1是切换模糊时滞系统的状态变化曲线, 表明在原本不可稳的模糊时滞系统中引入切换机制确实可带来稳定性.

5 结束语(Conclusion)

本文研究了状态不可观测的切换模糊时滞系统的稳定性问题. 采用模糊平行分布补偿算法(PDC), 设计了切换模糊观测器和反馈控制器. 对观测误差和观测器状态分别采用共同Lyapunov函数方法和多Lyapunov函数方法, 给出了使切换模糊时滞系统渐近稳定的充分条件和切换律设计. 通过仿真算例, 验证了结论的有效性.

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