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设计高阶PI观测器对线性系统故障作鲁棒检测

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摘要: 针对具有干扰的线性定常系统, 研究了基于高阶PI观测器的鲁棒故障检测设计问题. 基于Sylvester矩阵方程的参数化解, 给出了干扰与残差解耦的充要条件, 并提出了基于高阶PI观测器的线性系统鲁棒故障检测参数化设计方法. 数值算例及仿真分析表明所提鲁棒故障检测参数化设计方法是有效的.

关键词: 线性系统; 高阶PI观测器; 鲁棒故障检测; 参数化设计

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Design of higher-order PI observers for robust fault detection in linear time-invariant systems

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Abstract: We investigate the design of a higher-order PI observer for robustly detecting the faults in the linear time-invariant system with disturbances. Based on the parametric solutions of Sylvester matrix equation, a necessary and sufficient condition for decoupling the disturbances from the residuals is proposed, and a parameterized method for designing the higher-order PI observers for robust fault-detection is established. The simulation of a numerical example validates the efficacy of the proposed parameterized design method for robust fault-detection.

Key words: linear systems; higher-order PI observers; robust fault detection; parameterized design

1 引言(Introduction)

随着工业过程规模和复杂程度的不断扩大, 如何提高运行系统的安全性、可靠性, 提高产品质量并降低生产成本, 防止和杜绝影响系统正常运行的故障的发生和发展成为急需解决的重要问题之一[1]. 提高复杂系统安全性、可靠性的方法有多种, 其中基于观测器的故障检测方法是基于模型方式的故障检测方法研究热点之一[2–6].

根据控制系统对偶性, 将积分环节引入观测器, 使得观测器进行估计时不仅用到当前状态, 也能利用过去状态. 这类观测器使用了比例和积分信息, 因此称为比例–积分(PI)观测器[7]. 因为通过积分回路可以引入更多的自由度, 使得PI观测器备受研究者的广泛关注. 在文献[8–9]中, PI观测器用来提高系统存在参数变化和有阶跃干扰时的鲁棒性. 在文献[10]中通过使用PI观测器积分回路提供的自由度设计了一个抗系统不确定性的鲁棒控制系统. 文献[11]指出, PI观测器可以估计有表现为未知输入, 非线性以及未建模动态的任意外部输入系统的状态. 文献[12]考虑了基于PI观测器的扰动抑制和故障检测问题. 另外, 文献[13]给出了PI观测器的参数化设计方法, 该方法能提供所有设计自由度, 为系统的进一步设计带来方便. 从扰动抑制的观点看, PI观测器仅对阶跃扰动有效. 为提高抑制扰动的有效性, 文献[14]最先将多积分环节引入连续系统的观测器设计中, 提出了高阶积分观测器, 文中的仿真例子也显示了该类观测器的有效性. 文献[15]给出了此类高阶积分观测器的一个参数化设计方法.

本文在系统存在干扰情况下, 针对线性定常系统给出了基于高阶PI观测器的鲁棒故障检测参数化设计方法. 该设计方法的优点在于: 采用高阶PI观测器能够提高抑制扰动的有效性的同时, 还能够给出具有鲁棒故障检测功能的所有高阶PI观测器的表达式, 从而为控制系统设计提供了更多自由度.

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2 问题提出(Problem statement)

考虑如下线性定常系统:

$$\begin{cases}\dot{x}=Ax+Bu+Dd+Ff,\\ y=Cx,\end{cases} \tag{1}$$

其中: $x\in\mathbb{R}^n$和$u\in\mathbb{R}^r$分别为系统(1)的状态向量和输入向量; $d\in\mathbb{R}^q$, $f\in\mathbb{R}^\tau$和$y\in\mathbb{R}^m$分别为未知扰动向量、故障向量和输出向量; 适当维数的已知矩阵A, B, C, D和F为满足下述假设条件:

假设 1 矩阵B, D和F均列满秩, 矩阵C行满秩.

假设 2 $\operatorname{rank}\begin{bmatrix}sI-A\\ C\end{bmatrix}=n, \forall s\in\mathbb{C}.$

定义 1 当$d=0,\ f=0$时, 如果

$$\lim_{t\to\infty}(\hat{x}-x)=0,\ \lim_{t\to\infty}\omega_i(t)=0,\ i=1,2,\cdots,l$$

成立, 则系统(1)的高阶PI观测器定义为

$$\begin{cases}\dot{\hat{x}}=A\hat{x}+Bu+L(y-C\hat{x})+\tilde{F}\sum\limits_{i=1}^{l}\omega_i,\\ \omega_i^{(i)}=K_i(y-C\hat{x}),\ i=1,2,\cdots,l,\end{cases} \tag{2}$$

其中: $\hat{x}\in\mathbb{R}^n$为系统(1)中状态向量x的观测向量, $\omega_i\in\mathbb{R}^p$为代表加权输出估计误差的积分, 矩阵L, $\tilde{F}$和K_i为观测器(2)的待求增益矩阵, L为比例增益矩阵, $\tilde{F}$为积分增益矩阵, K_i为对应于i次积分的输出估计误差加权矩阵.

定义误差为$e=\hat{x}-x$, 将式(1)和式(2)相减可得

$$\begin{cases}\dot{e}=(A-LC)e+\tilde{F}\sum\limits_{i=1}^{l}\omega_i-Dd-Ff,\\ \omega_i^{(i)}=-K_iCe,\ i=1,2,\cdots,l.\end{cases} \tag{3}$$

令

$$\eta_i=\sum_{j=0}^{i-1}\omega_{l-j}^{(l-i-1)},\ i=1,2,\cdots,l. \tag{4}$$

综合式(3)和式(4), 可得

$$\begin{cases}\dot{e}=(A-LC)e+\tilde{F}\eta_l-Dd-Ff,\\ \dot{\eta}_1=-K_lCe,\\ \dot{\eta}_i=-K_{l-i+1}Ce+\eta_{i-1},\ i=2,3,\cdots,l.\end{cases} \tag{5}$$

将方程(5)写成矩阵形式为

$$\varOmega=\begin{bmatrix}A-LC & 0 & 0 & \cdots & 0 & \tilde{F}\\ -K_lC & 0 & 0 & \cdots & 0 & 0\\ -K_{l-1}C & I & 0 & \cdots & 0 & 0\\ -K_{l-2}C & 0 & I & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ -K_1C & 0 & 0 & \cdots & I & 0\end{bmatrix}\begin{bmatrix}\dot{e}\\ \dot{\eta}_1\\ \dot{\eta}_2\\ \dot{\eta}_3\\ \vdots\\ \dot{\eta}_l\end{bmatrix}=$$

$$\varOmega\begin{bmatrix}e\\ \eta_1\\ \eta_2\\ \eta_3\\ \vdots\\ \eta_l\end{bmatrix}-\begin{bmatrix}D\\ 0\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}d-\begin{bmatrix}F\\ 0\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}f. \tag{6}$$

为简化方程(6), 令

$$A_{\rm o}=\begin{bmatrix}A-LC & 0 & 0 & \cdots & 0 & \tilde{F}\\ -K_lC & 0 & 0 & \cdots & 0 & 0\\ -K_{l-1}C & I & 0 & \cdots & 0 & 0\\ -K_{l-2}C & 0 & I & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ -K_1C & 0 & 0 & \cdots & I & 0\end{bmatrix},$$

$$D'=[D\ \ 0\ \ 0\ \ 0\ \cdots\ 0]^{\rm T},$$

$$F'=[F\ \ 0\ \ 0\ \ 0\ \cdots\ 0]^{\rm T}.$$

当$d=0,\ f=0$时, 方程(6)则变为

$$\begin{bmatrix}\dot{e}\\ \dot{\eta}_1\\ \dot{\eta}_2\\ \dot{\eta}_3\\ \vdots\\ \dot{\eta}_l\end{bmatrix}=A_o\begin{bmatrix}e\\ \eta_1\\ \eta_2\\ \eta_3\\ \vdots\\ \eta_l\end{bmatrix}. \tag{7}$$

如果系统(7)稳定, 即矩阵$A_{\rm o}$的所有特征值具有负实部, 由定义1可知式(2)为系统(1)的一个高阶PI观测器.

定义残差向量为

$$r=G(y-C\hat{x}), \tag{8}$$

其中$0\neq G\in\mathbb{R}^{\sigma\times m}$为待定的加权矩阵. 当扰动向量$d$和故障向量$f$不存在且系统(5)稳定时, 残差$r\to 0$. 但当扰动向量$d$和故障向量$f$存在时, 此残差向量不再趋向于零. 式(8)可写为

$$r=-G[C\ \ 0\ \ 0\ \ 0\cdots 0]\begin{bmatrix}e\\ \eta_1\\ \eta_2\\ \eta_3\\ \vdots\\ \eta_l\end{bmatrix}. \tag{9}$$

令

$$C'=[C\ \ 0\ \ 0\ \ 0\cdots 0].$$

由式(6)和式(9)可得频域关系式

$$r=GC'(sI-A_{\rm o})^{-1}D'd(s)+ GC'(sI-A_{\rm o})^{-1}F'f(s). \tag{10}$$

在故障检测中, 残差信号用作故障的指示器. 为了避免错误的故障警告, 应该消除扰动对残差的影响, 这就要求

$$GC'(sI-A_{\rm o})^{-1}D'=0. \tag{11}$$

此外, 非退化矩阵有较低的特征值灵敏度, 故要求矩阵$A_{\rm o}$是非退化的.

因此, 基于高阶PI观测器的线性定常系统鲁棒故障检测设计问题可由问题RFD(robust fault detection)如下描述.

问题RFD(robust fault detection): 给定满足假设1和2的线性定常系统(1), 如果存在形如(2)的高阶PI观测器, 求解增益矩阵L, $\tilde{F}$, $K_i(i=1,2,3,\cdots,l)$和适当的加权矩阵G使得下列条件成立:

1) 矩阵$A_{\rm o}$稳定且非退化;

2) 鲁棒故障检测条件(11)成立.

注 1 当式(11)成立时, 残差信号对扰动信号没有响应. 残差信号作为信号的指示器, 而残差信号必须对故障信号有响应, 这样必有

$$GC'(sI-A_{\rm o})^{-1}F'\neq 0. \tag{12}$$

条件(12)容易满足, 因此该条件没必要包含在问题RFD中.

3 高阶PI观测器增益的参数化解(Parametric solutions of gain matrices for higher-order PI observers)

下面求解问题RFD中条件1), 简记为问题PI.

问题PI 给定满足假设1和2的系统(1), 求取高阶PI观测器(2)中增益矩阵L, $\tilde{F}$和K_i, $i=1,2,3,\cdots,l$使得矩阵$A_{\rm o}$稳定且非退化. 记矩阵$A_{\rm o}$的所有特征值构成的Jordan对角阵为

$$\varLambda=\operatorname{diag}\{s_1,s_2,\cdots,s_d\}, \tag{13}$$

其中$d=n+lp$. 记矩阵$A_{\rm o}$的左特征向量矩阵为

$$T=\begin{bmatrix}T_e\\T_l\\T_{l-1}\\\vdots\\T_1\end{bmatrix},\ T_{\rm e}\in\mathbb{C}^{n\times d},\ T_i\in\mathbb{C}^{p\times d}. \tag{14}$$

由左特征向量的定义有

$$\operatorname{rank}T=d, \tag{15}$$

$$T^{\rm T}A_{\rm o}=\varLambda T^{\rm T}. \tag{16}$$

方程(16)可分解为

$$T_{\rm e}^{\rm T}(A-LC)-\sum_{i=1}^{l}T_i^{\rm T}K_iC=\varLambda T_{\rm e}^{\rm T}, \tag{17}$$

$$\begin{cases}T_{\rm e}^{\rm T}\tilde{F}=\varLambda T_1^{\rm T},\\T_i^{\rm T}=\varLambda T_{i+1}^{\rm T},\ i=1,2,3,\cdots,l-1.\end{cases} \tag{18}$$

记

$$-Z^{\rm T}=T_{\rm e}^{\rm T}L+\sum_{i=1}^{l}T_i^{\rm T}K_i, \tag{19}$$

则式(19)可变为

$$T_{\rm e}^{\rm T}A+Z^{\rm T}C=\varLambda T_{\rm e}^{\rm T}. \tag{20}$$

3.1 求解矩阵$T_{\rm e}$和Z(Solutions of matrices $T_{\rm e}$ and Z)

因假设1和2成立, 故有

$$(sI-A^{\rm T})^{-1}C^{\rm T}=N(s)D^{-1}(s),\ \forall s\in\mathbb{C},$$

其中$N(s)\in\mathbb{R}^{n\times m}[s]$和$D(s)\in\mathbb{R}^{m\times m}[s]$为右互质矩阵.

引理 1[16–18] 若假设1和2成立, 则方程(20)中矩阵$T_{\rm e}$和Z的参数化解分别为

$$T_{\rm e}=[N(s_1)g_1\ \ N(s_2)g_2\ \cdots\ N(s_d)g_d], \tag{21}$$

$$Z=[D(s_1)g_1\ \ D(s_2)g_2\ \cdots\ D(s_d)g_d], \tag{22}$$

其中$g_i\in\mathbb{C}^m(i=1,\cdots,d)$为自由参量.

3.2 求解矩阵T_i和$\tilde{F}$(Solutions of matrices T_i and $\tilde{F}$)

由式(18), 可得

$$T_{\rm e}^{\rm T}\tilde{F}=\varLambda^iT_i^{\rm T}, \tag{23}$$

因为矩阵$\varLambda$的可逆性, 上式可写为

$$T_i=\tilde{F}^{\rm T}T_{\rm e}\varLambda^{-i}. \tag{24}$$

利用式(21), 由式(24)可得

$$T_i=[s_1^{-i}\tilde{F}^{\rm T}N(s_1)g_1\ \cdots\ s_d^{-i}\tilde{F}^{\rm T}N(s_d)g_d], \tag{25}$$

其中$\tilde{F}$可以看作T_i的参数. 由式(14)(21)(25)可得左特征向量矩阵T的参数化解为

$$T=\begin{bmatrix}N(s_1)g_1&\cdots&N(s_d)g_d\\s_1^{-l}\tilde{F}^{\rm T}N(s_1)g_1&\cdots&s_d^{-l}\tilde{F}^{\rm T}N(s_d)g_d\\s_1^{-l+1}\tilde{F}^{\rm T}N(s_1)g_1&\cdots&s_d^{-l+1}\tilde{F}^{\rm T}N(s_d)g_d\\\vdots&&\vdots\\s_1^{-1}\tilde{F}^{\rm T}N(s_1)g_1&\cdots&s_d^{-1}\tilde{F}^{\rm T}N(s_d)g_d\end{bmatrix}. \tag{26}$$

这样式(15)可转化为由参数$\tilde{F}$, s_i, $g_i(i=1,\cdots,d)$表示的如下约束:

约束 1 $\det[T(\tilde{F},s_i,\,g_i,\,i=1,\cdots,d)]\neq 0.$

3.3 求解矩阵L和K_i(Solutions of matrices L and K_i)

由式(19)可得

$$T^{\rm T}\tilde{L}=-Z^{\rm T}, \tag{27}$$

其中

$$\tilde{L}=[L\ \ K_l\ \ K_{l-1}\ \cdots\ K_1]^{\rm T} \tag{28}$$

在约束1成立时, 可知观测器增益阵$\tilde{L}$为

$$\tilde{L}=-(ZT^{-1})^{\rm T}, \tag{29}$$

其中Z和T分别由式(22)和式(26)给出. 为保证观测器增益矩阵(29)为实阵, 需满足下述约束条件:

约束 2 $s_i=\bar{s}_j\Leftrightarrow g_i=\bar{g}_j, i,j=1,\cdots,d.$

综上, 通过下述定理可给出问题PI的解.

定理 1 给定满足假设1和2的系统(1), 问题PI的高阶PI观测器(2)中增益矩阵L, $\tilde{F}$和$K_i(i=1,2,3,\cdots,l)$的参数化解可通过式(22)(26)和式(28)由式(29)给出, 其中参数$\tilde{F}$, s_i, $g_i(i=1,\cdots,d)$需满足约束1–2. 此外, 矩阵$A_{\rm o}$的左特征向量矩阵T由式(26)给出.

4 求解问题RFD(Solutions to problem RFD)

利用式(14)和式(16), 将式(11)的左端写为

$$\begin{aligned}&GC'(sI-A_{\rm o})^{-1}D'=\\&GC'T^{-{\rm T}}(sI-T^{\rm T}A_{\rm o}T^{-{\rm T}})^{-1}T^{\rm T}D'=\\&GC'T^{-{\rm T}}(sI-\varLambda)^{-1}T^{\rm T}D'=\\&GC'T^{-{\rm T}}(sI-\varLambda)^{-1}T_{\rm e}^{\rm T}D.\end{aligned}\tag{30}$$

令

$$P=[p_1\cdots p_d]=GC'T^{-{\rm T}},\ p_i\in\mathbb{C}^{\sigma}.\tag{31}$$

由式(19)(30)–(31), 可得

$$\begin{aligned}&GC'(sI-A_{\rm o})^{-1}D'=\\&[p_1\ \cdots\ p_d]{\rm diag}\{\frac{1}{s-s_1},\frac{1}{s-s_2},\cdots,\frac{1}{s-s_d}\}T_{\rm e}^{\rm T}D=\\&(\sum_{i=1}^{d}\frac{p_it_{{\rm e}i}^{\rm T}}{s-s_i})D=\sum_{i=1}^{d}\frac{p_ig_i^{\rm T}N^{\rm T}(s_i)D}{s-s_i}.\end{aligned}$$

其中

$$\begin{aligned}T_{\rm e}=&[t_{\rm e1}\ t_{\rm e2}\ \cdots\ t_{{\rm e}d}]=\\&[N(s_1)g_1\,N(s_2)g_2\cdots N(s_d)g_d].\end{aligned}$$

由于变量s的任意性, 鲁棒故障检测条件(11)等价于下述约束:

约束 3　$p_ig_i^{\rm T}N^{\rm T}(s_i)D=0,\ i=1,\cdots,d.$

由式(14)和式(31)可得

$$G[C\ \ 0\ \ 0\ \ 0\ \cdots\ 0]=P[T_{\rm e}^{\rm T}\ \ T_l^{\rm T}\ \ T_{l-1}^{\rm T}\cdots T_1^{\rm T}],$$

其等价于

$$GC=PT_{\rm e}^{\rm T},PT_i^{\rm T}=0,i=1,2,3,\cdots,l.$$

应用式(21), 上式中第1式等价于

$$GC=\sum_{i=1}^{d}p_ig_i^{\rm T}N^{\rm T}(s_i).\tag{32}$$

应用式(25), 上式中第2式可转化为如下约束:

约束 4　$\sum_{j=1}^{l}(\sum_{i=1}^{d}s_i^{-j}p_ig_i^{\rm T}N^{\rm T}(s_i)\tilde{F})=0.$

为保证由式(32)求出实矩阵G, 需满足如下约束条件:

约束 5　$p_i=\bar{p}_j\Leftrightarrow s_i=\bar{s}_j.$

约束 6　${\rm rank}C={\rm rank}\begin{bmatrix}C\\\sum_{i=1}^{d}p_ig_i^{\rm T}N^{\rm T}(s_i)\end{bmatrix}.$

因为矩阵C行满秩, 由式(32)算得

$$G=\sum_{i=1}^{d}p_ig_i^{\rm T}N^{\rm T}(s_i)C^{\rm T}(CC^{\rm T})^{-1}.\tag{33}$$

由上述推导, 可得求解问题RFD的如下定理.

定理 2　给定满足假设1和假设2的系统(1), 问题RFD有解的充要条件是存在满足约束1–6的参数$\tilde{F}$和$s_i\in\mathbb{C}$, $p_i\in\mathbb{C}^{\sigma}$, $g_i\in\mathbb{C}^m$, $i=1,\cdots,d$. 此时, 高阶PI观测器(2)的增益阵L, $\tilde{F}$和$K_i(i=1,2,3,\cdots,l$通过式(22)(26)和式(28)由式(29)给出, 以及加权矩阵G由式(33)给出.

5 实例分析(Example analysis)

考虑线性系统

$$\begin{cases}\dot{x}=Ax+Bu(t)+Ff(t)+Dd(t),\\y=Cx,\end{cases}$$

其参数矩阵为

$$A=\begin{bmatrix}-10&10&0\\28&-1&0\\0&0&-\frac{8}{3}\end{bmatrix},\ B=\begin{bmatrix}1&0\\-1&1\\0&1\end{bmatrix},$$

$$C=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix},\ F=\begin{bmatrix}1&-1\\-0.5&1\\0.5&0\end{bmatrix},\ D=\begin{bmatrix}1\\2\\1\end{bmatrix}.$$

利用定理2设计$p=1$, $l=2$的高阶PI观测器, 且设定加权矩阵$p_i(i=1,2,\cdots,5)$为行分量, 即$\sigma=1$.

1) 算得矩阵

$$[sI-A^{\rm T}\ C^{\rm T}]=\begin{bmatrix}10+s&-28&0&1&0&0\\-10&1+s&0&0&-1&0\\0&0&s+\frac{8}{3}&0&0&1\end{bmatrix}$$

的秩为3, 满足高阶PI观测器设计要求.

2) 计算满足右互质分解的多项式矩阵为

$$N(s)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\ D(s)=\begin{bmatrix}10+s&-28&0\\10&-1-s&0\\0&0&\frac{8+3s}{3}\end{bmatrix}.$$

3) 假设闭环极点s_1,s_2,s_3,s_4,s_5是负实数, 且限制向量$g_i(i=1,2,3,4,5)$为实, 故约束2自然成立. 记

$$g_i=\begin{bmatrix}g_{i1}\\g_{i2}\\g_{i3}\end{bmatrix},\ i=1,2,3,4,5,\ \tilde{F}^{\rm T}=[f_1\ \ f_2\ \ f_3].$$

因

$$\begin{aligned}&N(s_i)g_i=g_i,\ i=1,\cdots,5,\\&s_i^{-1}\tilde{F}^{\rm T}N(s_i)g_i=\frac{f_1g_{i1}+f_2g_{i2}+f_3g_{i3}}{s_i},\ i=1,\cdots,5,\\&s_i^{-2}\tilde{F}^{\rm T}N(s_i)g_i=\frac{f_1g_{i1}+f_2g_{i2}+f_3g_{i3}}{s_i^2},\ i=1,\cdots,5.\end{aligned}$$

令

$$f_1g_{i1}+f_2g_{i2}+f_3g_{i3}=\mu_i,\ i=1,2,3,4,5,$$

则由式(26)可得

$$t_i=[g_{i1}\ \ g_{i2}\ \ g_{i3}\ \ \frac{\mu_i}{s_i^2}\ \ \frac{\mu_i}{s_i}]^{\rm T},\ i=1,2,3,4,5,$$

$$z_i=\begin{bmatrix}g_{i1}(s_i+10)-28g_{i2}\\10g_{i1}-g_{i2}(s_i+1)\\g_{i3}(s+\frac{8}{3})\end{bmatrix},\ i=1,2,3,4,5,$$

则约束1为

$$\det T = \det \begin{bmatrix} g_{11} & g_{21} & g_{31} & g_{41} & g_{51} \\ g_{12} & g_{22} & g_{32} & g_{42} & g_{52} \\ g_{13} & g_{23} & g_{33} & g_{43} & g_{53} \\ \frac{\mu_1}{s_1^2} & \frac{\mu_2}{s_2^2} & \frac{\mu_3}{s_3^2} & \frac{\mu_4}{s_4^2} & \frac{\mu_5}{s_5^2} \\ \frac{\mu_1}{s_1} & \frac{\mu_2}{s_2} & \frac{\mu_3}{s_3} & \frac{\mu_4}{s_4} & \frac{\mu_5}{s_5} \end{bmatrix} \neq 0.$$

而矩阵A_o满秩的充分必要条件是

$$\operatorname{rank}\begin{bmatrix} A & \widetilde{F} \\ C & 0 \end{bmatrix} = 4,$$

其等价于f_1, f_2, f_3中至少有一个不为0.

约束3为

$$p_i(g_{i1} + 2g_{i2} + g_{i3}) = 0,\ i = 1,2,3,4,5.$$

约束4为

$$\sum_{i=1}^{5}((s_i^{-1} + s_i^{-2})p_i\mu_i) = 0.$$

不妨限定参数$p_i(i = 1,2,3,4,5)$为实数, 则约束5自然满足. 而约束6为

$$\operatorname{rank}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ \sum_{i=1}^{5} p_i g_{i1} & \sum_{i=1}^{5} p_i g_{i2} & \sum_{i=1}^{5} p_i g_{i3} \end{bmatrix} = 3.$$

容易看出, 无论矩阵左侧的参数取何值, 上述条件均成立.

4) 算得满足约束1–6的一组参数为

$$s_1 = -3,\ s_2 = -1,\ s_3 = -\frac{8}{3},\ s_4 = -10,$$
$$s_5 = -6,\ p_1 = 1,\ p_2 = 2,\ p_3 = p_4 = 0,$$
$$p_5 = -1,\ f_1 = -1,\ f_2 = 0,\ f_3 = 1,$$
$$g_1 = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix},\ g_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix},\ g_3 = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix},$$
$$g_4 = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix},\ g_5 = \begin{bmatrix} \frac{11}{5} \\ 1 \\ -\frac{21}{5} \end{bmatrix}.$$

5) 基于步骤4)的参数, 由式(22)和式(26)分别算得

$$Z = \begin{bmatrix} -21 & 37 & -13.3333 & -56 & -19.2 \\ 12 & 10 & 21.6667 & 8 & 27 \\ 1 & 1.6667 & 0 & -36.6667 & 14 \end{bmatrix},$$

$$T = \begin{bmatrix} 1 & 1 & 2 & -1 & 2.2 \\ 1 & -1 & 1 & 2 & 1 \\ -3 & 1 & 3 & 5 & -4.2 \\ -0.444 & 0 & 0.1406 & 0.06 & -0.1778 \\ 1.333 & 0 & -0.375 & -0.6 & 1.0667 \end{bmatrix}.$$

进一步, 由式(29)算得

$$L = \begin{bmatrix} -5.6978 & 2.9142 & 15.7532 \\ 29.0021 & 0.0245 & 0.6358 \\ -2.3001 & 12.8898 & 14.7223 \end{bmatrix},$$
$$K_1 = [2.4548\ \ -251.3606\ \ 198.1541],$$
$$K_2 = [-6.0851\ \ -123.9928\ \ 109.7646],$$

以及加权阵

$$G = [-3.2\ \ 2\ \ -0.8].$$

此时

$$GCF = [-2.6\ \ 1.2] \neq 0.$$

因此鲁棒故障检测条件(11)满足.

为进一步验证定理2的有效性, 选取扰动信号和故障分别为

$$d(t) = 10^3 \mathrm{e}^{-t},\ f(t) = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}.$$

仿真时间为0∼10 s, 在4 s时加入扰动信号和故障信号进行干扰, 仿真效果图如图1–5所示. 图1–3为干扰前后误差信号中3个分量的响应图, 从图中易看到从4 s开始干扰对误差信号产生的影响, 但到10 s附近误差分量又趋近于0.

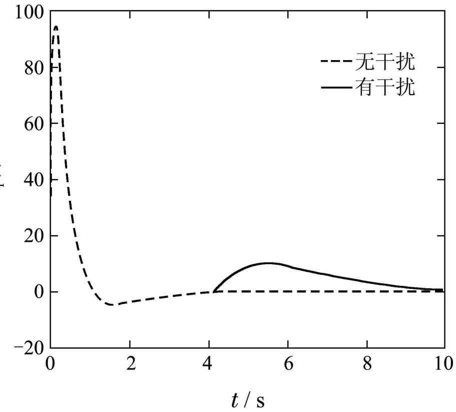

图 1 干扰前后误差信号的第1个分量$e_1(t)$的响应图

Fig. 1 Response diagrams of the first component $e_1(t)$ for the error signal before and after disturbance

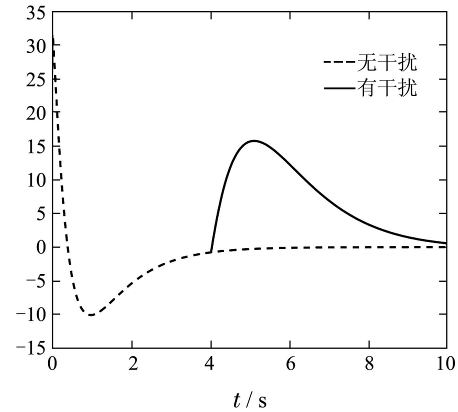

图 2 干扰前后误差信号的第2个分量$e_2(t)$的响应图

Fig. 2 Response diagrams of the second component $e_2(t)$ for the error signal before and after disturbance

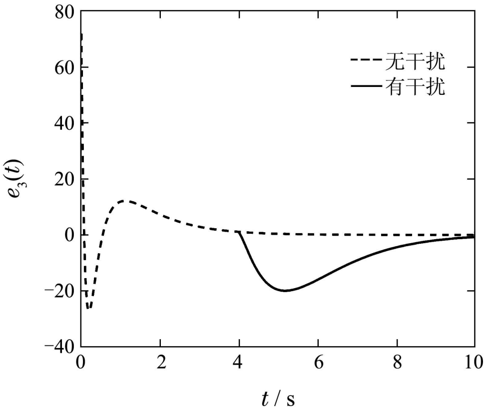

图 3　干扰前后误差信号的第3个分量$e_3(t)$的响应图

Fig. 3　Response diagrams of the third component $e_3(t)$ for the error signal before and after disturbance

图4为干扰信号作用下系统残差信号$r(t)$的响应图, 从图中易看到4 s开始干扰对残差信号产生的影响, 但到8 s附近残差信号趋近于0. 图5为故障信号作用下系统残差信号$r(t)$的响应图, 从图中易看到4 s开始故障对残差信号产生的影响, 其值显然不为0, 从而可以判断系统在4 s开始存在故障. 上述仿真结果说明本文所提鲁棒故障检测设计方法的有效性.

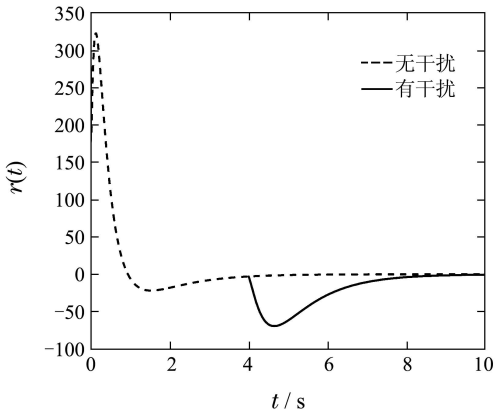

图 4　干扰前后系统残差信号$r(t)$的响应图

Fig. 4　Response diagrams of the residual signal $r(t)$ for the system before and after disturbance

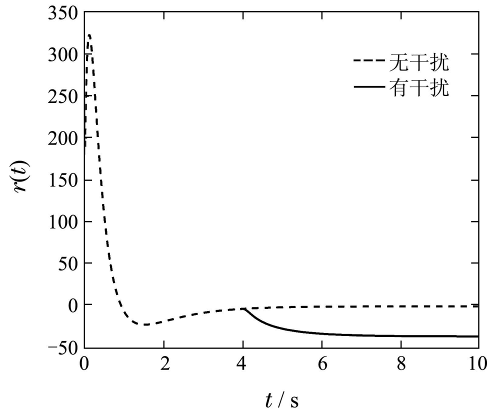

图 5　故障前后系统残差信号$r(t)$的响应图

Fig. 5　Response diagrams of the residual signal $r(t)$ for the system before and after fault

6　结论(Conclusions)

针对具有外界干扰的线性定常系统, 对其进行故障检测时, 消除干扰的影响是一个关键问题, 也是一个提高系统故障检测的鲁棒性问题. 本文基于一类Sylvester矩阵方程的解, 给出了同时具有消除干扰和故障检测功能的高阶PI观测器参数化设计方法. 数值算例及其仿真分析表明, 本文给出的基于高阶PI观测器的线性系统鲁棒故障检测参数化设计方法在消除干扰提高系统的鲁棒性以及故障检测中是有效的.

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