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中立型时滞系统的新的绝对稳定性的法则

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摘要: 研究了具有扇形有界非线性的中立型时滞系统的绝对稳定性. 利用新的Lyapunov-Krasovskii泛函, 基于线性矩阵不等式得到了保守性低的稳定性结果. 从而给出了该系统的新的绝对稳定的法则. 最后用数值算例来演示所得到的结论的可行性与较低的保守性. 文中也考虑了不确定性中立型时滞系统, 他们的不确定性必须是范数有界的, 但可以是时变的.

关键词: 绝对稳定性; 中立型时滞系统; 不确定性; 扇形有界的非线性

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A novel criterion for absolute stability of neutral delay systems

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Abstract: The absolute stability of neutral delay systems with sector-bounded nonlinearity is studied. Using new Lyapunov-Krasovskii functional, we derive the less conservative stability conditions for such systems based on linear matrix inequalities(LMIs), and present a novel absolute stability criterion for these systems. A numerical example is provided to show that the proposed criterion significantly extends the allowable upper limit of the delay size over the corresponding limits in some other existing criteria in the literature. We also consider the uncertain neutral delay systems, for which the uncertainties are norm-bounded but parameters are allowed to be time varying.

Key words: absolute stability; neutral delay systems; uncertainty; sector-bounded nonlinearity

1 引言(Introduction)

带有扇形有界非线性的滞后型时滞系统的绝对稳定问题已经在文献[1]中得到研究. 本文在文献[1]中的基础上推广了带有扇形有界非线性的中立型时滞系统的绝对稳定性问题.

基于LMI方法, 利用改进的Lyapunov-Krasovskii泛函, 得到了此类系统的新的绝对稳定性法则. 在最后, 用两个数值算例来验证了本文所得到的结论的可行性与较低的保守性. 而且在本文中, 还考虑了不确定中立型时滞系统的绝对稳定性问题. 所考虑的不确定性是范数有界的且可能是时变的.

2 问题描述(Problem statement)

考虑下列带有扇形有界非线性的中立型时滞系统:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h) + C\dot{x}(t-h) + \\ \qquad D\omega(t), \forall t \geqslant t_0, \\ \omega(t) = -\varphi(t, z(t)), \\ z(t) = Mx(t) + Nx(t-h), \\ x(t_0+\theta) = \phi(\theta), \forall\theta \in [-h, 0]. \end{cases} \tag{1}$$

这里: $x(t) \in \mathbb{R}^n$是状态向量, $\omega(t) \in \mathbb{R}^n$是输入向量, $z(t) \in \mathbb{R}^n$是系统的输出向量. $h > 0$为延时. $\phi(\cdot)$是初始条件, A, B, C, D, M和N是已知的实矩阵. $\varphi(t, z(t))$是一个可记忆的, 可以是时变的非线性向量函数, 且对于t是分段连续的, 对于$z(t)$是全局Lipschitz的, $\varphi(t, 0) = 0$并且对于$t \in [t_0, \infty), \forall z(t) \in \mathbb{R}^n$来说满足下列扇形条件:

$$[\varphi(t, z(t)) - K_1 z(t)]^{\mathrm{T}}[\varphi(t, z(t)) - K_2 z(t)] \leqslant 0. \tag{2}$$

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这里: $K_1\in\mathbb{R}^n$和$K_2\in\mathbb{R}^n$是常矩阵, $K=K_2-K_1$是一个正定对称矩阵. 在文献[2]中, 这样的一个非线性函数$\varphi(t,z(t))$被称为属于扇形$[K_1,K_2]$. 滞后型时滞系统的绝对稳定性的定义如文献[1]所示. 在这里将引入中立型时滞系统的绝对稳定性的定义:

定义 1　如果系统(1)对于任意满足式(2)的非线性函数$\varphi(\cdot)$ 都是全局一致渐近稳定的, 那么该系统被称为在扇形$[K_1,K_2]$ 中是绝对稳定的.

在陈述主要结论之前, 首先介绍下列引理.

引理 1[3](S-procedure)　令$F_i=F_i^{\mathrm{T}}\in\mathbb{R}^{n\times n}$, $i=0,1,2,\cdots,p$. 如果存在标量$\epsilon_i\geqslant 0, i=0,1,2,\cdots,p$使得$F_0-\sum\limits_{i=1}^{p}\epsilon_iF_i>0$ 成立, 那么对于所有满足$\xi^{\mathrm{T}}F_i\xi>0$的$\xi\neq 0$, 这个式子$\xi^{\mathrm{T}}F_0\xi>0$是成立的. 进一步, 对于$p=1$, 这两个式子是等价的.

引理 2[4]　对于任意对称矩阵$Q\in\mathbb{R}^{m\times m}, Q=Q^{\mathrm{T}}>0$, 当有$\gamma>0$时, 向量函数$\omega:[-\gamma,0]\to\mathbb{R}^n$有下列积分, 那么

$$\gamma\int_{-\gamma}^{0}\omega^{\mathrm{T}}(\beta)Q\omega(\beta)\,\mathrm{d}\beta\leqslant$$
$$(\int_{-\gamma}^{0}\omega(\beta)\,\mathrm{d}\beta)^{\mathrm{T}}Q(\int_{-\gamma}^{0}\omega(\beta)\,\mathrm{d}\beta).$$

引理 3[5]　对于给定的具有适当维数的矩阵$Q=Q^{\mathrm{T}}, H, E$和$R=R^{\mathrm{T}}>0$, 对于所有满足$F^{\mathrm{T}}F\leqslant R$的矩阵$F$,

$$Q+HFE+E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}}<0$$

成立, 当且仅当存在$\epsilon>0$使得

$$Q+\epsilon HH^{\mathrm{T}}+\epsilon^{-1}E^{\mathrm{T}}RE<0.$$

3 主要结论(Main results)

首先, 将给出下列的限制条件:

假设 1　矩阵C的所有特征值都在单位圆内.

在这首先考虑当非线性函数$\varphi(t,z(t))$属于特殊的扇形$[0,K]$ 时的条件. 因此可以获得下列不等式:

$$\varphi^{\mathrm{T}}(t,z(t))[\varphi(t,z(t))-Kz(t)]\leqslant 0. \tag{3}$$

定理 1　在假设1的情况下, 对于给定的$h>0$, 如果存在$\epsilon>0$, 正定对称矩阵

$$Q=\begin{bmatrix}Q_{11}&Q_{12}&Q_{13}&Q_{14}\\ *&Q_{22}&Q_{23}&Q_{24}\\ *&*&Q_{33}&Q_{34}\\ *&*&*&Q_{44}\end{bmatrix}, R=\begin{bmatrix}R_{11}&R_{12}&R_{13}\\ *&R_{22}&R_{23}\\ *&*&R_{33}\end{bmatrix},$$

$$W=\begin{bmatrix}W_{11}&W_{12}\\ *&W_{22}\end{bmatrix}, S=\begin{bmatrix}S_{11}&S_{12}&S_{13}&S_{14}&S_{15}\\ *&S_{22}&S_{23}&S_{24}&S_{25}\\ *&*&S_{33}&S_{34}&S_{35}\\ *&*&*&S_{44}&S_{45}\\ *&*&*&*&S_{55}\end{bmatrix},$$

P_1, P_2, 以及矩阵$Y, M_i(i=1,\cdots,8)$使得下列线性矩阵不等式成立:

$$\varPhi_0=\begin{bmatrix}\varPhi_{11}&\varPhi_{12}&\varPhi_{13}&\varPhi_{14}&\varPhi_{15}&\varPhi_{16}&\varPhi_{17}&\varPhi_{18}&\varPhi_{19}&-\epsilon M^{\mathrm{T}}K^{\mathrm{T}}+P_1D\\ *&\varPhi_{22}&\varPhi_{23}&\varPhi_{24}&\varPhi_{25}&0&\varPhi_{27}&S_{15}&-M_2&P_2D\\ *&*&\varPhi_{33}&\varPhi_{34}&\varPhi_{35}&\varPhi_{36}&\varPhi_{37}&\varPhi_{38}&\varPhi_{39}&-\epsilon N^{\mathrm{T}}K^{\mathrm{T}}\\ *&*&*&\varPhi_{44}&\varPhi_{45}&-Q_{24}&\varPhi_{47}&S_{25}&-M_4&0\\ *&*&*&*&\varPhi_{55}&\varPhi_{56}&-S_{35}^{\mathrm{T}}&-S_{55}&\varPhi_{59}&0\\ *&*&*&*&*&-Q_{44}&\varPhi_{67}&S_{45}&-M_6&0\\ *&*&*&*&*&*&-1/hR_{11}&-1/hR_{13}&\varPhi_{79}&0\\ *&*&*&*&*&*&*&-1/hR_{33}&\varPhi_{89}&0\\ *&*&*&*&*&*&*&*&-1/hR_{22}&0\\ *&*&*&*&*&*&*&*&*&-2\epsilon I\end{bmatrix}<0. \tag{4}$$

式中:

$$\varPhi_{11}=M_1+M_1^{\mathrm{T}}+P_1A+A^{\mathrm{T}}P_1+Q_{11}+hR_{11}+S_{13}+S_{13}^{\mathrm{T}},$$
$$\varPhi_{12}=S_{11}-P_1+A^{\mathrm{T}}P_2+M_2^{\mathrm{T}}+Q_{12}+hR_{12}+h^2W_{11},$$
$$\varPhi_{13}=P_1B-M_1+M_3^{\mathrm{T}}-S_{13}+S_{15}+S_{23}^{\mathrm{T}}+Q_{13}+hR_{13},$$
$$\varPhi_{14}=P_1C+M_4^{\mathrm{T}}+S_{12}+Q_{14}+hW_{12},$$
$$\varPhi_{15}=M_5^{\mathrm{T}}+S_{34}-S_{15},$$
$$\varPhi_{16}=M_6^{\mathrm{T}}+S_{14}-hW_{12}, \varPhi_{17}=M_7^{\mathrm{T}}+S_{33},$$
$$\varPhi_{18}=M_8^{\mathrm{T}}+S_{35}, \varPhi_{19}=-M_1-hW_{11},$$
$$\varPhi_{22}=-P_2-P_2^{\mathrm{T}}+Q_{22}+hR_{22},$$
$$\varPhi_{23}=P_2B-M_2+S_{12}+Q_{23}+hR_{23}+W_{12},$$
$$\varPhi_{24}=P_2C+Q_{24}, \varPhi_{25}=S_{14}-hW_{12},$$
$$\varPhi_{27}=S_{13}-hW_{11},$$
$$\varPhi_{33}=-Q_{11}-M_3-M_3^{\mathrm{T}}+Q_{33}+hR_{33}-$$

$S_{23}-S_{23}^{\mathrm{T}}+S_{25}+S_{25}^{\mathrm{T}}$,

$\Phi_{34}=-M_4^{\mathrm{T}}-Q_{12}+Q_{34}+W_{22}+S_{22}$,

$\Phi_{35}=-M_5^{\mathrm{T}}-S_{25}-S_{34}+S_{45}^{\mathrm{T}}-Q_{13}$,

$\Phi_{36}=S_{24}-M_6^{\mathrm{T}}-Q_{14}-W_{22}$,

$\Phi_{37}=-S_{33}+S_{35}^{\mathrm{T}}-M_7^{\mathrm{T}}$,

$\Phi_{38}=S_{55}-S_{35}-M_8^{\mathrm{T}}$,

$\Phi_{39}=-M_3-W_{12}^{\mathrm{T}}, \Phi_{44}=-Q_{22}+Q_{44}$,

$\Phi_{45}=S_{24}-Q_{23}-W_{22}$,

$\Phi_{47}=-W_{12}^{\mathrm{T}}+S_{23}$,

$\Phi_{55}=-Q_{33}-S_{45}-S_{45}^{\mathrm{T}}$,

$\Phi_{56}=-Q_{34}+S_{44}+W_{22}$,

$\Phi_{59}=-M_5-W_{12}^{\mathrm{T}}, \Phi_{67}=S_{34}^{\mathrm{T}}+W_{12}^{\mathrm{T}}$,

$\Phi_{79}=-1/hR_{12}-M_7+W_{11}$,

$\Phi_{89}=-1/h\times R_{23}^{\mathrm{T}}-M_8$.

证 注$x_t=x(t+\theta), -2h\leqslant\theta\leqslant 0$, 并且Lyapunov-Krasovskii 泛函如下:

$$V(x_t):=\begin{bmatrix} x(t)\\ x(t-h)\\ \int_{t-h}^{t}x(s)\,\mathrm{d}s\\ x(t-2h)\\ \int_{t-2h}^{t-h}x(s)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}} S \begin{bmatrix} x(t)\\ x(t-h)\\ \int_{t-h}^{t}x(s)\,\mathrm{d}s\\ x(t-2h)\\ \int_{t-2h}^{t-h}x(s)\,\mathrm{d}s\end{bmatrix} + \int_{-h}^{0}\int_{t+\theta}^{t}\begin{bmatrix} x(s)\\ \dot{x}(s)\\ x(s-h)\end{bmatrix}^{\mathrm{T}} R \begin{bmatrix} x(s)\\ \dot{x}(s)\\ x(s-h)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta+$$

$$\begin{bmatrix} \int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,\mathrm{d}s\mathrm{d}\theta\\ \int_{t-h}^{t}\dot{x}(s-h)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}} W \begin{bmatrix} \int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,\mathrm{d}s\mathrm{d}\theta\\ \int_{t-h}^{t}\dot{x}(s-h)\,\mathrm{d}s\end{bmatrix} + \int_{t-h}^{t}\begin{bmatrix} x(s)\\ \dot{x}(s)\\ x(s-h)\\ \dot{x}(s-h)\end{bmatrix}^{\mathrm{T}} Q \begin{bmatrix} x(s)\\ \dot{x}(s)\\ x(s-h)\\ \dot{x}(s-h)\end{bmatrix}\mathrm{d}s. \tag{5}$$

注意零初始条件, 即$x(t)=0, \forall t\leqslant 0$, 根据引理1与引理2, 计算$V(x_t)$沿着系统(1)的时间导数:

$$\dot{V}(x_t)\leqslant\begin{bmatrix} x(t)\\ x(t-h)\\ \int_{t-h}^{t}x(s)\,\mathrm{d}s\\ x(t-2h)\\ \int_{t-2h}^{t-h}x(s)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}} S \begin{bmatrix} \dot{x}(t)\\ \dot{x}(t-h)\\ x(t)-x(t-h)\\ \dot{x}(t-2h)\\ x(t-h)-x(t-2h)\end{bmatrix}+$$

$$2[x^{\mathrm{T}}(t)PM_1P+\dot{x}^{\mathrm{T}}(t)PM_2P+x^{\mathrm{T}}(t-h)PM_3P+\dot{x}^{\mathrm{T}}(t-h)PM_4P+x^{\mathrm{T}}(t-2h)PM_5P+$$

$$\dot{x}^{\mathrm{T}}(t-2h)PM_6P+(\int_{t-h}^{t}x(s)\,\mathrm{d}s)^{\mathrm{T}}PM_7P+(\int_{t-h}^{t}x(s)\,\mathrm{d}s)^{\mathrm{T}}PM_8P][x(t)-x(t-h)-\int_{t-2h}^{t-h}\dot{x}(s)\,\mathrm{d}s]+$$

$$2[x^{\mathrm{T}}(t)P_1+\dot{x}P_2][-\dot{x}(t)+Ax(t)+Bx(t-h)+C\dot{x}(t-h)+D\omega(t)]+$$

$$\begin{bmatrix} x(t)\\ \dot{x}(t)\\ x(t-h)\\ \dot{x}(t-h)\end{bmatrix}^{\mathrm{T}} Q \begin{bmatrix} x(t)\\ \dot{x}(t)\\ x(t-h)\\ \dot{x}(t-h)\end{bmatrix} - \begin{bmatrix} x(t-h)\\ \dot{x}(t-h)\\ x(t-2h)\\ \dot{x}(t-2h)\end{bmatrix}^{\mathrm{T}} Q \begin{bmatrix} x(t-h)\\ \dot{x}(t-h)\\ x(t-2h)\\ \dot{x}(t-2h)\end{bmatrix} + h\begin{bmatrix} x(t)\\ \dot{x}(t)\\ x(t-h)\end{bmatrix}^{\mathrm{T}} R \begin{bmatrix} x(t)\\ \dot{x}(t)\\ x(t-h)\end{bmatrix} -$$

$$\begin{bmatrix} \int_{t-h}^{t}x(s)\,\mathrm{d}s\\ \int_{t-h}^{t}\dot{x}(s)\,\mathrm{d}s\\ \int_{t-2h}^{t-h}x(s)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}} 1/hR \begin{bmatrix} \int_{t-h}^{t}x(s)\,\mathrm{d}s\\ \int_{t-h}^{t}\dot{x}(s)\,\mathrm{d}s\\ \int_{t-2h}^{t-h}x(s)\,\mathrm{d}s\end{bmatrix} 2\begin{bmatrix} hx(t)-\int_{t-h}^{t}x(s)\,\mathrm{d}s\\ x(t-h)-x(t-2h)\end{bmatrix}^{\mathrm{T}} W \begin{bmatrix} h\dot{x}(t)-\int_{t-h}^{t}\dot{x}(s)\,\mathrm{d}s\\ \dot{x}(t-h)-\dot{x}(t-2h)\end{bmatrix}=\zeta^{\mathrm{T}}\Omega\zeta,$$

式中:

$$\zeta=\Big[x^{\mathrm{T}}(t)\quad \dot{x}^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-h)\quad \dot{x}^{\mathrm{T}}(t-h)\quad x^{\mathrm{T}}(t-2h)$$

$$\dot{x}(t-2h)\int_{t-h}^{t}x^{\mathrm{T}}(s)\,\mathrm{d}s\quad \int_{t-h}^{t}x^{\mathrm{T}}(s)\,\mathrm{d}s\quad \int_{t-h}^{t}x^{\mathrm{T}}(s)\,\mathrm{d}s\quad \omega^{\mathrm{T}}(t)\Big]^{\mathrm{T}},$$

$$\varOmega=\begin{bmatrix}\varPhi_{11}&\varPhi_{12}&\varPhi_{13}&\varPhi_{14}&\varPhi_{15}&\varPhi_{16}&\varPhi_{17}&\varPhi_{18}&\varPhi_{19}&P_1D\\ *&\varPhi_{22}&\varPhi_{23}&\varPhi_{24}&\varPhi_{25}&0&\varPhi_{27}&S_{15}&-M_2&P_2D\\ *&*&\varPhi_{33}&\varPhi_{34}&\varPhi_{35}&\varPhi_{36}&\varPhi_{37}&\varPhi_{38}&\varPhi_{39}&0\\ *&*&*&\varPhi_{44}&\varPhi_{45}&-Q_{24}&\varPhi_{47}&S_{25}&-M_4&0\\ *&*&*&*&\varPhi_{55}&\varPhi_{56}&-S_{35}^{\mathrm{T}}&-S_{55}&\varPhi_{59}&0\\ *&*&*&*&*&-Q_{44}&\varPhi_{67}&S_{45}&-M_6&0\\ *&*&*&*&*&*&-1/hR_{11}&-1/hR_{13}&\varPhi_{79}&0\\ *&*&*&*&*&*&*&-1/hR_{33}&\varPhi_{89}&0\\ *&*&*&*&*&*&*&*&-1/hR_{22}&0\\ *&*&*&*&*&*&*&*&*&0\end{bmatrix}.$$

这里$\varPhi_{ij}(i,j=1,\cdots,9)$如式(4)所定义. 根据Lyapunov稳定性定理, 系统(1)的绝对稳定的充分条件为:

$$\dot V(x_t)\leqslant \zeta^{\mathrm{T}}\varOmega\zeta<0, \tag{6}$$

根据式(3), 可以获得下列不等式:

$$\omega^{\mathrm{T}}(t)\omega(t)+\omega^{\mathrm{T}}(t)[Mx(t)+Nx(t-h)]<0. \tag{7}$$

很容易发现, 如果存在$\epsilon>0$使得下列不等式存在时,

$$\zeta^{\mathrm{T}}\varOmega\zeta+2\epsilon\{-\omega^{\mathrm{T}}(t)\omega(t)-\omega^{\mathrm{T}}(t)[Mx(t)+Nx(t-h)]\}<0, \tag{8}$$

则由S-procedure, 即有

$$\zeta^{\mathrm{T}}\varPhi_0\zeta<0.$$

因此最终获得了系统(1)的绝对稳定性条件是不等式(4)成立.　　证毕.

注 1　在定理1中, 当$D=0$时, 就得到了稳定性条件.

考虑满足更普通的扇形条件$[K_1,K_2]$的扇形有界的非线性$\varphi(t,z(t))$, 在这里记$\widehat\varphi(t,z(t))$为:

$$\widehat\varphi(t,z(t))=\varphi(t,z(t))-K_1z(t), \tag{9}$$

因此利用式(9)的变换, 原始系统可以改写为下列形式:

$$\begin{cases}\dot x(t)=(A-DK_1M)x(t)+(B-DK_1N)x(t-\\ \qquad h)+C\dot x(t-h)+D\widehat\omega(t),\forall t\geqslant t_0,\\ \widehat\omega(t)=-\widehat\varphi(t,z(t)).\end{cases} \tag{10}$$

其余部分如公式(1)所示.

把扇形$[K_1,K_2]$换为$[0,K_2-K_1]$. 对于$\forall\in[0,\infty),\forall z(t)\in\mathbb{R}^n$,$\widehat\varphi(t,z(t))$满足下式:

$$\widehat\varphi^{\mathrm{T}}(t,z(t))[\widehat\varphi(t,z(t))-(K_2-K_1)z(t)]\leqslant 0, \tag{11}$$

考虑系统(10), 再根据定理1, 可以推得下列推论:

推论 1　在满足定理1的条件下, 有下列线性矩阵不等式成立:

$$\widehat\varPhi=\begin{bmatrix}\widehat\varPhi_{11}&\widehat\varPhi_{12}&\widehat\varPhi_{13}&\varPhi_{14}&\varPhi_{15}&\varPhi_{16}&\varPhi_{17}&\varPhi_{18}&\varPhi_{19}&-\epsilon M^{\mathrm{T}}(K_2-K_1)^{\mathrm{T}}+P_1D\\ *&\varPhi_{22}&\widehat\varPhi_{23}&\varPhi_{24}&\varPhi_{25}&0&\varPhi_{27}&S_{15}&-M_2&P_2D\\ *&*&\varPhi_{33}&\varPhi_{34}&\varPhi_{35}&\varPhi_{36}&\varPhi_{37}&\varPhi_{38}&\varPhi_{39}&-\epsilon N^{\mathrm{T}}(K_2-K_1)^{\mathrm{T}}\\ *&*&*&\varPhi_{44}&\varPhi_{45}&-Q_{24}&\varPhi_{47}&S_{25}&-M_4&0\\ *&*&*&*&\varPhi_{55}&\varPhi_{56}&-S_{35}^{\mathrm{T}}&-S_{55}&\varPhi_{59}&0\\ *&*&*&*&*&-Q_{44}&\varPhi_{67}&S_{45}&-M_6&0\\ *&*&*&*&*&*&-1/hR_{11}&-1/hR_{13}&\varPhi_{79}&0\\ *&*&*&*&*&*&*&-1/hR_{33}&\varPhi_{89}&0\\ *&*&*&*&*&*&*&*&-1/hR_{22}&0\\ *&*&*&*&*&*&*&*&*&-2\epsilon I\end{bmatrix}<0, \tag{12}$$

式中:

$$\widehat\varPhi_{11}=M_1+M_1^{\mathrm{T}}+P_1(A-DK_1M)+(A-DK_1M)^{\mathrm{T}}P_1+Q_{11}+hR_{11}+S_{13}+S_{13}^{\mathrm{T}},$$
$$\widehat\varPhi_{12}=S_{11}-P_1+(A-DK_1M)^{\mathrm{T}}P_2+M_2^{\mathrm{T}}+Q_{12}+hR_{12}+h^2W_{11},$$
$$\widehat\varPhi_{13}=P_1(B-DK_1N)-M_1+M_3^{\mathrm{T}}-S_{13}+S_{15}+S_{23}^{\mathrm{T}}+Q_{13}+hR_{13},$$

$$\widehat{\varPhi}_{23} = P_2(B - DK_1N) - M_2 + S_{12} + Q_{23} + hR_{23} + W_{12},$$

且其他的$\varPhi_{ij}(i,j = 1,\cdots,9)$ 如(4)所定义. 那么带有满足(2)的非线性的系统(10)是绝对稳定的.

考虑带有满足式(3)的扇形有界的非线性$\varphi(t,z(t))$的不确定中立型时滞系统, 如下所描述:

$$\begin{aligned}\dot{x}(t) = &(A+\Delta A(t))x(t)+(B+\Delta B(t))x(t-h)+\\ &(C+\Delta C(t))\dot{x}(t-h)+(D+\Delta D(t))\omega(t),\\ &\forall t \geqslant t_0,\end{aligned} \tag{13}$$

其余部分如公式(1)所示.

这里考虑如下所描述的范数有界的不确定性:

$$\begin{aligned}&[\Delta A(t)\quad \Delta B(t)\quad \Delta C(t)\quad \Delta D(t)] =\\ &HF(t)[\varPhi_a\quad \varPhi_b\quad \varPhi_c\quad \varPhi_d].\end{aligned} \tag{14}$$

这里$F(t)$是未知的可以是时不变的矩阵且满足下式:

$$\sigma_{\max}(F(t)) \leqslant 1, \tag{15}$$

且$H,\varPhi_a,\varPhi_b,\varPhi_c$和$\varPhi_d$是已知的实矩阵.

假设 2

$$\parallel C \parallel + \parallel H \parallel\parallel \varPhi_c \parallel \leqslant 1.$$

利用前面提到的方法和引理2, 能够推得下面的结论.

推论 2 在假设2的情况下, 在满足定理1的条件下, 如果存在$\lambda > 0$, 有下列线性矩阵不等式成立:

$$\bar{\varPhi}=\begin{bmatrix}
\bar{\varPhi}_{11} & \varPhi_{12} & \bar{\varPhi}_{13} & \bar{\varPhi}_{14} & \varPhi_{15} & \varPhi_{16} & \varPhi_{17} & \varPhi_{18} & \varPhi_{19} & -\epsilon M^{\mathrm{T}}K^{\mathrm{T}}+P_1D+\lambda\varPhi_a^{\mathrm{T}}\varPhi_d & P_1H\\
* & \varPhi_{22} & \varPhi_{23} & \varPhi_{24} & \varPhi_{25} & 0 & \varPhi_{27} & S_{15} & -M_2 & P_2D & P_2H\\
* & * & \bar{\varPhi}_{33} & \bar{\varPhi}_{34} & \varPhi_{35} & \varPhi_{36} & \varPhi_{37} & \varPhi_{38} & \varPhi_{39} & -\epsilon N^{\mathrm{T}}K^{\mathrm{T}}\lambda\varPhi_b^{\mathrm{T}}\varPhi_d & 0\\
* & * & * & \bar{\varPhi}_{44} & \varPhi_{45} & -Q_{24} & \varPhi_{47} & S_{25} & -M_4 & \lambda\varPhi_c^{\mathrm{T}}\varPhi_d & 0\\
* & * & * & * & \varPhi_{55} & \varPhi_{56} & -S_{35}^{\mathrm{T}} & -S_{55} & \varPhi_{59} & 0 & 0\\
* & * & * & * & * & -Q_{44} & \varPhi_{67} & S_{45} & -M_6 & 0 & 0\\
* & * & * & * & * & * & -1/hR_{11} & -1/hR_{13} & \varPhi_{79} & 0 & 0\\
* & * & * & * & * & * & * & -1/hR_{33} & \varPhi_{89} & 0 & 0\\
* & * & * & * & * & * & * & * & -1/hR_{22} & 0 & 0\\
* & * & * & * & * & * & * & * & * & -2\epsilon I+\lambda\varPhi_d^{\mathrm{T}}\varPhi_d & 0\\
* & * & * & * & * & * & * & * & * & * & -\lambda I
\end{bmatrix}<0, \tag{16}$$

式中:

$$\begin{aligned}&\bar{\varPhi}_{11} = \varPhi_{11} + \lambda\varPhi_a^{\mathrm{T}}\varPhi_a, \bar{\varPhi}_{13} = \varPhi_{13} + \lambda\varPhi_a^{\mathrm{T}}\varPhi_b,\\ &\bar{\varPhi}_{14} = \varPhi_{14} + \lambda\varPhi_a^{\mathrm{T}}\varPhi_c, \bar{\varPhi}_{33} = \varPhi_{33} + \lambda\varPhi_b^{\mathrm{T}}\varPhi_b,\\ &\bar{\varPhi}_{34} = \varPhi_{34} + \lambda\varPhi_b^{\mathrm{T}}\varPhi_c, \bar{\varPhi}_{44} = \varPhi_{44} + \lambda\varPhi_c^{\mathrm{T}}\varPhi_c.\end{aligned}$$

且其他的$\varPhi_{ij}(i,j = 1,\cdots,9)$如式(4)所定义. 那么带有满足(3)的非线性的系统(13)是绝对稳定的.

证 将不等式(4)中系统矩阵A,B,C与D分别用相应的$A+\Delta A(t), B+\Delta B(t), C+\Delta C(t)$与$D+\Delta D(t)$代替, 得到的不等式与下面的不等式等价:

$$\varPhi_0 + \eta F\theta + \theta^{\mathrm{T}}F^{\mathrm{T}}\eta^{\mathrm{T}} < 0. \tag{17}$$

这里:

$$\begin{aligned}&\eta = [H^{\mathrm{T}}P_1\ H^{\mathrm{T}}P_2\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0],\\ &\theta = [\varPhi_a\quad 0\quad \varPhi_b\quad \varPhi_c\quad 0\quad 0\quad 0\quad 0\quad 0\quad \varPhi_d].\end{aligned}$$

依据引理2, 对于(13)保证(17)成立的充分条件是存在一个正数$\lambda > 0$使得

$$\varPhi_0 + \lambda^{-1}\eta\eta^{\mathrm{T}} + \lambda\theta^{\mathrm{T}}\theta < 0. \tag{18}$$

根据Schur补性质[6], 式(18)与式(16)是等价的.

证毕.

4 数值算例(Numerical examples)

例 1 考虑式(13)中的不确定中立型时滞系统, 其中的系统矩阵如下所示:

$$A = \begin{bmatrix}-2 & -1\\ 0.5 & 0.2\end{bmatrix}, B = \begin{bmatrix}0.5 & 1\\ -0.1 & -0.8\end{bmatrix},$$

$$C = \begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, D = \begin{bmatrix}-0.5 & 0\\ 0 & 0.2\end{bmatrix},$$

$$M = \begin{bmatrix}0.4 & 0\\ 0 & 0.5\end{bmatrix}, N = \begin{bmatrix}0.2 & 0\\ 0 & 0.3\end{bmatrix},$$

$$\varPhi_a = [0\quad 0.5], \varPhi_b = [0.1\quad 0.2],$$

$$\varPhi_c = [0.1\quad 0], \varPhi_d = 0.1,$$

$$K=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix},H=\begin{bmatrix}0.1\\ 0\end{bmatrix}.$$

根据推论2, 可得此系统最大允许时滞为$h=1.16$. 这说明本文中所的结论是可行的.

例 2 考虑式(1)中$D=0$的中立型时滞系统

$$A=\begin{bmatrix}-1.7073 & 0.6856\\ 0.2279 & -0.6368\end{bmatrix},$$

$$B=\begin{bmatrix}-2.5026 & -1.0540\\ -0.1856 & -1.5715\end{bmatrix},$$

$$C=\begin{bmatrix}0.0558 & 0.0360\\ 0.2747 & -0.1084\end{bmatrix}.$$

根据文献[7,8]中所使用的方法, 最大的允许时滞相应的分别为0.6189, 0.5735, 而根据本文中的结论最大的允许时滞h是0.7510. 因此, 对于这个例子来说, 本文所得到的时滞相关的稳定性条件的保守性较低.

参考文献(References):

[1] HAN Q L. Absolute stability of time-delay systems with sector-bounded nonlinearity[J]. *Automatica*, 2005, 41(10): 2171 – 2176.

[2] KHALIL H K. *Nonlinear Systems*[M]. Upper Saddle River, New Jersey: Prentice-Hall, 1992.

[3] YAKUBOVICH VA. *Procedure in Nonlinear Control Theory*[M]. Vestnik Leningradskogo Universitete, Ser Matematika, 1(13): 62 – 77.

[4] GU K. An integral inequality in the stability problem of time-delay systems[C] //*Proceedings of the 39th IEEE Conference on Decision and Control*. Sydney, Australia: [s.n.], 2805 – 2810.

[5] XIE L. Output feedback control of systems with parameter uncertainty[J]. *International Journal of Control*, 1996, 63(9): 741 – 750.

[6] BOYD S, GHAOUI LE, FERON E, et al. *Linear Matrix Inequalities in Systems and Control*[M]. Phliladelphia: SIAM, 1994.

[7] HE Y, WANG GQ, LIN C, et al. Augmented Lyapunov functional and delay-dependent stability criteria for neutral systems[J]. *International Journal of Robust and Nonlinear Control*, 2005, 15(18): 923 – 933.

[8] XU S Y, LAM J, ZOU Y. Further results on delay-dependent robust stability conditions of uncertain neutral systems[J]. *International Journal of Robust and Nonlinear Control*, 2005, 15(5): 233 – 246.

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