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严格反馈互联系统分散式backstepping自适应迭代学习控制

孙丽丽[1,2†], 吴铁军[1]

(1. 浙江大学 控制科学与工程学系, 浙江 杭州 310027; 2. 盐城工学院 电气工程学院, 江苏 盐城 224051)

摘要: 基于Lyapunov分析方法, 针对具有严格反馈形式的非线性互联系统, 本文设计了一种分散式backstepping自适应迭代学习控制器. 子系统之间的互联项为所有子系统输出项线性有界, 为每个子系统设计的控制器仅采用该子系统的信息, 不需要子系统之间相互传递信息. 在控制器中, 引入在时间轴和迭代轴上同时更新的自适应参数, 以补偿子系统之间的互联项影响. 通过采用本文给出的控制器, 可使得每个子系统的输出跟踪相应的参考模型输出, 仿真结果验证了本文算法的有效性.

关键词: 分散式控制; 互联非线性系统; backstepping自适应迭代学习控制; 严格反馈互联系统

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Decentralized backstepping adaptive iterative learning control for a class of interconnected nonlinear systems with strict feedback form

SUN Li-li[1,2†], WU Tie-jun[1]

(1. Department of Control Science and Engineering, Zhejiang University, Hangzhou Zhejiang 310027, China;
2. School of Electrical Engineering, Yancheng Institute of Technology, Yancheng Jiangsu 224051, China)

Abstract: Based on the Lyapunov synthesis approach, a decentralized backstepping adaptive iterative learning control scheme is proposed for a class of interconnected nonlinear systems of strict feedback form. The interconnections among subsystems are considered to be bounded by first order polynomials of subsystem outputs. The proposed controller of each subsystem depends only on local state variables without any information exchange with other subsystems. The adaptive parameters are updated along both iteration axis and time one to counter the effects of the interconnections. It is shown that by using the proposed decentralized controller, the outputs of the subsystems can track the desired reference outputs iteratively. Our simulation results show that the output tracking error of each subsystem converges along the iterative axis.

Key words: decentralized control; interconnected nonlinear system; backstepping adaptive iterative learning control; strict feedback interconnected system

1 引言(Introduction)

在现实世界中, 许多物理系统如机械生产系统、钢铁冶炼系统和石油化工系统都是由低维子系统互相连接组成. 这种由互联子系统构成的大系统通常采用分散式控制方案, 分散式的控制是将控制器施加在子系统上, 不需要在各子系统之间传递信息. 这种分散式控制方案具有可靠性高, 对外部扰动反映快速和较少的系统调节复杂性等优点. 针对具有未知非线性动态的互联系统的分散式控制涌现大量研究成果. 如文献[1]针对具有未知非线性互联项的互联大系统设计了分散式自适应控制. 文献[2]针对具有未建模动态的非线性严格反馈互联系统, 采用引入修正动态变量处理未建模动态和非线性自适应控制处理互联项相结合的方法. 在文献[3]中, 在分散式控制器中引入了非线性参数化神经网络控制. 文献[4–6]针对具有输入非线性(死区非线性和磁滞非线性)的严格反馈互联系统, 采用分散式backstepping技术与自适应参数控制相结合的方法实现了对非线性互联系统的调节控制.

迭代学习控制是处理系统复杂性和不确定性的有效手段之一, 由于传统的P型或PD型迭代学习控制在处理系统不确定性时具有一定的局限性, 自适应迭代学习控制概念被提出并得到广泛应用. 互联大系统在工业现代化和人类社会生产生活实践中具有广泛的应用前景, 近年来, 国内外研究工作者提出采用迭代学习控制方法来分析和控制可重复运行互联大系统, 以改善互联系统的动态品质, 如抑制系统超调, 加快响应速度, 缩短过渡时间等等. 文献[7]采用λ–范数的分析方法给出了D型分散式迭代学习控制, 但并没有

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†通信作者. E-mail: sunlily@126.com; Tel.: +86 18761875525.

考虑非线性互联项的影响. 文献[8]给出了一种基于模糊系统的分散式自适应迭代学习控制, 其中的互联项由模糊系统近似, 通常需要根据经验设计大量的模糊规则. 传统的PD型和高阶PD型分散式迭代学习控制在文献[9]中提出, 但是其迭代学习控制参数的选择标准需要基于整个互联系统, 在实际互联系统中很难得到整个系统的所有信息. 文献[10]针对一类不确定互联系统提出了分散式模型参考自适应迭代学习控制. 文献[11–12]给出了基于backstepping技术的迭代学习控制算法, 但这两种方法均属于集中式算法, 不适用于具有严格反馈形式的非线性互联系统.

本文针对具有严格反馈形式的非线性互联系统, 设计了分散式backstepping自适应迭代学习控制器. 每个子控制器仅采用该子系统的状态和输出信息, 不需要子系统之间相互传递信息. 引入了在时间轴和迭代轴上同时更新的自适应参数, 以补偿子系统之间的互联项影响. 仿真结果表明, 使用本文提出分散式backstepping自适应迭代学习控制算法, 可使得每个子系统的输出在迭代方向上渐进跟踪相应的参考模型输出.

2 问题描述(Problem statement)

考虑如下由N个子系统互联组成的严格反馈非线性互联系统, 其中子系统i的动态方程为

$$\begin{cases} \dot{y}_i^k = \begin{bmatrix} \dot{y}_{i1}^k \\ \vdots \\ \dot{y}_{i,n_i-1}^k \\ \dot{y}_{i,n_i}^k \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} y_{i1}^k \\ \vdots \\ y_{i,n_i-1}^k \\ y_{i,n_i}^k \end{bmatrix} + \\ \qquad \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} (x_{i1}^k + \bar{\theta}_{i0}^{\mathrm{T}} \gamma(y_1^k, \cdots, y_N^k)) = \\ \qquad A_i y_i^k + B_i(x_{i1}^k + \bar{\theta}_{i0}^{\mathrm{T}} \gamma_{i0}(y_1^k, \cdots, y_N^k)), \\ \dot{x}_{i1}^k = x_{i2}^k + \bar{\theta}_{i1}^{\mathrm{T}} \gamma_{i1}(y_1^k, \cdots, y_N^k, x_{i1}^k), \\ \quad \vdots \\ \dot{x}_{in}^k = \upsilon(x_i^k, y_i^k) + \\ \qquad \bar{\theta}_{in}^{\mathrm{T}} \gamma_{in}(y_1^k, \cdots, y_N^k, x_{i1}^k, \cdots, x_{i,n}^k) + u_i^k, \end{cases} \tag{1}$$

其中: 上标k表示互联系统在时间段$t \in [0, T]$上进行第k次重复运行, $x_i^k = (x_{i1}^k, \cdots, x_{in}^k)^{\mathrm{T}} \in \mathbb{R}^n$为子系统的状态向量, $u_i^k \in \mathbb{R}$为子系统i的控制输入向量, $y_i^k = (y_{i1}^k, \cdots, y_{i,n_i}^k)^{\mathrm{T}} \in \mathbb{R}^{n_i}$为子系统$i$的控制输出向量; $\bar{\theta}_{ij} \in \mathbb{R}^{p_{ij}}$为子系统$i$的未知常数向量; $\gamma_{ij}(y_1^k, \cdots, y_N^k, x_{i1}^k, \cdots, x_{i,j}^k) \in \mathbb{R}^{p_{ij}}$为子系统$i$与其他子系统之间的非线性互联项. 子系统$i$的参考模型为

$$\begin{cases} \dot{y}_{\mathrm{m}i} = A_i y_{\mathrm{m}i} + B_i x_{\mathrm{m}i1}, \\ \dot{x}_{\mathrm{m}i1}^k = x_{\mathrm{m}i2}, \\ \quad \vdots \\ \dot{x}_{\mathrm{m}i,n}^k = K_i (y_{\mathrm{m}i}^{\mathrm{T}}, x_{\mathrm{m}i}^{\mathrm{T}})^{\mathrm{T}} + b_{\mathrm{m}i} r_{\mathrm{m}i}, \end{cases} \tag{2}$$

其中: $r_i \in \mathbb{R}^{m_i}$为有界参考输入, $x_{\mathrm{m}i} = (x_{\mathrm{m}i1}, \cdots, x_{\mathrm{m}i1})$. 定义系统在第$k$次重复运行时状态跟踪误差为$\tilde{x}_i^k = x_i^k - x_{\mathrm{m}i}$, 输出跟踪误差为$\tilde{y}_i^k = y_i^k - y_{\mathrm{m}i}$. 迭代学习控制目的是设计分散式控制器使得系统(1)的子系统输出y_i^k在迭代过程中跟踪其参考模型(2)的参考输出$y_{\mathrm{m}i}$. 由式(1)和式(2)可得跟踪误差方程:

$$\begin{cases} \dot{\tilde{y}}_i^k = \begin{bmatrix} \dot{\tilde{y}}_{i1}^k \\ \vdots \\ \dot{\tilde{y}}_{i,n_1-1}^k \\ \dot{\tilde{y}}_{i,n_i}^k \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \tilde{y}_{i1}^k \\ \vdots \\ \tilde{y}_{i,n_i-1}^k \\ \tilde{y}_{i,n_i}^k \end{bmatrix} + \\ \qquad \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} (\tilde{x}_{i1}^k + \bar{\theta}_{i0}^{\mathrm{T}} \gamma(y_1^k, \cdots, y_N^k)) = \\ \qquad A_i \tilde{y}_i^k + B_i(\tilde{x}_{i1}^k + \bar{\theta}_{i0}^{\mathrm{T}} \gamma(y_1^k, \cdots, y_N^k)), \\ \dot{\tilde{x}}_{i1}^k = \tilde{x}_{i2}^k + \bar{\theta}_{i1}^{\mathrm{T}} \gamma_{i1}(y_1^k, \cdots, y_N^k, x_{i1}^k), \\ \quad \vdots \\ \dot{\tilde{x}}_{in}^k = \upsilon(x_i^k, y_i^k) + \\ \qquad \bar{\theta}_{in}^{\mathrm{T}} \gamma_{in}(y_1^k, \cdots, y_N^k, x_{i1}^k, \cdots, x_{i,n}^k) + u_i^k - \\ \qquad K_i (y_{\mathrm{m}i}^{\mathrm{T}}, x_{\mathrm{m}i}^{\mathrm{T}})^{\mathrm{T}} - b_{\mathrm{m}i} r_{\mathrm{m}i}. \end{cases} \tag{3}$$

由式(1)可看出, 对于任意正定矩阵$Q_i \in \mathbb{R}^{n_i \times n_i}$, 存在唯一的正定对称矩阵$P_i \in \mathbb{R}^{n_i \times n_i}$满足如下的李雅普诺夫矩阵方程:

$$A_i^{\mathrm{T}} P_i + P_i A_i - 2\alpha_i P_i B_i B_i^{\mathrm{T}} P_i = -Q_i. \tag{4}$$

在给出控制器之前, 对系统(1)及其参考模型(2)做如下假设:

假设 1 子系统之间的非线性互联项$\gamma_{ij}(y_1^k, \cdots, y_N^k, x_{i1}^k, \cdots, x_{ij}^k)$为已知形式的非线性连续可微函数.

假设 2 每个子系统的初始状态误差满足可重复初始条件$y_i^k(0) = y_{\mathrm{m}i}(0)$, $x_{i1}^k(0) = x_{\mathrm{m}i1}(0)$, $x_{i2}^k(0) = x_{\mathrm{m}i2}(0), \cdots, x_{in}^k(0) = x_{\mathrm{m}i,n}(0)$.

假设 3 严格反馈互联系统(1)中各子系统之间的非线性互联项$\gamma_{ij}(y_1^k, \cdots, y_N^k, x_{i1}^k, \cdots, x_{ij}^k)$关于其他子系统输出向量$\tilde{y}_l^k$, $1 \leqslant l \leqslant N$有界, 即下式成立:

$$\|\bar{\theta}_{ij}^{\mathrm{T}} \gamma_{ij}(y_1^k, \cdots, y_N^k, x_{i1}^k, \cdots, x_{ij}^k) -$$

$$\bar{\theta}_{ij}^{\mathrm{T}}\gamma_{ij}(y_{\mathrm{m}1},\cdots,y_{\mathrm{m}N},x_{i1}^k,\cdots,x_{ij}^k)\| \leqslant \eta_{ij}(x_{i1}^k,\cdots,x_{ij}^k)\sum_{l=1}^{N}\zeta_{ijl}\|\tilde{y}_l^k\|, \tag{5}$$

其中$\eta_{i0}^k=1$.

3 迭代学习控制器设计(Iterative learning controller design)

为了简化后面章节中推导和陈述的篇幅, 定义了如下简化标记: $\gamma_{ij}^k=\gamma_{ij}(y_1^k,\cdots,y_N^k,x_{i1}^k,\cdots,x_{ij}^k)$, $\gamma_{\mathrm{m}ij}^k=\gamma_{ij}(y_{\mathrm{m}1},\cdots,y_{\mathrm{m}N},x_{i1}^k,\cdots,x_{ij}^k)$. 所提出的backstepping自适应迭代学习控制包括$n+1$步, 在第j步中, 设计出虚拟控制$u_{i,j+1}^k$, $0\leqslant j\leqslant n-1$, 在最后一步给出实际控制输入$u_i^k$.

第0步 设计第i个子系统的0阶分散式虚拟控制器u_{i1}^k:

$$u_{i1}^k=-\big(\alpha_iB_i^{\mathrm{T}}P_i\tilde{y}_i^k+(\hat{\theta}_{i0}^k)^{\mathrm{T}}f_{i0}^k+\hat{\varsigma}_{i0}^kg_{i0}^k\big), \tag{6}$$

其中: $\alpha_i>0$为设计参数, $f_{i0}^k=\gamma_{\mathrm{m}i0}^k$, $g_{i0}^k=2^{-1}B_i^{\mathrm{T}}P_i\tilde{y}_i^k$, $\hat{\theta}_{i0}^k$和$\hat{\varsigma}_{i0}^k$为参数θ_{i0}和ς的估计值. 定义

$$\tilde{z}_{ij}^k=\tilde{x}_{ij}^k-u_{ij}^k, \tag{7}$$

因此

$$\dot{\tilde{y}}_i^k=A_i\tilde{y}_i^k+B_i(\tilde{z}_{i1}^k+u_{i1}^k+\bar{\theta}_{i0}^{\mathrm{T}}\gamma_{i0}^k). \tag{8}$$

在本文中定义$\tilde{\theta}_{ij}^k=\hat{\theta}_{ij}^k-\theta_{ij}$和$\tilde{\varsigma}_{ij}^k=\hat{\varsigma}_{ij}^k-\varsigma$, $1\leqslant i\leqslant N$, $0\leqslant j\leqslant n$, 其中$\hat{\varsigma}_{ij}^k$为控制增益ς的估计参数, ς用来消除子系统之间互联项影响, 其定义将在后面的推导过程中给出. 设计跟踪误差方程(3)的0阶子系统组合Lyapunov函数

$$V_0^k(t)=\frac{1}{2}\sum_{i=1}^{N}(\tilde{y}_i^k)^{\mathrm{T}}\tilde{y}_i^k, \tag{9}$$

令$\theta_{i0}=\bar{\theta}_{i0}$, 得到$V_0^k$的导数

$$\dot{V}_0^k=\frac{1}{2}\sum_{i=1}^{N}\big[(\tilde{y}_i^k)^{\mathrm{T}}(A_i^{\mathrm{T}}P_i+P_iA_i-2\alpha_iP_iB_iB_i^{\mathrm{T}}P_i)\tilde{y}_i^k+2(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i(\tilde{z}_{i1}^k+\theta_{i0}^{\mathrm{T}}\gamma_{i0}^k-(\hat{\theta}_{i0}^k)^{\mathrm{T}}\gamma_{\mathrm{m}i0}^k)-\hat{\varsigma}_{i0}^k\,\big\|(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i\big\|^2\big]. \tag{10}$$

在式(10)中, 存在

$$\theta_{i0}^{\mathrm{T}}\gamma_{i0}^k-(\hat{\theta}_{i0}^k)^{\mathrm{T}}\gamma_{\mathrm{m}i0}^k=\theta_{i0}^{\mathrm{T}}\gamma_{i0}^k-\theta_{i0}^{\mathrm{T}}\gamma_{\mathrm{m}i0}^k+\theta_{i0}^{\mathrm{T}}\gamma_{\mathrm{m}i0}^k-(\hat{\theta}_{i0}^k)^{\mathrm{T}}\gamma_{\mathrm{m}i0}^k. \tag{11}$$

利用式(4)和式(11), 式(10)可写为

$$\dot{V}_0^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\big[-\lambda_{\mathrm{m}}(Q_i)\|\tilde{y}_i^k\|^2+2(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i\big(\tilde{z}_{i1}^k+\theta_{i0}^{\mathrm{T}}\gamma_{i0}^k-\theta_{i0}^{\mathrm{T}}\gamma_{\mathrm{m}i0}^k\big)-\hat{\varsigma}_{i0}^k\,\big\|(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i\big\|^2-2(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i(\tilde{\theta}_{i0}^k)^{\mathrm{T}}\gamma_{\mathrm{m}i0}^k\big], \tag{12}$$

其中$\lambda_{\mathrm{m}}(Q_i)$为矩阵$Q_i$的最小特征值. 根据式(5), 可知下式成立:

$$\big\|2\sum_{i=1}^{N}(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i\theta_{i0}^{\mathrm{T}}(\gamma_{i0}^k-\gamma_{\mathrm{m}i0}^k)\big\|\leqslant 2\sum_{i=1}^{N}\big[\|(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i\|\sum_{l=1}^{N}(\varsigma_l')^{\frac{1}{2}}(\varsigma_l')^{-\frac{1}{2}}\zeta_{i0l}\|\tilde{y}_l^k\|\big]\leqslant \sum_{l=1}^{N}\big[\varsigma\|(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i\|^2+(\varsigma_i')^{-1}d_{i0}\|\tilde{y}_i^k\|^2\big], \tag{13}$$

其中:

$$\varsigma=\sum_{l=1}^{N}\varsigma_l', \tag{14}$$

$$d_{i0}=\sum_{l=1}^{N}(\zeta_{l0i})^2. \tag{15}$$

把式(13)代入式(12)可得

$$\dot{V}_0^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\big\{-\big[\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}d_{i0}\big]\|\tilde{y}_i^k\|^2+2(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i\tilde{z}_{i1}^k-2(\tilde{\theta}_{i0}^k)^{\mathrm{T}}\tau_{i0}^k-2\tilde{\varsigma}_{i0}^k\epsilon_{i0}^k\big\}, \tag{16}$$

其中:

$$\tau_{i0}^k=(\tilde{y}_i^k)^{\mathrm{T}}P_iB_if_{i0}^k, \tag{17}$$

$$\epsilon_{i0}^k=(\tilde{y}_i^k)^{\mathrm{T}}P_iB_ig_{i0}^k. \tag{18}$$

在式(6)中, 参数$\hat{\theta}_{i0}^k$和$\hat{\varsigma}_{i0}^k$在迭代方向上更新律

$$\gamma\begin{bmatrix}\dot{\hat{\theta}}_{i0}^k\\ \dot{\hat{\varsigma}}_{i0}^k\end{bmatrix}=-\begin{bmatrix}\hat{\theta}_{i0}^k\\ \hat{\varsigma}_{i0}^k\end{bmatrix}+(1-\gamma)\begin{bmatrix}\hat{\theta}_{i0}^{k-1}\\ \hat{\varsigma}_{i0}^{k-1}\end{bmatrix}+\alpha\begin{bmatrix}\tau_{i0}^k\\ \epsilon_{i0}^k\end{bmatrix}, \tag{19}$$

其中: $0<\gamma<1$, $\alpha>0$为设计参数, $\hat{\theta}_{i0}^k(0)=0$, $\hat{\varsigma}_{i0}^k(0)=0$. 当$k=0$时, $\gamma\dot{\hat{\theta}}_{i0}^0=-\hat{\theta}_{i0}^0+\alpha\tau_{i0}^0$, $\gamma\dot{\hat{\varsigma}}_{i0}^0=-\hat{\varsigma}_{i0}^0+\alpha\epsilon_{i0}^0$. 关于变量$\tilde{y}_i^k$和$\tilde{z}_{i1}^k$的动态方程可写为

$$\dot{\tilde{y}}_i^k=A_i\tilde{y}_i^k+B_i(\tilde{z}_{i1}^k+u_{i1}^k+\bar{\theta}_{i0}^{\mathrm{T}}\gamma_{i0}^k), \tag{20}$$

$$\dot{\tilde{z}}_{i1}^k=\dot{\tilde{x}}_{i1}^k-\dot{u}_{i1}^k=\tilde{x}_{i2}^k+\bar{\theta}_{i1}^{\mathrm{T}}\gamma_{i1}^k-\frac{\partial u_{i1}^k}{\partial\tilde{y}_i^k}\dot{\tilde{y}}_i^k-\frac{\partial u_{i1}^k}{\partial\hat{\theta}_{i0}^k}\dot{\hat{\theta}}_{i0}^k-\frac{\partial u_{i1}^k}{\partial\hat{\varsigma}_{i0}^k}\dot{\hat{\varsigma}}_{i0}^k-(\hat{\theta}_{i0}^k)^{\mathrm{T}}\sum_{j=1}^{N}\big(\frac{\partial\gamma_{\mathrm{m}i0}^k}{\partial y_{\mathrm{m}j}}\big)\dot{y}_{\mathrm{m}j}=\tilde{z}_{i2}^k+u_{i2}^k+\sum_{\iota=0}^{1}\varphi_{i1\iota}^k(\bar{\theta}_{i\iota})^{\mathrm{T}}\gamma_{i\iota}^k+\nu_{i1}^k, \tag{21}$$

其中: $\varphi_{i10}^k=-\dfrac{\partial u_{i1}^k}{\partial\tilde{y}_i^k}B_i$和$\varphi_{i11}^k=1$.

第1步 考虑下面的组合Lyapunov函数

$$V_1^k(t)=V_0^k(t)+\frac{1}{2}\sum_{i=1}^{N}(\tilde{z}_{i1}^k)^2, \tag{22}$$

对式(22)求导可得

$$\dot{V}_1^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\big\{-\big[\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}d_{i0}\big]\|\tilde{y}_i^k\|^2-2(\tilde{\theta}_{i0}^k)^{\mathrm{T}}\tau_{i0}^k-2\tilde{\varsigma}_{i0}^k\epsilon_{i0}^k+2\tilde{z}_{i1}^k\big((\tilde{y}_i^k)^{\mathrm{T}}P_iB_i+\tilde{z}_{i2}^k+u_{i2}^k+\sum_{\iota=0}^{1}\varphi_{i1\iota}^k(\bar{\theta}_{i\iota})^{\mathrm{T}}\gamma_{i\iota}^k+\nu_{i1}^k\big)\big\}. \tag{23}$$

与式(11)类似, 可得到

$$\sum_{i=1}^{N}\sum_{\iota=0}^{1}2(\tilde{z}_{i1}^k)\varphi_{i1\iota}^k(\bar{\theta}_{i\iota})^{\mathrm{T}}\gamma_{i\iota}^k=\sum_{i=1}^{N}\sum_{\iota=0}^{1}2(\tilde{z}_{i1}^k)\varphi_{i1\iota}^k(\bar{\theta}_{i\iota})^{\mathrm{T}}(\gamma_{i\iota}^k-\gamma_{\mathrm{m}i\iota}^k+\gamma_{\mathrm{m}i\iota}^k),\tag{24}$$

同时, 下式成立:

$$\sum_{i=1}^{N}\sum_{\iota=0}^{1}2|\tilde{z}_{i1}^k\varphi_{i1\iota}^k|\|(\bar{\theta}_{i\iota})^{\mathrm{T}}(\gamma_{i\iota}^k-\gamma_{\mathrm{m}i\iota}^k)\|\leqslant \sum_{i=1}^{N}\sum_{l=1}^{N}\sum_{\iota=0}^{1}2|\tilde{z}_{i1}^k\varphi_{i1\iota}^k|(\varsigma_l')^{\frac{1}{2}}(\varsigma_l')^{-\frac{1}{2}}\eta_{i\iota}^k\zeta_{i\iota l}\|\tilde{y}_l^k\|\leqslant \sum_{i=1}^{N}\sum_{\iota=0}^{1}\left[\varsigma(\tilde{z}_{i1}^k)^2\varpi_{i\iota}^k(\varphi_{i1\iota}^k)^2+(\varsigma_i')^{-1}d_{i\iota}\|\tilde{y}_i^k\|^2\right],\tag{25}$$

在式(25)中, ς由式(14)给出, $\varpi_{i\iota}^k$和$d_{i\iota}$的定义如下:

$$\varpi_{i\iota}^k=(\eta_{i\iota}^k)^2,\tag{26}$$

$$d_{i\iota}=\sum_{l=1}^{N}(\zeta_{l0i})^2.\tag{27}$$

把式(25)代入式(23)得到

$$\dot{V}_1^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\Big\{-\left[\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}(2d_{i0}+d_{i1})\right]\|\tilde{y}_i^k\|^2-2(\tilde{\theta}_{i0}^k)^{\mathrm{T}}\tau_{i0}^k-2\tilde{\varsigma}_{i0}^k\epsilon_{i0}^k+2\tilde{z}_{i1}^k\Big((\tilde{y}_i^k)^{\mathrm{T}}P_iB_i+\tilde{z}_{i2}^k+u_{i2}^k+\sum_{\iota=0}^{1}\varphi_{i1\iota}^k(\bar{\theta}_{i\iota})^{\mathrm{T}}\gamma_{\mathrm{m}i\iota}^k+\frac{\varsigma}{2}\tilde{z}_{i1}^k\sum_{\iota=0}^{1}\varpi_{i\iota}^k(\varphi_{i1\iota}^k)^2+\nu_{i1}^k\Big)\Big\},\tag{28}$$

令$\theta_{i1}=(\bar{\theta}_{i0}^{\mathrm{T}},\bar{\theta}_{i1}^{\mathrm{T}})^{\mathrm{T}}$, $f_{i1}^k=((\varphi_{i10}^k\gamma_{\mathrm{m}i0}^k)^{\mathrm{T}},(\varphi_{i11}^k\gamma_{\mathrm{m}i1}^k)^{\mathrm{T}})^{\mathrm{T}}$, $g_{i1}^k=2^{-1}\tilde{z}_{i1}^k\sum_{\iota=0}^{1}\varpi_{i\iota}^k(\varphi_{i1\iota}^k)^2$, 式(28)可写为

$$\dot{V}_1^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\Big\{-\left[\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}(2d_{i0}+d_{i1})\right]\|\tilde{y}_i^k\|^2-2(\tilde{\theta}_{i0}^k)^{\mathrm{T}}\tau_{i0}^k-2\tilde{\varsigma}_{i0}^k\epsilon_{i0}^k+2\tilde{z}_{i1}^k\big((\tilde{y}_i^k)^{\mathrm{T}}P_iB_i+z_{i2}^k+u_{i2}^k+(\theta_{i1})^{\mathrm{T}}f_{i1}^k+\varsigma g_{i1}^k+\nu_{i1}^k\big)\Big\}.\tag{29}$$

设计第i个子系统的第1阶子系统分散式虚拟控制u_{i2}^k如下:

$$u_{i2}^k=-\big(c_{i1}\tilde{z}_{i1}^k+(\tilde{y}_i^k)^{\mathrm{T}}P_iB_i+(\hat{\theta}_{i1}^k)^{\mathrm{T}}f_{i1}^k+\hat{\varsigma}_{i1}^kg_{i1}^k+\nu_{i1}^k\big),\tag{30}$$

其中$c_{i1}>0$为设计参数. 把式(30)代入式(29)可得

$$\dot{V}_1^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\Big\{-\left[\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}(2d_{i0}+d_{i1})\right]\|\tilde{y}_i^k\|^2-2\sum_{s=0}^{1}\left[(\tilde{\theta}_{is}^k)^{\mathrm{T}}\tau_{is}^k+\tilde{\varsigma}_{is}^k\epsilon_{is}^k\right]-2c_{i1}(\tilde{z}_{i1})^2+2\tilde{z}_{i1}^kz_{i2}^k\Big\},\tag{31}$$

其中:

$$\tau_{i1}^k=\tilde{z}_{i1}^kf_{i1}^k,\tag{32}$$

$$\epsilon_{i1}^k=\tilde{z}_{i1}^kg_{i1}^k.\tag{33}$$

式(30)中参数$\hat{\theta}_{i1}^k$和$\hat{\varsigma}_{i1}^k$沿着迭代方向的更新律为

$$\gamma\begin{bmatrix}\dot{\hat{\theta}}_{i1}^k\\ \dot{\hat{\varsigma}}_{i1}^k\end{bmatrix}=-\begin{bmatrix}\hat{\theta}_{i1}^k\\ \hat{\varsigma}_{i1}^k\end{bmatrix}+(1-\gamma)\begin{bmatrix}\hat{\theta}_{i1}^{k-1}\\ \hat{\varsigma}_{i1}^{k-1}\end{bmatrix}+\alpha\begin{bmatrix}\tau_{i1}^k\\ \epsilon_{i1}^k\end{bmatrix},\tag{34}$$

其中: $\hat{\theta}_{i1}^k(0)=0, \hat{\varsigma}_{i1}^k(0)=0$. 当$k=0$时, $\gamma\dot{\hat{\theta}}_{i1}^0=-\hat{\theta}_{i1}^0+\alpha\tau_{i1}^0$, $\gamma\dot{\hat{\varsigma}}_{i1}^0=-\hat{\varsigma}_{i1}^0+\alpha\epsilon_{i1}^0$.

第$j(2\leqslant j\leqslant n)$步　变量$\tilde{z}_{ij}$的动态方程为

$$\dot{\tilde{z}}_{ij}^k=\tilde{z}_{i,j+1}^k+u_{i,j+1}^k+\sum_{\iota=0}^{j}\varphi_{ij\iota}^k(\bar{\theta}_{i\iota})^{\mathrm{T}}\gamma_{i\iota}^k+\nu_{ij}^k.\tag{35}$$

考虑如下组合Lyapunov函数:

$$V_j^k(t)=V_{j-1}^k(t)+\frac{1}{2}\sum_{i=1}^{N}(\tilde{z}_{ij}^k)^2,\tag{36}$$

对V_j^k求导可得到

$$\dot{V}_j^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\Big\{-\left[\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}(jd_{i0}+\cdots+d_{i,j-1})\right]\|\tilde{y}_i^k\|^2-2\sum_{s=1}^{j-1}c_{is}(\tilde{z}_{is}^k)^2-2\sum_{s=0}^{j-1}((\tilde{\theta}_{is}^k)^{\mathrm{T}}\tau_{is}^k+\tilde{\varsigma}_{is}^k\epsilon_{is}^k)+2\tilde{z}_{ij}^k(\tilde{z}_{i,j+1}^k+\tilde{z}_{i,j-1}^k+u_{i,j+1}^k+\sum_{\iota=0}^{j}\varphi_{ij\iota}^k(\bar{\theta}_{i\iota})^{\mathrm{T}}\gamma_{i\iota}^k+\nu_{ij}^k)\Big\},\tag{37}$$

类似式(24)−(25)和式(29), $\dot{V}_j^k$可写为

$$\dot{V}_j^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\Big[-\big\{\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}[(j+1)d_{i0}+\cdots+d_{ij}]\big\}\|\tilde{y}_i^k\|^2-2\sum_{s=1}^{j-1}c_{is}(\tilde{z}_{is}^k)^2-2\sum_{s=0}^{j-1}\big((\tilde{\theta}_{is}^k)^{\mathrm{T}}\tau_{is}^k+\tilde{\varsigma}_{is}^k\epsilon_{is}^k\big)+2\tilde{z}_{ij}^k\big(\tilde{z}_{i,j+1}^k+\tilde{z}_{i,j-1}^k+u_{i,j+1}^k+(\theta_{ij})^{\mathrm{T}}f_{ij}^k+\varsigma g_{ij}^k+\nu_{ij}^k\big)\Big],\tag{38}$$

令$\theta_{ij}=(\bar{\theta}_{i0}^{\mathrm{T}},\cdots,\bar{\theta}_{ij}^{\mathrm{T}})^{\mathrm{T}}$, $g_{ij}^k=2^{-1}\tilde{z}_{ij}^k\sum_{\iota=0}^{j}\varpi_{i\iota}^k(\varphi_{ij\iota}^k)^2$, $f_{ij}^k=((\varphi_{ij0}^k\gamma_{\mathrm{m}i0})^{\mathrm{T}},\cdots,(\varphi_{ijj}^k\gamma_{\mathrm{m}ij})^{\mathrm{T}})^{\mathrm{T}}$. 设计虚拟控制律$u_{i,j+1}^k$如下:

$$u_{i,j+1}^k=-\big(c_{ij}\tilde{z}_{ij}^k+\tilde{z}_{i,j-1}^k+(\hat{\theta}_{ij}^k)^{\mathrm{T}}f_{ij}^k+\hat{\varsigma}_{ij}^kg_{ij}^k+\nu_{ij}^k\big),\tag{39}$$

其中$c_{ij}>0$为设计参数. 把式(39)代入式(38), 可得到

$$\dot{V}_j^k\leqslant\frac{1}{2}\sum_{i=1}^{N}\Big[-\big\{\lambda_{\mathrm{m}}(Q_i)-(\varsigma_i')^{-1}[(j+1)d_{i0}+\cdots+d_{ij}]\big\}\|\tilde{y}_i^k\|^2-2\sum_{s=1}^{j}c_{is}(\tilde{z}_{is}^k)^2-2\sum_{s=0}^{j}\big((\tilde{\theta}_{is}^k)^{\mathrm{T}}\tau_{is}^k+\tilde{\varsigma}_{is}^k\epsilon_{is}^k\big)+2\tilde{z}_{ij}^k\tilde{z}_{i,j+1}^k\Big],\tag{40}$$

其中:

$$\tau_{ij}^k=\tilde{z}_{ij}^kf_{ij}^k,\tag{41}$$

$$\epsilon_{ij}^k=\tilde{z}_{ij}^kg_{ij}^k.\tag{42}$$

在式(39)中, 参数$\hat{\theta}_{ij}^k$和$\hat{\varsigma}_{ij}^k$沿着迭代方向上的控制律设计为

$$\gamma\begin{bmatrix}\dot{\hat{\theta}}_{ij}^k\\ \dot{\hat{\varsigma}}_{ij}^k\end{bmatrix}=-\begin{bmatrix}\hat{\theta}_{ij}^k\\ \hat{\varsigma}_{ij}^k\end{bmatrix}+(1-\gamma)\begin{bmatrix}\hat{\theta}_{ij}^{k-1}\\ \hat{\varsigma}_{ij}^{k-1}\end{bmatrix}+\alpha\begin{bmatrix}\tau_{ij}^k\\ \epsilon_{ij}^k\end{bmatrix}, \tag{43}$$

其中: $\hat{\theta}_{ij}^k(0)=0, \hat{\varsigma}_{ij}^k(0)=0$. 当 $k=0$ 时, $\gamma\dot{\hat{\theta}}_{ij}^0=-\hat{\theta}_{ij}^0+\alpha\tau_{ij}^0, \gamma\dot{\hat{\varsigma}}_{ij}^0=-\hat{\varsigma}_{ij}^0+\alpha\epsilon_{ij}^0$.

第n步 变量$\tilde{z}_{in}^k$的动态方程为

$$\dot{\tilde{z}}_{in}^k=u_i^k+\upsilon_i^k+\sum_{\iota=0}^{n}\varphi_{in\iota}^k(\bar{\theta}_{i\iota})^{\mathrm T}\gamma_{i\iota}^k+\nu_{in}^k-K_i[y_{\mathrm{m}i}^{\mathrm T}\ \ x_{\mathrm{m}i}^{\mathrm T}]^{\mathrm T}-b_{\mathrm{m}i}r_{\mathrm{m}i}. \tag{44}$$

考虑如下Lyapunov函数:

$$V_n^k(t)=V_{n-1}^k(t)+\frac{1}{2}\sum_{i=1}^{N}(\tilde{z}_{in}^k)^2, \tag{45}$$

对$V_n^k(t)$求导可得

$$\begin{aligned}\dot{V}_n^k\leqslant{}&\frac{1}{2}\sum_{i=1}^{N}\{-[\lambda_{\mathrm m}(Q_i)-(\varsigma_i')^{-1}(nd_{i0}+\cdots+d_{i,n-1})]\|\tilde{y}_i^k\|^2-2\sum_{s=1}^{n-1}c_{is}(\tilde{z}_{is}^k)^2-2\sum_{s=0}^{n-1}((\tilde{\theta}_{is}^k)^{\mathrm T}\tau_{is}^k+\\&\tilde{\varsigma}_{is}^k\epsilon_{is}^k)+2\tilde{z}_{in}^k(u_i^k+\tilde{z}_{i,n-1}^k+\upsilon_i^k+\nu_{in}^k-b_{\mathrm{m}i}r_{\mathrm{m}i}+\sum_{\iota=0}^{n}\varphi_{in\iota}^k(\bar{\theta}_{i\iota})^{\mathrm T}\gamma_{i\iota}^k-K_i(y_{\mathrm{m}i}^{\mathrm T},x_{\mathrm{m}i}^{\mathrm T})^{\mathrm T})\},\end{aligned} \tag{46}$$

类似式(38), $\dot{V}_n^k(t)$可写为

$$\begin{aligned}\dot{V}_n^k\leqslant{}&\frac{1}{2}\sum_{i=1}^{N}[-\{\lambda_{\mathrm m}(Q_i)-(\varsigma_i')^{-1}[(n+1)d_{i0}+\cdots+d_{in}]\}\|\tilde{y}_i^k\|^2-2\sum_{s=1}^{n-1}c_{is}(\tilde{z}_{is}^k)^2-2\sum_{s=0}^{n-1}((\tilde{\theta}_{is}^k)^{\mathrm T}\tau_{is}^k+\\&\tilde{\varsigma}_{is}^k\epsilon_{is}^k)+2\tilde{z}_{in}^k(u_i^k+\tilde{z}_{i,n-1}^k+\upsilon_i^k+(\theta_{in})^{\mathrm T}f_{in}^k+\varsigma g_{in}^k+\nu_{in}^k-K_i(y_{\mathrm{m}i}^{\mathrm T},x_{\mathrm{m}i}^{\mathrm T})^{\mathrm T}-b_{\mathrm{m}i}r_{\mathrm{m}i})],\end{aligned} \tag{47}$$

其中: $f_{in}^k=((\varphi_{in0}^k\gamma_{\mathrm{m}i0}^k)^{\mathrm T},\cdots,(\varphi_{inn}^k\gamma_{\mathrm{m}i,n}^k)^{\mathrm T})^{\mathrm T}, \theta_{in}=(\bar{\theta}_{i0}^{\mathrm T},\cdots,\bar{\theta}_{in}^{\mathrm T})^{\mathrm T}$, $g_{in}^k=2^{-1}\tilde{z}_{in}^k\sum_{\iota=0}^{n}\varpi_{i\iota}^k(\varphi_{in\iota}^k)^2$. 第$i$个子系统的分散式控制律为

$$u_i^k=-(c_{in}\tilde{z}_{in}^k+\tilde{z}_{i,n-1}^k+\upsilon_i^k+(\hat{\theta}_{in}^k)^{\mathrm T}f_{in}^k+\hat{\varsigma}_{in}^k g_{ij}^k+\nu_{in}^k-K_i(y_{\mathrm{m}i}^{\mathrm T},x_{\mathrm{m}i}^{\mathrm T})^{\mathrm T}-b_{\mathrm{m}i}r_{\mathrm{m}i}), \tag{48}$$

其中$c_{in}>0$为设计参数. 将式(48)代入式(47)可得到

$$\dot{V}_n^k\leqslant\frac{1}{2}\sum_{i=1}^{N}[-\{\lambda_{\mathrm m}(Q_i)-(\varsigma_i')^{-1}[(n+1)d_{i0}+\cdots+d_{in}]\}\|\tilde{y}_i^k\|^2-2\sum_{s=1}^{n}c_{is}(\tilde{z}_{is}^k)^2-2\sum_{s=0}^{n}((\tilde{\theta}_{is}^k)^{\mathrm T}\tau_{is}^k+\tilde{\varsigma}_{is}^k\epsilon_{is}^k)], \tag{49}$$

其中:

$$\tau_{in}^k=\tilde{z}_{in}^k f_{in}^k, \tag{50}$$

$$\epsilon_{in}^k=\tilde{z}_{in}^k g_{in}^k. \tag{51}$$

式(48)中, 参数$\hat{\theta}_{in}^k$和$\hat{\varsigma}_{in}^k$迭代更新律为

$$\gamma\begin{bmatrix}\dot{\hat{\theta}}_{in}^k\\ \dot{\hat{\varsigma}}_{in}^k\end{bmatrix}=-\begin{bmatrix}\hat{\theta}_{in}^k\\ \hat{\varsigma}_{in}^k\end{bmatrix}+(1-\gamma)\begin{bmatrix}\hat{\theta}_{in}^{k-1}\\ \hat{\varsigma}_{in}^{k-1}\end{bmatrix}+\alpha\begin{bmatrix}\tau_{in}^k\\ \epsilon_{in}^k\end{bmatrix}, \tag{52}$$

其中: $\hat{\theta}_{in}^k(0)=0, \hat{\varsigma}_{in}^k(0)=0$. 当$k=0$时, $\gamma\dot{\hat{\theta}}_{in}^0=-\hat{\theta}_{in}^0+\alpha\tau_{in}^0, \gamma\dot{\hat{\varsigma}}_{in}^0=-\hat{\varsigma}_{in}^0+\alpha\epsilon_{in}^0$. 在式(49)中, 定义

$$2c_{i0}=\lambda_{\mathrm m}(Q_i)-(\varsigma_i')^{-1}[(n+1)d_{i0}+\cdots+d_{in}]. \tag{53}$$

因此, 当选择$\varsigma_i'>((n+1)d_{i0}+\cdots+d_{in})/\lambda_{\mathrm m}(Q_i)$时, 可使得$c_{i0}>0$. 当参数$\varsigma_i'$选定后, ς可由式(14)计算得到. 把式(53)代入式(49), 可得

$$\dot{V}_n^k\leqslant-\sum_{i=1}^{N}[c_{i0}\|\tilde{y}_i^k\|^2+\sum_{s=1}^{n}c_{is}(\tilde{z}_{is}^k)^2+\sum_{s=0}^{n}((\tilde{\theta}_{is}^k)^{\mathrm T}\tau_{is}^k+\tilde{\varsigma}_{is}^k\epsilon_{is}^k)]. \tag{54}$$

4 收敛性分析(Convergence analysis)

定理 1 考虑严格反馈互联系统(1)及其参考模型(2), 满足假设1–3, 本文设计的分散式backstepping自适应控制律(48)以及参数更新律(19)(34)和(52)可保证$\lim\limits_{k\to\infty}\int_0^T\|\tilde{y}_i^k\|^2\mathrm d\tau\leqslant\epsilon$, $\lim\limits_{k\to\infty}\int_0^T\|\tilde{z}_{ij}^k\|^2\mathrm d\tau\leqslant\epsilon, \forall t\in[0,T]$, 其中$\epsilon=(\gamma C_{\mathrm p}+\delta)/\alpha, \forall\delta>0$.

定理1的证明分为3个部分. 在第1部分中, 给出与迭代次数k相关的Lyapunov函数$V^k(t)$, 同时推导出该Lyapunov函数序列$\{V^k(t)\}$与$V^0(t)$相关. 第2部分证明$V^0(t)$在时间域$t\in[0,T]$范围内有界. 最后采用反证法得出定理1的结论.

证 第1部分: 选择如下Lyapunov函数:

$$V^k(t)=\alpha V_n^k(t)+V_{\mathrm p}^k(t), \tag{55}$$

其中$V_n^k(t)$由式(45)给出.

$$V_{\mathrm p}^k(t)=\frac{1-\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t[\|\tilde{\theta}_{ij}^k\|^2+(\tilde{\varsigma}_{ij}^k)^2]\mathrm d\tau+\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}[\|\tilde{\theta}_{ij}^k(t)\|^2+(\tilde{\varsigma}_{ij}^k(t))^2]. \tag{56}$$

$V^k(t)$的差分由下式给出:

$$\Delta V^k(t)=\alpha\Delta V_n^k(t)+\Delta V_{\mathrm p}^k(t), \tag{57}$$

其中:

$$\Delta V_n^k(t)=V_n^k(t)-V_n^{k-1}(t), \tag{58}$$

$$\Delta V_{\mathrm p}^k=V_{\mathrm p}^k(t)-V_{\mathrm p}^{k-1}(t)=$$

$$\frac{1-\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[\|\tilde\theta_{ij}^k\|^2-\|\tilde\theta_{ij}^{k-1}\|^2+(\tilde\varsigma_{ij}^k)^2-(\tilde\varsigma_{ij}^{k-1})^2\big]\mathrm{d}\tau+\gamma\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[(\tilde\theta_{ij}^k)^{\mathrm T}\dot{\hat\theta}_{ij}^k+\tilde\varsigma_{ij}^k\dot{\hat\varsigma}_{ij}^k\big]\mathrm{d}\tau+\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^k(0)\|^2+|\tilde\varsigma_{ij}^k(0)|^2\big]-\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^{k-1}\|^2+(\tilde\varsigma_{ij}^{k-1})^2\big]\leqslant$$
$$\frac{1-\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[\|\tilde\theta_{ij}^k\|^2-\|\tilde\theta_{ij}^{k-1}\|^2+(\tilde\varsigma_{ij}^k)^2-(\tilde\varsigma_{ij}^{k-1})^2\big]\mathrm{d}\tau+\gamma\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[(\tilde\theta_{ij}^k)^{\mathrm T}\dot{\hat\theta}_{ij}^k+\tilde\varsigma_{ij}^k\dot{\hat\varsigma}_{ij}^k\big]\mathrm{d}\tau+\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\theta_{ij}\|^2+\varsigma^2\big]. \tag{59}$$

将$\dot{\hat\theta}_{ij}^k$和$\dot{\hat\varsigma}_{ij}^k$代入式(59)可得到

$$\Delta V_{\mathrm p}^k\leqslant\frac{1-\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[\|\tilde\theta_{ij}^k\|^2-\|\tilde\theta_{ij}^{k-1}\|^2+(\tilde\varsigma_{ij}^k)^2-(\tilde\varsigma_{ij}^{k-1})^2\big]\mathrm{d}\tau+\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[-\|\tilde\theta_{ij}^k\|^2+(1-\gamma)(\tilde\theta_{ij}^k)^{\mathrm T}\tilde\theta_{ij}^{k-1}+\alpha(\tilde\theta_{ij}^k)^{\mathrm T}\tau_{ij}^k-\gamma(\tilde\theta_{ij}^k)^{\mathrm T}\theta_{ij}-(\tilde\varsigma_{ij}^k)^2+(1-\gamma)(\tilde\varsigma_{ij}^k)\tilde\varsigma_{ij}^{k-1}+\alpha\tilde\varsigma_{ij}^k\epsilon_{ij}^k-\gamma(\tilde\varsigma_{ij}^k)\varsigma\big]\mathrm{d}\tau+\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\theta_{ij}\|^2+\varsigma^2\big], \tag{60}$$

利用$(\tilde\theta_{ij}^k)^{\mathrm T}\tilde\theta_{ij}^{k-1}\leqslant 2^{-1}(\|\tilde\theta_{ij}^k\|^2+\|\tilde\theta_{ij}^{k-1}\|^2)$, $(\tilde\theta_{ij}^k)^{\mathrm T}\theta_{ij}\leqslant 2^{-1}(\|\tilde\theta_{ij}^k\|^2+\|\theta_{ij}\|^2)$, $\tilde\varsigma_{ij}^k\tilde\varsigma_{ij}^{k-1}\leqslant 2^{-1}((\tilde\varsigma_{ij}^k)^2+(\tilde\varsigma_{ij}^{k-1})^2)$, $\tilde\varsigma_{ij}^k\varsigma\leqslant 2^{-1}((\tilde\varsigma_{ij}^k)^2+\varsigma^2)$, 式(60)可写为

$$\Delta V_{\mathrm p}^k\leqslant\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[-\|\tilde\theta_{ij}^k\|^2+\|\theta_{ij}\|^2-(\tilde\varsigma_{ij}^k)^2+\varsigma^2\big]\mathrm{d}\tau+\alpha\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[(\tilde\theta_{ij}^k)^{\mathrm T}\tau_{ij}^k+\tilde\varsigma_{ij}^k\epsilon_{ij}^k\big]\mathrm{d}\tau+\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big(\|\theta_{ij}\|^2+\varsigma^2\big)\leqslant$$
$$\frac{\gamma}{2}(T+1)\sum_{i=1}^{N}\sum_{j=0}^{n}\big(\|\theta_{ij}\|^2+\varsigma^2\big)+\alpha\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[(\tilde\theta_{ij}^k)^{\mathrm T}\tau_{ij}^k+\tilde\varsigma_{ij}^k\epsilon_{ij}^k\big]\mathrm{d}\tau\leqslant$$
$$\gamma C_{\mathrm p}+\alpha\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[(\tilde\theta_{ij}^k)^{\mathrm T}\tau_{ij}^k+\tilde\varsigma_{ij}^k\epsilon_{ij}^k\big]\mathrm{d}\tau, \tag{61}$$

其中$C_{\mathrm p}=2^{-1}(T+1)\sum_{i=1}^{N}\sum_{j=0}^{n}\big(\|\theta_{ij}\|^2+\varsigma^2\big)$. 从$V_n^k$的定义可知

$$V_n^k(t)-V_n^k(0)\leqslant\sum_{i=1}^{N}\int_0^t\dot V_n^k\mathrm{d}\tau. \tag{62}$$

根据假设2可得$\tilde y_i^k(0)=0$, $\tilde x_{ij}^k(0)=0$. 由于$\hat\theta_{i0}^k(0)=0$和$\hat\varsigma_{i0}^k(0)=0$, 由式(6)可得到$u_{i1}^k(0)=0$. 进一步可知$\tilde z_{i1}^k(0)=0$. 经过类似推理, 可知$\tilde z_{ij}^k(0)=0, 1\leqslant j\leqslant n$.

$$V_n^k(0)=\frac{1}{2}\sum_{i=1}^{N}\big[\tilde y_i^{k\mathrm T}(0)P_i\tilde y_i^k(0)+\sum_{j=1}^{N}(\tilde z_{ij}^k(0))^2\big]=0, \tag{63}$$

把式(63)代入式(62)并利用式(54)可得

$$\Delta V_n^k(t)\leqslant -V_n^{k-1}-\sum_{i=1}^{N}\int_0^t\big[c_{i0}\|\tilde y_i^k\|^2+\sum_{s=1}^{n}c_{is}(\tilde z_{is}^k)^2+\sum_{s=0}^{n}\big((\tilde\theta_{is}^k)^{\mathrm T}\tau_{is}^k+\tilde\varsigma_{is}^k\epsilon_{is}^k\big)\big]\mathrm{d}\tau, \tag{64}$$

利用式(61)和式(64), 式(57)可写为

$$\Delta V^k\leqslant -\alpha V_n^{k-1}-\alpha\sum_{i=1}^{N}\int_0^t\big[c_{i0}\|\tilde y_i^k\|^2+\sum_{s=1}^{n}c_{is}(\tilde z_{is}^k)^2\big]\mathrm{d}\tau+\gamma C_{\mathrm p}\leqslant -\alpha\sum_{i=1}^{N}\int_0^t\big[c_{i0}\|\tilde y_i^k\|^2+\sum_{s=1}^{n}c_{is}(\tilde z_{is}^k)^2\big]\mathrm{d}\tau+\gamma C_{\mathrm p}. \tag{65}$$

因此$V^k(t)$可写为

$$V^k(t)=V^0(t)+\sum_{m=1}^{k}\Delta V^m(t)\leqslant V^0(t)-\alpha\sum_{m=1}^{k}\sum_{i=1}^{N}\int_0^t\big[c_{i0}\|\tilde y_i^k\|^2+\sum_{s=1}^{n}c_{is}(\tilde z_{is}^k)^2\big]\mathrm{d}\tau+k\gamma C_{\mathrm p}, \tag{66}$$

对式(66)两边同时取极限可得到

$$\lim_{k\to\infty}V^k(t)=V^0(t)+\lim_{k\to\infty}\sum_{m=1}^{k}\Delta V^m(t)\leqslant V^0(t)-\alpha\lim_{k\to\infty}\sum_{m=1}^{k}\sum_{i=1}^{N}\int_0^t\big[c_{i0}\|\tilde y_i^k\|^2+\sum_{s=1}^{n}c_{is}(\tilde z_{is}^k)^2\big]\mathrm{d}\tau+\lim\nolimits_{k\to\infty}k\gamma C_{\mathrm p}. \tag{67}$$

第2部分: 根据式(55)可知

$$V^0(t)=\frac{\alpha}{2}\sum_{i=1}^{N}\big[(\tilde y_i^0)^{\mathrm T}P_i\tilde y_i^0+\sum_{j=1}^{N}(\tilde z_{ij}^0)^2\big]+\frac{1-\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\int_0^t\big[\|\tilde\theta_{ij}^0\|^2+(\tilde\varsigma_{ij}^0)^2\big]\mathrm{d}\tau+\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^0\|^2+(\tilde\varsigma_{ij}^0)^2\big], \tag{68}$$

对式(68)求导并利用式(54), $\dot V^0(t)$可写为

$$\dot V^0(t)\leqslant-\alpha\sum_{i=1}^{N}\big[c_{i0}\|\tilde y_i^0\|^2+\sum_{j=1}^{n}c_{ij}(\tilde z_{ij}^0)^2+\sum_{j=0}^{n}\big((\tilde\theta_{ij}^0)^{\mathrm T}\tau_{ij}^0+\tilde\varsigma_{ij}^0\epsilon_{ij}^0)\big]+\frac{1-\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^0\|^2+(\tilde\varsigma_{ij}^0)^2\big]+\gamma\sum_{i=1}^{N}\sum_{j=0}^{n}\big[(\tilde\theta_{ij}^0)^{\mathrm T}\dot{\tilde\theta}_{ij}^0+\tilde\varsigma_{ij}^0\dot{\tilde\varsigma}_{ij}^0(t)\big]. \tag{69}$$

根据式(43)可得到

$$\alpha\tau_{ij}^0=\gamma\dot{\hat\theta}_{ij}^0+\hat\theta_{ij}^0, \tag{70}$$

$$\alpha\epsilon_{ij}^0=\gamma\dot{\hat\varsigma}_{ij}^0+\hat\varsigma_{ij}^0, \tag{71}$$

将式(70)和式(71)代入式(69)可得

$$\dot V^0(t)\leqslant-\alpha\sum_{i=1}^{N}\big[c_{i0}\|\tilde y_i^0\|^2+\sum_{j=1}^{n}c_{ij}(\tilde z_{ij}^0)^2\big]-\sum_{i=1}^{N}\sum_{j=0}^{n}\big[(\tilde\theta_{ij}^0)^{\mathrm T}(\gamma\dot{\hat\theta}_{ij}^0+\hat\theta_{ij}^0)+\tilde\varsigma_{ij}^0(\gamma\dot{\hat\varsigma}_{ij}^0+\hat\varsigma_{ij}^0)\big]+\frac{1-\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^0\|^2+(\tilde\varsigma_{ij}^0)^2\big]+\gamma\sum_{i=1}^{N}\sum_{j=0}^{n}\big[(\tilde\theta_{ij}^0)^{\mathrm T}\dot{\tilde\theta}_{ij}^0+\tilde\varsigma_{ij}^0\dot{\tilde\varsigma}_{ij}^0\big]. \tag{72}$$

由于θ_{ij}和ς为常数, $\dot{\tilde\theta}_{ij}^0=\dot{\hat\theta}_{ij}^0$和$\dot{\tilde\varsigma}_{ij}^0=\dot{\hat\varsigma}_{ij}^0$. 利用$\hat\theta_{ij}^0=\theta_{ij}+\tilde\theta_{ij}^0$和$\hat\varsigma_{ij}^0=\varsigma+\tilde\varsigma_{ij}^0$, 式(72)可写为

$$\dot V^0(t)\leqslant-\alpha\sum_{i=1}^{N}\big[c_{i0}\|\tilde y_i^0\|^2+\sum_{j=1}^{n}c_{ij}(\tilde z_{ij}^0)^2\big]-\sum_{i=1}^{N}\sum_{j=0}^{n}\big[(\tilde\theta_{ij}^0)^{\mathrm T}\theta_{ij}+\tilde\varsigma_{ij}^0\varsigma\big]-\frac{1+\gamma}{2}\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^0\|^2+(\tilde\varsigma_{ij}^0)^2\big], \tag{73}$$

在式(73)中, $-(\tilde\theta_{ij}^0)^{\mathrm T}\theta_{ij}\leqslant c\|\tilde\theta_{ij}^0\|^2+\|\theta_{ij}\|^2/(4c)$ 和 $-\tilde\varsigma_{ij}^0\varsigma_{ij}\leqslant c(\tilde\varsigma_{ij}^0)^2+(\varsigma)^2/(4c)$成立, $\forall c>0$. 因此有

$$\dot V^0(t)\leqslant-\alpha\sum_{i=1}^{N}\big[c_{i0}\|\tilde y_i^0\|^2+\sum_{j=1}^{n}c_{ij}(\tilde z_{ij}^0)^2\big]+\frac{1}{4c}\sum_{i=1}^{N}\big[\sum_{j=0}^{n}\|\theta_{ij}\|^2+(n+1)\varsigma^2\big]-\Big(\frac{1+\gamma}{2}-c\Big)\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^0\|^2+(\tilde\varsigma_{ij}^0)^2\big]. \tag{74}$$

令$0<c<(1+\gamma)/2$. 由于参数θ_{ij}和ς有界, 存在$\|\theta_{ij}\|\leqslant\|\theta_{\mathrm M}\|$和$|\varsigma|\leqslant|\varsigma_{\mathrm M}|$, 当不等式(75)满足时$\dot V^0(t)\leqslant0$, 因此$V^0(t)$有界.

$$\alpha\sum_{i=1}^{N}\big[c_{i0}\|\tilde y_i^0\|^2+\sum_{j=1}^{n}c_{ij}(\tilde z_{ij}^0)^2\big]+\Big(\frac{1+\gamma}{2}-c\Big)\sum_{i=1}^{N}\sum_{j=0}^{n}\big[\|\tilde\theta_{ij}^0\|^2+(\tilde\varsigma_{ij}^0)^2\big]\geqslant N\frac{n+1}{4c}\big(\|\theta_{\mathrm M}\|^2+\varsigma_{\mathrm M}^2\big). \tag{75}$$

第3部分: 假设存在一个正整数K, 可使得当$m>K$时, $\sum\limits_{i=1}^{N}\int_0^T\big[c_{i0}\|\tilde y_i^m\|^2+\sum\limits_{s=1}^{n}c_{is}(\tilde z_{is}^m)^2\big]\mathrm d\tau>\epsilon$, 利用式(67)可得到

$$\begin{aligned}&\lim_{k\to\infty}V^k(T)\leqslant\\&V^0(T)-\alpha\sum_{m=1}^{K}\sum_{i=1}^{N}\int_0^T\big[c_{i0}\|\tilde y_i^m\|^2+\sum_{s=1}^{n}c_{is}(\tilde z_{is}^m)^2\big]\mathrm d\tau+K\gamma C_{\mathrm p}-\\&\alpha\lim_{k\to\infty}\sum_{m=K+1}^{k}\sum_{i=1}^{N}\int_0^t\big[c_{i0}\|\tilde y_i^m\|^2+\sum_{s=1}^{n}c_{is}(\tilde z_{is}^m)^2\big]\mathrm d\tau+\\&\lim_{k\to\infty}(k-K)\gamma C_{\mathrm p}\leqslant B-\\&\lim_{k\to\infty}\alpha(k-K)\epsilon+\lim_{k\to\infty}(k-K)\gamma C_{\mathrm p}\leqslant\\&B+\lim_{k\to\infty}(k-K)(\gamma C_{\mathrm p}-\alpha\epsilon),\end{aligned} \tag{76}$$

其中$B=V^0(T)+K\gamma C_{\mathrm p}-\alpha\sum\limits_{m=1}^{K}\sum\limits_{i=1}^{N}\int_0^t\big[c_{i0}\|\tilde y_i^m\|^2+\sum\limits_{s=1}^{n}c_{is}(\tilde z_{is}^m)^2\big]\mathrm d\tau$为有界变量. 对于任意$\delta>0$, 取$\epsilon=(\gamma C_{\mathrm p}+\delta)/\alpha$, 代入式(76)可得到

$$\lim_{k\to\infty}V^k(T)\leqslant B-\lim_{k\to\infty}(k-K)\delta. \tag{77}$$

由于B为有界变量, 不等式(77)的右侧趋于$-\infty$, 与右侧 $V^k(T)\geqslant0$ 相矛盾. 因此$\lim\limits_{k\to\infty}\int_0^T\|\tilde y_i^k\|^2\mathrm d\tau\leqslant\epsilon$, $\lim\limits_{k\to\infty}\int_0^T(\tilde z_{ij}^k)^2\mathrm d\tau\leqslant\epsilon$成立.

注 1 每个子系统的输出跟踪误差沿着迭代方向渐进收敛到一个可调节的留集ϵ.

注 2 在本文中互联参数ς不需要事先已知, 通过设计自适应更新律$\hat\varsigma_{ij}^k$克服了子系统之间的相互作用和影响.

注 3 本文将文献[11]中给出的backstepping自适应迭代学习控制方法拓展应用在具有严格反馈形式的非线性互联系统中.

5 仿真研究(Simulation study)

考虑如下具有严格反馈形式的非线性互联系统及其参考系统模型:

子系统1:

$$\begin{cases} \dot{y}_{11}^k = y_{12}^k, \\ \dot{y}_{12}^k = x_{11}^k + \bar{\theta}_1(a_{11}y_{11}^k + a_{12}y_{21}^k), \\ \dot{x}_{11}^k = u_1^k + \bar{\theta}_1 b_{11}x_{11}^k y_{21}^k. \end{cases} \tag{78}$$

子系统2:

$$\begin{cases} \dot{y}_{21}^k = x_{21}^k + \bar{\theta}_2(a_{21}y_{11}^k + a_{22}y_{21}^k), \\ \dot{x}_{21}^k = u_2^k + \bar{\theta}_2 b_{21}x_{21}^k y_{21}^k. \end{cases} \tag{79}$$

参考模型1:

$$\begin{bmatrix} \dot{y}_{\mathrm{m}11}^k \\ \dot{y}_{\mathrm{m}12}^k \\ \dot{x}_{\mathrm{m}11}^k \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -27 & -27 & 9 \end{bmatrix} \begin{bmatrix} y_{\mathrm{m}11}^k \\ y_{\mathrm{m}12}^k \\ x_{\mathrm{m}11}^k \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} r_1, \tag{80}$$

参考模型2:

$$\begin{bmatrix} \dot{y}_{\mathrm{m}21}^k \\ \dot{x}_{\mathrm{m}21}^k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -6 \end{bmatrix} \begin{bmatrix} y_{\mathrm{m}21}^k \\ x_{\mathrm{m}21}^k \end{bmatrix} + \begin{bmatrix} 0 \\ 20 \end{bmatrix} r_2, \tag{81}$$

其中: $r_1 = r_2 = \sin(3t), t \in [0, 10\ \mathrm{s}], \bar{\theta}_1 = 1, a_{11} = 0.5, a_{12} = 0.8, b_{11} = 1.2, \bar{\theta}_2 = 2, a_{21} = 0.4, a_{22} = 0.9, b_{21} = 1.3$. 在式(78)和式(79)中, 可知$\gamma_{10}^k = a_{11}y_{11}^k + a_{12}y_{21}^k$, $\gamma_{11}^k = b_{11}x_{11}^k y_{21}^k$, $\gamma_{20}^k = a_{21}y_{11}^k + a_{22}y_{21}^k$和$\gamma_{21}^k = b_{21}x_{21}^k y_{11}^k$. 在仿真过程中, 选择的控制参数为$\alpha_1 = \alpha_2 = 10$, $\alpha = 15$, $\gamma = 0.1$和$c_{11} = c_{21} = 6$, $Q_1 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, $Q_2 = 10$. 求解式(4), 可得到$P_1 = \begin{bmatrix} 10.9545 & 1 \\ 1 & 1.0954 \end{bmatrix}$, $P_2 = 1$. 应用本文给出的控制律, 得到输出跟踪误差$\tilde{y}_1^k = (\tilde{y}_{11}^k, \tilde{y}_{12}^k)^{\mathrm{T}}$和$\tilde{y}_2^k = \tilde{y}_{21}^k$的Euclidean范数随迭代次数变化如图1.

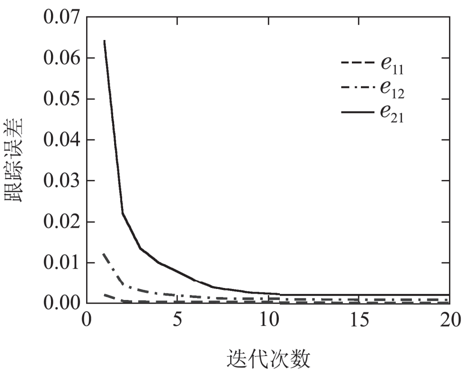

图 1　跟踪误差(e_{11}^k, e_{12}^k)和e_{21}^k的2–范数随迭代次数变化图

Fig. 1　2-norm of tracking errors (e_{11}^k, e_{12}^k) and e_{21}^k versus the number of iterations

从图1中可以看出, 输出跟踪误差Euclidean范数随迭代次数的增加而减少. 图2和图3分别给出了子系统1的输出$y_1 = (y_{11}, y_{12})^{\mathrm{T}}$及其参考输出$y_{\mathrm{m}1} = (y_{\mathrm{m}11}, y_{\mathrm{m}12})^{\mathrm{T}}$在第1次和第20次迭代时的轨迹图. 同样, 子系统2的输出y_2及其参考输出$y_{\mathrm{m}2}$在第1次和第20次迭代时的轨迹图由图4和图5给出. 从图3和图5中, 可以看出本文所给出的分散式backstepping自适应迭代学习控制律使得互联系统中的每个输出轨迹在经过20次迭代控制后有效地跟踪其相应的参考输出轨迹.

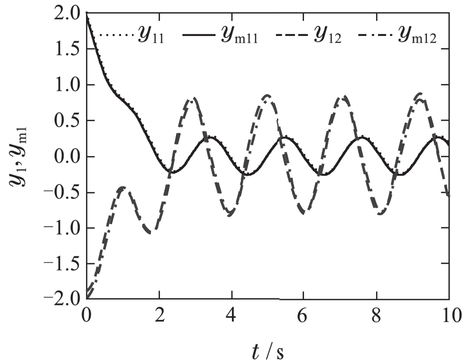

图 2　子系统1的输出(y_{11}, y_{12})及其参考输出$(y_{\mathrm{m}11}, y_{\mathrm{m}12})$在第1次迭代时的轨迹

Fig. 2　The trajectories of (y_{11}, y_{12}) and $(y_{\mathrm{m}11}, y_{\mathrm{m}12})$ at the 1st iteration

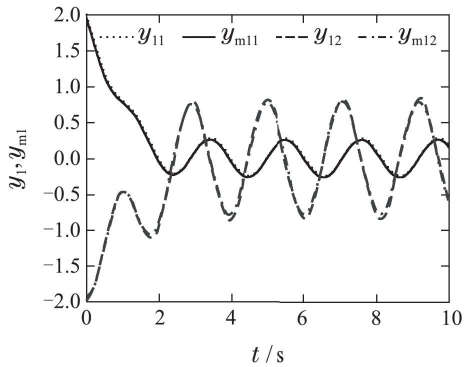

图 3　子系统1的输出(y_{11}, y_{12})及其参考输出$(y_{\mathrm{m}11}, y_{\mathrm{m}12})$在第20次迭代时的轨迹

Fig. 3　The trajectories of (y_{11}, y_{12}) and $(y_{\mathrm{m}11}, y_{\mathrm{m}12})$ at the 20th iteration

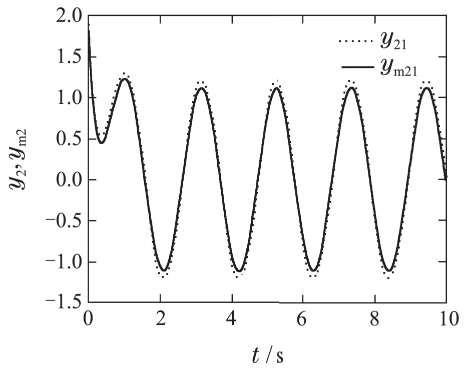

图 4　子系统2的输出y_{21}及其参考输出$y_{\mathrm{m}21}$在第1次迭代时的轨迹

Fig. 4　The trajectories of y_{21} and $y_{\mathrm{m}21}$ at the 1st iteration

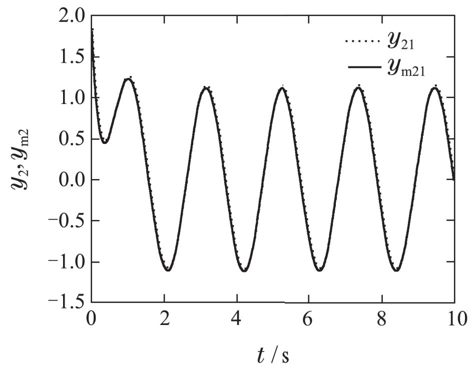

图 5 子系统2的输出y_{21}及其参考输出y_{m21}在第20次迭代时的轨迹

Fig. 5 The trajectories of y_{21} and y_{m21} at the 20th iteration

6 结论(Conclusions)

本文针对一类具有严格反馈形式的非线性互联系统设计了一种分散式backstepping自适应迭代学习控制器. 该控制律保证互联系统中的每个子系统输出跟踪误差沿着迭代方向渐进收敛到一个可调节的留集ϵ. 在每个子控制器中仅采用子系统信息, 不需要子系统之间相互传递信息. 通过仿真示例验证了本文所给出控制器的有效性.

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作者简介:

孙丽丽 (1976–), 女, 讲师, 博士, 研究方向为互联大系统控制、迭代学习控制、机器人控制系统, E-mail: sunlily@126.com;

吴铁军 (1950–), 男, 教授, 博士生导师, 研究方向为复杂系统控制与优化、智能机器人系统、导航制导与控制, E-mail: tjwu@iipc.zju.edu.cn.