第 28 卷第 5 期
2011 年 5 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 28 No. 5
May 2011

文章编号: 1000−8152(2011)05−0639−06

一类具有积分输入到状态稳定未建模动态的高阶非线性系统的状态反馈调节

段 纳[1,2], 王 璐[3], 赵丛然[2]

(1. 徐州师范大学 电气工程及自动化学院, 江苏 徐州 221116; 2. 曲阜师范大学 自动化研究所, 山东 曲阜 273165;

3. 海军工程大学 电子工程学院, 湖北 武汉 430033)

摘要: 本文研究了一类具有积分输入到状态稳定(iISS)未建模动态的高阶非线性系统的状态反馈调节问题. 基于反推技术和iISS的性质, 通过恰当地选取设计函数和参数修正律, 给出了一个状态反馈控制器的设计过程. 设计的控制器保证了闭环系统所有信号的有界性, 并且状态渐近调节到零. 仿真例子验证了该控制方案的有效性.

关键词: 高阶非线性系统; 状态反馈; 积分输入到状态稳定

中图分类号: TP273 **文献标识码**: A

State-feedback regulation for a class of higher-order nonlinear systems with integral input-to-state stability unmodeled dynamics

DUAN Na[1,2], WANG Lu[3], ZHAO Cong-ran[2]

(1. School of Electrical Engineering & Automation, Xuzhou Normal University, Xuzhou Jiangsu 221116, China;
2. Institute of Automation, Qufu Normal University, Qufu Shandong 273165, China;
3. College of Electronic Engineering, Naval University of Engineering, Wuhan Hubei 430033)

Abstract: We investigate the state-feedback regulation for a class of higher-order nonlinear systems with integral input-to-state stability(iISS) unmodeled dynamics. Based on the backstepping design technique and the properties of iISS, the design procedures for a state-feedback controller are developed by appropriately choosing the designed functions and the update laws of parameters. The designed controller guarantees that all the signals in the closed-loop system are bounded, and the states can be regulated asymptotically to zero. A simulation example demonstrates the effectiveness of the control scheme.

Key words: higher-order nonlinear systems; state-feedback; integral input-to-state stability (iISS)

1 引言(Introduction)

考虑如下的高阶非线性系统:

$$\begin{cases} \dot{\eta} = f_0(\eta, x_1), \\ \dot{x}_i = x_{i+1}^{p_i} + f_i(\eta, \bar{x}_i),\ i = 1, \cdots, n-1, \\ \dot{x}_n = u^{p_n} + f_n(\eta, x), \end{cases} \tag{1}$$

其中: $\eta \in \mathbb{R}^r, x = (x_1, \cdots, x_n) \in \mathbb{R}^n$和$u \in \mathbb{R}$分别是系统的未建模动态、可测状态和控制输入. 对任意的$i = 1, \cdots, n, p_i \geqslant 1$是奇数, 函数$f_0(\cdot)$和$f_i(\cdot)$是局部Lipschitz函数. $\bar{x}_i \triangleq (x_1, \cdots, x_i)(i = 1, \cdots, n), \bar{x}_n = x, x_i$表示$x$的第$i$个元素.

近年来, 上述系统的鲁棒性研究取得了较大进展[1,2], 然而未建模动态至少需要满足输入状态稳定(ISS). 文献[3]首次引入了一个较弱但非常有意义的概念: 积分输入到状态稳定(iISS), 它证明了iISS严格弱于ISS, 并给出了一个非常保守的Lyapunov型的充分条件. 文献[4]是第1篇深入研究系统iISS性质的文章, 它给出了一个基于Lyapunov函数的判断iISS性质的充要条件, 证明了iISS是非线性系统的一个固有性质. 文献[5]针对具有iISS未建模动态的非线性系统, 利用iISS的性质, 构造性地给出了全局鲁棒输出反馈控制器的统一设计和分析. 文献[6~8]进一步研究了iISS的性质, 并给出了iISS的小增益条件. 文献[9]将文献[5]推广到随机非线性系统.

本文针对具有iISS未建模动态的高阶非线性系统(1), 研究了状态反馈调节问题. 利用反推设计技术和iISS的性质, 通过恰当地选取设计函数和参数修正律, 设计了一个状态反馈控制器. 设计的控制器保

收稿日期: 2009−12−06; 收修改稿日期: 2010−05−18.
基金项目: 国家自然科学基金资助项目(10971256, 60974028); 高等学校博士学科点专项科研基金资助课题(20103705110002), 江苏省自然科学基金资助项目(BK2009083); 山东省自然科学基金资助项目(ZR2009GM008, ZR2009AL014).

证了闭环系统所有信号的有界性, 并且状态渐进调节到零. 仿真例子验证了这一控制方案的有效性.

2 预备知识(Preliminaries)

如下的符号将会在全文中使用: $\mathbb{R}_+$表示非负实数集, $\mathbb{R}^n$表示n维实空间. $|X|$表示向量X的欧氏范数. 对在原点的任一个小邻域内的s, 用$\sigma_1(s) = \mathrm{O}(\sigma_2(s))$表示$\sigma_1(s) \leqslant c\sigma_2(s)$, $c>0$为一常数.

考虑如下系统:

$$\dot{x} = f(x,u),\ x\in\mathbb{R}^n,\ u\in\mathbb{R}^m, \tag{2}$$

其中$f:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$是局部Lipschitz的.

定义 1[5] 如果存在$\tilde{\alpha}$, $\gamma\in\mathcal{K}_\infty$和$\beta_{\tilde{\alpha}}\in\mathcal{KL}$, 使得对每一个初始条件$x(0)\in\mathbb{R}^n$ 和任一可测、局部本质有界的函数$u:\mathbb{R}_+\to\mathbb{R}^m$, 任意的$t>0$, 式(2)的解$x(t)$存在且满足

$$\tilde{\alpha}(|x(t)|) \leqslant \beta_{\tilde{\alpha}}(|x(0)|,t) + \int_0^t \gamma(|u(\tau)|)\mathrm{d}\tau,$$

则称系统(2)关于输入u是积分输入到状态稳定的(iISS).

iISS性质可用Lyapunov函数等价表示.

引理 1[5] 系统(2)关于输入u是iISS的充分必要条件是存在一个正定、正则的iISS-Lyapunov函数$V(x)$, 使得对$\forall(x,u)\in\mathbb{R}^{n+m}$,

$$\begin{cases}\underline{\alpha}(|x|)\leqslant V(x)\leqslant\bar{\alpha}(|x|),\\ \dfrac{\partial V(x)}{\partial x}f(x,u)\leqslant-\alpha(|x|)+\gamma(|u|),\end{cases} \tag{3}$$

其中: α是一个正定连续函数, $\underline{\alpha},\ \bar{\alpha},\ \gamma\in\mathcal{K}_\infty$.

引理 2[5] 考虑iISS系统(2), $m>0$为任一整数, iISS-Lyapunov函数$V(x)$满足条件(3). 选取任一光滑函数ϕ满足$\phi^m(s)=\mathrm{O}(\alpha(s))$, 且当$\alpha$有界时, $\lim\limits_{s\to\infty}\sup\dfrac{\phi^m(s)}{\alpha(s)}<\infty$. 则存在正定函数$\sigma$和$\mathcal{K}_\infty$类函数$\varphi$, 使得

$$\int_0^t \phi^m(|x(\tau)|)\mathrm{d}\tau \leqslant \sigma(|x(0)|) + \int_0^t \varphi(|u(\tau)|)\mathrm{d}\tau.$$

进一步的, 若$\gamma(s)=\mathrm{O}(s^m)$, 则$\varphi(s)=\mathrm{O}(s^m)$.

3 状态反馈控制器的设计(Design of state-feedback controller)

对系统(1)作以下假设:

假设 1 η-子系统关于x_1是iISS的, 即存在一个iISS-Lyapunov函数V_0使得

$$\underline{\alpha}(|\eta|)\leqslant V_0(\eta)\leqslant\bar{\alpha}(|\eta|),$$
$$\frac{\partial V_0(\eta)}{\partial\eta}f_0(\eta,x_1)\leqslant-\alpha_0(|\eta|)+\gamma_0(|x_1|),$$

其中: α_0是一个正定连续函数, $\underline{\alpha},\ \bar{\alpha},\ \gamma_0\in\mathcal{K}_\infty$.

假设 2 对任意的$i=1,\cdots,n$, 存在未知常数$q_{i0}>0$, 非负光滑函数$f_{i0}(\cdot)$和$f_{ij}(\cdot)$, 使得

$$f_i(\eta,\bar{x}_i)|\leqslant q_{i0}f_{i0}^{p_i}(|\eta|)+\sum_{j=1}^{i}f_{ij}(\bar{x}_i)|x_j|^{p_i},$$

其中$j=1,\cdots,i$, $f_{i0}(0)=0$.

假设 3 $p=p_1\geqslant p_2\geqslant\cdots\geqslant p_n\geqslant1$是奇数.

下面利用反推方法设计控制器. 引进误差变量

$$z_i=x_i-\alpha_i(\bar{x}_{i-1},\bar{k}_{i-1}),\ \alpha_1=0, \tag{4}$$

$\alpha_i(\bar{x}_{i-1},\bar{k}_{i-1})(i=2,\cdots,n)$为待设计的控制律.

Step 1 由式(1)和式(4)得

$$\dot{z}_1=x_2^{p_1}+f_1(\eta,x_1). \tag{5}$$

选取$V_1(z_1)=\dfrac{1}{p-p_1+2}z_1^{p-p_1+2}$, 光滑函数$\psi_1(z_1)$满足

$$\psi_1(z_1)\geqslant(1+\bar{a}_1(z_1)+f_{11}(z_1))^{\frac{1}{p_1}} \tag{6}$$

和第1个虚拟控制律

$$\begin{cases}\alpha_2(x_1,k_1)=-c_1k_1\psi_1(z_1)z_1,\\ \dot{k}_1=d_1\psi_1^{p_1}(z_1)z_1^{p+1}.\end{cases} \tag{7}$$

应用假设2, Young不等式, 式(4)(5)得

$$\begin{aligned}\dot{V}_1\leqslant\ & z_1^{p-p_1+1}(x_2^{p_1}-\alpha_2^{p_1})+z_1^{p-p_1+1}\alpha_2^{p_1}+\\ & |z_1|^{p-p_1+1}(q_{10}f_{10}^{p_1}(|\eta|)+|z_1|^{p_1}f_{11}(z_1))\leqslant\\ & z_1^{p-p_1+1}(x_2^{p_1}-\alpha_2^{p_1})+z_1^{p-p_1+1}\alpha_2^{p_1}+\\ & q_{11}f_{10}^{p+1}(|\eta|)+z_1^{p+1}\bar{a}_1(z_1)+z_1^{p+1}f_{11}(z_1)\leqslant\\ & -\bar{c}_1k_1^{p_1}\psi_1^{p_1}z_1^{p+1}+\psi_1^{p_1}z_1^{p+1}+q_{11}f_{10}^{p+1}(|\eta|)+\\ & z_1^{p-p_1+1}(x_2^{p_1}-\alpha_2^{p_1}).\end{aligned} \tag{8}$$

其中:

$$q_{11}=a_1q_{10}^{(p+1)/p_1},$$
$$\bar{a}_1(z_1)=\frac{p-p_1+1}{p+1}\cdot(\frac{(p+1)a_1}{p_1})^{\frac{-p_1}{(p-p_1+1)}},$$

$a_1>0$为常数, $\bar{c}_1=c_1^{p_1}$, $c_1,d_1>0$是设计参数.

Step *i*($i=2,\cdots,n$) 假设在第$i-1$步已经得到虚拟控制律$\alpha_i(\bar{x}_{i-1},\bar{k}_{i-1})$和修正律$\dot{k}_{i-1}$形如:

$$\begin{cases}\alpha_i(\bar{x}_{i-1},\bar{k}_{i-1})=-c_{i-1}k_{i-1}\psi_{i-1}(\bar{z}_{i-1})z_{i-1},\\ \dot{k}_{i-1}=d_{i-1}\psi_{i-1}^{p_{i-1}}(\bar{z}_{i-1})z_{i-1}^{p+1},\end{cases} \tag{9}$$

使得$V_{i-1}(\bar{z}_{i-1})=\sum\limits_{j=1}^{i-1}\dfrac{1}{p-p_j+2}z_j^{p-p_j+2}$满足

$$\begin{aligned}\dot{V}_{i-1}\leqslant\ & -\sum_{j=1}^{i-1}\bar{c}_jk_j^{p_j}\psi_j^{p_j}z_j^{p+1}+\sum_{j=1}^{i-1}\psi_j^{p_j}z_j^{p+1}+\\ & \sum_{j=1}^{i-2}\lambda_{j,i-1}z_j^{p+1}+\sum_{j=1}^{i-1}q_{j,i-1}f_{j0}^{p+1}(|\eta|)+\\ & z_{i-1}^{p-p_{i-1}+1}(x_i^{p_{i-1}}-\alpha_i^{p_{i-1}}).\end{aligned} \tag{10}$$

其中对任意的$j=1,\cdots,i-1$, $\bar{c}_j=c_j^{p_j}>0$, c_j,

$d_j > 0$是设计参数, $q_{j,i-1}$是未知常数, $\psi_j(\bar z_j) \geqslant 1$为已知光滑函数. 下面证明对第$i$个Lyapunov函数

$$V_i(\bar z_i) = V_{i-1}(\bar z_{i-1}) + \frac{1}{p-p_i+2} z_i^{p-p_i+2}, \quad (11)$$

式(10)仍同样成立.

由式(1)(4)(7)和式(9)易得

$$\dot z_i = x_{i+1}^{p_i} + f_i(\eta, \bar x_i) - \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial x_j} f_j(\eta, \bar x_j) - \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial x_j} x_{j+1}^{p_j} - \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial k_j} d_j \psi_j^{p_j}(\bar z_j) z_j^{p+1}. \quad (12)$$

其中$x_{n+1} \triangleq u$. 利用式(10)~(12)可得

$$\begin{aligned}\dot V_i \leqslant & -\sum_{j=1}^{i-1} \bar c_j k_j^{p_j} \psi_j^{p_j} z_j^{p+1} + \sum_{j=1}^{i-1} \psi_j^{p_j} z_j^{p+1} + \\ & \sum_{j=1}^{i-2} \lambda_{j,i-1} z_j^{p+1} + \sum_{j=1}^{i-1} q_{j,i-1} f_{j0}^{p+1}(|\eta|) + \\ & z_i^{p-p_i+1}(x_{i+1}^{p_i} - \alpha_{i+1}^{p_i}) + z_i^{p-p_i+1} \alpha_{i+1}^{p_i} + \\ & z_{i-1}^{p-p_{i-1}+1}(x_i^{p_{i-1}} - \alpha_i^{p_{i-1}}) + z_i^{p-p_i+1}(f_i(\eta, \bar x_i) - \\ & \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial x_j} f_j(\eta, \bar x_j) - \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial x_j} x_{j+1}^{p_j} - \\ & \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial k_j} d_j \psi_j^{p_j} z_j^{p+1}).\end{aligned} \quad (13)$$

由式(4)(7)(9), 应用文献 [10]中的引理1, Young不等式, Hölder不等式及假设2, 3, 对任意的$m_1 = 1,2,3,4$, $m_2 = 1,\cdots,i$, $m_3 = 2,3,4$, $j = 1,\cdots,i-1$, 总存在正常数a_{i1}, ε_{i2j}, a_{i21}, $b_{i,m_3,j}$和非负光滑函数$\bar\varepsilon_{i2j}(\cdot)$, $\bar a_{i21}(\cdot)$, $\bar b_{i,m_3,j}(\cdot)$, $\gamma_{i2j}(\cdot)$, $\rho_{i,2,m_2}(\cdot)$, $\rho_{i,3,m_2}(\cdot)$, $\rho_{i4j}(\cdot)$, $\mu_{i,m_1}(\cdot)$满足

$$z_{i-1}^{p-p_{i-1}+1}(x_i^{p_{i-1}} - \alpha_i^{p_{i-1}}) \leqslant a_{i1} z_{i-1}^{p+1} + z_i^{p+1} \mu_{i1}(\bar z_i), \quad (14)$$

$$\begin{aligned} & z_i^{p-p_i+1}(f_i(\eta, \bar x_i) - \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial x_j} f_j(\eta, \bar x_j)) \leqslant \\ & |z_i|^{p-p_i+1}(q_{i0} f_{i0}^{p_i}(|\eta|) + \sum_{j=1}^{i} f_{ij}(\bar x_i)|x_j|^{p_j} + \\ & \sum_{j=1}^{i-1} |\frac{\partial \alpha_i}{\partial x_j}|(q_{j0} f_{j0}^{p_j}(|\eta|) + \sum_{l=1}^{j} f_{jl}(\bar x_j)|x_l|^{p_j})) \leqslant \\ & \sum_{j=1}^{i-1} q_{j0} f_{j0}^{p_j}(|\eta|)|z_i|^{p-p_j+1} \gamma_{i2j}(\bar z_i) + \\ & |z_i|^{p-p_i+1}(q_{i0} f_{i0}^{p_i}(|\eta|) + \sum_{j=1}^{i} \rho_{i2j}(\bar z_i)|z_j|^{p_i}) \leqslant \\ & \sum_{j=1}^{i-1} \varepsilon_{i2j} q_{j0}^{\frac{p+1}{p_j}} f_{j0}^{p+1}(|\eta|) + a_{i21} q_{i0}^{\frac{p+1}{p_i}} f_{i0}^{p+1}(|\eta|) + \\ & \sum_{j=1}^{i-1} b_{i2j} z_j^{p+1} + z_i^{p+1} \mu_{i2}(\bar z_i),\end{aligned} \quad (15)$$

$$\begin{aligned} & -z_i^{p-p_i+1} \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial x_j} x_{j+1}^{p_j} \leqslant \\ & \sum_{j=1}^{i} |z_i|^{p-p_i+1}|z_j|^{p_i} \rho_{i3j}(\bar z_i) \leqslant \\ & \sum_{j=1}^{i-1} b_{i3j} z_j^{p+1} + z_i^{p+1} \mu_{i3}(\bar z_i),\end{aligned} \quad (16)$$

$$\begin{aligned} & -z_i^{p-p_i+1} \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial k_j} d_j \psi_j^{p_j} z_j^{p+1} \leqslant \\ & \sum_{j=1}^{i-1} |z_i|^{p-p_i+1}|z_j|^{p_i} \rho_{i4j}(\bar z_i) \leqslant \\ & \sum_{j=1}^{i-1} b_{i4j} z_j^{p+1} + z_i^{p+1} \mu_{i4}(\bar z_i).\end{aligned} \quad (17)$$

其中:

$$\mu_{i2}(\cdot) \geqslant \sum_{j=1}^{i-1} \bar\varepsilon_{i2j} + \bar a_{i21} + \sum_{j=1}^{i-1} \bar b_{i2j} + \rho_{i2i},$$

$$\mu_{i3}(\cdot) \geqslant \sum_{j=1}^{i-1} \bar b_{i3j} + \rho_{i3i},\ \mu_{i4}(\cdot) \geqslant \sum_{j=1}^{i-1} \bar b_{i4j},$$

$$\bar\varepsilon_{i2j}(\bar z_i) \geqslant \frac{p-p_j+1}{p+1}\left(\frac{(p+1)\varepsilon_{i2j}}{p_j}\right)^{-\frac{p_j}{p-p_j+1}} \gamma_{i2j}^{\frac{p+1}{p-p_j+1}}(\bar z_i),$$

$$\bar a_{i21}(\bar z_i) \geqslant \frac{p-p_i+1}{p+1}\left(\frac{(p+1)a_{i21}}{p_i}\right)^{-\frac{p_i}{p-p_i+1}},$$

$$\bar b_{i,m_3,j}(\bar z_i) \geqslant \frac{p-p_i+1}{p+1}\left(\frac{(p+1)b_{i,m_3,j}}{p_i}\right)^{-\frac{p_i}{p-p_i+1}} \rho_{i,m_3,j}^{\frac{p+1}{p-p_i+1}}(\bar z_i).$$

选取光滑函数$\psi_i(\bar z_i)$满足

$$\psi_i(\bar z_i) \geqslant (1 + \mu_{i1}(\bar z_i) + \mu_{i2}(\bar z_i) + \mu_{i3}(\bar z_i) + \mu_{i4}(\bar z_i))^{\frac{1}{p_i}}, \quad (18)$$

第i个虚拟控制律

$$\begin{cases} \alpha_{i+1}(\bar x_i, \bar k_i) = -c_i k_i \psi_i(\bar z_i) z_i, \\ \dot k_i = d_i \psi_i^{p_i}(\bar z_i) z_i^{p+1}, \end{cases} \quad (19)$$

并将式(14)~(17)代入式(13)得

$$\begin{aligned}\dot V_i \leqslant & -\sum_{j=1}^{i} \bar c_j k_j^{p_j} \psi_j^{p_j} z_j^{p+1} + \sum_{j=1}^{i} \psi_j^{p_j} z_j^{p+1} + \\ & \sum_{j=1}^{i-1} \lambda_{ji} z_j^{p+1} + \sum_{j=1}^{i} q_{ji} f_{j0}^{p+1}(|\eta|) + \\ & z_i^{p-p_i+1}(x_{i+1}^{p_i} - \alpha_{i+1}^{p_i}).\end{aligned} \quad (20)$$

其中:

$$\lambda_{ji} = \lambda_{j,i-1} + b_{i2j} + b_{i3j} + b_{i4j},\ j = 1,\cdots,i-2,$$
$$\lambda_{i-1,i} = b_{i,2,i-1} + b_{i,3,i-1} + b_{i,4,i-1} + a_{i1},$$
$$q_{ji} = q_{j,i-1} + \varepsilon_{i2j} q_{j0}^{(p+1)/p_j},\ j = 1,\cdots,i-1,$$
$$q_{ii} = a_{i21} q_{i0}^{(p+1)/p_i},\ \bar c_i = c_i^{p_i},$$

$c_i, d_i > 0$是设计参数.

在第n步选取光滑函数$\psi_n(z)$和控制器分别满足:

$$\psi_n(z) \geqslant (1 + \mu_{n1}(z) + \mu_{n2}(z) + \mu_{n3}(z) +$$

$$\mu_{n4}(z))^{1/p_n}, \tag{21}$$

$$u(z)=-c_nk_n\psi_n(z)z_n,\ \dot{k}_n=d_n\psi_n^{p_n}(z)z_n^{p+1}. \tag{22}$$

由$\psi_i(\bar{z}_i)\geqslant 1(i=1,\cdots,n)$易得

$$\dot{V}_n\leqslant-\sum_{i=1}^{n}\bar{c}_ik_i^{p_i}\psi_i^{p_i}z_i^{p+1}+\sum_{i=1}^{n}\lambda_i\psi_i^{p_i}z_i^{p+1}+\sum_{i=1}^{n}q_if_{i0}^{p+1}(|\eta|), \tag{23}$$

其中: $V_n=\sum\limits_{i=1}^{n}\dfrac{1}{p-p_i+2}z_i^{p-p_i+2}$, $\bar{c}_n=c_n^{p_n}$, $c_n,d_n>0$为设计参数, 对任意的$1\leqslant i\leqslant n$, $\lambda_i=1+\lambda_{in}$, $\lambda_{nn}=0$, $q_i=q_{in}>0$为未知常数.

4 性能分析(Performance analysis)

下面给出本文的主要结果.

定理 1 若非线性系统(1)满足假设1～3, 假设1中$\gamma_0(s)=\mathrm{O}(s^{p+1})$, 且对任意的$i=1,\cdots,n$,

$$f_{i0}^{p+1}(s)=\mathrm{O}(\alpha_0(s)). \tag{24}$$

另外, 当$\alpha_0(s)$有界时,

$$\lim_{s\to\infty}\sup\frac{f_{i0}^{p+1}(s)}{\alpha_0(s)}<\infty. \tag{25}$$

通过恰当地选择光滑函数ψ_i, 则对任意的初始条件,

i) 闭环系统(1)和式(4)(7)(19)(22)的解在$[0,\infty)$上有定义且有界;

ii) $\lim\limits_{t\to\infty}(|x(t)|+|\eta(t)|+|u(t)|)=0$.

证 由式(24)(25)和引理2知存在正定函数σ_i和$\mathcal{K}_\infty$类函数$\bar{f}_{i0}$, 使得对任意的$i=1,\cdots,n$,

$$\int_0^t f_{i0}^{p+1}(|\eta(\tau)|)\mathrm{d}\tau\leqslant\sigma_i(|\eta(0)|)+\int_0^t\bar{f}_{i0}(|x_1(\tau)|)\mathrm{d}\tau. \tag{26}$$

且由假设$\gamma_0(s)=\mathrm{O}(s^{p+1})$知$\bar{f}_{i0}(|z_1|)=\mathrm{O}(z_1^{p+1})$. 对任意的$z_1\in\mathbb{R}$, 取光滑函数$\psi_1(z_1)$满足

$$\psi_1^{p_1}(z_1)z_1^{p+1}\geqslant\max_{1\leqslant i\leqslant n}\{\bar{f}_{i0}(|z_1|),\gamma_0(|z_1|)\}. \tag{27}$$

本文分两种情况讨论光滑函数$\psi_1(z_1)$的存在性:

1) 由$\gamma_0(s)=\mathrm{O}(s^{p+1})$和$\bar{f}_{i0}(|z_1|)=\mathrm{O}(z_1^{p+1})$知在$z_1=0$的某一邻域内, 一定存在正常数$c_0,\bar{c}_{0i}(i=1,\cdots,n)$使得$\gamma_0(|z_1|)\leqslant c_0z_1^{p+1}$, $\bar{f}_{i0}(|z_1|)\leqslant\bar{c}_{0i}z_1^{p+1}$, 从而在这一邻域内$\psi_1(z_1)$是存在的.

2) z_1在这一邻域外时, 满足式(27)的$\psi_1(z_1)$显然存在. 由于式(6)中的$\psi_1(z_1)$是存在的, 且式(6)中的$\psi_1(z_1)$和式(27)中的$\psi_1(z_1)$都是光滑函数, 故选取满足式(6)和式(27)的$\psi_1(z_1)$的最大者即可.

利用$\dot{k}_i$的定义, 对式(23)两边由0到t积分得

$$V_n(t)-V_n(0)\leqslant-\sum_{i=1}^{n}\int_0^t\frac{\bar{c}_i}{d_i}k_i^{p_i}(\tau)\dot{k}_i(\tau)\mathrm{d}\tau+\sum_{i=1}^{n}\int_0^t\frac{\lambda_i}{d_i}\dot{k}_i(\tau)\mathrm{d}\tau+\sum_{i=1}^{n}\int_0^t q_if_{i0}^{p+1}(|\eta(\tau)|)\mathrm{d}\tau. \tag{28}$$

应用式(7)(19)(22)(26)(27)和V_n的定义, 上式变为

$$-V_n(0)-\bar{a}_0\leqslant-\sum_{i=1}^{n}\frac{\bar{c}_i}{(p_i+1)d_i}k_i^{p_i+1}(t)+\sum_{i=1}^{n}\frac{\lambda_i}{d_i}k_i(t)+\sum_{i=1}^{n}\frac{q_i}{d_1}k_1(t), \tag{29}$$

其中$\bar{a}_0=\sum\limits_{i=1}^{n}\dfrac{\bar{c}_i}{(p_i+1)d_i}k_i^{p_i+1}(0)-\sum\limits_{i=1}^{n}\dfrac{\lambda_i}{d_i}k_i(0)+\sum\limits_{i=1}^{n}q_i\sigma_i(|\eta(0)|)-\sum\limits_{i=1}^{n}\dfrac{q_i}{d_1}k_1(0)$为常数.

假设闭环系统的解存在的最大定义区间为$t\in[0,t_{\mathrm{f}})$, 其中$0<t_{\mathrm{f}}\leqslant\infty$. 下面用反证法证明所有的$k_i(t)(1\leqslant i\leqslant n)$在$[0,t_{\mathrm{f}})$上是有界的. 不失一般性, 假定$k_1(t),\cdots,k_l(t)$无界, 而$k_{l+1}(t),\cdots,k_n(t)$有界, $1\leqslant l\leqslant n$. 由式(7)(19)和式(22)知对任意的$t\in[0,t_{\mathrm{f}})$和$i=1,\cdots,n$, $\dot{k}_i(t)\geqslant0$, 则$k_i(t)$是单调递增的. 由$k_1(t),\cdots,k_l(t)$无界, 则$\lim\limits_{t\to t_{\mathrm{f}}}k_j(t)=\infty$, $1\leqslant j\leqslant l$, 从而由式(29)的右式得

$$\lim_{t\to t_{\mathrm{f}}}(-\sum_{i=1}^{n}\frac{\bar{c}_i}{(p_i+1)d_i}k_i^{p_i+1}(t)+\sum_{i=1}^{n}\frac{\lambda_i}{d_i}k_i(t)+\sum_{i=1}^{n}\frac{q_i}{d_1}k_1(t))=\lim_{t\to t_{\mathrm{f}}}(-\sum_{i=1}^{n}\frac{\bar{c}_i}{(p_i+1)d_i}k_i^{p_i+1}(t))=\lim_{t\to t_{\mathrm{f}}}(-\sum_{j=1}^{l}\frac{\bar{c}_j}{(p_j+1)d_j}k_j^{p_j+1}(t))=-\infty,$$

而式(29)的左式恒为常数, 矛盾.

由式(28)和$k_1(t),\cdots,k_n(t)$的有界性知$V_n(t)$在$[0,\ t_{\mathrm{f}})$上是有界的, 从而$z_1(t),\cdots,z_n(t)$在$[0,t_{\mathrm{f}})$上是有界的. 又由式(7)(19)(22), k_i,z_i的有界性和ψ_i为光滑函数知$\alpha_2,\cdots,\alpha_n,u$是有界的. 由式(4)知$x_1=z_1$, 则$x_1$有界. 再由式(4)得$x_2=z_2+\alpha_2$, 由$z_2,\alpha_2$的有界性知$x_2$有界. 类似地依次递推可证$x_3(t),\cdots,x_n(t)$在$[0,t_{\mathrm{f}})$上是有界的. 由式(27)及$k_1$的有界性知

$$\int_0^t\gamma_0(|x_1(\tau)|)\mathrm{d}\tau\leqslant\int_0^t\frac{\dot{k}_1(\tau)}{d_1}\mathrm{d}\tau=\frac{1}{d_1}(k_1(t)-k_1(0))$$

在$[0,t_{\mathrm{f}})$上是有界的. 这一性质连同定义1, 引理1和假设1显得$\eta(t)$在$[0,t_{\mathrm{f}})$上是有界的. 因此, 闭环系统的所有信号在$[0,t_{\mathrm{f}})$都有界, 再由t_{f}的任意性知$t_{\mathrm{f}}=\infty$.

接下来证明结论ii). 由式(7)(19)(22)和k_i, x_i, z_i, $i=1,\cdots,n$, $\alpha_2,\cdots,\alpha_n,u$的有界性知$\dot{z}_i(i=1,\cdots,n)$有界. 由式(7)(19)(22), $\psi_i\geqslant1$及k_i的有界性知对任意的$i=1,\cdots,n$,

$$\lim_{t\to\infty}\int_0^t z_i^{p+1}(\tau)\mathrm{d}\tau \leqslant \lim_{t\to\infty}\int_0^t \frac{\dot{k}_i(\tau)}{d_i}\mathrm{d}\tau = \lim_{t\to\infty}\frac{1}{d_i}\left(k_i(t)-k_i(0)\right)<\infty,$$

即$z_i\in L_{p+1}$. 又由$z_i,\dot{z}_i\in L_\infty$, 利用Barbălat引理可得$z_1(t),\cdots,z_n(t)$收敛到零. 又由式(7)(19)(22)及$k_i,z_i$的有界性可知$\alpha_2(t),\cdots,\alpha_n(t),u(t)$收敛到零. 由式(4)知$x_1(t)$收敛到零. 再由式(4)得$x_2=z_2+\alpha_2$, 由$z_2,\alpha_2$的收敛性显然$x_2(t)$收敛到零. 类似地依次递推下去可证$x_3(t),\cdots,x_n(t)$收敛到零.

由假设1, 定义1和引理1知存在函数$\tilde{\alpha}\in\mathcal{K}_\infty$和$\beta_{\tilde{\alpha}}\in\mathcal{KL}$, 使得对$\forall t\geqslant t_0\geqslant 0$,

$$\tilde{\alpha}(|\eta(t)|)\leqslant\beta_{\tilde{\alpha}}(|\eta(t_0)|,t-t_0)+\int_{t_0}^t\gamma_0(|(x_1(\tau))|)\mathrm{d}\tau.$$

由t_0的任意性, 选取$t_0=t/2$, 由η的有界性知

$$\lim_{t\to\infty}\beta_{\tilde{\alpha}}(|\eta(t_0)|,t-t_0)=\lim_{t\to\infty}\beta_{\tilde{\alpha}}(|\eta(\frac{t}{2})|,\frac{t}{2})=0.$$

由$\int_0^\infty\gamma_0(|x_1(\tau)|)\mathrm{d}\tau$的有界性, 显然

$$\lim_{t\to\infty}\int_{t_0}^t\gamma_0(|(x_1(\tau))|)\mathrm{d}\tau=\lim_{t\to\infty}\int_{\frac{t}{2}}^t\gamma_0(|x_1(\tau)|)\mathrm{d}\tau=0,$$

从而$\lim\limits_{t\to\infty}\eta(t)=0$.

注 1　本文从两个方面同文献[5]相比较: 1) 由于文献[5]的非线性函数仅依赖未建模动态η和输出$y=x_1$, 因此文献[5]在反推设计中仅引入了一个参数修正律$\dot{k}$. 而本文研究的非线性函数不仅依赖η和x_1, 还依赖其他状态$x_2,\cdots,x_n$(请见假设2), 因此在反推设计的每一步, 除了需要恰当地选取ψ_i和α_i外, 还要选取n个参数修正律$\dot{k}_i$(见式(7)(9)(19)和式(22)), 这样利用引理2才可得到式(29). 然后根据iISS的性质证明了定理1的结论. 2) 由于研究的系统是高阶的(请见假设3), 同时满足更一般的假设2, 因此控制器的设计、稳定性和收敛性分析更加复杂.

5　仿真算例(Simulation example)

考虑如下的非线性系统:

$$\begin{cases}\dot{\eta}=f_0(\eta,x_1),\\ \dot{x}_1=x_2^3+f_1(\eta,x_1),\\ \dot{x}_2=u+f_2(\eta,x_1,x_2).\end{cases}\tag{30}$$

其中:

$$f_0(\eta,x_1)=-\frac{1}{2}\eta+\frac{1}{2}\eta x_1^2,$$

$$f_1(\eta,x_1)=\frac{1}{8}\sin^6\eta+f_{11}x_1^3,$$

$$f_2(\eta,x_1,x_2)=\frac{1}{2}\sin^2\eta+f_{21}x_1+f_{22}x_2,$$

$$p=3,\ q_{10}=q_{20}=1,\ f_{11}=f_{21}=f_{22}=0.1.$$

对于η-子系统, 由$V_0(\eta)=\ln(1+\eta^2)$可得

$$\dot{V}_0(\eta)\leqslant-\frac{\frac{1}{2}\eta^4+\eta^2}{(1+\eta^2)^2}+\frac{1}{2}x_1^4,$$

显然η-子系统满足iISS条件, 且假设1中

$$\alpha_0(|\eta|)=\frac{\frac{1}{2}\eta^4+\eta^2}{(1+\eta^2)^2},\ \gamma_0(|x_1|)=\frac{1}{2}x_1^4.$$

当选取$\eta(0)=1$, 将$x_1(t)\equiv\sqrt{3}$代入η-子系统得$\eta(t)=\mathrm{e}^t$, 从而$\lim\limits_{t\to\infty}\eta(t)=\infty$, 即$\eta$-子系统不满足ISS.

取$f_{10}(|\eta|)=f_{20}(|\eta|)=\frac{1}{2}\sin^2\eta$, 假设2满足, 且易证

$$f_{10}^4(|\eta|)\leqslant\lambda_{01}\alpha_0(|\eta|),$$
$$f_{20}^4(|\eta|)\leqslant\lambda_{02}\alpha_0(|\eta|),$$

其中$\lambda_{01}=\lambda_{02}=\frac{1}{4}$. 由引理2, 选取

$$\bar{f}_{10}(|x_1|)=\bar{f}_{20}(|x_1|)=\frac{1}{4}\gamma_0(|x_1|)=\frac{1}{8}x_1^4.$$

根据第3, 4节, 选取控制器

$$\begin{cases}z_1=x_1,\ z_2=x_2-\alpha_2,\\ \alpha_2(x_1,k_1)=-c_1k_1\psi_1(z_1)z_1,\\ \dot{k}_1=d_1\psi_1^3(z_1)z_1^4,\\ u(x_1,x_2,k_1,k_2)=-c_2k_2\psi_2(z_1,z_2)z_2,\\ \dot{k}_2=d_2\psi_2(z_1,z_2)z_2^4,\end{cases}\tag{31}$$

由光滑函数ψ_1满足

$$\psi_1(z_1)\geqslant(1+\bar{a}_1+f_{11})^{\frac{1}{3}}$$
$$\psi_1^3(z_1)z_1^4\geqslant\max\{\bar{f}_{10}(z_1),\bar{f}_{20}(z_1),\gamma_0(z_1)\}=\frac{1}{2}z_1^4,$$

进而选取

$$\psi_1=(1+\bar{a}_1+f_{11})^{\frac{1}{3}},\ \bar{a}_1=3^3\cdot4^{-4}a_1^{-3},$$
$$\psi_2(z_1,z_2)=1+\mu_{21}(z_1,z_2)+\mu_{22}(z_1,z_2)+\mu_{23}(z_1,z_2)+\mu_{24}(z_1,z_2),$$
$$\mu_{21}(\cdot)=l_{21}a_{211}^{-\frac{1}{3}}+\frac{9}{4}c_1^2\psi_1^2a_{212}^{-1}(1+k_1^2)+4^{-4}3^7c_1^8a_{213}^{-3}\psi_1^8k_1^8,$$
$$\mu_{22}(\cdot)=l_{22}c_1^4\varepsilon_{221}^{-3}\psi_1^4(1+k_1^2)^2z_2^8+l_{21}a_{221}^{-\frac{1}{3}}+l_{21}b_{221}^{-\frac{1}{3}}(f_{21}+c_1f_{11}\psi_1z_1^2(1+k_1^2)^{\frac{1}{2}}+c_1f_{22}\psi_1(1+k_1^2)^{\frac{1}{2}})^{\frac{4}{3}}+f_{22},$$
$$\mu_{23}(\cdot)=4c_1\psi_1z_2^2(1+k_1^2)^{\frac{1}{2}}+3b_{231}^{-\frac{1}{3}}c_1^{\frac{8}{3}}\psi_1^{\frac{8}{3}}(1+z_1^2)^{\frac{4}{3}}(1+k_1^2)^{\frac{4}{3}},$$
$$\mu_{24}(\cdot)=l_{21}b_{241}^{-\frac{1}{3}}c_1^{\frac{4}{3}}d_1^{\frac{4}{3}}\psi_1^{\frac{16}{3}}(1+z_1^2)^{\frac{8}{3}},$$
$$l_{21}=3\cdot4^{-\frac{4}{3}},\ l_{22}=3^3\cdot4^{-4}.$$

选取参数$a_1=a_{211}=a_{212}=a_{213}=\varepsilon_{221}=a_{221}=b_{221}=b_{231}=b_{241}=2$, $c_1=c_2=0.25$, $d_1=d_2=0.2$和初值$\eta(0)=5$, $x_1(0)=0.5$, $x_2(0)=-1.8$, $k_1(0)=0$, $k_2(0)=0.1$. 图1给出了闭环系统(30)(31)的响应, 仿真结果验证了控制方案的有效性.

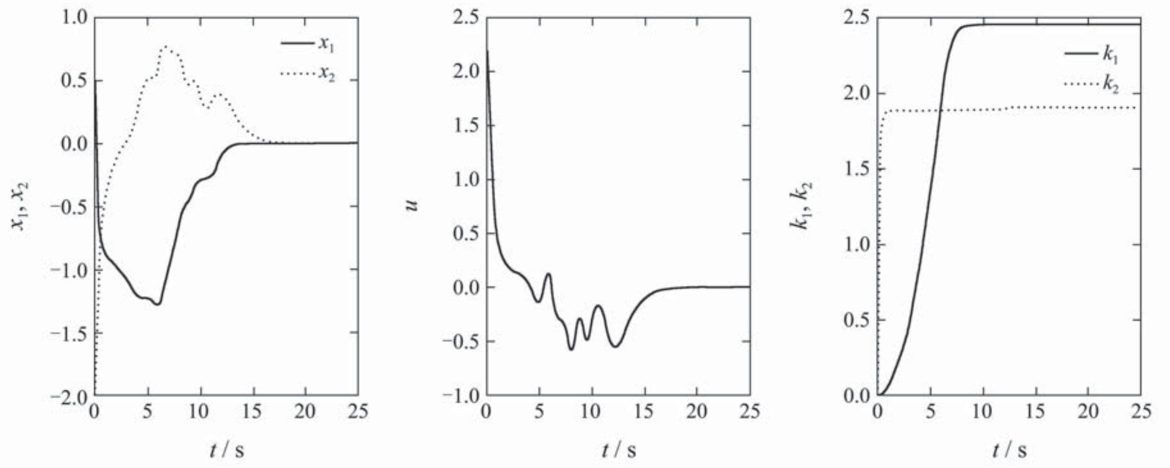

图 1 闭环系统(30)(31)的响应

Fig. 1 The responses of closed-loop system (30)(31)

6 结论(Conclusions)

本文针对具有iISS未建模动态的高阶非线性系统, 研究了状态反馈调节问题.

未来的工作包括: 利用文献[9]建立的随机稳定性理论, 研究具有随机iISS逆动态的高阶随机非线性系统的反馈控制问题; 找出这类系统的实际例子.

参考文献(References):

[1] LIN W, QIAN C J. Adaptive regulation of high-order lower-triangular systems: an adding a power integrator technique[J]. System and Control Letters, 2000, 39(5): 353 – 364.

[2] QIAN C J. *Global synthesis of nonlinear systems with uncontrollable linearization*[D]. Cleveland, USA: Case Western Reserve Universiy, 2001.

[3] SONTAG E D, WANG Y. Comments on integral variants of ISS[J]. *Systems and Control Letters*, 1998, 34(2): 93 – 100.

[4] ANGELI D, SONTAG E D, WANG Y. A characterization of integral input-to-state stability[J]. *IEEE Transactions on Automatic Control*, 2000, 45(6): 1082 – 1097.

[5] JIANG Z P, MAREELS I, HILL D J, et al. A unifying framework for global regulation via nonlinear output feedback: from ISS to iISS[J]. *IEEE Transactions on Automatic Control*, 2004, 49(4): 549 – 562.

[6] ITO H. State-dependent scaling problems and stability of interconnected iISS and ISS systems[J]. *IEEE Transactions on Automatic Control*, 2006, 51(10): 1626 – 1643.

[7] ITO H, JIANG Z P. Necessary and sufficient small gain conditions for integral input-to-state stable systems: a Lyapunov perspective[J]. *IEEE Transactions on Automatic Control*, 2009, 54(10): 2389 – 2404.

[8] ITO H. A Lyapunov approach to cascade interconnection of integral input-to-state stable systems[J]. *IEEE Transactions on Automatic Control*, 2010, 55(5): 702 – 708.

[9] YU X, XIE X J. Output feedback regulation of stochastic nonlinear systems with stochastic iISS inverse dynamics[J]. *IEEE Transactions on Automatic Control*, 2010, 55(2): 304 – 320.

[10] QIAN C J, LIN W. Almost disturbance decoupling for a class of high-order nonlinear systems[J]. *IEEE Transaction on Automatic Control*, 2000, 45(6): 1209 – 1214.

作者简介:

段 纳 (1981—), 女, 讲师, 主要从事非线性系统的自适应控制的研究, E-mail: duanna08@163.com;

王 璐 (1984—), 女, 博士研究生, 主要从事复杂系统分析建模与仿真的研究, E-mail: shuyingqingren@163.com;

赵从然 (1986—), 女, 硕士研究生, 主要从事随机非线性系统的研究, E-mail: congran369@126.com.