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具有扇形执行器的多机电力系统气门开度的时滞无关镇定

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摘要: 本文研究了具有扇形执行器的多机电力系统气门开度的时滞无关分散控制器和鲁棒分散控制器的设计问题. 首先利用三角变换把非线性关联函数变换为子系统状态变量的二次有界不等式形式, 然后基于Lyapunov稳定性理论, 推导出闭环多机电力系统及其参数不确定系统渐近稳定的线性矩阵不等式(LMI)充分条件, 最后以两机无穷大母线系统为例进行了仿真分析, 验证了所提方法的有效性.

关键词: 多机电力系统; 气门开度; 执行器饱和; 时滞无关; 线性矩阵不等式
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Delay-independent stabilization for steam valve opening of multi-machine power system with sector actuator

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Abstract: A time-independent decentralized controller and a robust decentralized controller have been developed for the steam valve opening of multi-machine power system with sector saturating actuator. The nonlinear interconnected function is first converted by trigonometric transformation into quadratically bounded inequalities in the subsystem states; and then, linear matrix inequality (LMI) sufficient conditions for the closed-loop multi-machine power system and uncertain system to be asymptomatically stable are derived based on Lyapunov stability theory. A two-machine infinite-bus system is considered as an application example. Simulation results demonstrate the effectiveness of the proposed method.

Key words: multi-machine power system; steam valve opening; saturating actuator; delay-independent; linear matrix inequality (LMI)

1 Introduction

Multi-machine power system is known as a complex interconnected large-scale system that is composed of many electric devices and mechanical components, so it's very common to have parameter uncertainty and time-delay which are often the source of instability. In the past years, scholars employed excitation control to enhance its stability[1–4]. However, application scope of this method is very limited for the constraints on the maximum value and rising rate of excitation current. Thus it's necessary to make further improvement on transient stability level by regulating the steam valve or water valve opening of prime mover to control the injected mechanical power. In addition, the control of steam value opening is more effective since the existence of water hammer effect weakens the efficiency of its improvement on stability of system[5]. At the same time, the physical limitations of the actuator are unavoidable in the operation of driving the actuator by signals emitted from the designed controllers, thus causing actuator saturation, which not only deteriorates the control system performance, but also leads to undesirable stability effects. In addition, in most cases, it is difficult to know the exact value of delays, and it is even not easy to estimate the bounds of the delays. Hence, it's significant to investigate time-independent robust steam valve opening control of large-sized steam turbine with actuator saturation to enhance stability level of multi-machine power system.

In the past decade, scholars paid close attention to the control of steam valve opening and several contributions had been published. They can be divided into two types: one was linear feedback controller, for example, by virtual of solving Ricatti equation[6], making use of LMI approach[7], based on state observer[8–9], etc.; the other was nonlinear feedback controller, for example, using backstepping method[10–12], availing itself of Hamiltonian control technique with dissipation and recursive method[13–15], etc. Among the above mentioned reports, only [12, 15] took actuator saturation into con-

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sideration; meanwhile, they had only thought over that the actuator reached its extreme limit. So far as we known, report on the control of steam vale opening for time-delay multi-machine power system have not been covered yet. And, though there are many useful theoretical results about linear uncertain time-delay systems with linear interconnection function considering actuator saturation[16–18], and also, some good results about special linear time-delay systems with constrained nonlinear interconnection function[19–20], there are no constructive results related to linear uncertain time-delay system with nonlinear interconnection function.

In this paper, the design of delay-independent decentralized controller and robust decentralized controller for steam valve opening of uncertain time-delay multi-machine power systems with actuator saturation are handled. The system parameter uncertainties are unknown but bounded and the delays are time-varying. Sufficient LMI conditions of uncertain multi-machine power system and its nominal system asymptomatic stability have been developed based on the Lyapunov stability theory combined with LMI technique, and the design algorithm of controller gain matrices and three corollaries are presented.

2 Model description

Considering delays, input constraints and the time-varying structured uncertainties, an N-machine power system with steam valve control, which is composed of turbine generator with reheater, is described by the interconnection of N subsystems as follows[5–6]:

$$\begin{cases}\dot{x}_i(t)= \\ [A_i+\Delta A_i(t)]x_i(t)+[B_i+\Delta B_i(t)]u_{si}(t)+ \\ \sum\limits_{j=1,j\neq i}^{N} p_{ij}[G_{ij}+\Delta G_{ij}(t)]g_{ij}(x_i(t),x_j(t-\tau_{ij})), \\ u_{si}(t)=\mathrm{sat}(u_i(t)), \\ x_i(t)=\phi_i(t),\ t\in[-\tau,0],\end{cases} \quad (1)$$

where $x_i(t)=[\Delta\delta_i(t)\ \ \omega_i(t)\ \ \Delta P_{\mathrm{M}i}(t)\ \ \Delta X_{\mathrm{E}i}(t)]^{\mathrm{T}}$ and $\Delta\delta_i(t)=\delta_i(t)-\delta_{i0}$, $\Delta P_{\mathrm{M}i}(t)=P_{\mathrm{M}i}(t)-P_{\mathrm{M}i0}$, $\Delta X_{\mathrm{E}i}(t)=X_{\mathrm{E}i}(t)-X_{\mathrm{E}i0}$. $u_i(t)$, which is generated from the designed controller, is the control input vector to the actuator, $u_{si}(t)$ is the control input vector to the plant. $g_{ij}(x_i(t),x_j(t-\tau_{ij}))$ is the nonlinear function vector characterizing the interconnection between the ith generator and jth generator and

$$g_{ij}(x_i(t),x_j(t-\tau_{ij}))= \\ \sin(\delta_i(t)-\delta_j(t-\tau_{ij}))-\sin(\delta_{i0}-\delta_{j0}), \quad (2)$$

τ_{ij} is the unknown time-varying signal transmission delay term between the ith generator and jth generator, and satisfied

$$0\leqslant\tau_{ij}\leqslant\tau\leqslant\infty. \quad (3)$$

The nominal system matrices are represented as follows:

$$A_i=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -\dfrac{D_i}{2H_i} & \dfrac{\omega_0}{2H_i}(1-F_{\mathrm{IP}_i}) & \dfrac{\omega_0}{2H_i}F_{\mathrm{IP}_i}\\ 0 & 0 & -\dfrac{1}{T_{\mathrm{M}i}} & \dfrac{K_{\mathrm{M}i}}{T_{\mathrm{M}i}}\\ 0 & -\dfrac{K_{\mathrm{E}i}}{T_{\mathrm{E}i}R_i\omega_0} & 0 & -\dfrac{1}{T_{\mathrm{E}i}}\end{bmatrix},$$

$$B_i=[0\ \ 0\ \ 0\ \ 1/T_{\mathrm{E}i}]^{\mathrm{T}},$$

$$G_{ij}=[0\ \ -\zeta_{ij}\ \ 0\ \ 0]^{\mathrm{T}},\ \zeta_{ij}=\omega_0 E'_{qi}E'_{qj}B_{ij}/2H_i,$$

where, p_{ij}: constant of either 1 or 0 ($p_{ij}=0$ means that the jth generator has no connection with ith generator); H_i: inertia constant for ith generator, in seconds; D_i: damping coefficient for ith generator, in pu; $F_{\mathrm{IP}i}$: fraction of the turbine power generated by the intermediate pressure section; $T_{\mathrm{M}i}$ and $K_{\mathrm{M}i}$: time constant and gain of ith machine's turbine; $T_{\mathrm{E}i}$ and $K_{\mathrm{E}i}$: time constant and gain of ith machine's speed governor; R_i: regulation constant of ith machine in pu; B_{ij}: ith row and jth column element of nodal susceptance matrix at the internal nodes after eliminated all physical buses, in pu; $P_{\mathrm{M}i}$: mechanical power for ith machine, in pu; $X_{\mathrm{E}i}$: steam valve opening for ith machine, in pu; ω_i: relative speed for ith machine, in rad/s; δ_i: rotor angle for ith machine, in rad; ω_0: the synchronous machine speed; E'_{qi} and E'_{qj}: internal transient voltage for ith and jth machine, in pu, which are assumed to be constant; $\delta_{i0}, P_{\mathrm{M}i0}$ and $X_{\mathrm{E}i0}$: the initial values of δ_i, $P_{\mathrm{M}i}$ and $X_{\mathrm{E}i}$, respectively. $\Delta A_i(t), \Delta B_i(t)$ and $\Delta G_{ij}(t)$ are real time varying parameter uncertainties and assumed to be of the following structure:

$$\begin{cases}[\Delta A_i(t)\ \ \Delta B_i(t)]=L_iF_i(t)[M_i\ N_i],\\ \Delta G_{ij}(t)=L_{ij}F_{ij}(t)E_{ij}\end{cases} \quad (4)$$

with $F_i(t)$ and $F_{ij}(t)$ being unknown matrix functions with Lebesgue measurable elements and satisfying

$$F_i^{\mathrm{T}}(t)F_i(t)\leqslant I_i,\ F_{ij}^{\mathrm{T}}(t)F_{ij}(t)\leqslant I_{ij}, \quad (5)$$

where L_i, M_i, N_i, L_{ij} and E_{ij} are known real constant matrices with appropriate dimensions.

The sector nonlinear function $u_{si}(t)$ is considered to be inside the sector $[a_i,\ 1]$ and is shown in Fig.1, where $0\leqslant a_i\leqslant 1$.

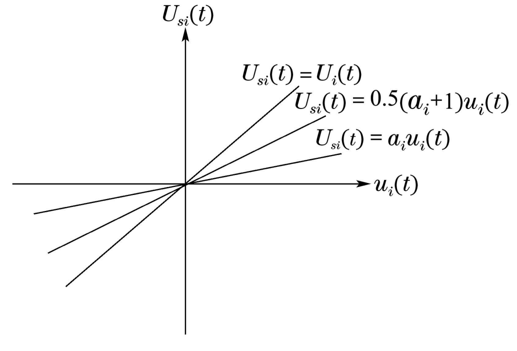


Fig. 1 Sector nonlinear function

3 Main results

For convenience, the nonlinear interconnection function $g_{ij}(x_i(t), x_j(t-\tau_{ij}))$ is converted into the following form via transformation of triangle functions:

$$\begin{aligned}&g_{ij}(x_i(t), x_j(t-\tau_{ij})) = \\ &-2\cos(\beta_i(\delta))\sin(\Delta\delta_i(t)-\Delta\delta_j(t-\tau_{ij}))/2 = \\ &-2\cos(\beta_i(\delta))\sin(x_{i1}(t)-x_{j1}(t-\tau_{ij}))/2,\end{aligned}$$

where $\beta_i(\delta)=[\delta_i(t)-\delta_j(t-\tau_{ij})+\delta_{i0}-\delta_{j0}]/2$.

It is immediately got that

$$\begin{aligned}&g_{ij}^{\mathrm{T}}(x_i(t), x_j(t-\tau_{ij}))g_{ij}(x_i(t), x_j(t-\tau_{ij})) = \\ &4\cos^2(\beta_i(\delta))\sin^2[x_{i1}(t)-x_{j1}(t-\tau_{ij})]/2 \leqslant \\ &[x_{i1}(t)-x_{j1}(t-\tau_{ij})]^2 \leqslant \\ &x_{i1}^{\mathrm{T}}(t)x_{i1}(t)+x_{j1}^{\mathrm{T}}(t-\tau_{ij})x_{j1}(t-\tau_{ij}). \quad (6)\end{aligned}$$

Remark 1 $x_{i1}(t)=W_ix_i(t)$, $x_{j1}(t-\tau_{ij})=W_jx_j(t-\tau_{ij})$, where $W_i=W_j=[1\ 0\ 0\ 0]$.

Remark 2 The deduced result of the nonlinear interconnected function is smaller than the assumption in [6].

In the sequel, some useful lemmas, which are needed to solve our problem, are introduced.

Lemma 1 Let D, E and F be real matrices of appropriate dimensions with $F^{\mathrm{T}}(t)F(t)\leqslant I$, then for any given $\varepsilon>0$, we have that

$$DFE+E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}}\leqslant \varepsilon^{-1}DD^{\mathrm{T}}+\varepsilon E^{\mathrm{T}}E. \quad (7)$$

Lemma 2 Consider the partitioned matrix $F=\begin{bmatrix} C & D\\ D^{\mathrm{T}} & E\end{bmatrix}$ where $C=C^{\mathrm{T}},\ E=E^{\mathrm{T}}$ with appropriate dimensions, then F is positive definite if and only if either the following conditions hold

$$\begin{aligned}&E>0 \text{ and } C-DE^{-1}D^{\mathrm{T}}>0 \text{ or}\\ &C>0 \text{ and } E-D^{\mathrm{T}}C^{-1}D>0. \quad (8)\end{aligned}$$

Next, the state feedback control law is designed as

$$u_i(t)=K_ix_i(t), \quad (9)$$

where K_i is control gain matrix.

Thus, the resulting closed-loop system is written as

$$\begin{aligned}&\dot{x}_i(t) = \\ &(A_{ic}+\Delta A_{ic}(t))x_i(t)+[B_i+\Delta B_i(t)]\eta_i(t)+ \\ &\sum_{j=1,j\neq i}^{N} p_{ij}[G_{ij}+\Delta G_{ij}(t)]g_{ij}(x_i(t), x_j(t-\tau_{ij})),\end{aligned} \quad (10)$$

where $A_{ic}=A_i+0.5(1+a_i)B_iK_i$, $\Delta A_{ic}(t)=\Delta A_i(t)+0.5(1+a_i)\Delta B_i(t)K_i$, $\eta_i(t)=u_{si}(t)-0.5(1+a_i)K_ix_i(t)$.

From Fig.1, it's obvious that vector function $\eta_i(t)$ satisfies the following inequality:

$$\eta_i^{\mathrm{T}}(t)\eta_i(t)\leqslant \frac{1}{4}(1-a_i)^2x_i^{\mathrm{T}}(t)K_i^{\mathrm{T}}K_ix_i(t). \quad (11)$$

The main results are given in the following theorems.

Theorem 1 The nominal system of (10) is time-delay independent robustly stable with feedback gain matrix $K_i=Y_iX_i^{-1}$ if there exists symmetric and positive-definite X_i and any matrix Y_i, such that the following LMI holds:

$$\begin{bmatrix} \Xi_i & T_i & Y_i^{\mathrm{T}} & X_iW_i^{\mathrm{T}}\\ T_i^{\mathrm{T}} & -R_i & 0 & 0\\ Y_i & 0 & -\zeta_i & 0\\ W_iX_i & 0 & 0 & -\xi_i\end{bmatrix}<0, \quad (12)$$

where $\Xi_i=A_iX_i+X_iA_i^{\mathrm{T}}+0.5(1+a_i)(B_iY_i+Y_i^{\mathrm{T}}B_i^{\mathrm{T}})+\beta_iB_iB_i^{\mathrm{T}}$, $\zeta_i=\dfrac{4\beta_i}{(1-a_i)^2}$, other variables are defined below.

Proof Choose the Lyapunov-Krasovskii functional candidate in the following form:

$$\begin{aligned}&V(x_i(t), x_i(t-\tau_{ij})) = \\ &\sum_{i=1}^{N}\{x_i^{\mathrm{T}}(t)P_ix_i(t)+ \\ &\sum_{j=1,j\neq i}^{N}\int_{t-\tau_{ij}}^{t} p_{ij}\mu_{ij}x_{j1}^{\mathrm{T}}(\xi)x_{j1}(\xi)\mathrm{d}\xi\}, \quad (13)\end{aligned}$$

where P_i is a positive definite symmetric matrix and μ_{ij} is any given positive constant.

Taking the time derivative of $V(x_i(t), x_i(t-\tau_{ij}))$ along the trajectory of closed-loop nominal system of (9) is given by

$$\begin{aligned}&\dot{V}(x_i(t), x_i(t-\tau_{ij})) = \\ &\sum_{i=1}^{N}\{2x_i^{\mathrm{T}}(t)P_i[A_{ic}x_i(t)+B_i\eta_i(t)+ \\ &\sum_{j=1,j\neq i}^{N} p_{ij}G_{ij}g_{ij}(x_i(t), x_j(t-\tau_{ij}))]+ \\ &\sum_{j=1,j\neq i}^{N} p_{ij}\mu_{ij}[x_{j1}^{\mathrm{T}}(t)x_{j1}(t)- \\ &x_{j1}^{\mathrm{T}}(t-\tau_{ij})x_{j1}(t-\tau_{ij})]\}. \quad (14)\end{aligned}$$

According to Lemma 1 and (10), it follows that

$$\begin{aligned}&x_i^{\mathrm{T}}(t)P_iB_i\eta_i(t)+\eta_i^{\mathrm{T}}(t)B_i^{\mathrm{T}}P_ix_i(t)\leqslant \\ &\beta_ix_i^{\mathrm{T}}(t)P_iB_iB_i^{\mathrm{T}}P_ix_i(t)+\beta_i^{-1}\eta_i^{\mathrm{T}}(t)\eta_i(t)\leqslant \\ &\beta_ix_i^{\mathrm{T}}(t)P_iB_iB_i^{\mathrm{T}}P_ix_i(t)+ \\ &\frac{(1-a_i)^2}{4\beta_i}x_i^{\mathrm{T}}(t)K_i^{\mathrm{T}}K_ix_i(t), \quad (15)\end{aligned}$$

where β_i is any given positive constant.

Substituting (6) and (15) into (14), we obtain that

$$\begin{aligned}&\dot{V}(x_i(t), x_i(t-\tau_{ij}))\leqslant \\ &\sum_{i=1}^{N}\{x_i^{\mathrm{T}}(t)\tilde{\Xi}_ix_i(t)+\sum_{j=1,j\neq i}^{N} p_{ij}[2x_i^{\mathrm{T}}(t)P_iG_{ij}\bar{g}_{ij}- \\ &\mu_{ij}x_{j1}^{\mathrm{T}}(t-\tau_{ij})x_{j1}(t-\tau_{ij})]\} =\end{aligned}$$

$$\sum_{i=1}^{N}\{x_i^{\mathrm T}(t)\tilde{\Xi}_i x_i(t)+\sum_{j=1,j\neq i}^{N}p_{ij}[2x_i^{\mathrm T}(t)P_iG_{ij}\bar g_{ij}-\mu_{ij}x_{j1}^{\mathrm T}(t-\tau_{ij})x_{j1}(t-\tau_{ij})+\gamma_{ij}\bar g_{ij}^{\mathrm T}\bar g_{ij}-\gamma_{ij}\bar g_{ij}^{\mathrm T}\bar g_{ij}]\}\leqslant \sum_{i=1}^{N}\{z_i(t)\Theta_i z_i(t)-\sum_{j=1,j\neq i}^{N}p_{ij}[\mu_{ij}-\gamma_{ij}]x_{j1}^{\mathrm T}(t-\tau_{ij})x_{j1}(t-\tau_{ij})\},\tag{16}$$

where

$$\bar g_{ij}=g_{ij}(x_i(t),x_j(t-\tau_{ij})),$$
$$\tilde\Xi_i=P_iA_{ic}+A_{ic}^{\mathrm T}P_i+\beta_iP_iB_iB_i^{\mathrm T}P_i+\frac{(1-a_i)^2}{4\beta_i}K_i^{\mathrm T}K_i+\sum_{j=1,j\neq i}^{N}p_{ij}\mu_{ji}W_i^{\mathrm T}W_i,$$
$$z_i(t)=[x_i^{\mathrm T}(t)\ \ \bar g_{i1}\ \cdots\ \bar g_{i(i-1)}\ \ \bar g_{i(i+1)}\ \cdots\ \bar g_{iN}]^{\mathrm T},$$
$$\Theta_i=\begin{bmatrix}\bar\Xi_i & P_iT_i\\ T_i^{\mathrm T}P_i & -R_i\end{bmatrix},$$
$$\bar\Xi_i=P_iA_{ic}+A_{ic}^{\mathrm T}P_i+\beta_iP_iB_iB_i^{\mathrm T}P_i+\frac{1}{4\beta_i}(1-a_i)^2K_i^{\mathrm T}K_i+\xi_i^{-1}W_i^{\mathrm T}W_i,$$
$$\xi_i^{-1}=\sum_{j=1,j\neq i}^{N}p_{ij}[\mu_{ji}+\gamma_{ji}],$$
$$T_i=[p_{i1}G_{i1}\ \cdots\ p_{i(i-1)}G_{i(i-1)}\ \ p_{i(i+1)}G_{i(i+1)}\ \cdots\ p_{iN}G_{iN}],$$
$$R_i=\mathrm{diag}\{\gamma_{i1},\cdots,\gamma_{i(i-1)},\gamma_{i(i+1)},\cdots,\gamma_{iN}\}.$$

For μ_{ij} and γ_{ij} are any given positive constants, it's reasonable to let $\mu_{ij}\geqslant\gamma_{ij}$. Hence, if $\Theta_i<0$ is satisfied, $\dot V(x_i(t),x_i(t-\tau_{ij}))<0$ holds and the nominal system of (10) is asymptotically stable. Because $\Theta_i<0$ is a bilinear matrix inequality (BMI), it's necessary to find a way to transform the inequality to a form which is affine in the unknown variables. To achieve this, letting the variables $X_i=P_i^{-1}$ and $Y_i=K_iX_i$, pre-multiplying and post-multiplying Θ_i by $\mathrm{diag}\{X_i,I\}$, we obtain that

$$\Theta_i<0\Longleftrightarrow\begin{bmatrix}\hat\Xi_i & T_i\\ T_i^{\mathrm T} & -R_i\end{bmatrix}<0,\tag{17}$$

where

$$\hat\Xi_i=A_iX_i+X_iA_i^{\mathrm T}+0.5(1+a_i)(B_iY_i+Y_i^{\mathrm T}B_i^{\mathrm T})+\beta_iB_iB_i^{\mathrm T}+\frac{(1-a_i)^2}{4\beta_i}Y_i^{\mathrm T}Y_i+\xi_i^{-1}X_iW_i^{\mathrm T}W_iX_i.$$

According to Lemma 2, it is obvious that LMI (12) is equivalent to (17) and (12) guarantees that the negativeness of $\dot V(x_i(t),x_i(t-\tau_{ij}))$ whenever $x_i(t)$ is not zero, which immediately implies that the asymptotic stability of the nominal system of (10). The proof of Theorem 1 is completed.

In the sequel, the robust time-delay independent LMI condition is investigated.

Theorem 2 The system (10) is time-delay independent robustly stable with feedback gain matrix $K_i=Y_iX_i^{-1}$ if there exists symmetric and positive-definite X_i and any matrix Y_i, such that the following LMI holds

$$\begin{bmatrix}\Upsilon_i & T_i & Y_i^{\mathrm T} & X_iW_i^{\mathrm T} & \Omega_i^{\mathrm T}\\ T_i^{\mathrm T} & -R_i & 0 & 0 & 0\\ Y_i & 0 & -\zeta_i & 0 & 0\\ W_iX_i & 0 & 0 & -\xi_i & 0\\ \Omega_i & 0 & 0 & 0 & -\theta_i\end{bmatrix}<0,\tag{18}$$

where $\Upsilon_i=A_iX_i+X_iA_i^{\mathrm T}+0.5(1+a_i)(B_iY_i+Y_i^{\mathrm T}B_i^{\mathrm T})+\Phi_i$, $\Omega_i=[M_iX_i+0.5(1+a_i)N_iY_i]$, other variables are the same as the ones in Theorem 1 or defined below.

Proof Choose the same Lyapunov-Krasovskii functional candidate as Theorem 1 and take the time derivative of $V(x_i(t),x_i(t-\tau_{ij}))$ along the trajectory of closed-loop system (10) to obtain that

$$\dot V(x_i(t),x_i(t-\tau_{ij}))\leqslant \sum_{i=1}^{N}\{2x_i^{\mathrm T}(t)P_i[(A_{ic}+\Delta A_{ic}(t))x_i(t)+(B_i+\Delta B_i(t))\eta_i(t)+\sum_{j=1,j\neq i}^{N}p_{ij}(G_{ij}+\Delta G_{ij}(t))\bar g_{ij}]+\sum_{j=1,j\neq i}^{N}p_{ij}\mu_{ij}[x_{j1}^{\mathrm T}(t)x_{j1}(t)-x_{j1}^{\mathrm T}(t-\tau_{ij})x_{j1}(t-\tau_{ij})]\}.\tag{19}$$

According to Lemma 1, it follows that

$$2x_i^{\mathrm T}(t)P_i\Delta A_{ic}(t)x_i(t)=2x_i^{\mathrm T}(t)P_iL_iF_i(t)[M_i+0.5(1+a_i)N_iK_i]x_i(t)\leqslant x_i^{\mathrm T}(t)(\theta_iP_iL_iL_i^{\mathrm T}P_i+\theta_i^{-1}\bar\Omega_i^{\mathrm T}\bar\Omega_i)x_i(t)\tag{20}$$

and

$$2x_i^{\mathrm T}(t)P_i\Delta B_i(t)\eta_i(t)=2x_i^{\mathrm T}(t)P_iL_iF_i(t)N_i\eta_i(t)\leqslant \vartheta_ix_i^{\mathrm T}(t)P_iL_iL_i^{\mathrm T}P_ix_i(t)+\vartheta_i^{-1}\eta_i^{\mathrm T}(t)N_i^{\mathrm T}N_i\eta_i(t)\leqslant \vartheta_ix_i^{\mathrm T}(t)P_iL_iL_i^{\mathrm T}P_ix_i(t)+\frac{\chi_i(1-a_i)^2}{4\vartheta_i}x_i^{\mathrm T}(t)K_i^{\mathrm T}K_ix_i(t)\tag{21}$$

and

$$2x_i^{\mathrm T}(t)P_i\sum_{j=1,j\neq i}^{N}p_{ij}\Delta G_{ij}(t)\bar g_{ij}=2x_i^{\mathrm T}(t)P_i\sum_{j=1,j\neq i}^{N}p_{ij}L_{ij}F_{ij}(t)E_{ij}\bar g_{ij}\leqslant \delta_i^{-1}x_i^{\mathrm T}(t)P_i\sum_{j=1,j\neq i}^{N}p_{ij}L_{ij}L_{ij}^{\mathrm T}P_ix_i(t)+\delta_i\sum_{j=1,j\neq i}^{N}p_{ij}\bar g_{ij}^{\mathrm T}E_{ij}^{\mathrm T}E_{ij}\bar g_{ij}\leqslant p_{ij}\{\delta_i^{-1}x_i^{\mathrm T}(t)P_i\sum_{j=1,j\neq i}^{N}L_{ij}L_{ij}^{\mathrm T}P_ix_i(t)+\delta_i\sum_{j=1,j\neq i}^{N}\lambda_{ij}[x_{i1}^{\mathrm T}(t)x_{i1}(t)+$$

$$x_{j1}^{\mathrm{T}}(t-\tau_{ij})x_{j1}(t-\tau_{ij})]\}, \tag{22}$$

where θ_i, ϑ_i and δ_i are any given positive constants, $\bar{\Omega}_i = M_i + 0.5(1+a_i)N_iK_i$, χ_i and λ_{ij} are the maximum eigenvalues of $N_i^{\mathrm{T}}N_i$ and $E_{ij}^{\mathrm{T}}E_{ij}$ respectively.

Substituting (6)(15) and (20)−(22) into (19), we obtain that

$$\begin{aligned}
&\dot{V}(x_i(t),x_i(t-\tau_{ij})) \leqslant \\
&\sum_{i=1}^{N}\{x_i^{\mathrm{T}}(t)\tilde{\Upsilon}_i x_i(t) + \sum_{j=1,j\neq i}^{N} p_{ij}[2x_i^{\mathrm{T}}(t)P_iG_{ij}\bar{g}_{ij} - \\
&(\mu_{ij}-\delta_i\lambda_{ij})x_{j1}^{\mathrm{T}}(t-\tau_{ij})x_{j1}(t-\tau_{ij})]\} = \\
&\sum_{i=1}^{N}\{x_i^{\mathrm{T}}(t)\tilde{\Upsilon}_i x_i(t) + \sum_{j=1,j\neq i}^{N} p_{ij}[2x_i^{\mathrm{T}}(t)P_iG_{ij}\bar{g}_{ij} - \\
&(\mu_{ij}-\delta_i\lambda_{ij})x_{j1}^{\mathrm{T}}(t-\tau_{ij})x_{j1}(t-\tau_{ij}) + \\
&\gamma_{ij}\bar{g}_{ij}^{\mathrm{T}}\bar{g}_{ij} - \gamma_{ij}\bar{g}_{ij}^{\mathrm{T}}\bar{g}_{ij}]\} \leqslant \\
&\sum_{i=1}^{N}\{z_i^{\mathrm{T}}(t)\Pi_i z_i(t) - \sum_{j=1,j\neq i}^{N} p_{ij}[\mu_{ij}-\delta_i\lambda_{ij}-\gamma_{ij}]\cdot \\
&x_{j1}^{\mathrm{T}}(t-\tau_{ij})x_{j1}(t-\tau_{ij})\},
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\tilde{\Upsilon}_i = {} & P_iA_{ic} + A_{ic}^{\mathrm{T}}P_i + P_i\Phi_iP_i + \theta_i^{-1}\bar{\Omega}_i^{\mathrm{T}}\bar{\Omega}_i + \\
& \frac{(1-a_i)^2}{4}(\frac{\chi_i}{\vartheta_i}+1)K_i^{\mathrm{T}}K_i + \\
& \sum_{j=1,j\neq i}^{N} p_{ij}\mu_{ji}W_i^{\mathrm{T}}W_i, \\
\Phi_i = {} & (\theta_i+\vartheta_i)L_iL_i^{\mathrm{T}} + \beta_iB_iB_i^{\mathrm{T}} + \\
& \delta_i^{-1}\sum_{j=1,j\neq i}^{N} p_{ij}L_{ij}L_{ij}^{\mathrm{T}},
\end{aligned}$$

$$\Pi_i = \begin{bmatrix} \bar{\Upsilon}_i & P_iT_i \\ T_i^{\mathrm{T}}P_i & -R_i \end{bmatrix},\ \bar{\Upsilon} = \tilde{\Upsilon} + \sum_{j=1,j\neq i}^{N} p_{ij}\gamma_{ji}W_i^{\mathrm{T}}W_i.$$

For μ_{ij}, δ_i and γ_{ij} are any given positive constants, it's reasonable to let $\mu_{ij} \geqslant \delta_i\lambda_{ij} + \gamma_{ij}$. Hence, if $\Pi_i < 0$ holds, the system (10) is robust asymptotically stable. Using the same manipulation as Theorem 1, we find that LMI (18) is equivalent to $\Pi_i < 0$ and (18) guarantees the negativeness of $\dot{V}(x_i(t), x_i(t-\tau_{ij}))$ whenever $x_i(t)$ is not zero, which implies that the robust asymptotic stability of the system (10). The proof of Theorem 2 is achieved.

Meanwhile, when some parameters take some special values, the following results are easily derived.

Corollary 1 When $a_i = 1$, if LMI (12) or (18) holds, the nominal system of (10) or system (10) is time-independent asymptotic stability without actuator saturation.

Corollary 2 When $\tau_{ij} = 0$ and $a_i = 1$, denoting $K_i = \iota_iB_i^{\mathrm{T}}P_i$, the following result is obtained that

$$\begin{aligned}
\bar{\Upsilon}_i < 0 \Leftrightarrow {} & P_iA_i + A_i^{\mathrm{T}}P_i + 2\iota_iP_iB_iB_i^{\mathrm{T}}P_i + \\
& P_i\Theta_iP_i + \bar{\Omega}_i + \bar{\nu}_i^{-1}W_i^{\mathrm{T}}W_i + Q_i = 0,
\end{aligned} \tag{24}$$

where $\bar{\Omega}_i = \eta_i^{-1}(M_i + \iota_iN_iB_i^{\mathrm{T}}P_i)^{\mathrm{T}}(M_i + \iota_iN_iB_i^{\mathrm{T}}P_i)$, $\bar{\nu}_i^{-1} = \sum\limits_{j=1,j\neq i}^{N} 2p_{ij}(\theta_i\lambda_{ij} + \vartheta_i)$ and Q_i is positive definite matrix.

It is obvious that Corollary 2 is the main result of [6], therefore Theorem 1 has wider range of application.

Corollary 3 When $\tau_{ij} = 0$ and $a_i = 1$, the LMI (12) can be simplified as follows:

$$\begin{bmatrix} \tilde{S}_i & X_iW_i^{\mathrm{T}} \\ W_iX_i & -\tilde{\nu}_iI_i \end{bmatrix} < 0, \tag{25}$$

where

$$\begin{aligned}
\tilde{S}_i = {} & A_iX_i + X_iA_i^{\mathrm{T}} + B_iY_i + Y_i^{\mathrm{T}}B_i^{\mathrm{T}} + \\
& \sum_{j=1,j\neq i}^{N} p_{ij}\gamma_{ij}G_{ij}G_{ij}^{\mathrm{T}}, \\
\tilde{\nu}_i^{-1} = {} & 2\sum_{j=1,j\neq i}^{N} \gamma_{ij}p_{ij}.
\end{aligned}$$

If letting A_{D}, B_{D}, K_{D} and G_{D} are the same as [7] and $G_i = \sum\limits_{j=1,j\neq i}^{N} p_{ij}G_{ij}g_{ij}(x_i(t), x_j(t))$, LMI (25) implies the main result in [7]. In addition, LMI (25) is simpler and more general than LMI (20) in [7] in description form.

4 Simulation results

In this section, a two-machine infinite bus example system which is shown in Fig.2 is chosen to demonstrate the design procedure and the effectiveness of the proposed decentralized controller. Since generator #3 is with an infinite bus, we have $\acute{E}_{q3} = 1\angle 0°$.

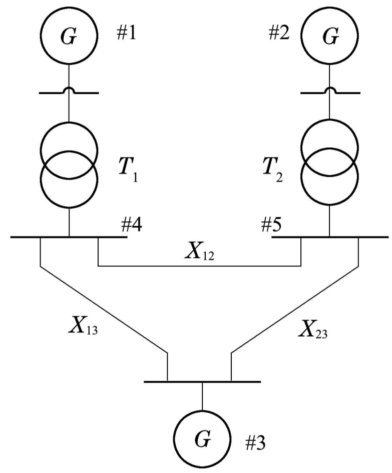


Fig. 2 A two-machine infinite bus example power system

From Fig.2, it's easily seen that $p_{12} = p_{21} = 1$ and $p_{13} = p_{23} = 1$.

The system parameters used in the simulation are as follows[6]:

$x_{\mathrm{d1}} = 1.863,\ \acute{x}_{\mathrm{d1}} = 0.257,\ x_{\mathrm{T1}} = 0.129,$

$\acute{T}_{\mathrm{d01}} = 16.9\,\mathrm{s},\ H_1 = 4\,\mathrm{s},\ D_1 = 5,\ k_{\mathrm{c1}} = 1,$

$$x_{d2} = 2.36,\ \acute{x}_{d2} = 0.319,\ x_{T2} = 0.11,$$
$$\acute{T}_{d02} = 7.96\,\text{s},\ H_2 = 5.1\,\text{s},\ D_2 = 3,$$
$$k_{c2} = 1,\ \omega_0 = 314.159,\ F_{IP1} = F_{IP2} = 0.3,$$
$$K_{M1} = K_{E1} = 1,\ K_{M1} = K_{M2} = 1\,\text{rad/s},$$
$$R_1 = R_2 = 0.05,\ T_{M1} = T_{M2} = 0.35\,\text{s},$$
$$T_{E1} = T_{E2} = 0.1\,\text{s},\ x_{12} = 0.55,\ x_{13} = 0.53,$$
$$x_{23} = 0.6,\ x_{ad1} = x_{ad2} = 1.712.$$

The matrices A_i, B_i and G_{ij} that describe the nominal system model are as follows:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.625 & 27.48 & 11.781 \\ 0 & 0 & -2.85 & 2.857 \\ 0 & -0.637 & 0 & -10 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.392 & 20.56 & 9.24 \\ 0 & 0 & -2.857 & 2.857 \\ 0 & -0.637 & 0 & -10 \end{bmatrix},$$

$$G_{12} = G_{13} = [0\ \ -27.49\ \ 0\ \ 0]^{\mathrm{T}},$$
$$G_{21} = G_{23} = [0\ \ -23.1\ \ 0\ \ 0]^{\mathrm{T}},$$
$$B_1 = [0\ \ 0\ \ 0\ \ 10]^{\mathrm{T}},\ B_2 = [0\ \ 0\ \ 0\ \ 10]^{\mathrm{T}}.$$

According to Theorem 1, we choose constants as $\beta_1 = \beta_2 = 10$, $\mu_{12} = \mu_{21} = 2$ and $\gamma_{12} = \gamma_{21} = 1$, the sector saturation coefficients are $a_1 = a_2 = 0.4$. It's obvious that $\mu_{12} > \gamma_{12}$ and $\mu_{21} > \gamma_{21}$ are satisfied. The feasible solutions of LMI (12) with $X_1 > 0$ and $X_2 > 0$ are as follows;

$$X_1 = \begin{bmatrix} 5.59 & -72.84 & -54.71 & 171.19 \\ -72.84 & 1371.23 & 1903.35 & -5577.53 \\ -54.71 & 1903.35 & 25112.00 & -61362.35 \\ 171.19 & -5577.53 & -61362.35 & 150819.20 \end{bmatrix},$$

$$Y_1 = [2.72\ \ -65.52\ \ -370.50\ \ 152.07],$$

$$X_2 = \begin{bmatrix} 5.29 & -75.06 & -199.63 & 519.99 \\ -75.06 & 1539.10 & 6161.61 & -15720.40 \\ -199.63 & 6161.61 & 62556.95 & -150569.67 \\ 519.99 & -15720.40 & -150569.67 & 363436.33 \end{bmatrix},$$

$$Y_2 = [3.26\ \ -80.55\ \ -541.35\ \ 550.17].$$

Hence, the feedback gain matrices are

$$K_1 = Y_1 X_1^{-1} = [-138.59\ \ -19.58\ \ -36.98\ \ -15.61],$$
$$K_2 = Y_2 X_2^{-1} = [-169.19\ \ -26.47\ \ -39.25\ \ -17.16].$$

With time-delay $\tau_{12} = \tau_{21} = 0.1\,\text{s}$, the state trajectories of generator #1 are shown in Fig.3, and those of generator #2 are shown in Fig.4.

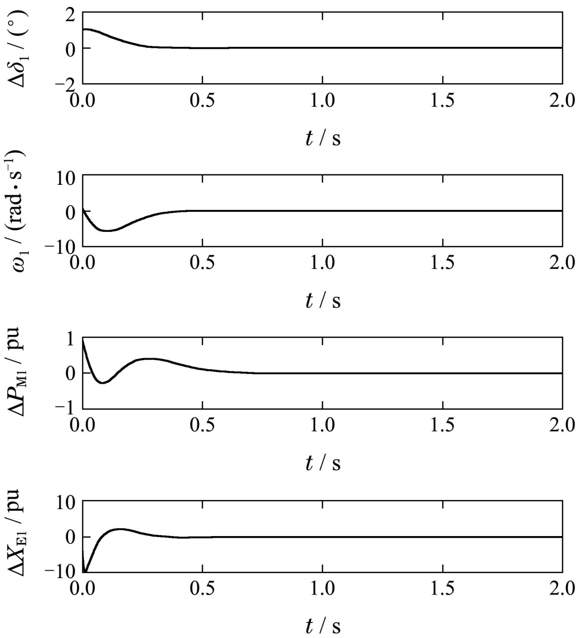


Fig. 3 The state trajectories for the #1 generator with the presented controller in Theorem 1

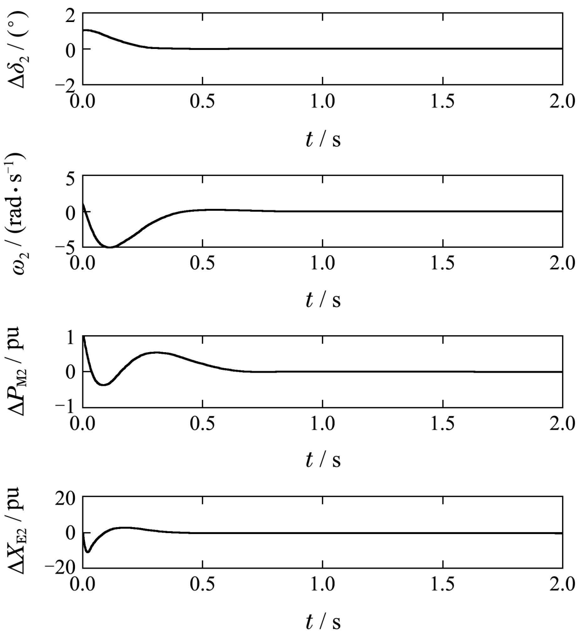
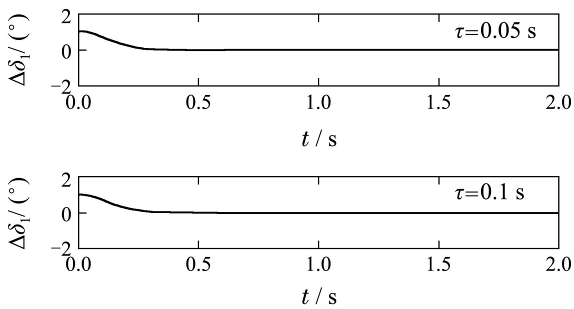


Fig. 4 The state trajectories for the #2 generator with the presented controller in Theorem 1

When the constant delay is varied and other parameters are the same, the increment of rotor angle trajectories of Generator #1 is shown in Fig.5.



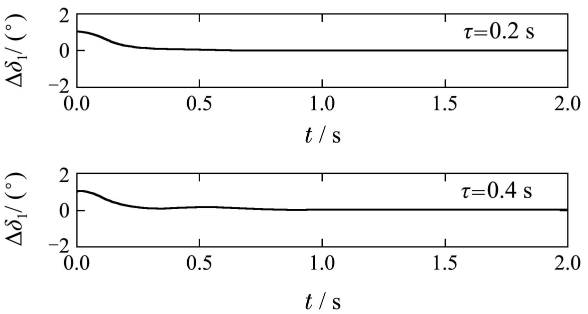


Fig. 5 The increment of rotor angle trajectories of generator #1 with various constant delays

It's easily seen from Figs.3 and 4 that the state trajectories of generator #1 and #2 have been quickly stabilized before $t = 0.8$ s with the controller in Theorem 1. What's more, the increment of rotor angle trajectories of generator #1 can be quickly stabilized before $t = 0.8$ s when taking various constant delays.

Next, Theorem 2 will be validated. For simplicity, only the parametric perturbation in $T_{\mathrm{M}i}$ is considered, which is used to emulate the time constant uncertainties in the high-pressure and low-pressure sections. Let $\rho_i(t) = 1/T_{\mathrm{M}i} - 1/(T_{\mathrm{M}i} - \Delta T_{\mathrm{M}i}) = 0.635\tilde{\phi}_i(t)$ with $|\tilde{\phi}_i(t)| \leqslant 1$, it follows that

$$\Delta A_i(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_i(t) & \rho_i(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The structure of parametric uncertainty is expressed as follows:
$L_i = [0\ \ 0\ \ 1.41|\rho_i(t)|_{\max}\ \ 0]^{\mathrm{T}}$, $M_i = \mathrm{diag}\{1,1,1,1\}$,
$F_i(t) = [0\ \ 0\ \ -0.707\rho_i(t)/|\rho_i(t)|_{\max}\ \ 0.707\rho_i(t)/|\rho_i(t)|_{\max}]$.

According to Theorem 2, we choose constants as $\beta_1 = \beta_2 = 10$, $\theta_1 = \theta_2 = 1$, $\mu_{12} = \mu_{21} = 2$ and $\gamma_{12} = \gamma_{21} = 1$, the sector saturation coefficients are $a_1 = a_2 = 0.4$. It's obvious that $\mu_{12} r_{12} > \gamma_{12}$ and $\mu_{21} > \gamma_{21}$ are satisfied. The feasible solutions of LMI (18) with $X_1 > 0$ and $X_2 > 0$ are as follows:

$$X_1 = \begin{bmatrix} 19.38 & -180.18 & -56.66 & -57.31 \\ -180.18 & 4525.51 & 653.23 & -3458.78 \\ -56.66 & 653.23 & 1605.53 & -2746.54 \\ -57.31 & -3458.78 & -2746.54 & 27006.93 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 847.16 & -70068.74 & -28089.99 & 63380.76 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 19.85 & -181.88 & -55.60 & -53.82 \\ -181.88 & 4115.46 & 593.07 & -3689.29 \\ -55.60 & 593.07 & 1785.93 & -2610.26 \\ -53.82 & -3689.29 & -2610.26 & 27523.69 \end{bmatrix},$$

$$Y_2 = [906.37\ \ -59921.38\ \ -28035.67\ \ 64633.41].$$

Hence, the feedback gain matrices are

$$K_1 = Y_1 X_1^{-1} = [-283.09\ \ -26.43\ \ -23.65\ \ -4.04],$$

$$K_2 = Y_2 X_2^{-1} = [-266.42\ \ -26.69\ \ -20.53\ \ -3.70].$$

With time-delay $\tau_{12} = \tau_{21} = 0.1$ s and parameter uncertainty $\gamma_1(t) = \gamma_2(t) = 0.625$, the state trajectories of generator #1 are shown in Fig.6, and that of generator #2 are shown in Fig.7.

It's easily seen from Figs.6 and 7 that the state trajectories of generator #1 and #2 have been quickly stabilized before $t = 1$ s with the controller in Theorem 2 when the operation parameters change in the given range.

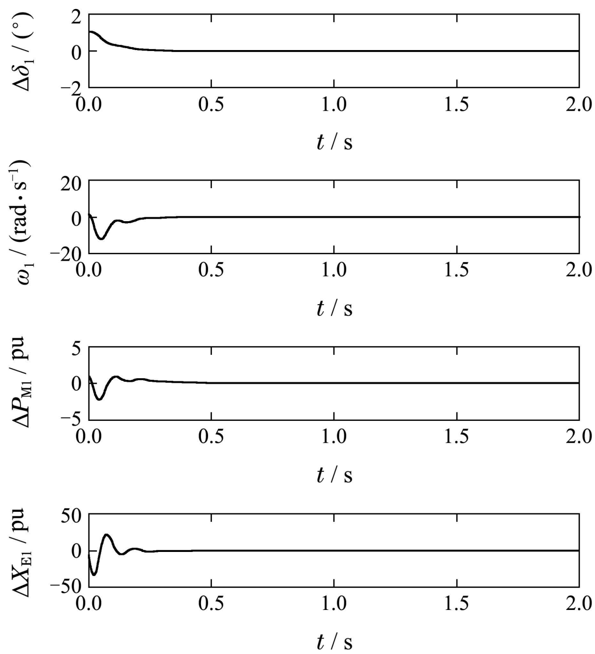
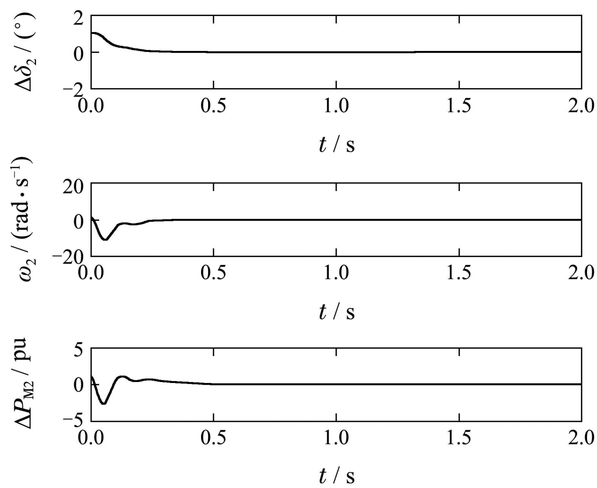


Fig. 6 The state trajectories for the generator #1 with the presented controller in Theorem 2



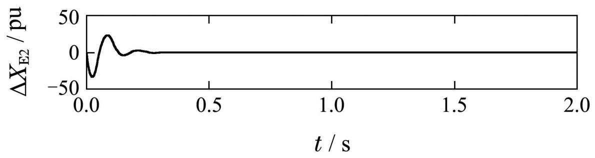


Fig. 7 The state trajectories for the generator #2 with the presented controller in Theorem 2

Numerous simulation results show that, though there are sector actuator saturation and time-delays in the nominal system of (10) or the uncertain system (10), the corresponding control schemes are time-independent and can still rapidly damp the oscillations and greatly enhance the stability of the multi-machine power system.

5 Conclusion

In this paper, a decentralized feedback control scheme has been proposed to enhance the transient stability of uncertain and time delay multi-machine power system with sector nonlinear saturating actuator. Sufficient conditions of asymptomatic stability for nominal and uncertain closed-loop power system have been presented. The LMI method has been used to compute the control gain matrices. It has been shown from the simulation results that the presented control schemes are efficient and permits the rapid stability of the closed-loop system. What's more, it's obvious that the results in [6] and [7] are special cases of Theorem 1 or 2 in our paper.

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