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一类非线性直升机模型的滑模降阶控制器设计

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摘要: 针对带有交叉耦合的多输入CE150直升机模型, 研究了一类多输入仿射非线性系统的控制设计问题, 基于滑模变结构控制理论, 采用了一种新的控制器设计方法: 滑模降阶方法, 即反复运用变结构控制理论, 对一类高阶的仿射非线性系统, 构造了合适的微分同胚变换函数, 把初始高阶系统降至低阶系统, 并构造了变结构控制律, 再利用当前级和上一级控制输入的映射关系反推出初始系统的控制输入. 通过CE150直升机模型仿真结果表明, 该方法有效可行.

关键词: CE150直升机模型; 仿射非线性系统; 滑模控制; 滑模降阶; 微分同胚变换
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Sliding-mode reduced-order controller design for a class of nonlinear helicopter model

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Abstract: We investigate the design of a multiple-input affine nonlinear system for the multiple-input CE150 helicopter model with cross coupling. A new method for designing the sliding-mode reduced-order controller is developed based on the sliding-mode control theory. For a type of higher order affine nonlinear system, we build appropriate diffeomorphism transformation functions to reduce the initial higher order system to a lower order system by repeatedly using variable structure control theory as well as building variable structure control rule. The control input to the initial system is backward deducted successively from the mapping relationship between the present system and the previous system. This method is feasible and effective according to the simulation results of CE150 helicopter model.

Key words: CE150 helicopter model; nonlinear affine systems; sliding-mode control; sliding-mode reduced-order; diffeomorphism transformation

1 引言(Introduction)

随着现代社会信息化、系统化的发展, 人类面临的各种控制系统规模越来越大, 由此导致系统模型及控制器的阶数也越来越高, 相应的对系统分析计算和综合实现复杂度也越来越高, 因此, 模型降阶一直是控制理论研究领域的热门课题[1–12]. 然而, 对系统模型进行降阶简化应遵循的基本原则是: 不能用忽略掉系统的关键性能为代价来得到具有结构和计算上优点的模型[1]. 目前传统的模型降阶方法主要包括: Hankel范数降阶法、因式分解降阶法、奇异摄动降阶法、平衡截断降阶法、Krylov子空间降阶法和频率加权降阶法等[3,6–9], 在上述的方法中, 频率加权模型降阶法得到更多的关注. 近年来, 有学者在提高模型降阶精度和结构化性能方面做了研究, 并取得了一定的成果[10–12]. 然而, 到目前为止, 众多的模型降阶方法基本上是针对线性系统而设计的, 更确切地说, 大多数降阶方法只适合于线性时不变系统, 而对于非线性系统, 模型降阶方法发展及其缓慢.

直升机模型是一个高阶、非线性、强耦合、多输入多输出的复杂系统, 在飞行过程中会遇到飞行条件、气动特性的变化及遭受多种不确定因素的干扰, 因此, 直升机需要设计鲁棒飞行控制系统. 目前直升机飞行控制器设计方法主要有: 特征结构配置(EA)、线性二次型高斯设计(LQG)、单通道分析与设计(ICAD)、定量反馈理论(QFT)、H_∞控制等[13–19]. 综合比较这些方法, 鲁棒H_∞控制方法更有潜力、更有发展前景.

本文针对CE150直升机模型, 采用了一种新的滑模降阶控制器设计方法, 通过构造合适的微分同胚

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变换函数, 将初始高阶系统降至低阶系统, 从而实现对复杂直升机系统模型降阶, 然后应用反推运算得到初始系统的控制输入. 值得一提的是, 从滑模变结构控制原理看, 用本方法实现了系统模型降阶且不损失系统任何信息, 而仅是系统结构上的分离. 本文控制目标是设计控制器使得直升机模型闭环系统渐近稳定, 且同时抑制外部扰动和削弱抖振.

2 直升机系统模型描述(Description of helicopter model)

研究如下用仿射非线性系统描述的CE150直升机模型(CE150 helicopter model)[20]:

$$\dot{x} = f(x) + G(x)(u + \mu), \tag{1}$$

其中:

$$f(x)=\begin{bmatrix} x_3 \\ x_4 \\ \alpha(x_1,x_2,x_3,x_7) \\ -\dfrac{1}{T_1^2}x_2 - \dfrac{2}{T_1}x_4 \\ x_7 \\ x_8 \\ \beta(x_1,x_6,x_7,x_9) \\ -\dfrac{1}{T_2^2}x_6 - \dfrac{2}{T_2}x_8 \\ -\dfrac{1}{T_{\rm pr}}x_9 \end{bmatrix},\ G(x)=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{1}{T_1^2} & 0 \\ 0 & 0 \\ 0 & 0 \\ b_{71}(x) & 0 \\ 0 & \dfrac{1}{T_2^2} \\ b_{79} & 0 \end{bmatrix},$$

$$\alpha(x_1,x_2,x_3,x_7) = \frac{1}{I_\psi}[a_1(x_2+\bar{u}_{\rm d1})^2 + b_1(x_2+\bar{u}_{\rm d1}) - B_\psi x_3 - h_{\rm g}\sin(x_1+\bar{\psi}+\theta) - K_{\rm G}\cos(x_1+\bar{\psi})(x_2+\bar{u}_{\rm d1})x_7],$$

$$\beta(x_1,x_6,x_7,x_9) = \frac{1}{I_\phi\sin(x_1+\bar{\psi})}[a_2(x_6+\bar{u}_{\rm d2})^2 + b_2(x_6+\bar{u}_{\rm d2}) - B_\phi x_7 + \frac{K_{\rm r}T_{\rm or}}{T_{\rm pr}}x_9 + K_{\rm r}\bar{u}_{\rm d1}],$$

$$b_{71}(x) = \frac{K_{\rm r}T_{\rm or}}{I_\phi\sin(x_1+\bar{\psi})T_{\rm pr}},\ b_{79} = \frac{1}{T_{\rm or}} - \frac{1}{T_{\rm pr}},$$

$u\in\mathbb{R}^2$为控制输入, $\mu\in\mathbb{R}^2$为外部扰动. 式中: 状态变量

$$x_1 = \psi - \bar{\psi},\ x_2 = u_{\rm d1} - \bar{u}_{\rm d1},\ x_3 = \frac{{\rm d}\psi}{{\rm d}t},$$
$$x_4 = \frac{{\rm d}u_{\rm d1}}{{\rm d}t},\ x_5 = \phi - \bar{\phi},\ x_6 = u_{\rm d2} - \bar{u}_{\rm d2},$$
$$x_7 = \frac{{\rm d}\phi}{{\rm d}t},\ x_8 = \frac{{\rm d}u_{\rm d2}}{{\rm d}t},\ x_9 = \varepsilon - \bar{\varepsilon},$$

其中: ψ和ϕ分别表示直升机模型的仰角和方位角; $u_{\rm d1}$和$u_{\rm d2}$分别表示主电机的电枢电压和副电机的电枢电压; $\bar{\psi},\bar{\phi}$分别表示直升机模型的理想仰角和方位角, 本文取理想仰角$\bar{\psi}=\pi/2$; $\bar{u}_{\rm d1},\bar{u}_{\rm d2}$分别表示主副电机的电枢理想电压; ε反应主螺旋桨对应方位角的旋转动态, 并且满足以下各式:

$$a_1\bar{u}_{\rm d1}^2 + b_1\bar{u}_{\rm d1} - h_{\rm g}\sin(\bar{\psi}+\theta) = 0, \tag{2}$$
$$a_2\bar{u}_{\rm d2}^2 + b_2\bar{u}_{\rm d2} + K_{\rm r}\bar{u}_{\rm d1} = 0, \tag{3}$$
$$\frac{1}{T_{\rm pr}}\bar{\varepsilon} = (\frac{1}{T_{\rm or}} - \frac{1}{T_{\rm pr}})\bar{u}_{\rm d1}, \tag{4}$$
$$a_1(x_2+\bar{u}_{\rm d1})^2 \simeq a_1\bar{u}_{\rm d1}^2 + 2a_1\bar{u}_{\rm d1}x_2, \tag{5}$$
$$-h_{\rm g}\sin(x_1+\bar{\psi}+\theta) \simeq -h_{\rm g}\sin(\bar{\psi}+\theta) - h_{\rm g}\cos(\bar{\psi}+\theta)x_1, \tag{6}$$
$$a_2(x_6+\bar{u}_{\rm d2})^2 \simeq a_2\bar{u}_{\rm d2}^2 + 2a_2\bar{u}_{\rm d2}x_6, \tag{7}$$
$$K_{\rm G}\cos(x_1+\bar{\psi})(x_2+\bar{u}_{\rm d1})x_7 \simeq K_{\rm G}\bar{u}_{\rm d1}\cos\bar{\psi}x_7 = 0, \tag{8}$$
$$I_\phi\sin(x_1+\bar{\psi}) \simeq I_\phi\sin\bar{\psi}. \tag{9}$$

由式(6)–(7)得

$$\alpha(x_1,x_2,x_3,x_7) \simeq \frac{1}{I_\psi}[a_1(x_2+\bar{u}_{\rm d1})^2 + b_1(x_2+\bar{u}_{\rm d1}) - B_\psi x_3 - h_{\rm g}\sin(\bar{\psi}+\theta) - h_{\rm g}\cos(\bar{\psi}+\theta)x_1 - K_{\rm G}\cos(x_1+\bar{\psi})(x_2+\bar{u}_{\rm d1})x_7],$$

$$\beta(x_1,x_6,x_7,x_9) \simeq \beta(x_6,x_7,x_9) = \frac{1}{I_\phi\sin\bar{\psi}}[a_2(x_6+\bar{u}_{\rm d2})^2 + b_2(x_6+\bar{u}_{\rm d2}) - B_\phi x_7 + \frac{K_{\rm r}T_{\rm or}}{T_{\rm pr}}x_9 + K_{\rm r}\bar{u}_{\rm d1}].$$

3 滑模降阶控制器设计方法(The method of sliding mode reduced order controller design)

不失一般性, 考虑系统(1)为n阶p输入系统, 即: 状态$x\in\mathbb{R}^n$, 控制输入$u\in\mathbb{R}^p$, $f(x),G(x)$为向量场, $\mu\subset\mathbb{R}^p$为外部扰动.

假设 1 存在微分同胚变换$\phi(\cdot)$, 其变换关系为$\begin{bmatrix}\varepsilon\\ \eta\end{bmatrix} = \phi(\cdot)$, 其中: $\varepsilon\in\mathbb{R}^p$, $\eta\in\mathbb{R}^{n-p\times k}$, 且$\phi(\cdot)$满足$\dfrac{\partial\phi(\cdot)}{\partial(\cdot)}G(\cdot) = \begin{bmatrix} I_{p\times p} \\ 0_{(n-p\times k)\times p}\end{bmatrix}$, 其中: I是$p\times p$的单位阵, k表示第k次降阶.

假设 2 外部扰动μ有上界, 即$0\leqslant\|\mu\|\leqslant b$, 其中$b$为未知参数.

3.1 第1次降阶(First order reduction)

3.1.1 坐标变换(Coordinate transformation)

根据假设1, 作微分同胚变换$\phi(x)$, 变换关系是$\begin{bmatrix}\varepsilon_1\\ \eta_1\end{bmatrix} = \phi(x)$, 其中: $\varepsilon_1\in\mathbb{R}^p$, $\eta_1\in\mathbb{R}^{n-p\times 1}$, 且$\phi(x)$满足$\dfrac{\partial\phi(x)}{\partial x}G(x) = \begin{bmatrix} I_{p\times p} \\ 0_{(n-p\times 1)\times p}\end{bmatrix}$. 记$\dfrac{\partial\phi(x)}{\partial x}f(x) =$

$\begin{bmatrix} f_{\mathrm{b}}(\eta_1,\varepsilon_1) \\ f_{\mathrm{a}}(\eta_1,\varepsilon_1) \end{bmatrix}$. 对$\phi(x)$求导得

$$\begin{bmatrix} \dot\varepsilon_1 \\ \dot\eta_1 \end{bmatrix} = \frac{\partial\phi(x)}{\partial x}f(x) + \frac{\partial\phi(x)}{\partial x}G(x)(u+\mu) = \begin{bmatrix} f_{\mathrm{b}}(\eta_1,\ \varepsilon_1) \\ f_{\mathrm{a}}(\eta_1,\ \varepsilon_1) \end{bmatrix} + \begin{bmatrix} I_{p\times p} \\ 0_{(n-p\times 1)\times p} \end{bmatrix}(u+\mu),$$

则变换后的系统为

$$\dot\eta_1 = f_{\mathrm{a}}(\eta_1,\ \varepsilon_1), \tag{10}$$

$$\dot\varepsilon_1 = f_{\mathrm{b}}(\eta_1,\ \varepsilon_1) + u + \mu. \tag{11}$$

3.1.2 设计滑动流形(Desgn of sliding manifold)

定义变量

$$s_1 = \varepsilon_1 - R(\eta_1). \tag{12}$$

取$s_1 = \varepsilon_1 - R(\eta_1) = 0$作为滑动流形. 当运动限制在该流形之上时, 服从$\varepsilon_1 = R(\eta_1)$, 代入式(10)得到降阶系统模型

$$\dot\eta_1 = f_{\mathrm{a}}(\eta_1, R(\eta_1)). \tag{13}$$

$\varepsilon_1 = R(\eta_1)$应作为一个控制输入进行设计, 使系统(13)渐近稳定, 且满足$R(0) = 0$.

3.1.3 接近态设计(Desgn of accessible state)

对式(12)求导得

$$\dot s_1 = f_{\mathrm{b}}(\eta_1,\varepsilon_1) + u + \mu - \frac{\partial R(\eta_1)}{\partial\eta_1}\dot\eta_1. \tag{14}$$

设计控制输入:

$$u = -f_{\mathrm{b}}(\eta_1,\varepsilon_1) + \frac{\partial R(\eta_1)}{\partial\eta_1}f_{\mathrm{a}}(\eta_1,\varepsilon_1) + \upsilon, \tag{15}$$

$$\upsilon = -h_1 s_1 - \hat b\,\mathrm{sgn}\, s_1, \tag{16}$$

$$\dot{\hat b} = r|s_1|, \tag{17}$$

其中: $h_1 > 0$, $\hat b$为b的估计值, r为给定参数. 为了消除抖振用饱和函数$\mathrm{sat}(s_1)$代替符号函数$\mathrm{sgn}\, s_1$, 其定义如下:

$$\mathrm{sat}(s_1) = \begin{cases} 1, & s_1 \geqslant \lambda, \\ -\dfrac{1}{\lambda^2}(s_1-\lambda)^2+1, & 0 \leqslant s_1 < \lambda, \\ \dfrac{1}{\lambda^2}(s_1+\lambda)^2-1, & -\lambda < s_1 < 0, \\ -1, & s_1 \leqslant -\lambda, \end{cases}$$

且$\lambda > 0$.

3.1.4 稳定性分析(Stability analysis)

把式(15)–(16)代入式(14)得

$$\dot s_1 = -h_1 s_1 - \hat b\,\mathrm{sat}(s_1) + \mu. \tag{18}$$

构造Lyapunov函数$V_1 = \dfrac{1}{2}s_1^{\mathrm T}s_1 + \dfrac{1}{2r}(b-\hat b)^2$, 对$V_1$求导得

$$\begin{aligned}\dot V_1 &= s_1^{\mathrm T}\dot s_1 - \frac{1}{r}(b-\hat b)\dot{\hat b} = \\ &-h_1 s_1^{\mathrm T}s_1 - s_1^{\mathrm T}\hat b\,\mathrm{sat}(s_1) + s_1^{\mathrm T}\mu - \|s_1\|(b-\hat b) \leqslant \\ &-h_1 s_1^{\mathrm T}s_1 - \|s_1\|\hat b + \|s_1\|b - \|s_1\|(b-\hat b) = \\ &-h_1 s_1^{\mathrm T}s_1 < 0.\end{aligned}$$

因为$V_1 > 0$, $\dot V_1 < 0$满足Lyapunov稳定性定理, 可见轨线向$s_1 = 0$收敛.

3.2 第2次降阶(Second order reduction)

为求$\varepsilon_1 = R(\eta_1)$使得闭环系统(13)渐近稳定, 对系统进行第2次降阶, 考虑系统(10)总能表示为

$$\dot\eta_1 = f_{\mathrm{a}}(\eta_1,\varepsilon_1) = f(\eta_1) + G(\eta_1)\varepsilon_1. \tag{19}$$

3.2.1 坐标变换(Coordinate transformation)

根据假设1, 作微分同胚变换$\phi(\eta_1)$, 变换关系是$\begin{bmatrix} \varepsilon_2 \\ \eta_2 \end{bmatrix} = \phi(\eta_1)$, 其中: $\varepsilon_2 \in \mathbb{R}^p$, $\eta_2 \in \mathbb{R}^{n-p\times 2}$, 且$\phi(\eta_1)$满足$\dfrac{\partial\phi(\eta_1)}{\partial\eta_1}G(\eta_1) = \begin{bmatrix} I_{p\times p} \\ 0_{(n-p\times 2)\times p} \end{bmatrix}$. 记$\dfrac{\partial\phi(\eta_1)}{\partial\eta_1}f(\eta_1) = \begin{bmatrix} f_{\mathrm{b}}(\eta_2,\varepsilon_2) \\ f_{\mathrm{a}}(\eta_2,\varepsilon_2) \end{bmatrix}$. 对$\phi(\eta_1)$求导得

$$\begin{bmatrix} \dot\varepsilon_2 \\ \dot\eta_2 \end{bmatrix} = \begin{bmatrix} f_{\mathrm{b}}(\eta_2,\varepsilon_2) \\ f_{\mathrm{a}}(\eta_2,\varepsilon_2) \end{bmatrix} + \begin{bmatrix} I_{p\times p} \\ 0_{(n-p\times 2)\times p} \end{bmatrix}\varepsilon_1,$$

则变换后的系统为

$$\dot\eta_2 = f_{\mathrm{a}}(\eta_2,\varepsilon_2), \tag{20}$$

$$\dot\varepsilon_2 = f_{\mathrm{b}}(\eta_2,\varepsilon_2) + \varepsilon_1. \tag{21}$$

3.2.2 设计滑动流形(Desgn of sliding manifold)

定义变量

$$s_2 = \varepsilon_2 - R(\eta_2). \tag{22}$$

取$s_2 = \varepsilon_2 - R(\eta_2) = 0$作为滑动流形, 当运动限制在该流形上时, 服从$\varepsilon_2 = R(\eta_2)$, 代入式(20)得到降阶系统模型

$$\dot\eta_2 = f_{\mathrm{a}}(\eta_2, R(\eta_2)). \tag{23}$$

同理, $\varepsilon_2 = R(\eta_2)$应作为一个控制输入进行设计, 使系统(23)渐近稳定, 且满足$R(0) = 0$.

3.2.3 接近态设计(Desgn of accessible state)

对式(22)求导得

$$\dot s_2 = f_{\mathrm{b}}(\eta_2,\varepsilon_2) + \varepsilon_1 - \frac{\partial R(\eta_2)}{\partial\eta_2}\dot\eta_2. \tag{24}$$

设计控制输入

$$\varepsilon_1 = -f_{\mathrm{b}}(\eta_2,\varepsilon_2) + \frac{\partial R(\eta_2)}{\partial\eta_2}f_{\mathrm{a}}(\eta_2,\varepsilon_2) - h_2 s_2, \tag{25}$$

其中$h_2 > 0$, 把控制输入式(25)代入式(24)可得$\dot s_2 = -h_2 s_2$, 则该系统为指数稳定.

3.3 第k次降阶(k-th order reduction)

为求$\varepsilon_{k-1} = R(\eta_{k-1})$使得闭环系统$\dot\eta_{k-1} =$

$f_{\rm a}(\eta_{k-1}, R(\eta_{k-1}))$渐近稳定, 对系统进行第$k$次降阶, 考虑系统总能表示为

$$\dot{\eta}_{k-1}=f_{\rm a}(\eta_{k-1},\varepsilon_{k-1})=f(\eta_{k-1})+G(\eta_{k-1})\varepsilon_{k-1}. \tag{26}$$

3.3.1 坐标变换(Coordinate transformation)

根据假设1, 作微分同胚变换$\phi(\eta_{k-1})$, 变换关系是$\begin{bmatrix}\varepsilon_k\\ \eta_k\end{bmatrix}=\phi(\eta_{k-1})$, 其中$\varepsilon_k\in\mathbb{R}^p$, $\eta_k\in\mathbb{R}^{n-p\times k}$, 且$\phi(\eta_{k-1})$满足$\dfrac{\partial\phi(\eta_{k-1})}{\partial\eta_{k-1}}G(\eta_{k-1})=\begin{bmatrix}I_{p\times p}\\ 0_{(n-p\times k)\times p}\end{bmatrix}$. 记$\dfrac{\partial\phi(\eta_{k-1})}{\partial\eta_{k-1}}f(\eta_{k-1})=\begin{bmatrix}f_{\rm b}(\eta_k,\varepsilon_k)\\ f_{\rm a}(\eta_k,\varepsilon_k)\end{bmatrix}$, 对$\phi(\eta_{k-1})$求导得

$$\begin{bmatrix}\dot{\varepsilon}_k\\ \dot{\eta}_k\end{bmatrix}=\begin{bmatrix}f_{\rm b}(\eta_k,\varepsilon_k)\\ f_{\rm a}(\eta_k,\varepsilon_k)\end{bmatrix}+\begin{bmatrix}I_{p\times p}\\ 0_{(n-p\times k)\times p}\end{bmatrix}\varepsilon_{k-1},$$

则变换后的系统为

$$\dot{\eta}_k=f_{\rm a}(\eta_k,\varepsilon_k), \tag{27}$$

$$\dot{\varepsilon}_k=f_{\rm b}(\eta_k,\varepsilon_k)+\varepsilon_{k-1}. \tag{28}$$

3.3.2 设计滑动流形(Desgn of sliding manifold)

定义变量

$$s_k=\varepsilon_k-R(\eta_k). \tag{29}$$

取$s_k=\varepsilon_k-R(\eta_k)=0$作为滑动流形, 当运动限制在该流形上时, 服从$\varepsilon_k=R(\eta_k)$, 代入式(27)得降阶系统模型:

$$\dot{\eta}_k=f_{\rm a}(\eta_k,R(\eta_k)). \tag{30}$$

同理, $\varepsilon_k=R(\eta_k)$应作为一个控制输入进行设计, 使得系统(30)渐近稳定, 且满足$R(0)=0$.

3.3.3 接近态设计(Desgn of accessible state)

对式(29)求导得

$$\dot{s}_k=f_{\rm b}(\eta_k,\varepsilon_k)+\varepsilon_{k-1}-\frac{\partial R(\eta_k)}{\partial\eta_k}\dot{\eta}_k. \tag{31}$$

设计控制输入

$$\varepsilon_{k-1}=-f_{\rm b}(\eta_k,\varepsilon_k)+\frac{\partial R(\eta_k)}{\partial\eta_k}f_{\rm a}(\eta_k,\varepsilon_k)-h_ks_k, \tag{32}$$

其中$h_k>0$, 把控制输入式(32)代入式(31)可得$\dot{s}_k=-h_ks_k$, 则该系统为指数稳定.

3.4 滑模降阶控制器设计分析(Sliding-mode reduced order controller design analysis)

引理 1(多输入滑模降阶可降至最低阶数引理) 在满足假设1的条件下, 对于一个n阶p输入系统: 1) 若p能被n整除, 可作$\dfrac{n-p}{p}$次降阶, 则系统最低可降至p阶; 2) 若p未能被n整除, 其余数为q, 可作$\dfrac{n-q}{p}$次降阶, 则系统最低可降至q阶.

证 由多输入滑模降阶理论推导规律可得: 对于一个n阶p输入系统, 做一次微分同胚变换可使系统降p阶. 1) 若p能被n整除, 当系统作$\dfrac{n-p}{p}$次降阶后, 所得系统阶数为

$$r=n-\frac{(n-p)}{p}\cdot p=p.$$

若系统作了$\dfrac{n-p}{p}+1$次降阶, 则所得系统阶数为

$$r=n-(\frac{n-p}{p}+1)\cdot p=0,$$

因此这种情况不可能出现, 所以当p能被n整除时, 系统最多可作$\dfrac{n-p}{p}$次降阶, 最低可降至p阶.

同理可得, 当p未能被n整除, 且余数为q时, 则系统最多可作$\dfrac{n-q}{p}$次降阶, 最低可降至q阶.

1) 若p能被n整除, 当$k=\dfrac{n-p}{p}$时, 即系统已进行了$\dfrac{n-p}{p}$次降阶, 记$\omega=\dfrac{n-p}{p}$, 则

$$\dot{\eta}_\omega=f_{\rm a}(\eta_\omega,\varepsilon_\omega), \tag{33}$$

其中: $\varepsilon_\omega\in\mathbb{R}^p$, $\eta_\omega\in\mathbb{R}^p$. 由于系统(33)是低阶系统, 容易得到控制输入$\varepsilon_\omega=R(\eta_\omega)$, 使得闭环系统$\dot{\eta}_\omega=f_{\rm a}(\eta_\omega,R(\eta_\omega))$渐近稳定, 为此容易得到当$k=\dfrac{n-2p}{p}$时$2p$阶系统的控制输入

$$\varepsilon_{\frac{n-2p}{p}}=-f_{\rm b}(\eta_\omega,\varepsilon_\omega)+\frac{\partial R(\eta_\omega)}{\partial\eta_\omega}f_{\rm a}(\eta_\omega,\varepsilon_\omega)-h_\omega s_\omega,$$

其中$h_\omega>0$. 以此类推, 可以求得$n-p$阶系统的控制输入$\varepsilon_1=R(\eta_1)$, 则初始系统的控制输入u可由式(15)–(17)得到.

2) 若p未能被n整除, 当$k=\dfrac{n-q}{p}$时, 即系统已进行了$\dfrac{n-q}{p}$次降阶, 基本分析同上.

因此，可以得出以下结论:

定理 1 对于满足假设1和假设2的系统(1), 利用本文的滑模降阶理论, 所得的非线性控制输入(15)–(17)可确保闭环系统所有信号全局有界, 且可以完全抑制匹配的外部干扰输入和削弱抖振, 使闭环系统状态变量渐近稳定.

4 数值算例(Numerical example)

为验证本文所提出的滑模降阶控制器设计方法的有效性, 对非线性直升机系统(1)做降阶处理并设计控制器. 系统(1)为九阶系统, 由引理1可知该系统可进行3次降阶得到一个3阶系统.

第1次降阶: 为方便寻找微分同胚变换函数, 将$G(x)$中非线性项线性化, 由式(9)得, 并记

$$B_{71}=b_{71}(x)=\frac{K_{\rm r}T_{\rm or}}{I_\phi\sin(x_1+\bar\psi)T_{\rm pr}}\simeq\frac{K_{\rm r}T_{\rm or}}{I_\phi T_{\rm pr}\sin\bar\psi}.$$

取微分同胚变换

$$\begin{bmatrix}\varepsilon_1\\ \eta_1\end{bmatrix}=\phi(x)=\begin{bmatrix}T_1^2x_4\\ T_2^2x_8\\ x_1\\ x_2\\ x_3\\ x_5\\ x_6\\ -T_1^2x_4+\dfrac{1}{B_{71}}x_7\\ -T_1^2x_4+\dfrac{1}{b_{79}}x_9\end{bmatrix}, \tag{34}$$

其中: $\varepsilon_1\in\mathbb{R}^2$, $\eta_1\in\mathbb{R}^7$, 可得$\dfrac{\partial\phi(x)}{\partial x}G(x)=[I_{2\times2}\ \ 0_{7\times2}]^{\rm T}$, 则假设1成立. 则

$$\frac{\partial\phi(x)}{\partial x}f(x)=\begin{bmatrix}f_{\rm b}(\eta_1,\varepsilon_1)\\ f_{\rm a}(\eta_1,\varepsilon_1)\end{bmatrix}=\begin{bmatrix}-x_2-2T_1x_4\\ -x_6-2T_2x_8\\ x_3\\ x_4\\ \alpha(x_1,x_2,x_3,x_7)\\ x_7\\ x_8\\ x_2+2T_1x_4+\dfrac{1}{B_{71}}\beta(x_1,x_6,x_7,x_9)\\ x_2+2T_1x_4-\dfrac{1}{b_{79}T_{\rm pr}}x_9\end{bmatrix}.$$

由式(10)–(11)可得变换后系统为

$$\dot\eta_1=f_{\rm a}(\eta_1,\varepsilon_1)=\begin{bmatrix}x_3\\ x_4\\ \alpha(x_1,x_2,x_3,x_7)\\ x_7\\ x_8\\ x_2+2T_1x_4+\dfrac{1}{B_{71}}\beta(x_6,x_7,x_9)\\ x_2+2T_1x_4-\dfrac{1}{b_{79}T_{\rm pr}}x_9\end{bmatrix}, \tag{35}$$

$$\dot\varepsilon_1=f_{\rm b}(\eta_1,\varepsilon_1)+u+\mu=\begin{bmatrix}-x_2-2T_1x_4\\ -x_6-2T_2x_8\end{bmatrix}+u+\mu. \tag{36}$$

第2次降阶: 系统(35)可写为

$$\dot\eta_1=f_{\rm a}(\eta_1,\varepsilon_1)=f(\eta_1)+G(\eta_1)\varepsilon_1=$$
$$\begin{bmatrix}\eta_{13}\\ 0\\ \alpha_1(\eta_{11},\eta_{12},\eta_{13},\eta_{16})\\ B_{71}\eta_{16}\\ 0\\ \eta_{12}+\beta_1(\eta_{15},\eta_{16},\eta_{17})\\ \eta_{12}-\dfrac{1}{T_{\rm pr}}\eta_{17}\end{bmatrix}+\begin{bmatrix}0&0\\ \dfrac{1}{T_1^2}&0\\ A_1&0\\ B_{71}&0\\ 0&\dfrac{1}{T_2^2}\\ A_2&0\\ A_3&0\end{bmatrix}\varepsilon_1, \tag{37}$$

其中:

$$x_4=\frac{1}{T_1^2}\varepsilon_{11},\ x_8=\frac{1}{T_2^2}\varepsilon_{12},\ x_1=\eta_{11},$$
$$x_2=\eta_{12},\ x_3=\eta_{13},\ x_5=\eta_{14},\ x_6=\eta_{15},$$
$$x_7=B_{71}\eta_{16}+B_{71}\varepsilon_{11},\ x_9=b_{79}\eta_{17}+b_{79}\varepsilon_{11},$$
$$\alpha_1(\eta_{11},\eta_{12},\eta_{13},\eta_{16})=$$
$$\frac{1}{I_\psi}[a_1(\eta_{12}+\bar u_{\rm d1})^2-B_\psi\eta_{13}+b_1(\eta_{12}+\bar u_{\rm d1})-$$
$$h_{\rm g}\sin(\bar\psi+\theta)-h_{\rm g}\cos(\bar\psi+\theta)\eta_{11}-$$
$$K_{\rm G}B_{71}\cos(\eta_{11}+\bar\psi)(\eta_{12}+\bar u_{\rm d1})\eta_{16}],$$
$$\beta_1(\eta_{15},\eta_{16},\eta_{17})=$$
$$\frac{1}{I_\phi B_{71}\sin\bar\psi}[a_2(\eta_{15}+\bar u_{\rm d2})^2+b_2(\eta_{15}+\bar u_{\rm d2})-$$
$$B_\phi B_{71}\eta_{16}+\frac{K_{\rm r}T_{\rm or}b_{79}}{T_{\rm pr}}\eta_{17}+K_{\rm r}\bar u_{\rm d1}].$$

由式(8)得

$$A_1=-K_GB_{71}\cos(\eta_{11}+\bar\psi)(\eta_{12}+\bar u_{\rm d1})\simeq-K_GB_{71}\bar u_{\rm d1}\cos(\bar\psi)=0,$$
$$A_2=\frac{2}{T_1}+\frac{-B_\phi B_{71}+\dfrac{K_{\rm r}T_{\rm or}b_{79}}{T_{\rm pr}}}{I_\phi B_{71}\sin\bar\psi},$$
$$A_3=\frac{2}{T_1}-\frac{1}{T_{\rm pr}}.$$

取微分同胚变换

$$\begin{bmatrix}\varepsilon_2\\ \eta_2\end{bmatrix}=\phi(\eta_1)=\begin{bmatrix}T_1^2\eta_{12}\\ T_2^2\eta_{15}\\ \eta_{11}\\ \eta_{13}\\ -T_1^2\eta_{12}+\dfrac{1}{B_{71}}\eta_{14}\\ -T_1^2\eta_{12}+\dfrac{1}{A_2}\eta_{16}\\ -T_1^2\eta_{12}+\dfrac{1}{A_3}\eta_{17}\end{bmatrix}, \tag{38}$$

其中: $\varepsilon_2\in\mathbb{R}^2$, $\eta_2\in\mathbb{R}^5$. 可得

$$\frac{\partial\phi(\eta_1)}{\partial\eta_1}G(\eta_1)=[I_{2\times2}\ \ 0_{5\times2}]^{\rm T},$$

则假设1成立. 则

$$\frac{\partial\phi(\eta_1)}{\partial\eta_1}f(\eta_1)=\begin{bmatrix}f_{\mathrm{b}}(\eta_2,\varepsilon_2)\\ f_{\mathrm{a}}(\eta_2,\varepsilon_2)\end{bmatrix}=\begin{bmatrix}0\\0\\ \eta_{13}\\ \alpha_1(\eta_{11},\eta_{12},\eta_{13},\eta_{16})\\ \eta_{16}\\ \frac{1}{A_2}\eta_{12}+\frac{1}{A_2}\beta_1(\eta_{15},\eta_{16},\eta_{17})\\ \frac{1}{A_3}\eta_{12}-\frac{1}{A_3T_{\mathrm{pr}}}\eta_{17}\end{bmatrix}.$$

由式(20)−(21)可得变换后系统为

$$\dot\eta_2=f_{\mathrm{a}}(\eta_2,\varepsilon_2)=\begin{bmatrix}\eta_{13}\\ \alpha_1(\eta_{11},\eta_{12},\eta_{13},\eta_{16})\\ \eta_{16}\\ \frac{1}{A_2}\eta_{12}+\frac{1}{A_2}\beta_1(\eta_{15},\eta_{16},\eta_{17})\\ \frac{1}{A_3}\eta_{12}-\frac{1}{A_3T_{\mathrm{pr}}}\eta_{17}\end{bmatrix},\tag{39}$$

$$\dot\varepsilon_2=f_{\mathrm{b}}(\eta_2,\varepsilon_2)+\varepsilon_1=\begin{bmatrix}0\\0\end{bmatrix}+\varepsilon_1,\tag{40}$$

其中$\varepsilon_1=R(\eta_1)$作为一个控制输入进行设计.

第3次降阶: 由式(2)(5)(8)得

$$\alpha_1(\eta_{11},\eta_{12},\eta_{13},\eta_{16})\simeq\alpha_1(\eta_{11},\eta_{12},\eta_{13})=\frac{1}{I_\psi}[(2a_1\bar u_{\mathrm{d1}}+b_1)\eta_{12}-B_\psi\eta_{13}-h_{\mathrm{g}}\cos(\bar\psi+\theta)\eta_{11}].$$

由式(3)(7)得

$$\beta_1(\eta_{15},\eta_{16},\eta_{17})=\frac{1}{I_\phi B_{71}\sin\bar\psi}[(2a_2\bar u_{\mathrm{d2}}+b_2)\eta_{15}-B_\phi B_{71}\eta_{16}+\frac{K_{\mathrm{r}}T_{\mathrm{or}}b_{79}}{T_{\mathrm{pr}}}\eta_{17}].$$

系统(39)可写为

$$\dot\eta_2=f_{\mathrm{a}}(\eta_2,\varepsilon_2)=f(\eta_2)+G(\eta_2)\varepsilon_2=\begin{bmatrix}\eta_{22}\\ \alpha_2(\eta_{21},\eta_{22})\\ A_2\eta_{24}\\ \beta_2(\eta_{24},\eta_{25})\\ -\frac{1}{T_{\mathrm{pr}}}\eta_{25}\end{bmatrix}+\begin{bmatrix}0&0\\A_4&0\\A_2&0\\A_5&A_6\\A_7&0\end{bmatrix}\varepsilon_2,\tag{41}$$

其中:

$$\eta_{12}=\frac{1}{T_1^2}\varepsilon_{21},\ \eta_{15}=\frac{1}{T_2^2}\varepsilon_{22},\ \eta_{11}=\eta_{21},$$
$$\eta_{13}=\eta_{22},\ \eta_{14}=B_{71}\eta_{23}+B_{71}\varepsilon_{21},$$
$$\eta_{16}=A_2\eta_{24}+A_2\varepsilon_{21},\ \eta_{17}=A_3\eta_{25}+A_3\varepsilon_{21},$$
$$\alpha_2(\eta_{21},\eta_{22})=\frac{1}{I_\psi}[-B_\psi\eta_{22}-h_{\mathrm{g}}\cos(\bar\psi+\theta)\eta_{21}],$$
$$\beta_2(\eta_{24},\eta_{25})=\frac{1}{A_2I_\phi B_{71}\sin\bar\psi}(-B_\phi B_{71}A_2\eta_{24}+\frac{K_{\mathrm{r}}T_{\mathrm{or}}b_{79}A_3}{T_{\mathrm{pr}}}\eta_{25}),$$
$$A_4=\frac{2a_1\bar u_{\mathrm{d1}}+b_1}{I_\psi T_1^2},$$
$$A_5=\frac{1}{A_2T_1^2}+\frac{-B_\phi B_{71}A_2+\frac{K_{\mathrm{r}}T_{\mathrm{or}}b_{79}A_3}{T_{\mathrm{pr}}}}{A_2I_\phi B_{71}\sin\bar\psi},$$
$$A_6=\frac{2a_2\bar u_{\mathrm{d2}}+b_2}{A_2I_\phi B_{71}T_2^2\sin\bar\psi},\ A_7=\frac{1}{A_3T_1^2}-\frac{1}{T_{\mathrm{pr}}}.$$

取微分同胚变换

$$\begin{bmatrix}\varepsilon_3\\ \eta_3\end{bmatrix}=\phi(\eta_2)=\begin{bmatrix}\frac{1}{A_2}\eta_{23}\\ -\frac{A_5}{A_2A_6}\eta_{23}+\frac{1}{A_6}\eta_{24}\\ \eta_{21}\\ \frac{1}{A_4}\eta_{22}-\frac{1}{A_2}\eta_{23}\\ -\frac{1}{A_2}\eta_{23}+\frac{1}{A_7}\eta_{25}\end{bmatrix},\tag{42}$$

其中: $\varepsilon_3\in\mathbb{R}^2$, $\eta_3\in\mathbb{R}^3$. 可得$\frac{\partial\phi(\eta_2)}{\partial\eta_2}G(\eta_2)=[I_{2\times2}\ 0_{3\times2}]^{\mathrm{T}}$, 则假设1成立. 则

$$\frac{\partial\phi(\eta_2)}{\partial\eta_2}f(\eta_2)=\begin{bmatrix}f_{\mathrm{b}}(\eta_3,\varepsilon_3)\\ f_{\mathrm{a}}(\eta_3,\varepsilon_3)\end{bmatrix}=\begin{bmatrix}\eta_{24}\\ -\frac{A_5}{A_6}\eta_{24}+\frac{1}{A_6}\beta_2(\eta_{24},\eta_{25})\\ \eta_{22}\\ \frac{1}{A_4}\alpha_2(\eta_{21},\eta_{22})-\eta_{24}\\ -\eta_{24}-\frac{1}{A_7T_{\mathrm{pr}}}\eta_{25}\end{bmatrix}.$$

变换后可得

$$\dot\eta_3=f_{\mathrm{a}}(\eta_3,\varepsilon_3)=\begin{bmatrix}\eta_{22}\\ \frac{1}{A_4}\alpha_2(\eta_{21},\eta_{22})-\eta_{24}\\ -\eta_{24}-\frac{1}{A_7T_{\mathrm{pr}}}\eta_{25}\end{bmatrix},\tag{43}$$

$$\dot\varepsilon_3=f_{\mathrm{b}}(\eta_3,\varepsilon_3)+\varepsilon_2=\begin{bmatrix}\eta_{24}\\ -\frac{A_5}{A_6}\eta_{24}+\frac{1}{A_6}\beta_2(\eta_{24},\eta_{25})\end{bmatrix}+\varepsilon_2,\tag{44}$$

其中$\varepsilon_2=R(\eta_2)$作为一个控制输入进行设计.

为求控制输入$\varepsilon_3=R(\eta_3)$使得3阶闭环系统$\dot\eta_3=f_{\mathrm{a}}(\eta_3,R(\eta_3))$渐近稳定, 系统(43)可写为

$$\dot\eta_3=f_{\mathrm{a}}(\eta_3,\varepsilon_3)=$$

$$\begin{bmatrix} A_4\eta_{32}+A_4\varepsilon_{31} \\ \alpha_3(\eta_{31},\eta_{32})-(\dfrac{B_\psi}{I_\psi}+A_5)\varepsilon_{31}-A_6\varepsilon_{32} \\ -\dfrac{1}{T_{\rm pr}}\eta_{33}-(A_5+\dfrac{1}{T_{\rm pr}})\varepsilon_{31}-A_6\varepsilon_{32} \end{bmatrix}, \tag{45}$$

其中:

$\eta_{23}=A_2\varepsilon_{31},\ \eta_{24}=A_5\varepsilon_{31}+A_6\varepsilon_{32},\ \eta_{21}=\eta_{31},$

$\eta_{22}=A_4\eta_{32}+A_4\varepsilon_{31},\ \eta_{25}=A_7\eta_{33}+A_7\varepsilon_{31},$

$\alpha_3(\eta_{31},\eta_{32})=\dfrac{1}{A_4I_\psi}[-B_\psi A_4\eta_{32}-h_{\rm g}\cos(\bar\psi+\theta)\eta_{31}],$

取$\varepsilon_3=[\varepsilon_{31}\ \ \varepsilon_{32}]^{\rm T}=[\eta_{33}\ \ 0]^{\rm T}$, 则系统(45)闭环指数稳定. 则$R(\eta_3)=[\eta_{33}\ \ 0]^{\rm T}$, 取$h_3=1$, 所以5阶系统控制输入

$$\varepsilon_2=-f_{\rm b}(\eta_3,\varepsilon_3)+\frac{\partial R(\eta_3)}{\partial\eta_3}f_{\rm a}(\eta_3,\varepsilon_3)-h_3s_3=\begin{bmatrix}-0.091\eta_{23}-2\eta_{24}+0.1914\eta_{25}\\ 0.00091\eta_{23}+0.0212\eta_{24}-0.0095\eta_{25}\end{bmatrix}. \tag{46}$$

将式(46)代入系统(41)可得此闭环系统指数稳定, 则

$$R(\eta_2)=\begin{bmatrix}-0.091\eta_{23}-2\eta_{24}+0.1914\eta_{25}\\ 0.00091\eta_{23}+0.0212\eta_{24}-0.0095\eta_{25}\end{bmatrix}.$$

取$h_2=1$, 所以7阶系统控制输入由式(25)得

$$\varepsilon_1=\begin{bmatrix}-0.072\eta_{12}-0.24\eta_{14}-25.15\eta_{15}-0.072\eta_{16}-0.289\eta_{17}+43.314\eta_{15}^2\\ 0.00036\eta_{12}+0.0024\eta_{14}+0.2253\eta_{15}+0.000718\eta_{16}+0.0036\eta_{17}-0.457\eta_{15}^2\end{bmatrix}. \tag{47}$$

将式(47)代入系统(37)可得此闭环系统指数稳定, 则

$$R(\eta_1)=\begin{bmatrix}-0.072\eta_{12}-0.24\eta_{14}-25.15\eta_{15}-0.072\eta_{16}-0.289\eta_{17}+43.314\eta_{15}^2\\ 0.00036\eta_{12}+0.0024\eta_{14}+0.2253\eta_{15}+0.000718\eta_{16}+0.0036\eta_{17}-0.457\eta_{15}^2\end{bmatrix}.$$

取$h_1=1$, 那么由式(15)–(17)可得初始系统的控制输入为

$$u=\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}0.567x_2+0.04941x_4-0.24x_5-25.15x_6-0.43x_7-25.15x_8-0.0837x_9+86.628x_6x_8+43.314x_6^2-0.072\gamma(x_{679})\\ 0.004678x_2+0.0012x_4+0.0024x_5+1.2253x_6+0.0043x_7+0.5853x_8+0.001x_9-0.914x_6x_8-0.457x_6^2+0.000718\gamma(x_{679})\end{bmatrix}-\hat b{\rm sat}(s_1),\ \dot{\hat b}=r\|s_1\|, \tag{48}$$

其中:

$$\gamma(x_{679})=1058.2(-0.4498x_6^2+0.2612x_6-0.003x_7+0.000945x_9),$$

$$s_1=\begin{bmatrix}s_{11}\\s_{12}\end{bmatrix}=\begin{bmatrix}0.072x_2+0.00639x_4+0.24x_5+25.15x_6+0.19x_7+0.0881x_9-43.314x_6^2\\ -0.00036x_2+0.00004138x_4-0.0024x_5-0.2253x_6-0.002x_7+0.04x_8-0.0011x_9+0.457x_6^2\end{bmatrix}.$$

在数值仿真中，直升机模型的参数和初始条件取为

$I_\psi=0.0028,\ B_\psi=0.0033,\ h_{\rm g}=0.0726,$

$\theta=-7^\circ,\ a_1=0.0296,\ b_1=0.0588,$

$T_1=0.1,\ K_{\rm G}=0.013,\ I_\phi=0.0025,$

$B_\phi=0.003,\ a_2=-0.4498,\ T_{\rm pr}=20,$

$b_2=-0.0798,\ T_2=0.2,\ K_{\rm r}=0.063,$

$T_{\rm or}=0.3,\ \bar\psi=\dfrac{\pi}{2},\ \bar\phi=\dfrac{5\pi}{9},\ \bar u_{\rm d1}=0.856,$

$u_{\rm d2}=-0.379,\ x_0=[0\ \ 0\ \ 0\ \ 0\ \ \dfrac{-5\pi}{18}\ \ 0\ \ 0\ \ 0\ \ 0].$

由此可得

$\eta_{10}=[0\ \ 0\ \ 0\ \ \dfrac{-5\pi}{18}\ \ 0\ \ 0\ \ 0],$

$\eta_{20}=[0\ \ 0\ \ \dfrac{-5\pi}{6.804}\ \ 0\ \ 0],$

$\eta_{30}=[0\ \ \dfrac{5\pi}{150.23}\ \ \dfrac{5\pi}{150.23}],$

$\hat b_0=[0.1],\ r=10^7,$

$\lambda=0.005$和外部扰动$\mu=[0.05\sin t\ \ 0.05\cos t]$. 仿真可得如下曲线图, 展示了闭环系统状态的动态轨迹, 从中可见闭环系统的良好稳态性能. 图1表示初始系统的状态轨线, 可以看到外部扰动被完全的抑制; 图2表示初始系统的控制输入和滑动流形曲线, 从图中可到看到控制输入抖振已被

一定程度的削弱。

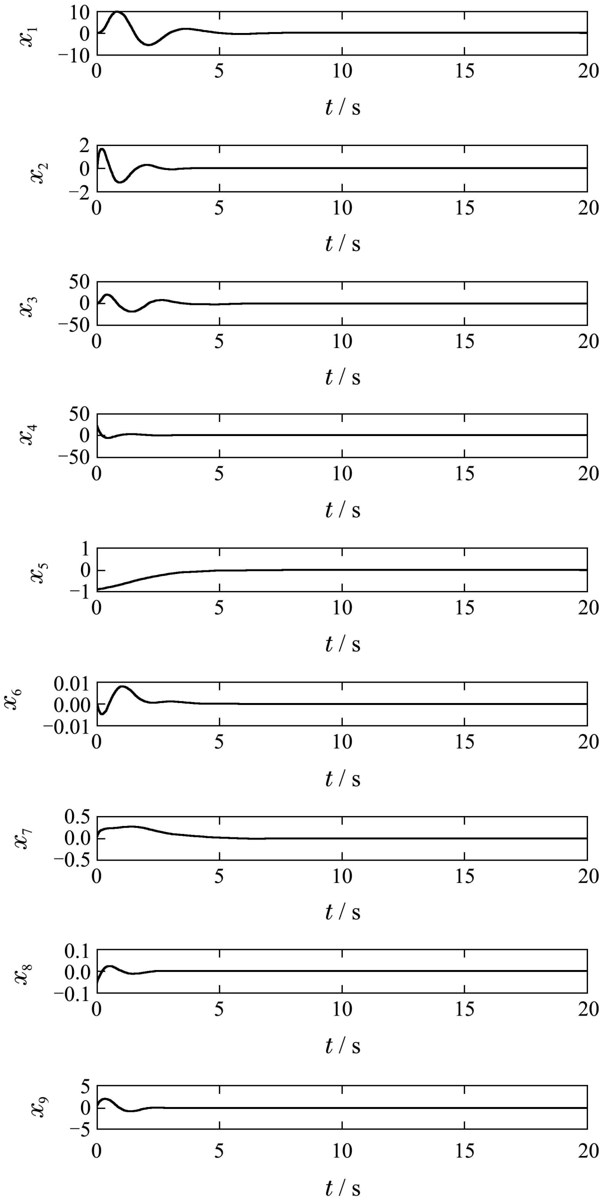

图 1 初始系统的状态轨线

Fig. 1 State trajectories of the original system

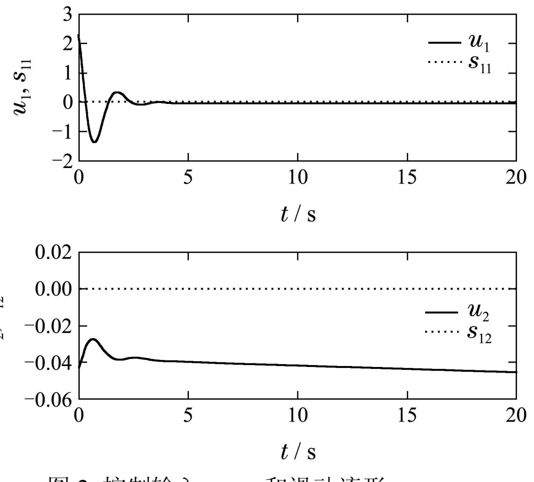

图 2 控制输入u_1, u_2和滑动流形s_{11}, s_{12}

Fig. 2 Control inputs u_1, u_2 and sliding manifolds s_{11}, s_{12}

5 结语(Conclusions)

本文基于滑模变结构控制理论, 采用了一种新的滑模降阶控制器设计方法, 针对CE150直升机非线性模型, 通过构造出3个微分同胚变换函数, 将初始系统降至3阶系统, 并利用滑模降阶的映射关系反推出初始系统的控制输入, 通过仿真结果表明所提出控制器设计方法有效, 且具有鲁棒性. 与其他模型降阶方法相比, 本文方法具有如下优点:

1) 适用于非线性控制系统模型降阶;

2) 比起传统的降阶方法, 本方法不损失系统任何信息;

3) 滑动流形的设计具有一般性, 适用范围广.

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