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带有界扰动的一类大型互联非线性系统的鲁棒分散控制

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摘要: 研究带有界扰动的一类大型互联非线性系统的鲁棒分散控制问题, 该系统的第i个子系统的标称模型具有相对阶r_i及指数稳定的零动态, 且每个子系统的互联项满足匹配条件. 通过子系统状态的线性变换得到鲁棒分散状态反馈控制器, 当该控制律作用于系统时, 系统的状态能够收敛到原点的一个小邻域内, 并给出仿真算例说明该结论的可行性和有效性.

关键词: 大型互联系统; 状态反馈; 分散控制

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Robust decentralized control for a class of large-scale interconnected nonlinear systems with bounded disturbances

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Abstract: The robust decentralized control is considered for a class of large-scale interconnected nonlinear systems with bounded disturbances. The nominal model of the i-th subsystem has a relative degree r_i and a zero dynamics with exponential stability; and the interconnection between subsystems satisfies the matching condition. Applying the linear transformation to the states of each subsystem, we obtain the robust decentralized state feedback controllers which ensure the states of the systems to converge to a small region around the origin. A simulation example also shows the feasibility and effectiveness of the conclusion.

Key words: large-scale interconnected systems; state feedback; decentralized control

1 引言(Introduction)

大型互联系统的分散控制设计是近年来控制理论研究的热点问题, 关于此问题的研究论文有很多, 其中利用系统的状态来进行控制设计的有文[1∼4], 包含了鲁棒控制设计和自适应控制设计等; 利用系统的输出来进行控制设计的有文[5∼8]. 文[1∼3]研究了一类大型互联系统的状态反馈控制设计问题, 该系统每个子系统的标称模型是全状态线性化系统, 其中文[1,2]中系统的互联项满足匹配条件, 而文[3]中系统的互联项不满足匹配条件. 当子系统的标称模型是部分状态线性化系统, 如何进行控制设计, 文[4]对此进行了研究, 其讨论的系统的互联项不满足匹配条件, 但系统需符合较强的几何假设条件[3,8∼10].

本文研究带有界扰动的一类大型互联非线性系统的鲁棒分散状态反馈控制问题, 该系统是由满足通常假设条件(关于相对阶和零动态)的仿射非线性系统互联而成, 其互联项满足匹配条件, 且每个子系统均含有有界扰动项, 笔者提出一种新的控制设计方法, 当状态反馈控制律作用于该系统, 扰动为零时, 原点是闭环系统的指数稳定平衡点; 扰动不为零时, 从原点的某邻域出发的系统的解有界, 且收敛到原点的一个小邻域内, 该邻域与扰动的界有关, 扰动的界越小, 该邻域越小. 笔者得到的状态反馈控制律与扰动的界无关, 因此, 扰动的界可以是未知的.

2 问题描述(Problem description)

考虑如下形式的大型互联非线性不确定系统:

$$\begin{cases} \dot{x}_i = f_i(x_i) + q_i(x) + g_i(x_i)u_i + d_i(x,t), \\ y_i = h_i(x_i). \end{cases} \tag{1}$$

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这里: $i=1,\cdots,N$表示各个子系统; $x_i\in\mathbb{R}^{n_i}$, $u_i\in\mathbb{R}$, $y_i\in\mathbb{R}$分别是第i个子系统的状态、输入和输出; 而$x=[x_1^{\mathrm T}\ \ x_2^{\mathrm T}\ \ \cdots\ \ x_N^{\mathrm T}]^{\mathrm T}$, $q_i(x)\in\mathbb{R}^{n_i}$是未知的互联项; $d_i(x,t)\in\mathbb{R}^{n_i}$是有界扰动; $f_i(0)=0$,$h_i(0)=0$, $f_i(x_i)\in\mathbb{R}^{n_i}$, $g_i(x_i)\in\mathbb{R}^{n_i}$, $h_i(x_i)\in\mathbb{R}$是足够光滑的. 定义范数$\|\cdot\|$为通常的2–范数, 即$\|x\|=\sqrt{\sum\limits_{i=1}^{n}x_i^2}$, $\|A\|=\sqrt{\rho(A^{\mathrm T}A)}$.

假设 1 互联项$q_i(x)$是满足匹配条件的, 即

$$q_i(x)=g_i(x_i)w_i(x),$$

$w_i(x)\in\mathbb{R}$是未知的, 且$w_i(0)=0$.

由此, 式(1)的标称系统为

$$\begin{cases}\dot x_i=f_i(x_i)+g_i(x_i)u_i+g_i(x_i)w_i(x),\\ y_i=h_i(x_i).\end{cases}\tag{2}$$

再作如下假设:

假设 2 系统(2)的第i个子系统在$x_i=0$处具有相对阶r_i[8], 其中$1\leqslant r_i\leqslant n_i(1\leqslant i\leqslant n)$.

引理 1[8] 假设2成立, 令

$$\varphi_{i1}^{(1)}(x_i)=h_i(x_i),\ \varphi_{i2}^{(1)}(x_i)=L_{f_i}h_i(x_i),$$
$$\vdots$$
$$\varphi_{ir_i}^{(1)}(x_i)=L_{f_i}^{r_i-1}h_i(x_i),$$

若r_i严格小于n_i, 则总可找到n_i-r_i个函数$\varphi_{i(r_i+1)}^{(2)}(x_i),\cdots,\varphi_{in_i}^{(2)}(x_i)$, 且

$$L_{g_i}\varphi_{ij}^{(2)}(x_i)=0.$$

对$r_i+1\leqslant j\leqslant n_i$和在$x_i=0$附近的所有$x_i$成立, 并使得映射

$$\varphi_i(x_i)=\begin{pmatrix}\varphi_{i1}^{(1)}(x_i)\\ \vdots\\ \varphi_{ir_i}^{(1)}(x_i)\\ \varphi_{i(r_i+1)}^{(2)}(x_i)\\ \vdots\\ \varphi_{in_i}^{(2)}(x_i)\end{pmatrix}$$

在$x_i=0$处有非奇异的雅可比矩阵, 由此该雅可比矩阵可以作为在$x_i=0$的一个邻域内的一个局部坐标变换.

引理 2 假设1,2成立, 则存在定义在原点的某邻域U_i内的局部微分同胚

$$\begin{pmatrix}z_i\\ \eta_i\end{pmatrix}=\begin{pmatrix}\varphi_i^{(1)}(x_i)\\ \varphi_i^{(2)}(x_i)\end{pmatrix}=\varphi_i(x_i),\ x_i\in U_i,$$

使得系统(1)化为

$$\begin{cases}\dot z_i=\\ A_iz_i+B_i(b_i(z_i,\eta_i)+a_i(z_i,\eta_i)u_i)+\\ \xi_i^{(1)}(z,\eta,t)+B_ia_i(z_i,\eta_i)w_i(\varphi^{-1}(z,\eta)),\\ \dot\eta_i=p_i(z_i,\eta_i)+\xi_i^{(2)}(z,\eta,t),\\ y_i=c_iz_i.\end{cases}\tag{3}$$

其中:

$$a_i(z_i,\eta_i)=L_{g_i}L_{f_i}^{r_i-1}h_i(\varphi_i^{-1}(z_i,\eta_i)),$$
$$b_i(z_i,\eta_i)=L_{f_i}^{r_i}h_i(\varphi_i^{-1}(z_i,\eta_i)),$$
$$\xi_i^{(1)}(z,\eta,t)=\frac{\partial\varphi_i^{(1)}(x_i)}{\partial x_i}d_i(x,t)|_{x=\varphi^{-1}(z,\eta)},$$
$$\xi_i^{(2)}(z,\eta,t)=\frac{\partial\varphi_i^{(2)}(x_i)}{\partial x_i}d_i(x,t)|_{x=\varphi^{-1}(z,\eta)},$$
$$p_i(z_i,\eta_i)=\frac{\partial\varphi_i^{(2)}(x_i)}{\partial x_i}f_i(x_i)|_{x=\varphi_i^{-1}(z_i,\eta_i)},$$
$$A_i=\begin{pmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\ \vdots&\vdots&\ddots&\ddots&\vdots\\0&0&0&\ddots&1\\0&0&0&\cdots&0\end{pmatrix}_{r_i\times r_i},\ B_i=\begin{pmatrix}0\\0\\ \vdots\\0\\1\end{pmatrix}_{r_i\times 1},$$
$$c_i=[1\ 0\ \cdots\ 0]_{1\times r_i},\ z=[z_1^{\mathrm T}\ z_2^{\mathrm T}\ \cdots\ z_N^{\mathrm T}]^{\mathrm T},$$
$$\eta=[\eta_1^{\mathrm T}\ \eta_2^{\mathrm T}\ \cdots\ \eta_N^{\mathrm T}]^{\mathrm T},\ \varphi=[\varphi_1^{\mathrm T}\ \varphi_2^{\mathrm T}\ \cdots\ \varphi_N^{\mathrm T}]^{\mathrm T}.$$

证 取

$$\varphi_i^{(1)}(x_i)=\begin{pmatrix}\varphi_{i1}^{(1)}(x_i)\\ \vdots\\ \varphi_{ir_i}^{(1)}(x_i)\end{pmatrix},\ \varphi_i^{(2)}(x_i)=\begin{pmatrix}\varphi_{i(r_i+1)}^{(2)}(x_i)\\ \vdots\\ \varphi_{in_i}^{(2)}(x_i)\end{pmatrix},$$

即$\varphi_i(x_i)$为引理1中的$\varphi_i(x_i)$. 取U_i为引理1中$x_i=0$的一个邻域, 则由引理1,

$$x_i\rightarrow\begin{pmatrix}z_i\\ \eta_i\end{pmatrix}$$

是U_i内的局部微分同胚. 再由相对阶的定义可知

$$\frac{\partial\varphi_i^{(1)}(x_i)}{\partial x_i}f_i(x_i)=A_iz_i+B_ib_i(z_i,\eta_i),$$
$$\frac{\partial\varphi_i^{(1)}(x_i)}{\partial x_i}g_i(x_i)=B_ia_i(z_i,\eta_i),$$

所以

$$\begin{aligned}\dot z_i=&\frac{\partial\varphi_i^{(1)}(x_i)}{\partial x_i}(f_i(x_i)+g_i(x_i)u_i+\\&g_i(x_i)w_i(x)+d_i(x,t))=\\&A_iz_i+B_i(b_i(z_i,\eta_i)+a_i(z_i,\eta_i)u_i)+\\&\xi_i^{(1)}(z,\eta,t)+B_ia_i(z_i,\eta_i)w_i(\varphi^{-1}(z,\eta)).\end{aligned}$$

由引理1, $\dfrac{\partial\varphi_i^{(2)}(x_i)}{\partial x_i}g_i(x_i)=0$, 由此

$$\begin{aligned}\dot{\eta}_i=&\frac{\partial\varphi_i^{(2)}(x_i)}{\partial x_i}(f_i(x_i)+g_i(x_i)u_i+\\&g_i(x_i)w_i(x)+d_i(x,t))=\\&p_i(z_i,\eta_i)+\xi_i^{(2)}(z,\eta,t).\end{aligned}$$

证毕.

注 1　存在正数r, 使得$\|z_i\|<r$,$\|\eta_i\|<r$时,

$$\begin{pmatrix}z_i\\\eta_i\end{pmatrix}\in\varphi_i(U_i),$$

其中$\varphi_i(U_i)$是U_i的象区域($i=1,\cdots,N$).

假设 3

$$\begin{aligned}&|w_i(\varphi^{-1}(z,\eta))|\leqslant\\&\sum_{j=1}^{N}\sum_{k=1}^{p_{ij}}\beta_{ij}^k\|z_j\|^k+\sum_{j=1}^{N}\sum_{k=1}^{q_{ij}}\gamma_{ij}^k\|\eta_j\|^k,\end{aligned}$$

其中: p_{ij},q_{ij}是已知的大于等于1的正整数, $\beta_{ij}^k,\gamma_{ij}^k$是已知的正实数.

注 2　由文[1,2], 假设3是通常的假设条件, 当$r_i=n_i,d_i(x,t)=0$时, 系统(3)可化为文[1,2]研究的系统.

注 3　由$\varphi_i^{(1)}(x_i)$的足够光滑性可知, 矩阵$(\dfrac{\partial\varphi_i^{(1)}}{\partial x_i})^{\mathrm T}(\dfrac{\partial\varphi_i^{(1)}}{\partial x_i})$的每个元素在$U_i$均有界, 由盖尔圆定理[11], $\|\dfrac{\partial\varphi_i^{(1)}}{\partial x_i}\|$在$U_i$有界, 同理可得$\|\dfrac{\partial\varphi_i^{(2)}}{\partial x_i}\|$在$U_i$有界, 所以存在正常数$k_i^{(1)},k_i^{(2)}$, 使得

$$\begin{aligned}&\|\xi_i^{(1)}(z,\eta,t)\|\leqslant k_i^{(1)}\|d_i(x,t)\|,\\&\|\xi_i^{(2)}(z,\eta,t)\|\leqslant k_i^{(2)}\|d_i(x,t)\|.\end{aligned}$$

再由$f_i(x_i),g_i(x_i),h_i(x_i)$的足够光滑性可知, $a_i(z_i,\eta_i)=L_{g_i}L_{f_i}^{r_i-1}h_i(x_i)$在$U_i$有界, 即存在正常数$m_i$, 使得

$$|a_i(z_i,\eta_i)|\leqslant m_i.$$

假设 4　系统(2)的零动态$\dot{\eta}_i=p_i(0,\eta_i)$在$\eta_i=0$为(局部)指数稳定的.

假设 5　$\|d_i(x,t)\|\leqslant d_i$, d_i是常数(可能未知).

注 4　考虑系统是小的扰动的情况, 即扰动的界d_i小于某个正数ρ_i, 这个正数后面给出.

引理 3[12]　在假设4的条件下, 存在函数$V_i(\eta_i)$和正常数$a_i^{(1)},a_i^{(2)},a_i^{(3)},a_i^{(4)}$, 使得对于$\forall\eta_i\in B_\delta^i$, $\forall t\geqslant 0$, 有

$$a_i^{(1)}\|\eta_i\|^2\leqslant V_i(\eta_i)\leqslant a_i^{(2)}\|\eta_i\|^2,\tag{4}$$

$$\frac{\partial V_i}{\partial\eta_i}p_i(0,\eta_i)\leqslant -a_i^{(3)}\|\eta_i\|^2,\tag{5}$$

$$\|\frac{\partial V_i}{\partial\eta_i}\|\leqslant a_i^{(4)}\|\eta_i\|.\tag{6}$$

其中B_δ^i是n_i-r_i维空间中以原点为球心、半径为δ的球内.

不妨设$\delta=r$.

引理 4[8]　存在r_i阶对称正定阵, r_i维Hurwitz向量K_i, 使得

$$P_i(A_i-B_iK_i)+(A_i-B_iK_i)^{\mathrm T}P_i=-I_i\tag{7}$$

成立($i=1,\cdots,N$), 其中I_i为r_i阶单位阵.

系统(3)的鲁棒分散状态反馈控制问题: 设计一个反馈控制律$u_i=\alpha_i(z_i,\eta_i)$, $\alpha_i(0,0)=0$使得

1) 当$d_i(x,t)=0$时, 原点$\begin{pmatrix}z\\\eta\end{pmatrix}=0$是闭环系统

$$\begin{aligned}\dot{z}_i=&A_iz_i+B_i(b_i(z_i,\eta_i)+a_i(z_i,\eta_i)u_i)+\\&B_ia_i(z_i,\eta_i)w_i(\varphi^{-1}(z,\eta)),\\\dot{\eta}_i=&p_i(z_i,\eta_i),\\u_i=&\alpha_i(z_i,\eta_i)\end{aligned}$$

的指数稳定平衡点($i=1,\cdots,N$).

2) 当$d_i(x,t)\neq 0$, 且$\|d_i(x,t)\|\leqslant d_i$时, 从原点的某邻域内出发的系统

$$\begin{aligned}\dot{z}_i=&A_iz_i+B_i(b_i(z_i,\eta_i)+a_i(z_i,\eta_i)u_i)+\\&\xi_i^{(1)}(z,\eta,t)+B_ia_i(z_i,\eta_i)w_i(\varphi^{-1}(z,\eta)),\\\dot{\eta}_i=&p_i(z_i,\eta_i)+\xi_i^{(2)}(z,\eta,t),\\u_i=&\alpha_i(z_i,\eta_i)\end{aligned}$$

的解

$$\begin{pmatrix}z(t)\\\eta(t)\end{pmatrix}$$

有界, 且收敛到原点的一个小邻域内, 该邻域与扰动的界有关, 扰动的界越小, 该邻域越小($i=1,\cdots,N$).

3　主要结论(Main results)

利用文[13]的思想方法, 对系统(3)的每个子系统的状态进行线性变换, 记$E_{l_i}=\mathrm{diag}\{l_i^{r_i-1},l_i^{r_i-2},\cdots,l_i,1\}$, l_i为待定正常数, 作状态的线性变换

$$\begin{pmatrix}\bar{z}_i\\\bar{\eta}_i\end{pmatrix}=\begin{pmatrix}E_{l_i}&0\\0&I\end{pmatrix}\begin{pmatrix}z_i\\\eta_i\end{pmatrix},$$

注意到$E_{l_i}B_i=B_i$, $E_{l_i}A_iE_{l_i}^{-1}=l_iA_i$, 则系统(3)化为

$$\begin{cases}\dot{\bar{z}}_i=l_iA_i\bar{z}_i+B_i(b_i(E_{l_i}^{-1}\bar{z}_i,\bar{\eta}_i)+\\\quad a_i(E_{l_i}^{-1}\bar{z}_i,\bar{\eta}_i)u_i)+E_{l_i}\xi_i^{(1)}(E^{-1}\bar{z},\bar{\eta},t)+\\\quad B_ia_i(E_{l_i}^{-1}\bar{z}_i,\bar{\eta}_i)w_i(\varphi^{-1}(E^{-1}\bar{z},\bar{\eta})),\\\dot{\bar{\eta}}_i=p_i(E_{l_i}^{-1}\bar{z}_i,\bar{\eta}_i)+\xi_i^{(2)}(E^{-1}\bar{z},\bar{\eta},t),\end{cases}\tag{8}$$

其中: $E=\mathrm{diag}\{E_{l_1},E_{l_2},\cdots,E_{l_N}\}$, $\overline{z}=[\overline{z}_1^{\mathrm{T}}\ \ \overline{z}_2^{\mathrm{T}}\ \ \cdots\ \ \overline{z}_N^{\mathrm{T}}]^{\mathrm{T}}$, $\overline{\eta}=[\overline{\eta}_1^{\mathrm{T}}\ \overline{\eta}_2^{\mathrm{T}}\ \cdots\ \overline{\eta}_N^{\mathrm{T}}]^{\mathrm{T}}$. 由变换的等价性, 下对系统(8)进行控制设计, 有下面的主要结论:

定理 1 假设3,4,5成立, 则系统(8)的分散反馈控制律为

$$u_i=\frac{1}{a_i(E_{l_i}^{-1}\overline{z}_i,\overline{\eta}_i)}(-b_i(E_{l_i}^{-1}\overline{z}_i,\overline{\eta}_i)-l_iK_i\overline{z}_i),\tag{9}$$

其中K_i是式(7)中的K_i.

证 构造Lyapunov函数

$$V(\overline{z},\overline{\eta})=\sum_{i=1}^{N}(\overline{z}_i^{\mathrm{T}}P_i\overline{z}_i+V_i(\overline{\eta}_i)),$$

则由式(7)~(9)得

$$\begin{aligned}\dot V=&\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+2\overline{z}_i^{\mathrm{T}}P_iE_{l_i}\xi_i^{(1)}(E^{-1}\overline{z},\overline{\eta},t)+\\&2\overline{z}_i^{\mathrm{T}}P_iB_ia_i(E_{l_i}^{-1}\overline{z}_i,\overline{\eta}_i)w_i(\varphi^{-1}(E^{-1}\overline{z},\overline{\eta}))+\\&\frac{\partial V_i}{\partial\eta_i}p_i(E_{l_i}^{-1}\overline{z}_i,\overline{\eta}_i)+\frac{\partial V_i}{\partial\eta_i}\xi_i^{(2)}(E^{-1}\overline{z},\overline{\eta},t).\end{aligned}$$

要求$l_i\geqslant1$, 则$\|E_{l_i}\|=l_i^{r_i-1}$,$\|E_{l_i}^{-1}\|=1$, 由注3及假设5得

$$\begin{aligned}\dot V\leqslant&\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+2\|\overline{z}_i\|\|P_i\|l_i^{r_i-1}k_i^{(1)}d_i+\\&2m_i\|\overline{z}_i\|\|P_i\||w_i(\varphi^{-1}(E^{-1}\overline{z},\overline{\eta}))|+\\&\frac{\partial V_i}{\partial\eta_i}p_i(0,\overline{\eta}_i)+\|\frac{\partial V_i}{\partial\eta_i}\|k_i^{(2)}d_i+\\&\frac{\partial V_i}{\partial\eta_i}(p_i(E_{l_i}^{-1}\overline{z}_i,\overline{\eta}_i)-p_i(0,\overline{\eta}_i))),\end{aligned}$$

而

$$\begin{aligned}&\|p_i(E_{l_i}^{-1}\overline{z}_i,\overline{\eta}_i)-p_i(0,\overline{\eta}_i)\|\leqslant\|\frac{\partial p_i}{\partial z_i}E_{l_i}^{-1}\overline{z}_i\|\leqslant\\&\|\frac{\partial p_i}{\partial z_i}\|\|E_{l_i}^{-1}\|\|\overline{z}_i\|\leqslant k_i^{(3)}\|\overline{z}_i\|,\end{aligned}\tag{10}$$

$k_i^{(3)}$是与l_i无关的正常数, 由假设3及$\|E_{l_i}^{-1}\|=1$得

$$\begin{aligned}&|w_i(\varphi^{-1}(E^{-1}\overline{z},\overline{\eta}))|\leqslant\\&\sum_{j=1}^{N}\sum_{k=1}^{p_{ij}}\beta_{ij}^k\|E_{l_j}^{-1}\overline{z}_j\|^k+\sum_{j=1}^{N}\sum_{k=1}^{q_{ij}}\gamma_{ij}^k\|\overline{\eta}_j\|^k\leqslant\\&\sum_{j=1}^{N}\sum_{k=1}^{p_{ij}}\beta_{ij}^k\|\overline{z}_j\|^k+\sum_{j=1}^{N}\sum_{k=1}^{q_{ij}}\gamma_{ij}^k\|\overline{\eta}_j\|^k.\end{aligned}$$

由式(5)(6)(10)及基本不等式得

$$\begin{aligned}&\dot V\leqslant\\&\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+\|\overline{z}_i\|^2\|P_i\|^2(k_i^{(1)})^2+\\&l_i^{2r_i-2}d_i^2+2m_i\|\overline{z}_i\|\|P_i\|(\sum_{j=1}^{N}\sum_{k=1}^{p_{ij}}\beta_{ij}^k\|\overline{z}_j\|^k+\\&\sum_{j=1}^{N}\sum_{k=1}^{q_{ij}}\gamma_{ij}^k\|\overline{\eta}_j\|^k)-a_i^{(3)}\|\overline{\eta}_i\|^2+\\&a_i^{(4)}k_i^{(2)}d_i\|\overline{\eta}_i\|+a_i^{(4)}k_i^{(3)}\|\overline{z}_i\|\|\overline{\eta}_i\|)=\\&\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+\|\overline{z}_i\|^2\|P_i\|^2(k_i^{(1)})^2+l_i^{2r_i-2}d_i^2+\\&2m_i\|P_i\|(\sum_{j=1}^{N}\|\overline{z}_i\|\|\overline{z}_j\|\sum_{k=1}^{p_{ij}}\beta_{ij}^k\|\overline{z}_j\|^{k-1}+\\&\sum_{j=1}^{N}\|\overline{z}_i\|\|\overline{\eta}_j\|\sum_{k=1}^{q_{ij}}\gamma_{ij}^k\|\overline{\eta}_j\|^{k-1})-a_i^{(3)}\|\overline{\eta}_i\|^2+\\&a_i^{(4)}k_i^{(2)}d_i\|\overline{\eta}_i\|+a_i^{(4)}k_i^{(3)}\|\overline{z}_i\|\|\overline{\eta}_i\|).\end{aligned}$$

设

$$\|\overline{z}_j\|<r,\ \|\overline{\eta}_j\|<r,\tag{11}$$

则

$$\|z_j\|=\|E_{l_j}^{-1}\overline{z}_j\|\leqslant\|\overline{z}_j\|<r,\ \|\eta_j\|=\|\overline{\eta}_j\|<r,$$

且

$$\begin{aligned}&\sum_{k=1}^{p_{ij}}\beta_{ij}^k\|\overline{z}_j\|^{k-1}\leqslant\sum_{k=1}^{p_{ij}}\beta_{ij}^kr^{k-1}=c_{ij},\\&\sum_{k=1}^{q_{ij}}\gamma_{ij}^k\|\overline{\eta}_j\|^{k-1}\leqslant\sum_{k=1}^{q_{ij}}\gamma_{ij}^kr^{k-1}=b_{ij}.\end{aligned}$$

由此及基本不等式得

$$\begin{aligned}&\dot V\leqslant\\&\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+\|\overline{z}_i\|^2\|P_i\|^2(k_i^{(1)})^2+\\&l_i^{2r_i-2}d_i^2+m_i\|P_i\|(\sum_{j=1}^{N}(\|\overline{z}_i\|^2+\|\overline{z}_j\|^2)c_{ij}+\\&\sum_{j=1}^{N}(\frac{1}{\varepsilon_j}\|\overline{z}_i\|^2+\varepsilon_j\|\overline{\eta}_j\|^2)b_{ij})-a_i^{(3)}\|\overline{\eta}_i\|^2+\\&\frac14a_i^{(3)}\|\overline{\eta}_i\|^2+\frac{(a_i^{(4)})^2(k_i^{(2)})^2}{a_i^{(3)}}d_i^2+\\&\frac14a_i^{(3)}\|\overline{\eta}_i\|^2+\frac{(a_i^{(4)})^2(k_i^{(3)})^2}{a_i^{(3)}}\|\overline{z}_i\|^2)=\\&\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+\|\overline{z}_i\|^2\|P_i\|^2(k_i^{(1)})^2+\\&(m_i\|P_i\|\sum_{j=1}^{N}(c_{ij}+\frac{b_{ij}}{\varepsilon_j}))\|\overline{z}_i\|^2+\\&\frac{(a_i^{(4)})^2(k_i^{(3)})^2}{a_i^{(3)}}\|\overline{z}_i\|^2-\frac12a_i^{(3)}\|\overline{\eta}_i\|^2+\\&(l_i^{2r_i-2}+\frac{(a_i^{(4)})^2(k_i^{(2)})^2}{a_i^{(3)}})d_i^2)+\\&\sum_{i=1}^{N}m_i\|P_i\|\sum_{j=1}^{N}c_{ij}\|\overline{z}_j\|^2+\end{aligned}$$

$$\sum_{i=1}^{N} m_i\|P_i\|\sum_{j=1}^{N}\varepsilon_j b_{ij}\|\overline{\eta}_j\|^2 =$$
$$\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+\|\overline{z}_i\|^2\|P_i\|^2(k_i^{(1)})^2+$$
$$(m_i\|P_i\|\sum_{j=1}^{N}(c_{ij}+\frac{b_{ij}}{\varepsilon_j}))\|\overline{z}_i\|^2+$$
$$\frac{(a_i^{(4)})^2(k_i^{(3)})^2}{a_i^{(3)}}\|\overline{z}_i\|^2-\frac{1}{2}a_i^{(3)}\|\overline{\eta}_i\|^2+$$
$$(l_i^{2r_i-2}+\frac{(a_i^{(4)})^2(k_i^{(2)})^2}{a_i^{(3)}})d_i^2)+$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ji}m_j\|P_j\|\|\overline{z}_i\|^2+$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_i b_{ji}\|P_j\|m_j\|\overline{\eta}_i\|^2=$$
$$\sum_{i=1}^{N}(-l_i\|\overline{z}_i\|^2+\|\overline{z}_i\|^2\|P_i\|^2(k_i^{(1)})^2+$$
$$(m_i\|P_i\|\sum_{j=1}^{N}(c_{ij}+\frac{b_{ij}}{\varepsilon_j}))\|\overline{z}_i\|^2+$$
$$\frac{(a_i^{(4)})^2(k_i^{(3)})^2}{a_i^{(3)}}\|\overline{z}_i\|^2+(\sum_{j=1}^{N}c_{ji}m_j\|P_j\|)\|\overline{z}_i\|^2-$$
$$\frac{1}{2}a_i^{(3)}\|\overline{\eta}_i\|^2+(\sum_{j=1}^{N}\varepsilon_i b_{ji}\|P_j\|m_j)\|\overline{\eta}_i\|^2+$$
$$(l_i^{2r_i-2}+\frac{(a_i^{(4)})^2(k_i^{(2)})^2}{a_i^{(3)}})d_i^2).$$

选取正数$\varepsilon_1,\cdots,\varepsilon_N$, 使得

$$\varepsilon_i=\frac{a_i^{(3)}}{4\sum\limits_{j=1}^{N}b_{ji}\|P_j\|m_j},\ i=1,\cdots,N,$$

再取

$$l_i=\|P_i\|^2(k_i^{(1)})^2+m_i\|P_i\|\sum_{j=1}^{N}(c_{ij}+\frac{b_{ij}}{\varepsilon_j})+$$
$$\frac{(a_i^{(4)})^2(k_i^{(3)})^2}{a_i^{(3)}}+\sum_{j=1}^{N}c_{ji}m_j\|P_j\|+1,\tag{12}$$

则满足$l_i>1$, 且记

$$l_i^{2r_i-2}+\frac{(a_i^{(4)})^2(k_i^{(2)})^2}{a_i^{(3)}}=k_i^{(4)},$$

则

$$\dot V\leqslant\sum_{i=1}^{N}(-\|\overline{z}_i\|^2-\frac{1}{4}\|\overline{\eta}_i\|^2+k_i^{(4)}d_i^2).$$

记$\overline{\lambda}_i,\hat\lambda_i$分别是$P_i$的最小和最大特征值,

$$2\lambda_1=\min\{\frac{1}{\hat\lambda_1},\cdots,\frac{1}{\hat\lambda_N},\frac{1}{4a_1^{(2)}},\cdots,\frac{1}{4a_N^{(2)}}\},$$

则由式(4)得

$$\dot V\leqslant-2\lambda_1V+\sum_{i=1}^{N}k_i^{(4)}d_i^2,\tag{13}$$

所以, 当扰动为零时, 即$d_i=0(i=1,\cdots,N)$, 由式(13), 闭环系统是指数稳定的. 当扰动不为0时, 由式(13)及文[14]中引理1得

$$V(t)\leqslant V(0)\exp(-2\lambda_1t)+\frac{\sum\limits_{i=1}^{N}k_i^{(4)}d_i^2}{2\lambda_1}.\tag{14}$$

记

$$\lambda_2=\min\{\overline{\lambda}_1,\cdots,\overline{\lambda}_N,a_1^{(1)},\cdots,a_N^{(1)}\},$$
$$\lambda_3=\max\{\hat\lambda_1,\cdots,\hat\lambda_N,a_1^{(2)},\cdots,a_N^{(2)}\},$$
$$\frac{\lambda_3}{\lambda_2}=\lambda_4,\ \frac{k_i^{(4)}}{2\lambda_1\lambda_2}=k_i^{(5)}.$$

再由式(4)及式(14)得

$$\sum_{i=1}^{N}(\|\overline{z}_i\|^2+\|\overline{\eta}_i\|^2)\leqslant$$
$$\lambda_4(\sum_{i=1}^{N}(\|\overline{z}_i(0)\|^2+\|\overline{\eta}_i(0)\|^2))\exp(-2\lambda_1t)+$$
$$\sum_{i=1}^{N}k_i^{(5)}d_i^2.\tag{15}$$

由此解$\begin{pmatrix}\overline{z}(t)\\ \overline{\eta}(t)\end{pmatrix}$有界, 且收敛到原点的一个小邻域内, 该邻域与扰动的界有关, 扰动的界越小, 该邻域越小. 因变换的等价性, 解$\begin{pmatrix}z(t)\\ \eta(t)\end{pmatrix}$, 也如此.

下面讨论初值$\begin{pmatrix}z(0)\\ \eta(0)\end{pmatrix}$的允许选取范围, 记$\lambda_5=\max\{l_1^{2r_1-2},\cdots,l_N^{2r_N-2}\}$, $\lambda_4\lambda_5=\lambda_6$. 由$\frac{\lambda_3}{\lambda_2}=\lambda_4$可知$\lambda_4\geqslant1$, 而$\lambda_5>1$, 所以$\lambda_6>1$. 由式(15)可得

$$\|\overline{z}_j\|^2\leqslant\sum_{i=1}^{N}(\|\overline{z}_i\|^2+\|\overline{\eta}_i\|^2)\leqslant$$
$$\lambda_4\sum_{i=1}^{N}(\|\overline{z}_i(0)\|^2+\|\overline{\eta}_i(0)\|^2)+\sum_{i=1}^{N}k_i^{(5)}d_i^2=$$
$$\lambda_4\sum_{i=1}^{N}(\|E_{l_i}z_i(0)\|^2+\|\eta_i(0)\|^2)+\sum_{i=1}^{N}k_i^{(5)}d_i^2\leqslant$$
$$\lambda_4\sum_{i=1}^{N}(l_i^{2r_i-2}\|z_i(0)\|^2+\|\eta_i(0)\|^2)+\sum_{i=1}^{N}k_i^{(5)}d_i^2\leqslant$$
$$\lambda_4\sum_{i=1}^{N}l_i^{2r_i-2}(\|z_i(0)\|^2+\|\eta_i(0)\|^2)+\sum_{i=1}^{N}k_i^{(5)}d_i^2\leqslant$$
$$\lambda_4\lambda_5\sum_{i=1}^{N}(\|z_i(0)\|^2+\|\eta_i(0)\|^2)+\sum_{i=1}^{N}k_i^{(5)}d_i^2=$$
$$\lambda_6\|\begin{pmatrix}z(0)\\ \eta(0)\end{pmatrix}\|^2+\sum_{i=1}^{N}k_i^{(5)}d_i^2,\tag{16}$$

同样

$$\|\bar{\eta}_j\|^2 \leqslant \lambda_6 \| \begin{pmatrix} z(0) \\ \eta(0) \end{pmatrix} \|^2 + \sum_{i=1}^{N} k_i^{(5)} d_i^2. \qquad (17)$$

证明过程得以能够进行, 用到了假设条件式(11), 为了使该条件成立, 由式(16)(17), 要求

$$\lambda_6 \| \begin{pmatrix} z(0) \\ \eta(0) \end{pmatrix} \|^2 + \sum_{i=1}^{N} k_i^{(5)} d_i^2 < r^2, \qquad (18)$$

所以取

$$\| \begin{pmatrix} z(0) \\ \eta(0) \end{pmatrix} \| < \frac{r}{\sqrt{2\lambda_6}}, d_i < \frac{r}{\sqrt{2Nk_i^{(5)}}} = \rho_i. \ (19)$$

因为$\lambda_6 > 1$, 所以$\dfrac{r}{\sqrt{2\lambda_6}} < r$, 故初值的选取范围定为

$$\| \begin{pmatrix} z(0) \\ \eta(0) \end{pmatrix} \| < \frac{r}{\sqrt{2\lambda_6}} = M,$$

则从原点的邻域

$$U = \left\{ \begin{pmatrix} z \\ \eta \end{pmatrix} \middle| \, \| \begin{pmatrix} z \\ \eta \end{pmatrix} \| < M \right\}$$

内出发的解满足控制设计要求. 证毕.

注 5 从式(18)可知, d_i的界ρ_i与初值$\| \begin{pmatrix} z(0) \\ \eta(0) \end{pmatrix} \|$的界$M$的选取可以相互调配, 当$M$变大时, 则$\rho_i$要变小, 反之亦然, 可谓有得有失.

注 6 在证明过程中, 作者要求待定常数$l_i \geqslant 1$,这是因为当$l_i \geqslant 1$时, 有$\|E_{l_i}^{-1}\| = 1$, 所以$\|z_i\| \leqslant \|\bar{z}_i\|$, 也就是说, 可以用$\|\bar{z}_i\|$ 的大小来控制$\|z_i\|$, 这一点对定理的证明和初值范围的确定是至关重要的.

注 7 具体的控制设计过程如下: 先由局部微分同胚将系统(1)化为系统(3), 然后取定正常数r满足注1的要求, 再由式(12)确定待定常数l_i, 最后由式(19)确定允许选取的初值和扰动的范围.

4 仿真算例(Simulation example)

取$N = 2, n = 2$, 由文[8]中例4.3.4来构造如下的互联系统:

$$\begin{cases} \dot{x}_{11} = x_{13} - x_{12}^3, \\ \dot{x}_{12} = -x_{12} - u_1 - w_1(x), \\ \dot{x}_{13} = x_{11}^2 - x_{13} + u_1 + w_1(x), \\ y_1 = x_{11}, \\ \dot{x}_{21} = x_{23} - x_{22}^3, \\ \dot{x}_{22} = -x_{22} - u_2 - w_2(x), \\ \dot{x}_{23} = x_{21}^2 - x_{23} + u_2 + w_2(x), \\ y_2 = x_{21}, \end{cases} \qquad (20)$$

其中:

$$\begin{aligned} & w_1(x) = \\ & (\sin(x_{21} + x_{23} - x_{22}^3))\sqrt{x_{11}^2 + (x_{13} - x_{12}^3)^2} + \\ & (\cos(x_{11} + x_{13} - x_{12}^3))\sqrt{x_{21}^2 + (x_{23} - x_{22}^3)^2} + \\ & (\sin(x_{22} + x_{23}))(x_{12} + x_{13}) + \\ & (\cos(x_{12} + x_{13}))(x_{22} + x_{23}), \\ & w_2(x) = \\ & (\sin(x_{11} + x_{13} - x_{12}^3))\sqrt{x_{21}^2 + (x_{23} - x_{22}^3)^2} + \\ & (\cos(x_{21} + x_{23} - x_{22}^3))\sqrt{x_{11}^2 + (x_{13} - x_{12}^3)^2} + \\ & (\sin(x_{12} + x_{13}))(x_{22} + x_{23}) + \\ & (\cos(x_{22} + x_{23}))(x_{12} + x_{13}). \end{aligned}$$

该系统相对阶为$r_1 = r_2 = 2$, 如文[8], 取全局定义的坐标变换:

$$\begin{aligned} & z_{11} = x_{11},\ z_{12} = x_{13} - x_{12}^3, \\ & \eta_1 = x_{12} + x_{13},\ 21 = x_{21}, \\ & z_{22} = x_{23} - x_{22}^3,\ \eta_2 = x_{22} + x_{23}. \end{aligned}$$

记$z_1 = \begin{pmatrix} z_{11} \\ z_{12} \end{pmatrix}$, $z_2 = \begin{pmatrix} z_{21} \\ z_{22} \end{pmatrix}$, 则系统(20)可化为

$$\begin{aligned} & \dot{z}_{11} = z_{12}, \\ & \dot{z}_{12} = x_{11}^2 - x_{13} + 3x_{12}^3 + (1 + 3x_{12}^2)u_1 + \\ & \quad (1 + 3x_{12}^2)(\sin(z_{21} + z_{22}))\|z_1\| + \\ & \quad (\cos(z_{11} + z_{12}))\|z_2\| + \\ & \quad (\sin\ \eta_2)\eta_1 + (\cos\ \eta_1)\eta_2, \\ & \dot{\eta}_1 = z_{11}^2 - \eta_1,\ y_1 = z_{11},\ \dot{z}_{21} = z_{22}, \\ & \dot{z}_{22} = x_{21}^2 - x_{23} + 3x_{22}^3 + (1 + 3x_{22}^2)u_2 + \\ & \quad (1 + 3x_{22}^2)(\sin(z_{11} + z_{12}))\|z_2\| + \\ & \quad (\cos(z_{21} + z_{22}))\|z_1\| + \\ & \quad (\sin\ \eta_1)\eta_2 + (\cos\ \eta_2)\eta_1, \\ & \dot{\eta}_2 = z_{21}^2 - \eta_2,\ y_2 = z_{21}. \end{aligned}$$

取$U_i = \{(x_{i1}, x_{i2}, x_{i3}) \mid |x_{ij}| < 1, j = 1, 2, 3\}(i = 1, 2)$, 由此可算得控制设计中所需的常数: $k_i^{(1)} = 3\sqrt{2}, k_i^{(2)} = \sqrt{2}, k_i^{(3)} = 2, m_i = 4, (i = 1, 2)$; 再由计算可得: $a_i^{(1)} = a_i^{(2)} = 1, a_i^{(3)} = a_i^{(4)} = 2, K_i = [2, 3]$,

$$P_i = \begin{pmatrix} \frac{5}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

由式(12)(9)得分散反馈控制律

$$u_1 = \frac{1}{1 + 3x_{12}^2}(-x_{11}^2 + x_{13} -$$

$$3x_{12}^3-(\frac{595}{4}+\frac{235}{4})((\frac{595}{2}+\frac{235}{2})x_{11}+3x_{13}-3x_{12}^3)),$$

$$u_2=\frac{1}{1+3x_{22}^2}(-x_{21}^2+x_{23}-3x_{22}^3-(\frac{595}{4}+\frac{235}{4})((\frac{595}{2}+\frac{235}{2})x_{21}+3x_{23}-3x_{22}^3)).$$

再由式(19)取初值: $x_{11}(0)=x_{12}(0)=x_{13}(0)=x_{21}(0)=x_{22}(0)=x_{23}(0)=0.0001$, 则系统(20)的仿真结果为图1至图6.

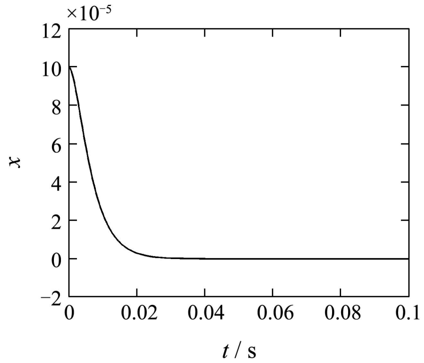

图 1　x_{11}的轨迹

Fig. 1　Trajectory of x_{11}

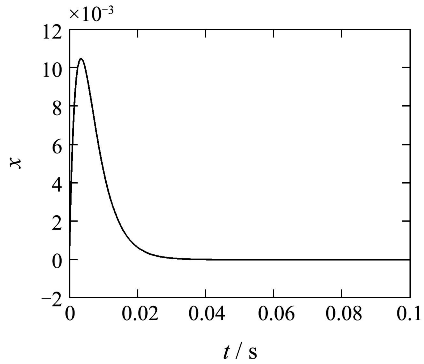

图 2　x_{12}的轨迹

Fig. 2　Trajectory of x_{12}

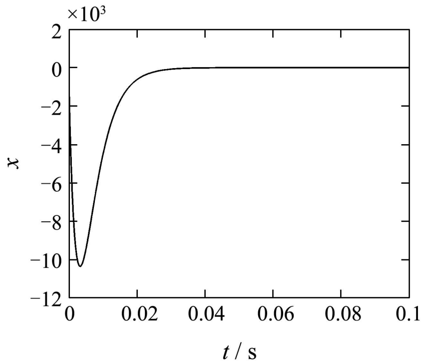

图 3　x_{13}的轨迹

Fig. 3　Trajectory of x_{13}

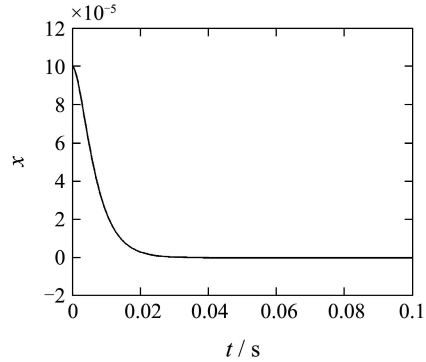

图 4　x_{21}的轨迹

Fig. 4　Trajectory of x_{21}

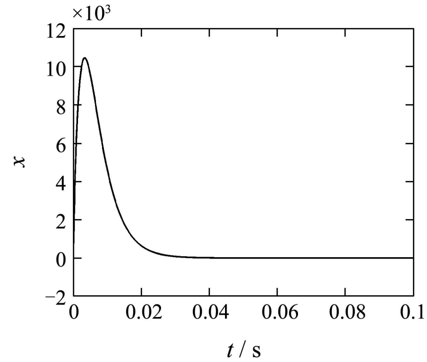

图 5　x_{22}的轨迹

Fig. 5　Trajectory of x_{22}

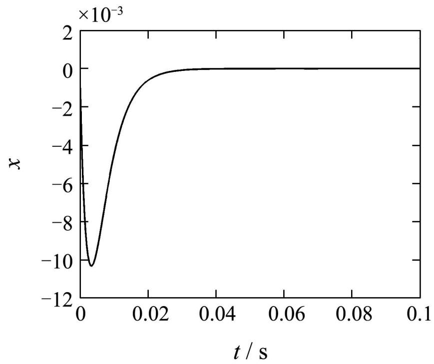

图 6　x_{23}的轨迹

Fig. 6　Trajectory of x_{23}

5　结论(Conclusion)

本文在通常的假设条件(相对阶、零动态及 $|w_i(\varphi^{-1}(z,\eta))|\leqslant\sum_{j=1}^{N}\sum_{k=1}^{p_{ij}}\beta_{ij}^k\|z_j\|^k+\sum_{j=1}^{N}\sum_{k=1}^{q_{ij}}\gamma_{ij}^k\|\eta_j\|^k$)下, 考虑了带有界扰动的一类大型互联仿射非线性系统的鲁棒分散反馈控制问题. 与文[2]相比, 本文增加了η_i动态和有界扰动项, 通过系统状态的线性变换对系统进行了控制设计, 最终得到了很好的结果, 仿真结果也说明如此.

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