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具有未知输出函数的非线性系统全局输出反馈控制

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摘要: 研究了一类具有未知输出函数的非线性系统全局输出反馈控制问题. 由于输出函数未知, 传统的观测器将无法实现. 为解决这个问题, 首先设计了一个与输出函数无关的状态补偿器, 使得标称线性系统全局渐近稳定. 然后, 应用齐次控制方法通过适当选择增益参数, 使得不确定非线性系统在有限时间内全局渐近稳定. 数值算例表明该算法的有效性.

关键词: 未知输出函数; 全局稳定; 齐次控制; 有限时间
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Global output feedback control of nonlinear systems with unknown output function

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Abstract: This paper addresses the problem of global output feedback stabilization of a class of nonlinear systems with unknown output function. Since the output function is not precisely known, traditional observers based on the output is not implementable. To tackle the problem, a state compensator independent of the output function is designed at first and the compensator states are used to globally stabilize the nominal linear system without the perturbing nonlinearities. Then, the homogeneous domination approach is applied to design a scaled homogeneous compensator and controller with an appropriately selected gain to make the uncertain nonlinear system globally finite-time stable. The numerical example shows the validity of the proposed method.

Key words: unknown output function; global stabilization; homogeneous domination; finite-time

1　引言(Introduction)

本文考虑一类具有未知输出函数的非线性系统

$$\begin{cases}\dot{z}_i = z_{i+1} + \phi_i(z_1, \cdots, z_i),\ i = 1, \cdots, n-1,\\ \dot{z}_n = \nu + \phi_n(z_1, \cdots, z_n),\\ y = h(z_1).\end{cases} \tag{1}$$

其中: $z = (z_1, \cdots, z_n)^{\mathrm{T}} \in \mathbb{R}^n$, $y \in \mathbb{R}$, $\nu \in \mathbb{R}$分别是系统的状态, 输出和控制输入. 非线性函数$\phi_i(\cdot)$, $i = 1, \cdots, n$是下三角形式, $h(\cdot)$是连续可微函数且满足$h(0) = 0$. 本文目的是设计如下输出反馈控制器:

$$\dot{\chi} = \Gamma(\chi, u),\ \nu = u(\chi, y), \tag{2}$$

且满足$\Gamma(0, 0) = 0$, $u(0, 0) = 0$, 使得闭环系统(1)(2)全局渐近稳定.

非线性系统的输出反馈控制问题是非线性控制领域最重要的问题之一[1~3]. 不同于线性系统, 非线性系统一般不满足分离原理[4]. 近年来, 研究一类不可测状态非线性系统的输出反馈控制问题已受到广泛关注. 众所周知, 由于有限时间逃逸现象, 对系统(1)的全局输出反馈镇定的条件是非线性函数$\phi_i(\cdot)$不能增长过快. 文献[5~7]对不可测状态的全局输出反馈镇定的一般假设条件是Lipschitz或线性增长. 文献[8]将这一条件扩展到输出增长率是一个多项式的情形. 上述条件的共同特点是非线性函数$\phi_i(\cdot)$中不可测状态至少是线性增长的. 文献 [2]研究了既不满足线性增长条件也不满足Lipschitz条件, 对于不可测状态的下三角形式的非线性系统. 然而, 文献 [1~3, 8]中构造的观测器需要z_1可测. 由于系统(1)中$h(\cdot)$是未知函数, 故此方法不可实现. 本文目标是解决这一具有挑战性的问题, 并给出未知输出函数的非线性系统全局输出反馈镇定问题的解决方案. 首先, 构造无z_1的状态补偿器及相应的控制器, 使得标称线性系统全局渐近稳定. 然后, 运用齐次控制方法来处理非线性项$\phi_i(\cdot)$. 通过引入高增益到补

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偿器, 当增益足够大时可保证该闭环系统全局渐近稳定. 此外, 由于齐次补偿器的度小于零, 可以证明在有限时间内所有状态将收敛到原点, 并一直停留在原点. 最后通过示例验证本文设计控制器方法的有效性.

2 预备知识(Preliminaries)

本节中列出了一些有用的定义和引理.

首先回顾一下齐次系统的定义和性质.

加权齐次性: 对实数$r_i > 0$, $i = 1, \cdots, n$, 固定坐标$(x_1, \cdots, x_n) \in \mathbb{R}^n$和$\forall \varepsilon > 0$,

1) 扩张$\Delta_\varepsilon(x)$定义为$\Delta_\varepsilon(x) = (\varepsilon^{r_1} x_1, \cdots, \varepsilon^{r_n} x_n)$, 其中$r_i$称为分量$x_i$的权, 简记$\Delta = (r_1, \cdots, r_n)$.

2) 若存在$\tau \in \mathbb{R}$, 使得函数$V \in C(\mathbb{R}^n, \mathbb{R})$, 对$\forall x \in \mathbb{R}^n \backslash \{0\}$, $V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1, \cdots, x_n)$, 则称$V$是齐次的且度为$\tau$.

3) 若存在$\tau \in \mathbb{R}$, 使得向量场$f \in C(\mathbb{R}^n, \mathbb{R}^n)$, 对$\forall x \in \mathbb{R}^n \backslash \{0\}$, $f_i(\Delta_\varepsilon(x)) = \varepsilon^{\tau + r_i} f_i(x_1, \cdots, x_n)$, $i = 1, \cdots, n$, 则称f是齐次的且度为τ.

4) 齐次p–范数: 对$p \geqslant 1$, 定义

$$\|x\|_{\Delta,p} = (\sum_{i=1}^{n} |x_i|^{p/r_i})^{1/p}, \ \forall x \in \mathbb{R}^n.$$

为了简便, 当$p = 2$时, 记$\|x\|_{\Delta,2}$为$\|x\|_\Delta$.

引理 1[9]　若扩张Δ_ε齐次系统

$$\dot{x} = f(x), \ f(0) = 0 \tag{3}$$

的平凡解$x = 0$是全局稳定, 则存在Δ_ε–齐次正定的Lyapunov函数V, 使得

$$\dot{V}|_{(3)} = \frac{\partial V}{\partial x} f(x) < 0, \ \forall x \neq 0.$$

引理 2[9]　设f是$\mathbb{R}^n$上的连续向量场, 原点为局部渐近稳定的平衡点. 设f是齐次的且度为k, 任意$p \in \mathbb{N}^*$和$m > p \times \max\limits_i \{r_i\}$, 系统(3)存在正定的齐次$C^p$ Lyapunov函数V度为m, 则$\dot{V}$是齐次的且度为$m + k$.

引理 3[10]　设$V: \mathbb{R}^n \to \mathbb{R}$是关于$\Delta$齐次的且度为$\tau$函数, 则

A) $\dfrac{\partial V}{\partial x_i}$的度为$\tau - r_i$, 其中$r_i$为$x_i$的权;

B) 存在常数c, 使得

$$V(x) \leqslant c \|x\|_\Delta^\tau. \tag{4}$$

若$V(x)$是正定的, 则

$$\underline{c} \|x\|_\Delta^\tau \leqslant V(x), \ \underline{c} > 0. \tag{5}$$

引理 4[11]　对任意$x \in \mathbb{R}$, $y \in \mathbb{R}$和$p \geqslant 1$, 下列不等式成立;

$$\begin{cases} |x + y|^p \leqslant 2^{p-1} |x^p + y^p|, \\ \big(|x| + |y|\big)^{\frac{1}{p}} \leqslant |x|^{\frac{1}{p}} + |y|^{\frac{1}{p}} \leqslant 2^{\frac{p-1}{p}} \big(|x| + |y|\big)^{\frac{1}{p}}. \end{cases} \tag{6}$$

若$p \geqslant 1$是奇数或两个奇数比, 则

$$\begin{cases} |x - y|^p \leqslant 2^{p-1} |x^p - y^p|, \\ |x^{\frac{1}{p}} - y^{\frac{1}{p}}| \leqslant 2^{1 - \frac{1}{p}} |x - y|^{\frac{1}{p}}. \end{cases} \tag{7}$$

引理 5[12]　设c和d为正实数, 对任何实值函数$\gamma(x, y) > 0$, 则

$$|x|^c |y|^d \leqslant \frac{c}{c+d} \gamma(x, y) |x|^{c+d} + \frac{d}{c+d} \gamma^{-\frac{c}{d}}(x, y) |y|^{c+d}. \tag{8}$$

3 具有未知输出函数的线性系统输出反馈控制器设计(Output feedback stabilization of nominal linear system with unknown output function)

首先, 考虑具有未知输出函数的线性系统

$$\begin{cases} \dot{x}_i = x_{i+1}, \ i = 1, \cdots, n-1, \ \dot{x}_n = u, \\ y = h(x_1), \end{cases} \tag{9}$$

其中$h(x_1)$是未知的非线性输出函数, 且满足如下条件:

假设 1　存在两个常数$\underline{\theta}$和$\bar{\theta}$, 有

$$0 < \underline{\theta} \leqslant \frac{\partial h(x_1)}{\partial x_1} \leqslant \bar{\theta}, \ \forall x_1 \in \mathbb{R}. \tag{10}$$

注 1　假设1包含了几类输出函数, 如线性输出函数$h(x_1) = \theta x_1$且未知参数θ的上界和下界是已知的. 又如非线性输出函数$h(x_1) = 3x_1 + \sin x_1$.

定理 1　在假设1下, 可通过输出反馈对系统(9)设计有限时间全局稳定控制器.

证　即使输出y是可测的, 但状态x_1仍未知. 设计如下无x_1的补偿器:

$$\begin{cases} \dot{\hat{x}}_i = f_i(\hat{x}_1, \hat{x}_{i+1}) = \hat{x}_{i+1} - a_i \hat{x}_1^{r_{i+1}}, \\ \dot{\hat{x}}_n = f_n(\hat{x}_1, u) = u - a_n \hat{x}_1^{r_{n+1}}, \end{cases} \tag{11}$$

其中$i = 1, \cdots, n-1$.

取$r_1 = 1, r_{i+1} = r_i + \tau$, 其中$\tau \in (-\dfrac{1}{n}, 0)$和系数$a_i, i = 1, \cdots, n$使得齐次系统

$$\begin{cases} \dot{e}_i = e_{i+1} - a_i e_1^{r_{i+1}}, \ i = 1, \cdots, n-1, \\ \dot{e}_n = -a_n e_1^{r_{n+1}} \end{cases} \tag{12}$$

全局渐近稳定.

注 2　为了简单起见, 设$\tau = -q/p$, 其中q是偶数, p为奇数. 故$r_i = 1 + (i-1)\tau$, $i = 1, \cdots, n+1$, 分子和分母均是奇数.

由引理2和引理3, 可知系统(12)存在度为2的正定Lyapunov函数$V(e)$, 使得

$$\dot{V}(e)\big|_{(12)} \leqslant -\sum_{i=1}^{n} e_i^{\frac{2+\tau}{r_i}}, \ e = (e_1, \cdots, e_n). \tag{13}$$

根据引理3, 得

$$|\frac{\partial V}{\partial e_i}| \leqslant c_i(|e_1|^{2-r_i} + \cdots + |e_n|^{\frac{2-r_i}{r_n}}), \tag{14}$$

其中$c_i > 0$, $i = 1,\cdots,n$. 定义状态估计误差$e_i = x_i - \hat{x}_i$, $i = 1,\cdots,n$. 由式(9)和式(11), 得

$$\begin{cases} \dot{e}_i = e_{i+1} - a_i e_1^{r_{i+1}} + a_i[\hat{x}_1^{r_{i+1}} + (x_1 - \hat{x}_1)^{r_{i+1}}], \\ \qquad i = 1,\cdots,n-1, \\ \dot{e}_n = -a_n e_1^{r_{n+1}} + a_n[\hat{x}_1^{r_{n+1}} + (x_1 - \hat{x}_1)^{r_{n+1}}]. \end{cases} \tag{15}$$

由引理4, 存在常数$d_i > 0(i = 1,\cdots,n)$使得

$$\left|\hat{x}_1^{r_{i+1}} + (x_1 - \hat{x}_1)^{r_{i+1}}\right| \leqslant d_i\,|x_1|^{r_{i+1}}. \tag{16}$$

由式(13)~(16)得

$$\begin{aligned} &\dot{V}(e)|_{(15)} = \\ &\dot{V}(e)|_{(12)} + \frac{\partial V}{\partial e}[a_1(\hat{x}_1^{r_2} + (x_1 - \hat{x}_1)^{r_2}) \ \cdots \\ &a_n(\hat{x}_1^{r_{n+1}} + (x_1 - \hat{x}_1)^{r_{n+1}})]^{\mathrm{T}} \leqslant \\ &-\sum_{i=1}^{n} e_i^{\frac{2+\tau}{r_i}} + \sum_{i=1}^{n}(a_i c_i d_i(\sum_{j=1}^{n}|e_j|^{\frac{2-r_i}{r_j}})|x_1|^{r_{i+1}}) \leqslant \\ &-\frac{3}{4}\sum_{i=1}^{n} e_i^{\frac{2+\tau}{r_i}} + Mx_1^{2+\tau}, \end{aligned} \tag{17}$$

其中$M > 0$. 式(17)最后不等式由引理5得到.

接下来, 具体给出控制器设计步骤.

Step 1 选取Lyapunov函数$V_1 = V(e) + y^2/2$, 则求V_1的导数并将式(17)代入, 得

$$\begin{aligned} \dot{V}_1 = \dot{V}(e) + y\dot{y} \leqslant -\frac{3}{4}\sum_{i=1}^{n} e_i^{(2+\tau)/r_i} + Mx_1^{2+\tau} + \\ y\frac{\partial h}{\partial x_1}(\hat{x}_2 + e_2). \end{aligned} \tag{18}$$

根据假设1, 有

$$(\bar{\theta})^{-(2+\tau)}y^{2+\tau} \leqslant x_1^{2+\tau} \leqslant \underline{\theta}^{-(2+\tau)}y^{2+\tau}, \tag{19}$$

其证明过程详见附录.

由引理5和假设1, 则存在$g_0 > 0$, 使得

$$y\frac{\partial h}{\partial x_1}e_2 \leqslant |y|\bar{\theta}|e_2^{1/r_2}|^{1+\tau} \leqslant \frac{1}{4}e_2^{\frac{2+\tau}{r_2}} + g_0 y^{2+\tau}. \tag{20}$$

将式(19)(20)代入式(18), 得

$$\begin{aligned} \dot{V}_1 \leqslant& -\frac{3}{4}e_1^{2+\tau} - \frac{1}{2}e_2^{(2+\tau)/r_2} - \frac{3}{4}\sum_{i=3}^{n} e_i^{(2+\tau)/r_i} + \\ &(M\underline{\theta}^{-(2+\tau)} + g_0)y^{2+\tau} + y\frac{\partial h}{\partial x_1}(\hat{x}_2 - \hat{x}_2^*) + \\ &y\frac{\partial h}{\partial x_1}\,\hat{x}_2^*. \end{aligned} \tag{21}$$

构造虚拟控制器

$$\hat{x}_2^* = -\beta_1 y^{r_2},\ \beta_1 = (n + M\underline{\theta}^{-(2+\tau)} + g_0)/\underline{\theta}, \tag{22}$$

由此可得

$$\begin{aligned} \dot{V}_1 \leqslant& -\frac{3}{4}e_1^{2+\tau} - \frac{1}{2}e_2^{\frac{2+\tau}{r_2}} - \frac{3}{4}\sum_{i=3}^{n} e_i^{\frac{2+\tau}{r_i}} - \\ &n|\xi_1|^{2+\tau} + y\frac{\partial h}{\partial x_1}(\hat{x}_2 - \hat{x}_2^*), \end{aligned} \tag{23}$$

其中$\xi_1 = y$.

Step 2 取Lyapunov函数

$$V_2 = V_1 + W_2 = V_1 + \int_{\hat{x}_2^*}^{\hat{x}_2}(s^{1/r_2} - \hat{x}_2^{*1/r_2})^{2-r_2}\mathrm{d}s, \tag{24}$$

定义$\xi_2 = \hat{x}_2^{1/r_2} - \hat{x}_2^{*1/r_2}$. 求$V_2$的导数并将式(23)代入, 得

$$\begin{aligned} &\dot{V}_2 = \\ &\dot{V}_1 + \xi_2^{2-r_2}(\hat{x}_3 - a_2\hat{x}_1^{r_3}) - \\ &(2-r_2)\frac{\partial \hat{x}_2^{*1/r_2}}{\partial y}\int_{\hat{x}_2^*}^{\hat{x}_2}(s^{\frac{1}{r_2}} - \hat{x}_2^{*\frac{1}{r_2}})^{1-r_2}\mathrm{d}s\cdot\dot{y} \leqslant \\ &-\frac{3}{4}e_1^{2+\tau} - \frac{1}{2}e_2^{\frac{2+\tau}{r_2}} - \frac{3}{4}\sum_{i=3}^{n} e_i^{\frac{2+\tau}{r_i}} - n|\xi_1|^{2+\tau} + \\ &y\frac{\partial h}{\partial x_1}(\hat{x}_2 - \hat{x}_2^*) + \xi_2^{2-r_2}(\hat{x}_3 - a_2\hat{x}_1^{r_3}) - \\ &(2-r_2)\frac{\partial \hat{x}_2^{*\frac{1}{r_2}}}{\partial y}\int_{\hat{x}_2^*}^{\hat{x}_2}(s^{\frac{1}{r_2}} - \hat{x}_2^{*\frac{1}{r_2}})^{1-r_2}\mathrm{d}s\cdot\dot{y}. \end{aligned} \tag{25}$$

接下来, 具体分析式(25)右边的最后3项. 由引理5, 得

$$\begin{aligned} &y\frac{\partial h}{\partial x_1}(\hat{x}_2 - \hat{x}_2^*) \leqslant \\ &2^{1-r_2}\bar{\theta}|y||\xi_2|^{r_2} \leqslant \frac{1}{3}|\xi_1|^{2+\tau} + g_1|\xi_2|^{2+\tau}, \end{aligned} \tag{26}$$

其中$g_1 > 0$. 同理可得

$$\begin{aligned} &-a_2\xi_2^{2-r_2}\hat{x}_1^{r_3} \leqslant \\ &a_2|\xi_2|^{2-r_2}(|x_1|^{r_3} + |e_1|^{r_3}) \leqslant \\ &\frac{1}{3}|\xi_1|^{2+\tau} + \frac{1}{2n}|e_1|^{2+\tau} + g_2|\xi_2|^{2+\tau}, \end{aligned} \tag{27}$$

其中$g_2 > 0$. 对式(25)的最后一项的估计, 有

$$\begin{aligned} &-(2-r_2)\frac{\partial \hat{x}_2^{*1/r_2}}{\partial y}\int_{\hat{x}_2^*}^{\hat{x}_2}(s^{1/r_2} - \hat{x}_2^{*1/r_2})^{1-r_2}\mathrm{d}s\cdot\dot{y} \leqslant \\ &(2-r_2)|\frac{\partial \hat{x}_2^{*\frac{1}{r_2}}}{\partial y}||\xi_2|^{1-r_2}|\hat{x}_2 - \hat{x}_2^*||\frac{\partial h}{\partial x_1}||\hat{x}_2 + e_2| \leqslant \\ &(2-r_2)2^{1-r_2}|\frac{\partial \hat{x}_2^{*\frac{1}{r_2}}}{\partial y}||\xi_2\bar{\theta}|(\xi_2 - \beta_1^{1/r_2}y)^{r_2} + e_2| \leqslant \\ &\frac{1}{3}|\xi_1|^{2+\tau} + \frac{1}{2n}|e_2|^{(2+\tau)/r_2} + g_3|\xi_2|^{2+\tau}, \end{aligned} \tag{28}$$

其中$g_3 > 0$. 将式(26)~(28)代入式(25), 得

$$\begin{aligned} &\dot{V}_2 \leqslant \\ &-(\frac{3}{4} - \frac{2}{4n})e_1^{2+\tau} - (\frac{1}{2} - \frac{2}{4n})e_2^{\frac{2+\tau}{r_2}} - \end{aligned}$$

$$\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}-(n-1)|\xi_1|^{2+\tau}+\xi_2^{2-r_2}(\hat{x}_3-\hat{x}_3^*)+$$
$$(g_1+g_2+g_3)\xi_2^{2+\tau}+\xi_2^{2-r_2}\hat{x}_3^*. \tag{29}$$

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$$\hat{x}_3^*=-\beta_2\xi_2^{r_3},\ \beta_2=n-1+g_1+g_2+g_3.$$

再由式(29), 得

$$\dot{V}_2\leqslant-(\frac{3}{4}-\frac{2}{4n})e_1^{2+\tau}-(\frac{1}{2}-\frac{2}{4n})e_2^{\frac{2+\tau}{r_2}}-$$
$$\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}-(n-1)(|\xi_1|^{2+\tau}+$$
$$|\xi_2|^{2+\tau})+\xi_2^{2-r_2}(\hat{x}_3-\hat{x}_3^*). \tag{30}$$

归纳步骤 假设第$k-1$步, 存在正定的Lyapunov函数$V_{k-1}(e,y,\hat{x}_2,\cdots,\hat{x}_{k-1})$和虚拟控制器$\hat{x}_2^*,\cdots,\hat{x}_k^*$, 定义为

$$\begin{cases}\hat{x}_1^*=0,\ \xi_1=y^{1/r_1}-\hat{x}_1^{*1/r_1},\\ \hat{x}_2^*=-\beta_1\xi_1^{r_2},\ \xi_2=\hat{x}_2^{1/r_2}-\hat{x}_2^{*1/r_2},\\ \quad\vdots\\ \hat{x}_k^*=-\beta_{k-1}\xi_{k-1}^{r_k},\ \xi_k=\hat{x}_k^{1/r_k}-\hat{x}_k^{*1/r_k},\end{cases} \tag{31}$$

其中$\beta_1>0,\cdots,\beta_{k-1}>0$, 有

$$\dot{V}_{k-1}\leqslant$$
$$-(\frac{3}{4}-\frac{k-1}{4n})e_1^{2+\tau}-(\frac{1}{2}-\frac{k-1}{4n})e_2^{\frac{2+\tau}{r_2}}-$$
$$\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}+\xi_{k-1}^{2-r_{k-1}}(\hat{x}_k-\hat{x}_k^*)-$$
$$(n-k+2)(|\xi_1|^{2+\tau}+\cdots+|\xi_{k-1}|^{2+\tau}). \tag{32}$$

显然, 当$k=3$时式(32)即为式(30). 接下来证明当第k步时式(32)仍然成立. 取Lyapunov函数

$$\begin{cases}V_k(e,y,\hat{x}_2,\cdots,\hat{x}_k)=\\ V_{k-1}(e,y,\hat{x}_2,\cdots,\hat{x}_{k-1})+W_k(y,\hat{x}_2,\cdots,\hat{x}_k),\\ W_k(y,\hat{x}_2,\cdots,\hat{x}_k)=\int_{\hat{x}_k^*}^{\hat{x}_k}(s^{\frac{1}{r_k}}-\hat{x}_k^{*\frac{1}{r_k}})^{2-r_k}\mathrm{d}s.\end{cases} \tag{33}$$

其V_k导数为

$$\dot{V}_k\leqslant-(\frac{3}{4}-\frac{k-1}{4n})e_1^{2+\tau}-(\frac{1}{2}-\frac{k-1}{4n})e_2^{\frac{2+\tau}{r_2}}-$$
$$\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}+\xi_k^{2-r_k}(\hat{x}_{k+1}-a_k\hat{x}_1^{r_{k+1}})-$$
$$(n-k+2)\sum_{j=1}^{k-1}|\xi_j|^{2+\tau}+\xi_{k-1}^{2-r_{k-1}}(\hat{x}_k-\hat{x}_k^*)+$$
$$\sum_{l=2}^{k-1}\frac{\partial W_k}{\partial\hat{x}_l}\dot{\hat{x}}_l+\frac{\partial W_k}{\partial y}\dot{y}. \tag{34}$$

接下来, 估计式(34)右边的各项. 首先与式(26)处理相似, 由引理4和引理5, 得

$$\xi_{k-1}^{2-r_{k-1}}(\hat{x}_k-\hat{x}_k^*)\leqslant$$
$$2^{1-r_k}|\xi_{k-1}|^{2-r_{k-1}}|\xi_k|^{r_k}\leqslant$$
$$\frac{1}{2}|\xi_{k-1}|^{2+\tau}+g_4|\xi_k|^{2+\tau}, \tag{35}$$

其中$g_4>0$. 与式(27)类似, 得

$$-a_k\xi_k^{2-r_k}\hat{x}_1^{r_{k+1}}\leqslant\frac{1}{3}|\xi_1|^{2+\tau}+\frac{1}{8n}|e_1|^{2+\tau}+g_5|\xi_k|^{2+\tau}, \tag{36}$$

其中$g_5>0$.

另外, 存在正常数g_6, 有

$$\sum_{l=2}^{k-1}\frac{\partial W_k}{\partial\hat{x}_l}\dot{\hat{x}}_l\leqslant\frac{1}{2}\sum_{l=2}^{k-1}\xi_l^{2+\tau}+\frac{1}{3}|\xi_1|^{2+\tau}+$$
$$\frac{1}{8n}|e_1|^{2+\tau}+g_6|\xi_k|^{2+\tau}, \tag{37}$$

其证明过程详见附录.

对式(34)的最后一项的估计, 有

$$\frac{\partial W_k}{\partial y}\dot{y}=-(2-r_k)\frac{\partial\hat{x}_k^{*1/r_k}}{\partial y}\times$$
$$\int_{\hat{x}_k^*}^{\hat{x}_k}(s^{\frac{1}{r_k}}-\hat{x}_k^{*\frac{1}{r_k}})^{1-r_k}\mathrm{d}s\cdot\dot{y}\leqslant$$
$$(2-r_k)|\frac{\partial\hat{x}_k^{*1/r_k}}{\partial y}||\xi_k||\frac{\partial h}{\partial x_1}||\hat{x}_2+e_2|\leqslant$$
$$\frac{1}{2}|\xi_2|^{2+\tau}+\frac{1}{3}|\xi_1|^{2+\tau}+\frac{1}{4n}|e_2|^{\frac{2+\tau}{r_2}}+$$
$$g_7|\xi_k|^{2+\tau}, \tag{38}$$

其中$g_7>0$. 将式(35)~(38)代入式(34), 得

$$\dot{V}_k\leqslant-(\frac{3}{4}-\frac{k}{4n})e_1^{2+\tau}-(\frac{1}{2}-\frac{k}{4n})e_2^{\frac{2+\tau}{r_2}}-$$
$$\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}-(n-k+1)\sum_{j=1}^{k-1}|\xi_j|^{2+\tau}+$$
$$\xi_k^{2-r_k}(\hat{x}_{k+1}-\hat{x}_{k+1}^*)+\xi_k^{2-r_k}\hat{x}_{k+1}^*+$$
$$(g_4+g_5+g_6+g_7)|\xi_k|^{2+\tau}. \tag{39}$$

构造虚拟控制器

$$\hat{x}_{k+1}^*=-\beta_k\xi_k^{r_{k+1}},$$

其中$\beta_k=n-k+1+g_4+g_5+g_6+g_7$, 得

$$\dot{V}_k\leqslant-(\frac{3}{4}-\frac{k}{4n})e_1^{2+\tau}-(\frac{1}{2}-\frac{k}{4n})e_2^{\frac{2+\tau}{r_2}}-$$
$$\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}-(n-k+1)\sum_{j=1}^{k}|\xi_j|^{2+\tau}+$$
$$\xi_k^{2-r_k}(\hat{x}_{k+1}-\hat{x}_{k+1}^*). \tag{40}$$

这样就完成了归纳证明. 当$k=n$时, 由式(39)存在Lyapunov函数$V_n(e,y,\hat{x}_2,\cdots,\hat{x}_n)$和常数$\hat{c}$, 使得

$$\dot{V}_n\leqslant$$

$$-\frac{1}{2}e_1^{2+\tau}-\frac{1}{4}e_2^{\frac{2+\tau}{r_2}}-\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}-\sum_{j=1}^{n-1}|\xi_j|^{2+\tau}+$$
$$\xi_n^{2-r_n}(u-\hat{x}_{n+1}^*)+\xi_n^{2-r_n}\hat{x}_{n+1}^*+\hat{c}|\xi_n|^{2+\tau}, \quad (41)$$

其中$\hat{c}=g_4+g_5+g_6+g_7$. 取$\hat{x}_{n+1}^*=-b_n\xi_n^{r_n+\tau}$, $b_n\geqslant \hat{c}+1$, 则

$$\dot{V}_n\leqslant -\frac{1}{2}e_1^{2+\tau}-\frac{1}{4}e_2^{\frac{2+\tau}{r_2}}-\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}-\sum_{j=1}^{n}|\xi_j|^{2+\tau}+\xi_n^{2-r_n}(u-\hat{x}_{n+1}^*). \quad (42)$$

由式(42), 得到最终的控制器

$$u=\hat{x}_{n+1}^*=-b_n(\hat{x}_n^{\frac{1}{r_n}}+\beta_{n-1}^{\frac{1}{r_n}}(\hat{x}_{n-1}^{\frac{1}{r_{n-1}}}+\cdots+\beta_2^{\frac{1}{r_3}}(\hat{x}_2^{\frac{1}{r_2}}+\beta_1^{\frac{1}{r_2}}y)))^{r_n+\tau}, \quad (43)$$

进而

$$\dot{V}_n\leqslant-\frac{1}{2}e_1^{2+\tau}-\frac{1}{4}e_2^{\frac{2+\tau}{r_2}}-\frac{3}{4}\sum_{i=3}^{n}e_i^{\frac{2+\tau}{r_i}}-\sum_{j=1}^{n}|\xi_j|^{2+\tau}. \quad (44)$$

式(44)的右端是负定的. 因此, 闭环系统(9)(11)(43)是全局渐近稳定. 由齐次系统理论可知, 当$\tau<0$时, 闭环系统的状态在有限时间内趋于原点.

注 从V_n的构造, 不难证明V_n是关于

$$X:=[e_1\ \cdots\ e_n\ \ y\ \ \hat{x}_2\ \cdots\ \hat{x}_n]^{\mathrm{T}} \quad (45)$$

正定的且径向无界.

取扩张权系数

$$\varDelta=(r_1,\cdots,r_{2n})=(\varDelta_e,\varDelta_{\hat{x}}), \quad (46)$$

其中:

$$\varDelta_e=(1,\tau+1,\cdots,(n-1)\tau+1),$$
$$\varDelta_{\hat{x}}=(1,\tau+1,\cdots,(n-1)\tau+1).$$

可以看出

$$V_n(X)=V(e)+y^2/2+\sum_{k=2}^{n}\int_{\hat{x}_k^*}^{\hat{x}_k}(s^{\frac{1}{r_k}}-\hat{x}_k^{*\frac{1}{r_k}})^{2-r_k}\mathrm{d}s \quad (47)$$

是关于$\varDelta$, 度为2的齐次函数. 由引理3, 存在常数$\hat{c}_1>0$, 有

$$V_n(X)\leqslant\hat{c}_1\|X\|_\varDelta^2, \quad (48)$$

其中$\|X\|_\varDelta=\sqrt{\sum_{i=1}^{2n}|X_i|^{2/r_i}}$.

因此式(44)的右端的度为$2+\tau$. 由引理3, 存在常数$\hat{c}_2>0$, 有

$$\dot{V}_n(X)\leqslant-\frac{1}{2}e_1^{2+\tau}-\frac{1}{4}e_2^{(2+\tau)/r_2}-\frac{3}{4}\sum_{i=3}^{n}e_i^{(2+\tau)/r_i}-\sum_{j=1}^{n}|\xi_j|^{2+\tau}\leqslant-\hat{c}_2\|X\|_\varDelta^{2+\tau}. \quad (49)$$

4 系统(1)的全局输出反馈稳定(Global output feedback stabilization of system (1))

根据第3节的结论可解决系统(1)全局输出反馈稳定问题. 非线性项$\phi_i(\cdot)$满足如下条件:

假设 2 对任意的$t\geqslant0$, 存在常数$c>0$和$\tau\in(-\frac{1}{n},0)$, 对$i=1,\cdots,n$, 有

$$|\phi_i(t,z_1,\cdots,z_i)|\leqslant c(|z_1|^{r_i+\tau}+\cdots+|z_i|^{\frac{r_i+\tau}{r_i}}). \quad (50)$$

在前面构造的齐次控制器下, 通过输出反馈运用齐次控制方法可实现非线性系统(1)的全局稳定.

定理 2 在假设1和假设2下, 系统(1)的全局输出反馈稳定问题可解.

证 通过引入高增益$L\geqslant1$到所设计的输出反馈控制器, 若增益L足够大, 可使得闭环系统全局渐近稳定. 对系统(1)作坐标变换

$$x_i=\frac{z_i}{L^{i-1}},\ i=1,\cdots,n,\ u=\frac{\nu}{L^n}, \quad (51)$$

则系统(1)转化为

$$\begin{cases}\dot{x}_i=Lx_{i+1}+\dfrac{\phi_i(\cdot)}{L^{i-1}},\ i=1,\cdots,n-1.\\ \dot{x}_n=\dfrac{\nu}{L^{n-1}}+\dfrac{\phi_n(\cdot)}{L^{n-1}}=Lu+\dfrac{\phi_n(\cdot)}{L^{n-1}}.\end{cases} \quad (52)$$

构造一个具有增益L的补偿器

$$\begin{cases}\dot{\hat{x}}_i=L(\hat{x}_{i+1}-a_i\hat{x}_1^{r_{i+1}}),\ i=1,\cdots,n-1,\\ \dot{\hat{x}}_n=L(u-a_n\hat{x}_1^{r_{n+1}}).\end{cases} \quad (53)$$

此外, 用与式(43)相同的构造法设计ν, 即

$$\begin{cases}\nu=L^nu,\\ u=-b_n(\hat{x}_n^{\frac{1}{r_n}}+\beta_{n-1}^{\frac{1}{r_n}}(\hat{x}_{n-1}^{\frac{1}{r_{n-1}}}+\cdots+\\ \quad\beta_2^{\frac{1}{r_3}}(\hat{x}_2^{\frac{1}{r_2}}+\beta_1^{\frac{1}{r_2}}y)))^{r_n+\tau}.\end{cases} \quad (54)$$

采用前一节构造的Lyapunov函数V_n, 可得

$$\dot{V}_n\leqslant-L\hat{c}_2\|X\|_\varDelta^{2+\tau}+\frac{\partial V_n}{\partial X}(\phi_1(\cdot),\cdots,\frac{\phi_n(\cdot)}{L^{n-1}},\frac{\partial h}{\partial x_1}\phi_1,0,\cdots,0)^{\mathrm{T}}. \quad (55)$$

经坐标变换后, 假设2转化为

$$|\phi_i(t,z_1,\cdots,z_i)|\leqslant c(|x_1|^{r_i+\tau}+\cdots+|L^{i-1}x_i|^{\frac{r_i+\tau}{r_i}}). \quad (56)$$

因$L\geqslant1$, 得

$$|\frac{\phi_i(t,z_1,\cdots,z_i)}{L^{i-1}}|\leqslant cL^{1-1/r_i}\sum_{j=1}^{i}|x_j|^{\frac{r_i+\tau}{r_j}}=$$
$$cL^{1-1/r_i}(|x_1|^{r_i+\tau}+\cdots+|\hat{x}_i+e_i|^{\frac{r_i+\tau}{r_i}})\leqslant$$

$$\tilde{c}L^{1-1/r_i}(|y|^{r_i+\tau}+|\hat{x}_2|^{\frac{r_i+\tau}{r_2}}+|e_2|^{\frac{r_i+\tau}{r_2}}+\cdots+|\hat{x}_i|^{\frac{r_i+\tau}{r_i}}+|e_i|^{\frac{r_i+\tau}{r_i}}),\tag{57}$$

其中$\tilde{c}>0$是常数. 由假设1, 得

$$|\frac{\partial h}{\partial x_1}\phi_1|\leqslant\bar{\theta}|\phi_1|\leqslant\bar{\theta}\tilde{c}\|X\|_{\Delta}^{\tau+r_1}.\tag{58}$$

由引理3, 得

$$\begin{cases}|\dfrac{\partial V_n}{\partial X_i}||\dfrac{\phi_i}{L^{i-1}}|\leqslant k_iL^{1-1/r_i}\|X\|_{\Delta}^{2+\tau},\ i=1,\cdots,n,\\ |\dfrac{\partial V_n}{\partial y}||\dfrac{\partial h}{\partial x_1}\phi_1|\leqslant\\ k_{n+1}L^{1-1/r_1}\|X\|_{\Delta}^{2+\tau}=k_{n+1}\|X\|_{\Delta}^{2+\tau},\end{cases}\tag{59}$$

其中$k_i>0,\ i=1,\cdots,n+1$. 将式(59)代入式(55)得

$$\begin{aligned}&\dot{V}_n\leqslant\\&-L\hat{c}_2\|X\|_{\Delta}^{2+\tau}+\sum_{i=1}^{n}k_iL^{1-\frac{1}{r_i}}\|X\|_{\Delta}^{2+\tau}+\\&k_{n+1}\|X\|_{\Delta}^{2+\tau}\leqslant\\&-L(\hat{c}_2-\sum_{i=1}^{n}k_iL^{-\frac{1}{r_i}}-k_{n+1}L^{-1})\|X\|_{\Delta}^{2+\tau}.\end{aligned}\tag{60}$$

显然, 若选择足够大的增益L, 系统(1)的状态在齐次控制器下将在有限时间内趋于原点.

下面给出一个例子来验证本文所提算法的有效性. 考虑如下系统:

$$\begin{cases}\dot{z}_1=z_2+z_1^{9/11},\ \ \dot{z}_2=z_3+z_2^{7/9},\\ \dot{z}_3=\nu,\ \ y=cz_1+\sin z_1,\end{cases}\tag{61}$$

其中$1<c\leqslant3$是未知常数. 取$\tau=-2/11$, 易验证ϕ_1,ϕ_2满足假设2. 由定理2, 构造如下输出反馈控制器:

$$\begin{cases}\dot{\hat{x}}_1=L(\hat{x}_2-a_1\hat{x}_1^{r_2}),\\ \dot{\hat{x}}_2=L(\hat{x}_3-a_2\hat{x}_1^{r_3}),\\ \dot{\hat{x}}_3=L(u-a_3\hat{x}_1^{r_4}),\\ u=-\beta_3(\hat{x}_3^{1/r_3}+\beta_2^{1/r_3}(\hat{x}_2^{1/r_2}+\beta_1^{1/r_2}y))^{r_4},\end{cases}\tag{62}$$

其中: $a_1,a_2,a_3,\beta_1,\beta_2$和$\beta_3$均为常数, $\nu=L^3u$. 通过齐次控制方法, 使输出反馈控制器(62)中的增益L足够大, 可使系统(61)的状态在有限时间内趋于原点. 仿真中参数:

$$c=2,\ a_1=6,\ a_2=11,\ a_3=6,$$
$$\beta_1=2,\ \beta_2=7,\ \beta_3=9,\ L=4.$$

在初始条件为

$$[z_1(0)\ \ z_2(0)\ \ z_3(0)\ \ \hat{x}_1(0)\ \ \hat{x}_2(0)\ \ \hat{x}_3(0)]=[2\ \ 1\ \ 0.5\ \ 1\ \ 0\ \ 0]$$

下的仿真如图1所示.

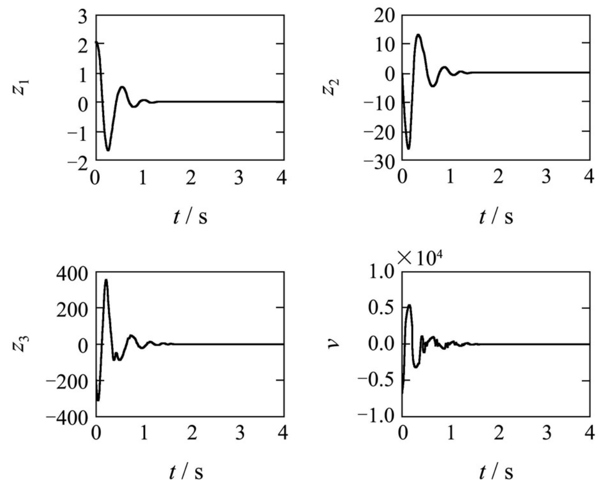

图 1 闭环系统的状态和控制输入曲线

Fig. 1 The states and control input of the closed-loop system

5 结论(Conclusions)

论文对具有未知输出函数的非线性系统设计了一种新的补偿器. 首先, 对标称线性系统设计输出反馈控制器; 然后应用齐次控制方法通过选择适当的增益, 使得不确定非线性系统在有限时间内全局稳定.

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附录(Appendix)

A) 式(19)的证明.

首先, 考虑$x_1 \geqslant 0$情形. 由假设1, 有

$$0 < \underline{\theta} \leqslant \frac{\partial h(x_1)}{\partial x_1} \leqslant \bar{\theta},$$

两边同时对区间$[0, x_1]$积分, 得

$$0 < \underline{\theta} x_1 \leqslant h(x_1) - h(0) \leqslant \bar{\theta} x_1. \tag{A1}$$

因为$h(0) = 0$, 所以

$$0 < \underline{\theta} x_1 \leqslant y \leqslant \bar{\theta} x_1.$$

又因$\tau = -\dfrac{q}{p} \in (-\dfrac{1}{n}, 0)$, 故$1 < 2 + \tau < 2$, 且分子为偶数, 分母为奇数. 从而得到

$$(\bar{\theta})^{-(2+\tau)} y^{2+\tau} \leqslant x_1^{2+\tau} \leqslant \underline{\theta}^{-(2+\tau)} y^{2+\tau}. \tag{A2}$$

同样, 当$x_1 < 0$时, 可以得到相同的结论式(A2).

B) 式(37)的证明.

证 通过W_k的形式, 有

$$\sum_{l=2}^{k-1} \frac{\partial W_k}{\partial \hat{x}_l} \dot{\hat{x}}_l =$$

$$\sum_{l=2}^{k-1} (r_k - 2) \frac{\partial \hat{x}_k^{*\frac{1}{r_k}}}{\partial \hat{x}_l} \int_{\hat{x}_k^*}^{\hat{x}_k} (s^{\frac{1}{r_k}} - \hat{x}_k^{*\frac{1}{r_k}})^{1-r_k} \mathrm{d}s \cdot \dot{\hat{x}}_l \leqslant$$

$$a_k |\xi_k| |\sum_{l=2}^{k-1} \frac{\partial \hat{x}_k^{*1/r_k}}{\partial \hat{x}_l} \dot{\hat{x}}_l| \leqslant$$

$$a_k |\xi_k| |\sum_{l=2}^{k-1} \frac{\partial(\hat{\beta}_{k-1} \hat{x}_{k-1}^{1/r_{k-1}} + \cdots + \hat{\beta}_1 y)}{\partial \hat{x}_l} (\hat{x}_{l+1} - a_l \hat{x}_1^{r_{l+1}})| \leqslant$$

$$\hat{a}_k |\xi_k| |\sum_{l=2}^{k-1} \hat{x}_l^{1/r_l - 1} (\hat{x}_{l+1} - a_l \hat{x}_1^{r_{l+1}})|, \tag{A3}$$

其中$a_k, \hat{a}_k, \hat{\beta}_{k-1}, \cdots, \hat{\beta}_1$是正数.

由坐标变换(31), 有

$$\hat{x}_{l+1} = (\xi_{l+1} - \beta_l^{1/r_{l+1}} \xi_l)^{r_{l+1}} \leqslant h(|\xi_{l+1}| + |\xi_l|)^{r_{l+1}}, \tag{A4}$$

其中$h = \max\{1, \beta_1, \cdots, \beta_n\}$. 将式(A4)代入式(A3), 得

$$\sum_{l=2}^{k-1} \frac{\partial W_k}{\partial \hat{x}_l} \dot{\hat{x}}_l \leqslant \check{a}_k |\xi_k| \sum_{l=2}^{k-1} (|\xi_l| + |\xi_{l-1}|)^{1-r_l} \times \\ \{(|\xi_{l+1}| + |\xi_l|)^{r_{l+1}} + (|e_1| + |y|)^{r_{l+1}}\},$$

其中$\check{a}_k > 0$. 由引理5, 存在正常数g_5,

$$\sum_{l=2}^{k-1} \frac{\partial W_k}{\partial \hat{x}_l} \dot{\hat{x}}_l \leqslant \sum_{l=2}^{k-1} \frac{\xi_l^{2+\tau}}{2} + \frac{1}{3} |\xi_1|^{2+\tau} + \\ \frac{1}{8n} |e_1|^{2+\tau} + g_5 |\xi_k|^{2+\tau}. \tag{A5}$$

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