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带未知有色观测噪声的自校正融合Kalman滤波器

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摘要: 对于带未知有色观测噪声的多传感器线性离散定常随机系统, 未知模型参数和噪声方差的一致的融合估值器用递推增广最小二乘法(RELS)和求解相关函数方程得到. 将这些估值器代入到最优解耦融合Kalman滤波器中, 得出了自校正解耦融合Kalman滤波器, 并用动态方差误差系统分析(DVESA)和动态误差分析(DESA)方法证明了它收敛于最优解耦融合Kalman滤波器, 因而具有渐近最优性. 一个带3传感器跟踪系统的仿真例子说明了其有效性.

关键词: 多传感器信息融合; 解耦融合; 未知有色观测噪声; 辨识; 自校正Kalman滤波器; 收敛性

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Self-tuning fusion Kalman filter with unknown colored observation noises

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Abstract: For the multisensor linear discrete time-invariant stochastic system with unknown colored observation noises, the consistent fused estimators of unknown model parameters and noise variances are obtained by using the recursive extended-least-squares (RELS) method and solving the correlation function equations. Substituting them into the optimal decoupled fused Kalman filter, we obtain a self-tuning decoupled fused Kalman filter. By means of the dynamic variance error system analysis (DVESA) method and the dynamic error system analysis (DESA) method, this filter is proved to be convergent to the optimal decoupled fusion Kalman filter with asymptotic optimality. A simulation example for a target-tracking system with 3 sensors shows its effectiveness.

Key words: multisensor information fusion; decoupled fusion; unknown colored measurement noises; identification; self-tuning Kalman filter; convergence

1 引言(Introduction)

多传感器最优信息融合Kalman滤波[1−2]广泛应用于许多高技术领域, 包括军事、国防、制导、跟踪、GPS定位、机器人等. 目前有两种常用的信息融合方法: 一种方法是状态融合方法, 一种是观测融合方法[3]. 状态融合方法又可分为集中式Kalman滤波和分布式[4−5]滤波. 用加权局部Kalman滤波器得到最优融合Kalman滤波器的方法[6]是一种重要的分布式融合Kalman滤波方法, 但其缺点是要求系统数学模型和噪声统计是精确已知的. 对于带白色观测噪声且未知噪声方差的多传感器系统, 基于噪声方差的信息融合估计[7], 文献[8]分别提出了自校正融合Wiener滤波器和Kalman滤波器, 并用文献[8]提出的动态误差系统分析(dynamic error system analysis, DESA)方法证明了自校正融合滤波器的收敛性. 文献[10]提出的动态方差误差系统分析(dynamic variance error system analysis, DVESA)方法证明了自校正Riccati方程的收敛性. 对带白色观测噪声, 且同时含未知模型参数和噪声方差的多传感器系统, 文献[11–12]提出了自校正Kalman融合器, 其中未知参数估值器用递推辅助变量(RIV)算法得到. 文献[13–14]对带有色观测噪声的多传感器系统, 但噪声统计和有色噪声模型参数均已知时, 用标量或矩阵加权准则提出了最优信息融合稳态Kalman滤波器.

本文对带有色观测噪声的多传感器系统, 当有色噪声模型参数和噪声方差未知时, 用按对角阵加权准则, 提出了自校正解耦融合Kalman滤波器, 其中用递推增广最小二乘法[15–16]可得到模型参数的一致估值器, 用取局部噪声方差估值器的算术平均方法[7,10]可得到融合噪声方差估值器. 用DVESA[10]方法和DESA[8]方法证明了自校正Kalman融合器收敛于相应的最优融合器, 因而具有渐近最优性.

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2 问题阐述(Problem represent)

考虑带有色观测噪声的多传感器离散随机系统

$$x(t+1) = \Phi x(t) + \Gamma w(t), \tag{1}$$

$$z_i(t) = H_{0i}x(t) + \eta_i(t), \tag{2}$$

$$\eta_i(t+1) = P_i\eta_i(t) + \xi_i(t), \tag{3}$$

其中: $x(t) \in \mathbb{R}^n$, $z_i(t) \in \mathbb{R}^{m_i}$, $w(t) \in \mathbb{R}^r$, $\eta_i(t) \in \mathbb{R}^{m_i}$分别为状态、第$i$个传感器的观测、输入噪声和观测噪声, $\Phi, \Gamma, H_{0i}, P_i$为适当维数的常矩阵, $i = 1, \cdots, L$.

假设 1 $w(t) \in \mathbb{R}^r, \xi_i(t) \in \mathbb{R}^{m_i}$为适当维数的带零均值, 方差为$Q_w$和$Q_{\xi_i}$的不相关白噪声, 即

$$\mathrm{E}\{\begin{bmatrix} w(t) \\ \xi_i(t) \end{bmatrix} [w^{\mathrm{T}}(k) \quad \xi_j^{\mathrm{T}}(k)]\} = \begin{bmatrix} Q_w & 0 \\ 0 & Q_{\xi_i}\delta_{ij} \end{bmatrix} \delta_{tk}, \tag{4}$$

其中: E为均值号, T为转置号, $\delta_{ii} = 1$, $\delta_{ij} = 0 (i \neq j)$.

假设 2 Φ, Γ, H_{0i}已知, Q_w, Q_{ξ_i}, P_i未知.

问题是基于观测$(z_i(t), z_i(t-1), \cdots)$, $i = 1, \cdots, L$, 求状态$x(t)$的自校正解耦融合Kalman滤波器.

引入观测变换

$$y_i(t) = (I_{m_i} - q^{-1}P_i)z_i(t+1) = z_i(t+1) - P_iz_i(t). \tag{5}$$

由式(1)和式(2)可知

$$y_i(t) = H_ix(t) + v_i(t), \tag{6}$$

$$H_i = H_{0i}\Phi - P_iH_{0i}, \tag{7}$$

$$v_i(t) = H_{0i}\Gamma w(t) + \xi_i(t), \tag{8}$$

于是原系统化为带相关白噪声系统

$$x(t+1) = \Phi x(t) + \Gamma w(t), \tag{9}$$

$$y_i(t) = H_ix(t) + v_i(t), i = 1, \cdots, L, \tag{10}$$

其中:

$$\mathrm{E}\{\begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix} [w^{\mathrm{T}}(k) \quad v_i^{\mathrm{T}}(k)]\} = \begin{bmatrix} Q_w & S_i \\ S_i^{\mathrm{T}} & R_{ii} \end{bmatrix} \delta_{tk},$$

$$\mathrm{E}[v_i(t)v_j^{\mathrm{T}}(k)] = R_{ij}\delta_{tk}, \ i \neq j,$$

且由式(4)和式(8)有

$$R_{ii} = H_{0i}\Gamma Q_w\Gamma^{\mathrm{T}}H_{0i}^{\mathrm{T}} + Q_{\xi_i}, \tag{11}$$

$$R_{ij} = H_{0i}\Gamma Q_w\Gamma^{\mathrm{T}}H_{0i}^{\mathrm{T}}, \tag{12}$$

$$S_i = Q_w\Gamma^{\mathrm{T}}H_{0i}^{\mathrm{T}}. \tag{13}$$

假设 3 系统式(9)(10)是完全可观, 完全可控的.

由式(5)可知, 由$(z_i(t+1), z_i(t), \cdots)$与由$(y_i(t), y_i(t-1), \cdots)$张成的线性流形相同, 基于观测$(z_i(t), z_i(t-1), \cdots)$的Kalman滤波器$\hat{x}_{z_i}(t|t)$就等价于基于观测$(y_i(t-1), y_i(t-2), \cdots)$的Kalman预报器$\hat{x}_i(t|t-1)$, 即

$$\hat{x}_{z_i}(t|t) = \hat{x}_i(t|t-1).$$

3 解耦融合稳态最优Kalman滤波器(Decoupled fusion state-steady optimal Kalman filter)

多传感器系统式(9)(10)的第i个子系统有局部稳态Kalman预报器[16]

$$\hat{x}_i(t+1|t) = \Psi_{pi}\hat{x}_i(t|t-1) + K_{pi}y_i(t), \tag{14}$$

其中:

$$J_i = \Gamma S_iR_{ii}^{-1}, \ \bar{\Phi} = \Phi - J_iH, \tag{15}$$

$$\Psi_{pi} = \bar{\Phi} - \bar{K}_{pi}H_i = \bar{\Phi}(I_n - K_{fi}H_i), \tag{16}$$

$$K_{pi} = \bar{K}_{pi} + J_i, \ \bar{K}_{pi} = \bar{\Phi}K_{fi}, \tag{17}$$

$$K_{fi} = \Sigma_iH_i^{\mathrm{T}}(H_i\Sigma_iH_i^{\mathrm{T}} + R_{ii})^{-1}. \tag{18}$$

稳态预报误差方差阵满足稳态Riccati方程

$$\Sigma_i = \bar{\Phi}[\Sigma_i - \Sigma_iH_i^{\mathrm{T}}(H_i\Sigma_iH_i^{\mathrm{T}} + R_{ii})^{-1}H_i\Sigma_i]\bar{\Phi}^{\mathrm{T}} + \Gamma Q\Gamma^{\mathrm{T}}, \tag{19}$$

$$Q = Q_w - S_iR_{ii}^{-1}S_i^{\mathrm{T}}. \tag{20}$$

稳态预报误差互协方差阵满足Lyapunov方程

$$\Sigma_{ij} = \Psi_{pi}\Sigma_{ij}\Psi_{pj}^{\mathrm{T}} + \Delta_{pij}, \tag{21}$$

其中

$$\Delta_{pij} = \Gamma Q_w\Gamma^{\mathrm{T}} - K_{pi}S_i^{\mathrm{T}}\Gamma^{\mathrm{T}} - \Gamma S_jK_{pj}^{\mathrm{T}} + K_{pi}R_{ij}K_{pj}^{\mathrm{T}}. \tag{22}$$

稳态最优解耦融合Kalman预报器[16]

$$\hat{x}_0(t+1|t) = \sum_{i=1}^{L} A_i\hat{x}_i(t+1|t), \tag{23}$$

其中加权阵

$$A_i = \mathrm{diag}\{a_{i1}, \cdots, a_{in}\}. \tag{24}$$

最优加权系数向量

$$[a_{1j}, a_{2j}, \cdots, a_{Lj}] = \frac{e^{\mathrm{T}}(\Sigma^{jj})^{-1}}{e^{\mathrm{T}}(\Sigma^{jj})^{-1}e}, \ j = 1, \cdots, n, \tag{25}$$

其中: 定义$e^{\mathrm{T}} = (1, \cdots, 1)$, $L \times L$矩阵Σ^{jj}为

$$\Sigma^{jj} = (\Sigma_{kr}^{(jj)}), \ k, r = 1, \cdots, L, \tag{26}$$

其中: $\Sigma_{kr}^{(jj)}$是Σ^{jj}的第(k, r)元素, 并且是Σ_{kr}的第(j, j)对角元素, 融合误差方差阵P_0的迹为

$$\mathrm{tr}\, P_0 = \sum_{j=1}^{n} e^{\mathrm{T}}(\Sigma^{jj})^{-1}e. \tag{27}$$

4 未知参数及噪声方差估计方法(Estimators of unknown parameters and noise variances)

由式(1)–(3)有

$$z_i(t)=H_{0i}(I_n-q^{-1}\Phi)^{-1}\Gamma w(t-1)+(I_{m_i}-q^{-1}P_i)^{-1}\xi_i(t-1). \tag{28}$$

应用求逆公式

$$(I_n-q^{-1}\Phi)^{-1}=\mathrm{adj}(I_n-q^{-1}\Phi)/\phi(q^{-1}),$$

其中

$$\phi(q^{-1})=\det(I_n-q^{-1}\Phi). \tag{29}$$

式(28)两边左乘$\phi(q^{-1})$, 由矩阵$\phi(q^{-1})I_{m_i}$与$(I_{m_i}-q^{-1}P_i)^{-1}$乘积的可交换性有

$$\begin{aligned}&\phi(q^{-1})I_{m_i}z_i(t)=\\&(H_{0i}\mathrm{adj}(I_n-q^{-1}\Phi)\Gamma w(t-1)+\\&(I_{m_i}-q^{-1}P_i)^{-1}\phi(q^{-1})I_{m_i}\xi_i(t-1).\end{aligned} \tag{30}$$

对式(30)两边左乘$(I_{m_i}-q^{-1}P_i)$有

$$\begin{aligned}&(I_{m_i}-q^{-1}P_i)\phi(q^{-1})z_i(t)=\\&(I_{m_i}-q^{-1}P_i)H_{0i}\mathrm{adj}(I_n-q^{-1}\Phi)\Gamma w(t-1)+\\&\phi(q^{-1})\xi_i(t-1).\end{aligned} \tag{31}$$

引入新的观测

$$\underline{z}_i(t)=\phi(q^{-1})z_i(t), \tag{32}$$

则有ARMA新息模型为

$$(I_{m_i}-q^{-1}P_i)\underline{z}_i(t)=D_i(q^{-1})\varepsilon_i(t), \tag{33}$$

其中

$$D_i(q^{-1})=I_{m_i}+D_{i1}q^{-1}+\cdots+D_{in_\mathrm{d}}q^{-n_\mathrm{d}},$$

且

$$\begin{aligned}&D_i(q^{-1})\varepsilon_i(t)=\\&(I_{m_i}-q^{-1}P_i)H_{0i}\mathrm{adj}(I_n-q^{-1}\Phi)\times\\&\Gamma w(t-1)+\phi(q^{-1})\xi_i(t-1),\end{aligned} \tag{34}$$

其中$n_\mathrm{d}\leqslant n$. 对ARMA新息模型式(33)用多维RELS算法[16]可辨识P_i, D_i及Q_{ε_i}.

由RELS算法辨识的一致性[15]有$t\to\infty$时,

$$\hat{P}_i(t)\to P_i,\ \hat{Q}_{\varepsilon_i}(t)\to Q_{\varepsilon_i},\ \hat{D}_i(t)\to D_i,\ \text{w.p.1.} \tag{35}$$

用解相关函数矩阵方程组的方法[7,10]得到未知噪声方差阵的一致的融合估计, 即

$$\hat{Q}_{wf}(t)\to Q_w,\ \hat{Q}_{\xi_{if}}(t)\to Q_{\xi_i},\ t\to\infty,\text{w.p.1.} \tag{36}$$

5 解耦融合自校正Kalman滤波器(Decoupled fusion self-tuning Kalman filter)

将在时刻t处所得估值$\hat{P}_i(t),\hat{Q}_{wf}(t),\hat{Q}_{\xi_{if}}(t)$带入上述有关定义和公式可得在时刻$t$处估值$\hat{H}_i(t)$, $\hat{R}_{ii}(t)$, $\hat{R}_{ij}(t)$, $\hat{S}_i(t)$, $\hat{J}_i(t)$, $\hat{\bar{\Phi}}(t)$, $\hat{\Psi}_{pi}(t)$, $\hat{K}_{pi}(t)$, $\hat{K}_{fi}(t)$, $\hat{y}_i(t)$, $\hat{Q}(t)$, 且预报误差方差阵估值满足Riccati方程

$$\begin{aligned}&\hat{\Sigma}_i(t+1|t)=\\&\hat{\bar{\Phi}}(t)[\hat{\Sigma}_i(t|t-1)-\hat{\Sigma}_i(t|t-1)\hat{H}_i^\mathrm{T}(t)\times\\&(\hat{H}_i(t)\hat{\Sigma}_i(t|t-1)\hat{H}_i^\mathrm{T}(t)+\hat{R}_{ii}(t))^{-1}\times\\&\hat{H}_i(t)\hat{\Sigma}_i(t|t-1)]\hat{\bar{\Phi}}(t)^\mathrm{T}+\Gamma\hat{Q}(t)\Gamma^\mathrm{T},\end{aligned} \tag{37}$$

且预报误差互协方差阵满足自校正Lyapunov方程

$$\begin{aligned}&\hat{\Sigma}_{ij}(t+1|t)=\\&\hat{\Psi}_{pi}(t)\hat{\Sigma}_{ij}(t|t-1)\hat{\Psi}_{pj}^\mathrm{T}(t)+\hat{\Delta}_{pij}(t),\end{aligned} \tag{38}$$

$$\begin{aligned}&\hat{\Delta}_{pij}(t)=\\&\Gamma\hat{Q}_{wf}(t)\Gamma^\mathrm{T}-\hat{K}_{pi}(t)\hat{S}_i^\mathrm{T}(t)\Gamma^\mathrm{T}-\\&\Gamma\hat{S}_j(t)\hat{K}_{pj}^\mathrm{T}(t)+\hat{K}_{pi}(t)\hat{R}_{ij}(t)\hat{K}_{pj}^\mathrm{T}(t).\end{aligned} \tag{39}$$

由式(23)–(26), 自校正解耦融合预报器为

$$\hat{x}_0^s(t+1|t)=\sum_{i=1}^{L}\hat{A}_i(t)\hat{x}_i^s(t+1|t), \tag{40}$$

其中加权阵

$$\hat{A}_i(t)=\mathrm{diag}\{\hat{a}_{i1}(t),\cdots,\hat{a}_{in}(t)\}. \tag{41}$$

自校正加权系数向量

$$[\hat{a}_{1j}(t)\ \ \hat{a}_{2j}(t)\ \ \cdots\ \ \hat{a}_{Lj}(t)]=\frac{e^\mathrm{T}(\hat{\Sigma}^{jj}(t+1|t))^{-1}}{e^\mathrm{T}(\hat{\Sigma}^{jj}(t+1|t))^{-1}e}, \tag{42}$$

其中定义

$$\hat{\Sigma}^{jj}(t+1|t)=(\hat{\Sigma}_{kr}^{(jj)}(t+1|t)),$$

$\hat{\Sigma}_{kr}^{jj}(t+1|t)$是$\hat{\Sigma}_{kr}(t+1|t)$的$(j,j)$对角元素.

6 自校正局部和融合Kalman预报器收敛性(Self-tuning local and fused Kalman predictor and its convergence)

式(35)和式(36)可证明[17]: 自校正Riccati方程式(37)收敛于稳态最优Riccati方程(19), 即

$$[\hat{\Sigma}_i(t+1|t)-\Sigma_i]\to 0,\ t\to\infty,\text{w.p.1.} \tag{43}$$

下述引理1为解决协方差阵收敛性分析的动态方差误差系统分析(DVESA)方法[10], 引理2为解决滤波器收敛性分析的动态误差系统分析(DESA)方法.

引理 1[10]　假设$n\times n$维矩阵$P(t)$满足时变Lyapunov方程

$$P(t)=F_1(t)P(t-1)F_2^\mathrm{T}(t)+U(t), \tag{44}$$

其中: $t\geqslant 0$, $F_1(t),F_2(t)$是一致渐近稳定的[18], 即存在常数$c_i>0$, $0<\rho_i<1$, 有

$$\|F_i(t,k)\|\leqslant c_i\rho_i^{t-k},\ \forall t\geqslant k\geqslant 0,$$

其中: $\|\cdot\|$为矩阵范数, $F_i(t,k)=F_i(t)\cdots F_i(k+1)$.

如果$U(t)$是有界的, 则$P(t)$是有界的, 如果$U(t) \to 0$, 则$P(t) \to 0,\ t \to \infty$.

引理 2[8,10] 考虑离散动态误差系统

$$\delta(t) = F(t)\delta(t-1) + u(t), \tag{45}$$

其中: $t \geqslant 0$, $F(t)$是一致渐近稳定的. 如果$u(t)$是有界的, 则$\delta(t)$是有界的, 如果$u(t) \to 0$, 则$\delta(t) \to 0,\ t \to \infty$.

应用式(36)和式(43)容易证明

$$\hat{K}_{pi}(t) \to K_{pi},\ \hat{\Psi}_{pi}(t) \to \Psi_{pi},\ \hat{\Delta}_{ij}(t) \to \Delta_{ij}. \tag{46}$$

令$\hat{\Psi}_{pi}(t) = \Psi_{pi} + \Delta\hat{\Psi}_{pi}(t)$, 则有$\hat{\Psi}_{pi}(t) \to 0$. 从式(37)中减式(19), 记

$$E_{ij}(t) = \hat{\Sigma}_{ij}(t+1|t) - \Sigma_{ij},$$

引出Lyapunov方程

$$E_{ij}(t) = \Psi_{pi}E_{ij}(t-1)\Psi_{pj}^{\mathrm{T}} + U_{ij}(t), \tag{47}$$

其中

$$\begin{aligned} U_{ij}(t) = {} & \Delta\hat{\Psi}_{pi}(t)\hat{\Sigma}_{ij}(t|t-1)\Psi_{pj}^{\mathrm{T}} + \\ & \Psi_{pi}\hat{\Sigma}_{ij}(t|t-1)\Delta\hat{\Psi}_{pj}(t) + \\ & \Delta\hat{\Psi}_{pi}(t)\hat{\Sigma}_{ij}(t|t-1)\Delta\hat{\Psi}_{pj}(t) + \\ & \hat{\Delta}_{ij}(t) - \Delta_{ij}. \end{aligned} \tag{48}$$

由于$\hat{\Psi}_{pi}(t)$是一致渐近稳定的[10,18], 对式(38)应用引理1得到$\hat{\Sigma}_{ij}(t|t-1)$是有界的, 从而

$$U_{ij}(t) \to 0,\ t \to \infty,\ \text{w.p.1},$$

由Ψ_{pi}和Ψ_{pj}是稳定的[18], 则也是一致渐近稳定的, 再应用引理1, 有$E_{ij}(t) \to 0$, 即

$$\hat{\Sigma}_{ij}(t) \to \Sigma_{ij},\ t \to \infty,\ \text{w.p.1}. \tag{49}$$

定理 1 对于多传感器系统式(9)和式(10)在假设1, 2, 3下, 如果观测过程$y_i(t)$以概率1有界, 则局部自校正Kalman预报器$\hat{x}_i^s(t+1|t)$以概率1收敛于局部稳态Kalman预报器$\hat{x}_i(t+1|t)$, 即

$$[\hat{x}_i^s(t+1|t) - \hat{x}_i(t+1|t)] \to 0,\ t \to \infty,\ \text{w.p.1}. \tag{50}$$

证 令$\hat{K}_{pi}(t) = K_{pi} + \Delta\hat{K}_{pi}(t)$, 由式(46)有

$$\Delta\hat{K}_{pi}(t) \to 0,\ t \to \infty, \text{w.p.1}.$$

由式(35)和$\hat{y}_i(t)$的表达式, 则

$$[\hat{y}_i(t) - y_i(t)] \to 0,\ t \to \infty,\ \text{w.p.1}. \tag{51}$$

由式(46)引出$\hat{K}_{pi}(t)$有界, 由$y_i(t)$的有界性引出$\hat{y}_i(t)$和$\hat{K}_{pi}(t)\hat{y}_i(t)$有界, 由于$\hat{\Psi}_{pi}(t)$是一致渐近稳定的, 应用引理2引出$\hat{x}_i^s(t+1|t)$是有界的.

令

$$\delta_i(t) = \hat{x}_i^s(t+1|t) - \hat{x}_i(t+1|t),$$

有动态误差系统

$$\delta_i(t) = \Psi_{pi}(t)\delta_i(t-1) + u_i(t), \tag{52}$$

其中

$$\begin{aligned} u_i(t) = {} & \Delta\hat{\Psi}_{pi}(t)\hat{x}_i^s(t|t-1) + K_{pi}(t)[\hat{y}_i(t) - \\ & y_i(t)] + \Delta\hat{K}_{pi}(t)\hat{y}_i(t). \end{aligned} \tag{53}$$

由$\hat{x}_i^s(t+1|t)$和$\hat{y}_i(t)$的有界性, 有

$$u_i(t) \to 0, \quad t \to \infty, \text{w.p.1}. \tag{54}$$

对式(52)应用引理2,

$$\delta_i(t) \to 0,\ t \to \infty,\ \text{w.p.1},$$

即

$$[\hat{x}_i^s(t+1|t) - \hat{x}_i(t+1|t)] \to 0,\ t \to \infty,\ \text{w.p.1}. \tag{55}$$

定理 2 对于多传感器系统式(9)和式(10)在假设1–3下, 如果观测过程$y_i(t)$以概率1有界, 则自校正解耦融合Kalman预报器$\hat{x}_0^s(t+1|t)$以概率1收敛于最优解耦融合稳态Kalman预报器$\hat{x}_0(t+1|t)$, 即

$$[\hat{x}_0^s(t+1|t) - \hat{x}_0(t+1|t)] \to 0,\ t \to \infty,\ \text{w.p.1}. \tag{56}$$

证 由式(25)(42)(43)(49)可知

$$\hat{a}_{ij}(t) \to a_{ij},\ t \to \infty,\ \text{w.p.1}. \tag{57}$$

由式(24)(41)引出$\hat{A}_i(t) \to A_i$, 令

$$\hat{A}_i(t) = A_i + \Delta\hat{A}_i(t),$$

则

$$\Delta\hat{A}_i(t) \to 0,\ t \to \infty,\ \text{w.p.1}, \tag{58}$$

则式(40)减去式(23)引出

$$\begin{aligned} & \hat{x}_0^s(t+1|t) - \hat{x}_0(t+1|t) = \\ & \sum_{i=1}^{L} A_i[\hat{x}_i^s(t+1|t) - \hat{x}_i(t+1|t)] + \\ & \sum_{i=1}^{L} \Delta\hat{A}_i(t)\hat{x}_i^s(t+1|t). \end{aligned} \tag{59}$$

由式(50)(58), 有式(56)成立. 证毕.

注 1 由于按概率1收敛引出按实现收敛性[8], 定理1和定理2中$y_i(t)$以概率1有界的条件减弱为观测数据$y_i(t)$有界(即$y_i(t)$得一个实现是有界的), 此时有$\hat{x}_i^s(t+1|t)$按实现收敛于$\hat{x}_i(t+1|t)$, $\hat{x}_0^s(t+1|t)$按实现收敛于$\hat{x}_0(t+1|t)$.

7 仿真例子(Simulation examples)

考虑带3传感器和带一维观测的二维跟踪系统

$$x(t+1) = \Phi x(t) + \Gamma w(t), \tag{60}$$

$$z_i(t) = H_{0i}x(t) + \eta_i(t), \tag{61}$$

$$\eta_i(t+1) = p_i\eta_i(t) + \xi_i(t),\ i = 1, 2, 3, \tag{62}$$

$$\varGamma=\begin{bmatrix}0.5T_0^2\\ T_0\end{bmatrix},\ H_{0i}=[1\ \ 0],\ \varPhi=\begin{bmatrix}1 & T_0\\ 0 & 1\end{bmatrix},\tag{63}$$

其中: T为采样周期, $x(t)=[s(t)\ \ \dot{s}(t)]^{\mathrm{T}}$和$\dot{s}(t)$分别为目标在时刻$tT_0$的位置和速度, $w(t)$及$\xi_i(t)$为零均值, 方差为Q_w, Q_{ξ_i}的独立高斯白噪声.

下面求当p_i, Q_w, Q_{ξ_i}未知时, 自校正解耦融合Kalman预报器

$$\hat{x}_0^s(t+1|t)=[\hat{s}_0^s(t+1|t)\ \ \hat{\dot{s}}_0^s(t+1|t)]^{\mathrm{T}}.$$

仿真中取

$$T_0=2.5,\ p_1=0.25,\ p_2=0.1,$$
$$p_3=0.2,\ Q_w=1,\ Q_{\xi_1}=0.64,$$
$$Q_{\xi_2}=1.69,\ Q_{\xi_3}=0.81.$$

用RELS算法和相关方法, 仿真结果如图1–4所示. 其中在图1和图2中, 直线代表真实值, 曲线代表估值. 可看到参数和噪声方差估值收敛于相应的真实值. 由图3和图4看到最优和自校正估值误差曲线呈漏斗状, 表明自校正融合器$\hat{x}_0^s(t+1|t)$收敛于最优融合器

$$\hat{x}_0(t+1|t)=[\hat{s}_0(t+1|t)\ \ \hat{\dot{s}}_0(t+1|t)]^{\mathrm{T}}.$$

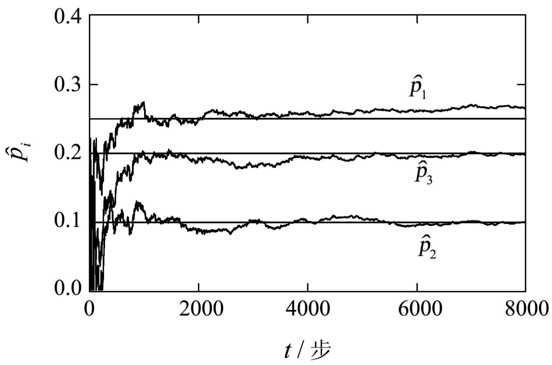

图 1 模型参数$\hat{p}_i$的收敛曲线

Fig. 1 The convergence curves of model parameters $\hat{p}_i$

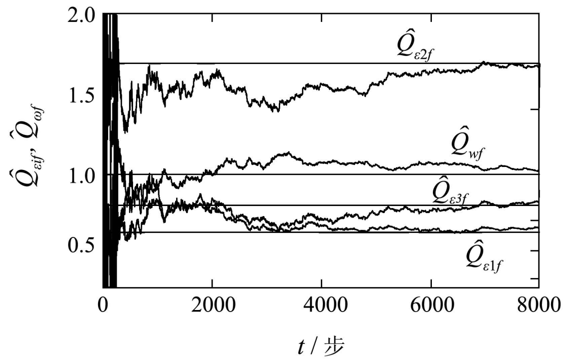

图 2 噪声方差融合估值$\hat{Q}_{\xi if}$的收敛曲线

Fig. 2 The convergence curves of fusion estimators $\hat{Q}_{\xi if}$ of noise variances

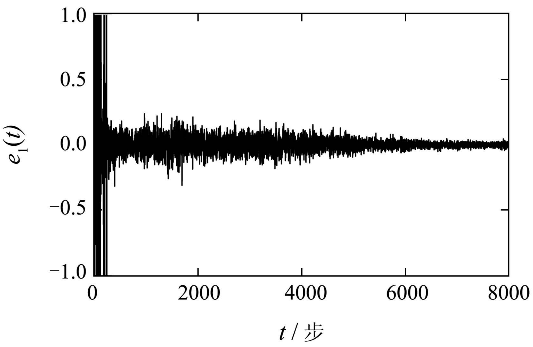

图 3 位置误差$e_1(t)=\hat{s}_0^s(t+1|t)-\hat{s}_0(t+1|t)$曲线

Fig. 3 The position error $e_1(t)=\hat{s}_0^s(t+1|t)-\hat{s}_0(t+1|t))$ curves

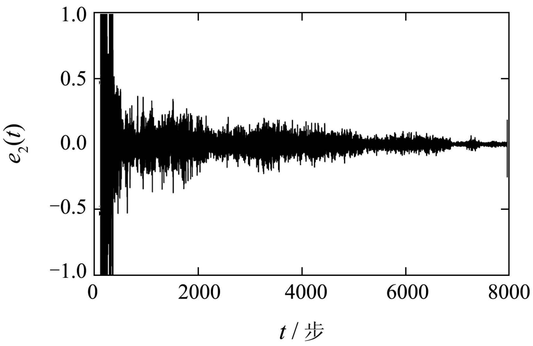

图 4 速度误差$e_2(t)=\hat{\dot{s}}_0^s(t+1|t)-\hat{\dot{s}}_0(t+1|t)$曲线

Fig. 4 The velocity error $e_2(t)=\hat{\dot{s}}_0^s(t+1|t)-\hat{\dot{s}}_0(t+1|t)$ curves

8 结论(Conclusion)

本文提出了带未知有色观测噪声系统自校正解耦融合Kalman滤波器. 用递推增广最小二乘法和解相关函数方程方法, 提出了未知有色观测噪声模型参数和噪声方差阵的一致的融合估值器. 用DVESA和DESA方法严格证明了所提出的自校正Kalman融合器的收敛于最优Kalman融合器, 因而它具有渐近最优性.

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