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互联系统的分布输出反馈自适应动态面控制

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摘要: 本文所提方案具有如下特点: 1) 通过使用初始化技术, 给出了互联系统中各个子系统跟踪误差的$\mathcal{L}_\infty$性能指标; 2) 采用动态面控制方法, 避免了反推法带来的"微分爆炸"问题; 3) 通过估计未知参数向量范数, 极大减轻了控制系统的计算负担; 4) 引入了一种高增益观测器, 保证了互联系统中各个子系统跟踪误差可以收敛到任意小, 仿真结果验证了所提方案的有效性.

关键词: 输出反馈; 高增益观测器; 动态面控制

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Output-feedback adaptive dynamic surface decentralized control for interconnected systems

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Abstract: The proposed scheme has features: 1) by using dynamic surface control technique, it eliminates the problem of differentiation explosion which is inherent to the backstepping design; 2) by estimating the norm of unknown parameter vectors, it reduces the computational burden greatly; 3) high gain observers are introduced, which forces the tracking error of each subsystem to converge to an arbitrarily small residual set. Simulation results are presented to illustrate the effectiveness of the proposed scheme.

Key words: output feedback; high-gain observers; dynamic surface control

1　引言(Introduction)

对于存在诸多不确定性和外部扰动的互联系统, 自适应分布控制策略是对其行之有效的控制方法. 但是由于互联系统中各个子系统间存在相互作用, 使得研究互联系统的控制问题变得非常困难. 因此, 与此问题相关的研究成果并不是很多. 文献[1]在假定各个子系统相对阶数不超过2的情况下, 对互联系统的分布自适应控制方法进行了研究. 近年来, 由于反推控制成为控制领域的研究热点, 也有学者采用反推法进行分布自适应控制, 如文献[2], 此方法与文献[3]中的方法相比, 改善了系统的过渡过程. 文献[4]最先提出了基于反推方法的分布自适应控制, 且对互联系统的各个子系统的相对阶无任何要求. 自此以后, 很多基于反推方法的分布自适应控制的研究成果相继出现. 比较有代表性的为文献[5]的研究成果, 但是子系统间相互作用的函数必须确切知道且满足全Lipschitz条件. 文献[6–7]解决了上述问题, 但无法保证互联系统中各个子系统跟踪误差可以收敛到任意小, 同时也无法克服反推法反复对所设计的虚拟控制信号微分而产生的"微分爆炸"问题.

另外, 磁滞环节也是最重要的非平滑非线性环节之一, 广泛存在于实际物理系统和装置中[8–11]. 当一个装置受到磁滞非线性环节影响时, 控制系统将产生诸如准确性降低、系统发生抖震甚至变得不稳定等严重问题[12]. 工程应用中, 磁滞现象的建模与控制是一个极具挑战性的问题. 近年来, 随着含有磁滞现象的智能材料的广泛使用, 该问题亦受到研究人员的广泛重视[13–16].

本文采用分布动态面控制方法来实现对非线性互联系统的分布跟踪控制, 通过使用初始化技术, 给出了互联系统中各个子系统跟踪误差的$\mathcal{L}_\infty$性能指标; 避免了采用反推法而带来的"微分爆炸"问题: 与文献[7, 17]针对非线性互联系统使用的普通K观测器相比, 本文所设计的高增益K观测器, 保证了互联系统

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中各个子系统跟踪误差可以收敛到任意小; 且通过对未知参数向量范数的估计来代替对整个未知参数向量本身的估计, 极大减轻了控制系统的计算负担.

2 系统描述(Problem statement)

考虑如下由N个子系统组成的互联系统:

$$\begin{cases}\dot{x}_i = A_i x_i + \varPhi_i(y_i)\theta_i + b_i w_i + \\ \qquad \sum\limits_{j=1}^{N} f_{ij}(t, y_j) + d_i(t), \\ y_i = x_{i,1},\ i = 1, \cdots, N,\end{cases} \tag{1}$$

其中:

$$A_i = \begin{bmatrix} 0 \cdots & I_{n_i-1} \\ \vdots & \vdots \\ 0 \cdots & 0 \end{bmatrix},\ \varPhi_i(y_i) = \begin{bmatrix} \varPhi_{i,1}(y_i) \\ \vdots \\ \varPhi_{i,n_i}(y_i) \end{bmatrix},$$

$$b_i = \begin{bmatrix} 0_{\rho_i-1\times 1} \\ b_{i,n_i-\rho_i} \\ \vdots \\ b_{i,0} \end{bmatrix},\ f_{ij}(y_i) = \begin{bmatrix} f_{ij,1}(y_i) \\ \vdots \\ f_{ij,n_i}(y_i) \end{bmatrix},$$

$$d_i(t) = \begin{bmatrix} d_{i,1}(t) \\ \vdots \\ d_{i,n_i}(t) \end{bmatrix},$$

$x_i = (x_{i1}, x_{i2}, \cdots, x_{in_i}) \in \mathbb{R}^{n_i}$是系统的状态向量, $\varPhi_i(y_i) \in \mathbb{R}^{n_i\times r_i}$为已知的光滑函数; $\theta_i \in \mathbb{R}^{r_i}$, $b_i \in \mathbb{R}^{n_i-\rho_i+1}$为未知常数向量, ρ_i为第i个子系统的相对阶数. 当$i \neq j$时, $f_{ij}(y_i) \in \mathbb{R}^{n_i}$表示第$j$个子系统对第$i$个子系统的非线性作用; 当$i = j$时, $f_{ij}(y_i) \in \mathbb{R}^{n_i}$表示第$i$个子系统的未建模部分; $d_i(t) \in \mathbb{R}^{n_i}$表示外部扰动; y_i和u_i为第i个子系统的输出和输入; $w_i \in \mathbb{R}$为第i个子系统的磁滞输出. 该磁滞现象用类间隙磁滞模型来描述. 其数学描述如下:

$$\frac{\mathrm{d}w_i}{\mathrm{d}t} = \alpha\left|\frac{\mathrm{d}u_i}{\mathrm{d}t}\right|(\lambda u_i - w_i) + \psi\frac{\mathrm{d}u_i}{\mathrm{d}t}, \tag{2}$$

其中: α, ψ为未知常数, λ为正常数, 且满足$\lambda > \psi$. 由$\dot{u}$的分段单调性, 对式(2)求解可得

$$w_i(t) = \lambda u_i(t) + d(u_i), \tag{3}$$

其中

$$\begin{aligned} &d_i(u) = \\ &(w_{i0} - \lambda u_{i0}) \exp\{-\alpha(u_i - u_{i0})\mathrm{sgn}\,\dot{u}_i\} + \\ &\exp\{-\alpha u_i \mathrm{sgn}\,\dot{u}_i\} \int_{u_0}^{u} (\psi - \lambda) \exp\{\alpha\xi \mathrm{sgn}\,\dot{u}_i\}\mathrm{d}\xi, \end{aligned}$$

从式(3)可知, 对于任何$u \in \mathbb{R}$, $d(u_i)$是有界的, 这里笔者假设$|d(u_i)| \leqslant D$. 将式(3)代入式(1)可得

$$\begin{aligned} &\dot{x}_i = A_i x_i + \varPhi_i(y_i)\theta_i + \beta_i u_i + b_i d(u_i) + \\ &\qquad \sum_{j=1}^{N} f_{ij}(t, y_j) + d_i(t), \\ &y_i = x_{i,1},\ i = 1, \cdots, N, \end{aligned} \tag{4}$$

其中$\beta_i = b_i\lambda$.

针对上述系统, 笔者做如下假设:

假设 1 对于任意i, $1 \leqslant i \leqslant N$, 多项式$N_i(s) := \beta_{i,n_i-\rho_i} s^{n_i-\rho_i} + \cdots + \beta_{i,1}s + \beta_{i,0}$是Hurwitz的, 其中: $\beta_{i,n_i-\rho_i}$的符号和相对阶数ρ_i已知; 不失一般性, 假定$\beta_{i,n_i-\rho_i} > 0$.

假设 2 未知非线性函数$f_{ij}(t, y_j)$满足

$$\|f_{ij}(t, y_j)\| \leqslant \bar{\kappa}_{ij}|\psi_j(y_j)|, \tag{5}$$

其中: $\|\cdot\|$表示Euclidean范数, $\bar{\kappa}_{ij}$为未知常数子系统间表示相互作用的强度, $\psi_j(y_j)$为已知非线性函数且至少ρ_i次可导.

假设 3 参考信号(或理想跟踪信号)$y_{\mathrm{r}i}$是二阶可导的, 并且$y_{\mathrm{r}}(0)$可以设置; $(y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}, \ddot{y}_{\mathrm{r}})^{\mathrm{T}}$属于一已知紧集.

控制目的是设计一个分散输出反馈自适应动态面控制方案, 使各个子系统的输出y_i能够跟踪参考信号$y_{\mathrm{r}i}$且保证闭环控制系统所有信号一致有界.

注 1 文献[21]指出, 很多实际的物理系统可以转换成式(1)的形式. 对于系统(1), 文献[22–23]采用反推控制方法来处理其他非平滑的未知非线性环节, 相应的必然会产生"微分爆炸"问题.

注 2 假设2表示第j个子系统对第i个子系统的非线性作用是有界的且可用一个关于y_j的高阶非线性函数来界定, 这样的假设是对文献[4,25]关于其线性有界条件的极大放宽, 且与文献[7,17]的假设一致, 因此假设2是合理的.

3 基于高增益观测器的输出反馈分布自适应动态面控制器设计(Adaptive dynamic surface controller design by using high-gain observer)

3.1 高增益状态观测器的构造(Construction of high-gain observer)

为了构造一个高增益观测器, 将式(4)写成如下的形式:

$$\begin{aligned} &\dot{x}_i = A_{i,0}x_i + q_i y_i + \varPhi_i(y_i)\theta_i + \beta_i u_i + b_i d(u_i) + \\ &\qquad \sum_{j=1}^{N} f_{ij}(t, y_j) + d_i(t), \\ &y_i = x_{i,1},\ i = 1, \cdots, N, \end{aligned} \tag{6}$$

$$A_{i,0} = \begin{bmatrix} -q_{i,1} & 1 & & \\ -q_{i,2} & & \ddots & \\ \vdots & & & 1 \\ -q_{i,n_i} & 0 & \cdots & 0 \end{bmatrix},\ q_i = \begin{bmatrix} q_{i,1} \\ q_{i,2} \\ \vdots \\ q_{i,n_i} \end{bmatrix}, \tag{7}$$

且可通过适当调整参数q_i使$A_{i,0}$是稳定的. 受到文献[20]的启发, 笔者设计高增益观测器来抑制系统中

的磁滞现象和各个子系统间的相互作用:

$$\dot{v}_{i,q} = k_i A_{i,0} v_{i,q} + \Psi_i^{-1} e_{n_i, n_i-\rho_i} u_i,\quad q = 0, 1, \cdots, n_i - \rho_i, \tag{8}$$

$$\dot{\xi}_{i,0} = k_i A_{i,0} \xi_{i,0} + k_i q_i y_i, \tag{9}$$

$$\dot{\Xi}_i = k_i A_{i,0} \Xi_i + \Psi_i^{-1} \Phi_i(y_i),\ i = 1, \cdots, N, \tag{10}$$

其中: $k_i \geqslant 1$为正设计参数, $e_{n_i, n_i-\rho_i}$为第i个子系统的第$n_i - \rho_i$个坐标向量, 且

$$\Psi_i = \mathrm{diag}\{1, k_i, k_i^2, \cdots, k_i^{n_i-1}\}, \tag{11}$$

则状态向量的估计值为如下形式:

$$\hat{x}_i = \Psi_i \xi_{i,0} + \sum_{j=0}^{n_i-\rho_i} \beta_{i,j} \Psi_i v_{i,j} + \Psi_i \Xi_i \theta_i, \tag{12}$$

定义估计误差为如下形式:

$$\epsilon_i = x_i - \hat{x}_i. \tag{13}$$

可以证明

$$\dot{\epsilon}_i = A_i \epsilon_i - k_i \Psi_i q_i \epsilon_{i,1} + b_i d(u_i) + \sum_{j=1}^{N} f_{ij}(t, y_j) + d_i(t), \tag{14}$$

其中$\epsilon_{i,1}$为向量ϵ_i的第1个元素.

3.2 分布自适应动态面控制器的设计(Decentralized adaptive dynamic surface controller design)

第1步$(i = 1, \cdots, N)$　定义第1组面误差

$$S_{i,1} = x_{i,1} - y_{\mathrm{r}i}, \tag{15}$$

其中$y_{\mathrm{r}i}$是参考信号. 对式(15)求导, 可得

$$\begin{aligned}\dot{S}_{i,1} = {} & k_i \xi_{i,(0,2)} + \sum_{j=0}^{n_i-\rho_i} \beta_{i,j} k_i v_{i,(j,2)} + (k_i \Xi_{i,2} + \Phi_{i,1}(y_i))\theta_i + \sum_{j=1}^{N} f_{ij,1}(t, y_j) + \epsilon_{i,2} + d_{i,1} - y_{\mathrm{r}i} = \\ & -l_{i,1} S_{i,1} + \beta_{i,n_i-\rho_i} k_i [v_{i,(n_i-\rho_i,2)} - \bar{v}_{i,(n_i-\rho_i,2)}] + \beta_{i,n_i-\rho_i} k_i \bar{v}_{i,(n_i-\rho_i,2)} + \\ & \beta_{i,n_i-\rho_i} \Theta_i^{\mathrm{T}} \omega_i + \epsilon_{i,2} + \sum_{j=1}^{N} f_{ij,1}(t, y_j) + d_{i,1} - y_{\mathrm{r}i},\end{aligned} \tag{16}$$

其中: $l_{i,1}$为正设计参数, $\bar{v}_{i,(n_i-\rho_i,2)}$为待设计的虚拟控制律,

$$\begin{aligned}&\Theta_i = [\frac{1}{\beta_{i,n_i-\rho_i}}, \frac{\beta_{i,0}}{\beta_{i,n_i-\rho_i}}, \cdots, \frac{\beta_{i,n_i-\rho_i-1}}{\beta_{i,n_i-\rho_i}}, \theta_i]^{\mathrm{T}} \\ &\omega_i = [k_i \xi_{i,(0,2)} - y_{\mathrm{r}i} + l_{i,1} S_{i,1}, k_i v_{i,(0,2)}, \cdots, k_i v_{i,(n_i-\rho_i-1,2)}, k_i \Xi_{i,2} + \Phi_{i,1}(y_i)].\end{aligned} \tag{17}$$

根据下述不等式:

$$S_{i,1} \Theta_i^{\mathrm{T}} \omega_i \leqslant \frac{\alpha_i S_{i,1}^2 \nu_i^* \omega_i^{\mathrm{T}} \omega_i}{2} + \frac{1}{2\alpha_i}, \tag{18}$$

其中: $\nu_i^* = \Theta_i^{\mathrm{T}} \Theta_i$, α_i为正设计参数, 可得

$$\begin{aligned}S_{i,1} \dot{S}_{i,1} \leqslant {} & S_{i,1}(-l_{i,1} S_{i,1} + \beta_{i,n_i-\rho_i} k_i (v_{i,(n_i-\rho_i,2)} - \bar{v}_{i,(n_i-\rho_i,2)}) + \beta_{i,n_i-\rho_i} k_i \bar{v}_{i,(n_i-\rho_i,2)} + \\ & \frac{\beta_{i,n_i-\rho_i} \alpha_i S_{i,1} \nu_i^* \omega_i^{\mathrm{T}} \omega_i}{2} + \epsilon_{i,2} + d_{i,1} + \sum_{j=1}^{N} f_{ij,1}(t, y_j)) + \frac{\beta_{i,n_i-\rho_i}}{2\alpha_i}.\end{aligned} \tag{19}$$

根据式(19), 虚拟控制律为

$$\bar{v}_{i,(n_i-\rho_i,2)} = -\frac{\alpha_i S_{i,1} \hat{\nu}_i \omega_i^{\mathrm{T}} \omega_i}{2k_i} - \frac{l_i^* S_{i,1} \phi(S_{i,1}) \psi_i^2(y_i)}{k_i}, \tag{20}$$

其中$\rho_i - 1$阶可导函数$\phi(\cdot)$定义为如下形式:

$$\phi(x) = \begin{cases} \dfrac{1}{x^2}, & |x| \geqslant \delta_i, \\ \dfrac{1}{(\delta_i^2 - x^2)^{\rho_i} + x^2}, & |x| < \delta_i, \end{cases} \tag{21}$$

$\hat{\nu}_i$为ν_i^*的估计值, 其调参律为

$$\dot{\hat{\nu}}_i = \gamma_{\nu_i} (\frac{\alpha_i S_{i,1}^2 \omega_i^{\mathrm{T}} \omega_i}{2} - \eta_i \hat{\nu}_i), \tag{22}$$

其中η_i为正设计参数. 令$\bar{v}_{i,(n_i-\rho_i,2)}$通过一阶低通滤波器, 得到新变量$z_{i,2}$:

$$\begin{aligned}&\tau_{i,2} \dot{z}_{i,2} + z_{i,2} = \bar{v}_{i,(n_i-\rho_i,2)}, \\ &z_{i,2}(0) = \bar{v}_{i,(n_i-\rho_i,2)}(0),\end{aligned} \tag{23}$$

$\tau_{i,2}$为一阶低通滤波器的时间常数.

第q步$(q = 2, \cdots, \rho_i - 1,\ i = 1, \cdots, N)$　定义第q组面误差:

$$S_{i,q} = v_{i,(n_i-\rho_i,q)} - z_{i,q}. \tag{24}$$

由式(24) 可知, $S_{i,q}$对时间的导数为

$$\begin{aligned}\dot{S}_{i,q} = {} & -k_i q_{i,q} v_{i,(n_i-\rho_i,1)} + k_i v_{i,(n_i-\rho_i,q+1)} - \dot{z}_{i,q} = \\ & -k_i q_{i,q} v_{i,(n_i-\rho_i,1)} + k_i (v_{i,(n_i-\rho_i,q+1)} - \bar{v}_{i,(n_i-\rho_i,q+1)}) + k_i \bar{v}_{i,(n_i-\rho_i,q+1)} - \dot{z}_{i,q}.\end{aligned} \tag{25}$$

由式(25), 虚拟控制律$\bar{v}_{i,(n_i-\rho_i,q+1)}$为

$$\bar{v}_{i,(n_i-\rho_i,q+1)} = \frac{-l_{i,q} S_{i,q} + k_i q_{i,q} v_{i,(n_i-\rho_i,1)} + \dot{z}_{i,q}}{k_i}, \tag{26}$$

其中$l_{i,q}(q = 2, \cdots, \rho_i - 1)$为正设计参数. 本文令$\bar{v}_{i,(n_i-\rho_i,q+1)}$通过一阶低通滤波器, 得到新变量得新变量$z_{i,q+1}$:

$$\begin{aligned}&\tau_{i,q+1} \dot{z}_{i,q+1} + z_{i,q+1} = \bar{v}_{i,(n_i-\rho_i,q+1)}, \\ &z_{i,q+1}(0) = \bar{v}_{i,(n_i-\rho_i,q+1)}(0),\end{aligned} \tag{27}$$

$\tau_{i,q+1}$为一阶低通滤波器的时间常数.

第ρ_i步$(i = 1, \cdots, N)$　定义第ρ_i组面误差

$$S_{i,\rho_i} = v_{i,(n_i-\rho_i,\rho_i)} - z_{i,\rho_i}, \tag{28}$$

对式(28)求导可得

$$\dot{S}_{i,\rho_i} = -k_i q_{i,\rho_i} v_{i,(n_i-\rho_i,1)} + k_i v_{i,(n_i-\rho_i,\rho_i+1)} + k_i^{1-\rho_i} u_i - \dot{z}_{i,\rho_i}, \tag{29}$$

因此, 实际控制律可设计为

$$u_i = k_i^{\rho_i-1}(-l_{i,\rho_i} S_{i,\rho_i} + k_i q_{i,\rho_i} v_{i,(n_i-\rho_i,1)} - k_i v_{i,(n_i-\rho_i,\rho_i+1)} + \dot{z}_{i,\rho_i}). \tag{30}$$

注 3 与文献[7,17]提出的反推控制相比, 自适应律只出现在设计的第1步, 且采用估计范数的形式替代估计未知参数的向量, 因此, 极大地减轻了系统的计算负担. 由于一阶低通滤波器的使用, 使得所设计的控制律变得很简单. 代价是系统的跟踪误差不能收敛到零.

4 系统稳定性分析(Systems stability analysis)

首先, 定义如下误差变量:

$$\bar{y}_{i,q} = z_{i,q} - \bar{v}_{i,(n_i-\rho_i,q)},\ q = 2,\cdots,\rho_i. \tag{31}$$

由式(20)(24)可知式(19)可变为

$$S_{i,1}\dot{S}_{i,1} \leqslant S_{i,1}(-l_{i,1}S_{i,1} + \beta_{i,n_i-\rho_i}k_i(S_{i,2} - \bar{y}_{i,2}) - \frac{\beta_{i,n_i-\rho_i}\alpha_i S_{i,1}\tilde{\nu}_i\omega_i^{\mathrm{T}}\omega_i}{2} + \epsilon_{i,2} - l_i^* S_{i,1}\phi(S_{i,1})\psi_i^2(y_i) + \frac{\beta_{i,n_i-\rho_i}}{2\alpha_i} + \sum_{j=1}^{N} f_{ij,1}(t,y_j) + d_{i,1}), \tag{32}$$

其中$\tilde{\nu}_i = \hat{\nu}_i - \nu_i^*$. 同理可得

$$\dot{S}_{i,q} = -l_{i,q}S_{i,q} + k_i S_{i,q+1} + k_i\bar{y}_{i,q+1}, \tag{33}$$

$$\dot{S}_{i,\rho_i} = -l_{i,\rho_i}S_{i,\rho_i}. \tag{34}$$

另一方面, 由式(31)–(34)可得到如下不等式:

$$|\dot{\bar{y}}_{i,2} + \frac{\bar{y}_{i,2}}{\tau_{i,2}}| \leqslant B_2(S_{i,1}, S_{i,2}, \bar{y}_{i,2}, \tilde{\nu}_i, y_{\mathrm{r}i}, \dot{y}_{\mathrm{r}i}, \ddot{y}_{\mathrm{r}i}, \epsilon_i), \tag{35}$$

$$\bar{y}_{i,q+1} + \frac{\bar{y}_{i,q+1}}{\tau_{i,q+1}} \leqslant B_{i,q+1}(S_{i,1}\cdots, S_{i,q+1}, \bar{y}_{i,2},\cdots, \bar{y}_{i,q+1}, \tilde{v}_i, \tilde{\nu}_i, y_{\mathrm{r}i}, \dot{y}_{\mathrm{r}i}, \ddot{y}_{\mathrm{r}i}, \epsilon_i), \tag{36}$$

其中$B_{i,q+1}(q=2,\cdots,\rho_i-1)$是连续函数. 关于闭环系统式(15)(24)(32)–(34), 令Lyapunov函数定义为如下形式:

$$V = \sum_{i=1}^{N} V_i, \tag{37}$$

且

$$V_i = \frac{1}{2}(\sum_{q=1}^{\rho_i} S_{i,q}^2 + \sum_{q=1}^{\rho_i}\bar{y}_{i,q+1}^2 + \frac{\beta_{i,n_i-\rho_i}\tilde{\nu}_i^2}{2\gamma_{\nu_i}} + V_{\epsilon_i}), \tag{38}$$

其中V_{ϵ_i}为关于估计误差ϵ_i的二次型方程, 其具体形式为

$$V_{\epsilon_i} = \epsilon_i^{\mathrm{T}} P_i \epsilon_i, \tag{39}$$

其中: $P_i = (\varPsi_i^{-1})^{\mathrm{T}}\bar{P}_i\varPsi_i^{-1}$, $\bar{P}_i = \bar{P}_i^{\mathrm{T}} > 0$且满足

$$\bar{P}_i A_{i,0} + A_{i,0}^{\mathrm{T}}\bar{P}_i = -2I, \tag{40}$$

$A_{i,0}$为Hurwitz矩阵且由式(7)给出. 做如下误差变换:

$$\hat{\epsilon}_i = \varPsi_i^{-1}\epsilon_i. \tag{41}$$

考虑式(14), 转换后的误差信号$\hat{\epsilon}_i$满足

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\epsilon}_i = k_i A_{i,0}\hat{\epsilon}_i + \varPsi_i^{-1}b_i d(u_i) + \varPsi_i^{-1}d_i(t) + \varPsi_i^{-1}\sum_{j=1}^{N} f_{ij,1}(t,y_j). \tag{42}$$

由于$A_{i,0}$为Hurwitz矩阵, 因此存在一个对称、正定矩阵$\bar{P}_i$使得式(40)成立. 定义如下二次型方程

$$V_{\hat{\epsilon}_i} = \hat{\epsilon}_i^{\mathrm{T}}\bar{P}_i\hat{\epsilon}_i, \tag{43}$$

则利用Young's不等式, 式(39)对时间的导数满足

$$\begin{aligned}\dot{V}_{\hat{\epsilon}_i} \leqslant& -2k_i\|\hat{\epsilon}_i\|^2 + \frac{2\|\hat{\epsilon}_i\|}{k_i^{\rho_i-1}}(\|\bar{P}_i\|\|b_i\|D + \|\bar{P}_i d_i\| + \|\bar{P}_i\|\sum_{j=1}^{N}\|f_{ij}(t,y_j)\|) = \\ & -2k_i\|\hat{\epsilon}_i\|^2 + 2\|\hat{\epsilon}_i\|\sqrt{k_i} + [\frac{\sqrt{k_i}\|\bar{P}_i\|\|b_i\|D}{k_i^{\rho_i}} + \frac{\sqrt{k_i}\|\bar{P}_i\|\sum_{j=1}^{N}\|f_{ij}(t,y_j)\|}{k_i^{\rho_i}}] \leqslant \\ & -\frac{k_i}{2\lambda_{\max}(\bar{P}_i)}\hat{\epsilon}_i^{\mathrm{T}}\bar{P}_i\hat{\epsilon}_i + k_i[(\frac{\|\bar{P}_i\|\|b_i\|D}{k_i^{\rho_i}})^2 + (\frac{\|\bar{P}_i d_i\|}{k_i^{\rho_i}})^2 + \frac{2N\|\bar{P}_i\|^2}{k_i^{2\rho_i-1}}\sum_{j=1}^{N}\|f_{ij}(t,y_j)\|^2],\end{aligned} \tag{44}$$

其中$\lambda_{\max}(\bar{P}_i)$表示矩阵$\bar{P}_i$特征值的最大值. 令

$$\zeta_{k_i} = \frac{k_i}{2\lambda_{\max}(\bar{P}_i)}, \tag{45}$$

$$\delta_{k_i} = k_i[(\frac{\|\bar{P}_i\|\|b_i\|D}{k_i^{\rho_i}})^2 + (\frac{\|\bar{P}_i d_i\|}{k_i^{\rho_i}})^2] \tag{46}$$

可知, 对于任意$k_i \geqslant 1$,

$$\dot{V}_{\hat{\epsilon}_i} \leqslant -\zeta_{k_i}V_{\hat{\epsilon}_i} + \delta_{k_i} + \frac{2N\|\bar{P}_i\|^2}{k_i^{2\rho_i-1}}\sum_{j=1}^{N}\|f_{ij}(t,y_j)\|^2 \tag{47}$$

成立. 此外, 因$P_i = (\varPsi_i^{-1})^{\mathrm{T}}\bar{P}_i\varPsi_i^{-1}$, 由式(41)–(43)得

$$V_{\epsilon_i} = \epsilon_i^{\mathrm{T}}P_i\epsilon_i = \hat{\epsilon}_i^{\mathrm{T}}\bar{P}_i\hat{\epsilon}_i = V_{\hat{\epsilon}_i}. \tag{48}$$

下面给出如下定理:

定理 1 考虑由式(15)(24)(32)–(34)所构成的闭环系统以及由式(37)所定义的Lyapunov函数. 令ς_i和$K_{i,0}$为大于零的常数. 则对于$V(0) \leqslant \varsigma (= \sum_{i=1}^{N}\varsigma_i)$, $y_{\mathrm{r}i}^2 + \dot{y}_{\mathrm{r}i}^2 + \ddot{y}_{\mathrm{r}i}^2 \leqslant K_{i,0}, i = 1,\cdots,N$, 通过选择恰当的设计参数$k_i, l_{i,1},\cdots,l_{i,\rho_i}, \tau_{i,1},\cdots,\tau_{i,\rho_i}, \eta_i$和$\gamma_{\nu_i}$, 整个闭环系统的所有信号一致有界. 此外, 系统跟踪误

差满足

$$\lim_{t\to\infty}|S_{i,1}|\leqslant\lim_{t\to\infty}\sqrt{2V_i}\leqslant\sqrt{\frac{C_i}{\zeta_i}},\ i=1,\cdots,N,\tag{49}$$

其中ζ_i为大于零的常数,

$$C_i=\frac{1}{2}[\|d_{i,1}\|^2+(\frac{\beta_{i,n_i-\rho_i}}{2\alpha_i})^2+(\rho_i-1)\sigma_i+\beta_{i,n_i-\rho_i}\eta_i\nu_i^{*2}+2\delta_{k_i}]+H_i,\ i=1,\cdots,N,\tag{50}$$

σ_i,ζ_i为可调正设计参数, H_i将在接下来的定理证明过程中给出, 使得$\lim\limits_{t\to\infty}|S_{i,1}|$可以任意小. 另外, 若将滤波器式(8)–(10)和调参律式(22)的初始条件置为零, 并且令$\xi_{i,(0,1)}(0)=y_i(0), y_{\rm ri}(0)=y_i(0)$, 则$V_i(t)$的上界为

$$V_i(t)\leqslant\frac{C_i}{2\zeta_i}+\frac{1}{k_i^2}\lambda_{\max}(\bar{P}_i)\|\epsilon_i(0)\|^2,\ i=1,\cdots,N.\tag{51}$$

证　定义如下紧集:

$$\begin{aligned}&\Omega_{i,1}=\{(y_{\rm ri},\dot{y}_{\rm ri},\ddot{y}_{\rm ri}):y_{\rm ri}^2+\dot{y}_{\rm ri}^2+\ddot{y}_{\rm ri}^2\leqslant K_{i,0}\},\\&\Omega_{i,2}=\{\sum_{q=1}^{\rho_i}S_{i,q}^2+\sum_{q=2}^{\rho_i}\bar{y}_{i,q}^2+\beta_{i,n_i-\rho_i}\tilde{\nu}_i^2+\\&\qquad 2\epsilon_i^{\rm T}P_i\epsilon_i\leqslant 2\varsigma_i\}.\end{aligned}\tag{52}$$

显然, $\Omega_{i,1}\times\Omega_{i,2}$也为一个紧集. 因此式(35)–(36)中的连续函数$B_{i,q+1}$, 在紧集$\Omega_{i,1}\times\Omega_{i,2}$上存在最大值, 称作$M_{i,q+1}$. 由式(34)–(36)可得

$$\begin{aligned}&\bar{y}_{i,q+1}\dot{\bar{y}}_{i,q+1}\leqslant-\frac{\bar{y}_{i,q+1}^2}{\tau_{i,q+1}}+B_{i,q+1}|\bar{y}_{i,q+1}|,\\&|\bar{y}_{i,q+1}|B_{i,q+1}\leqslant|\bar{y}_{i,q+1}|M_{i,q+1}\leqslant\frac{\bar{y}_{i,q+1}^2M_{i,q+1}^2}{2\sigma_i}+\frac{\sigma_i}{2},\end{aligned}\tag{53}$$

其中: $q=1,\cdots,\rho_i-1$, σ_i为正设计常数. 又根据假设2, 存在常数$\kappa_{ij}=O(\bar{\kappa}_{ij}), i=1,\cdots,N, j=1,\cdots,N$(详见文献[7]), 使得

$$\frac{N}{2}\sum_{j=1}^N\|f_{ij,1}(t,y_j)\|^2+\frac{2N\|\bar{P}_i\|^2}{k_i^{2\rho_i-1}}\sum_{j=1}^N\|f_{ij}(t,y_j)\|^2\leqslant\sum_{j=1}^N\kappa_{ij}\psi_j(y_j)\tag{54}$$

成立.

考虑式(32)–(34)(52)–(54), 可得

$$\begin{aligned}&\dot{V}_i\leqslant\\&-(l_{i,1}-\frac{2+k_i^2}{2}-\beta_{i,n_i-\rho_i}k_i)S_{i,1}^2-\\&(\frac{k_i}{\lambda_{\max}(\bar{P}_i)}-\frac{1}{2\lambda_{\min}(\bar{P}_i)})V_{\epsilon_i}-(\frac{1}{\tau_{i,2}}-\frac{M_{i,2}^2}{2\sigma_i}-\\&\frac{\beta_{i,n_i-\rho_i}k_i}{2})\bar{y}_{i,2}^2-\frac{\beta_{i,n_i-\rho_i}\eta_i}{2}\tilde{\nu}_i^2-\zeta_{k_i}V_{\epsilon_i}-\\&\beta_{i,n_i-\rho_i}l_i^*S_{i,1}^2\phi(S_{i,1})\psi_i^2(y_i)+\sum_{j=1}^N\kappa_{ij}\psi_j(y_j)-\\&(\frac{1}{\tau_{i,3}}-\frac{M_{i,3}^2}{2\sigma_i}-\frac{k_i}{2})\bar{y}_{i,3}^2-\frac{\beta_{i,n_i-\rho_i}k_i}{2}S_{i,2}^2+\\&\frac{k_i}{2}S_{i,3}^2+\frac{\sigma_i}{2}+\sum_{q=3}^{\rho_i-1}[-(l_{i,q}-\frac{3k_i}{2})S_{i,q}^2-\\&(\frac{1}{\tau_{i,q+1}}-\frac{M_{i,q+1}^2}{2\sigma_i}-\frac{k_i}{2})\bar{y}_{i,q+1}^2+\\&\frac{k_i}{2}S_{i,3}^2+\frac{\sigma_i}{2}+\sum_{q=3}^{\rho_i-1}[-(l_{i,q}-\frac{3k_i}{2})S_{i,q}^2-\\&(\frac{1}{\tau_{i,q+1}}-\frac{M_{i,q+1}^2}{2\sigma_i}-\frac{k_i}{2})\bar{y}_{i,q+1}^2-\\&\frac{k_i}{2}S_{i,q}^2+\frac{k_i}{2}S_{i,q+1}^2+\frac{\sigma_i}{2}]-(l_{i,\rho_i}-\frac{k_i}{2})S_{i,\rho_i}^2-\\&\frac{k_i}{2}S_{i,\rho_i}^2+\frac{1}{2}(d_{i,1}^2+\frac{\beta_{i,n_i-\rho_i}}{\alpha_i}+\\&\sigma_i+\beta_{i,n_i-\rho_i}\eta_i\nu_i^{*2}+2\delta_{k_i}),\ i=1,\cdots,N.\end{aligned}\tag{55}$$

由于

$$\begin{aligned}&\sum_{i=1}^N[-\beta_{i,n_i-\rho_i}l_i^*S_{i,1}^2\phi(S_{i,1})\psi_i^2(y_i)+\sum_{j=1}^N\kappa_{ij}\psi_j(y_j)]\leqslant\\&\sum_{i=1}^N[-\beta_{i,n_i-\rho_i}l_i^*S_{i,1}^2\phi(S_{i,1})+\sum_{j=1}^N\kappa_{ij}]\psi_i^2(y_i).\end{aligned}\tag{56}$$

令$\beta_{i,n_i-\rho_i}l_i^*\geqslant\sum\limits_{j=1}^N\kappa_{ij},\ h_i=S_{i,1}^2\phi(S_{i,1})\psi_i^2(y_i)+\psi_i^2(y_i)$, 由于式(21)对$\phi(\cdot)$的定义表明$h_i$是有界的, 则

$$\begin{aligned}&\sum_{i=1}^N[-\beta_{i,n_i-\rho_i}l_i^*S_{i,1}^2\phi(S_{i,1})\psi_i^2(y_i)+\sum_{j=1}^N\kappa_{ij}\psi_j(y_j)]\leqslant\\&\sum_{i=1}^N l_i^*h_i\leqslant\sum_{i=1}^N H_i,\end{aligned}\tag{57}$$

其中H_i为$l_i^*h_i(i=1,\cdots,N)$的上界. 通过选取如下设计参数:

$$\begin{cases}k_i\geqslant 2\lambda_{\max}(\bar{P}_i)\zeta_i+\dfrac{\lambda_{\max}(\bar{P}_i)}{\lambda_{\min}(\bar{P}_i)},\\ l_{i,1}\geqslant\dfrac{2+k_i^2}{2}+\beta_{i,n_i-\rho_i}+\zeta_i,\\ l_{i,2}\geqslant k_i+\dfrac{\beta_{i,n_i-\rho_i}k_i}{2}+\zeta_i,\ l_{i,q}\geqslant\dfrac{3k_i}{2}+\zeta_i,\\ \dfrac{1}{\tau_{i,2}}\geqslant\dfrac{M_{i,2}^2}{2\sigma_i}+\dfrac{\beta_{i,n_i-\rho_i}k_i}{2}+\zeta_i,\ \eta_i\geqslant\dfrac{2\zeta_i}{\gamma_{\nu_i}},\\ \dfrac{1}{\tau_{i,q+1}}\geqslant\dfrac{M_{i,q+1}^2}{2\sigma_i}+\dfrac{k_i}{2}+\zeta_i,\ q=2,\cdots,\rho_i-1.\end{cases}\tag{58}$$

将式(56)(58)代入式(38)可得

$$\dot{V}_i\leqslant-2\zeta_iV_i+C_i,\ i=1,\cdots,N,\tag{59}$$

其中

$$C_i=\frac{1}{2}(d_{i,1}^2+\frac{\beta_{i,n_i-\rho_i}}{\alpha_i}+(\rho_i-1)\sigma_i+2\delta_{k_i}+\beta_{i,n_i-\rho_i}\eta_i\nu_i^{*2}).$$

由式(37)可知

$$\dot{V} \leqslant -2\zeta V + C, \tag{60}$$

其中:

$$\zeta = \min\{\zeta_i\},\ i = 1, \cdots, N,$$
$$C = \sum_{i=1}^{N} C_i, \tag{61}$$

令$\zeta \geqslant C/2\varsigma$, 则当$V = \varsigma$时, $\dot{V} \leqslant 0$, 这也意味着$V \leqslant \varsigma$为一个不变集. 因此对于给定的$V(0) \leqslant \varsigma$, 当$t \geqslant 0$时, $V(t) \leqslant \varsigma$恒成立.

由上述分析可知, 对于任意初始条件, 通过选择合适的设计参数$k_i, l_{i,1}, \cdots, l_{i,\rho_i}, \tau_{i,1}, \cdots, \tau_{i,\rho_i}, \eta_i$和$\gamma_{\nu_i}, i = 1, \cdots, N$, V有界. 因此$S_{i,1}, \cdots, S_{i,\rho_i}, \bar{y}_{i,2}, \cdots, y_{i,\rho_i}, \tilde{\nu}_i, \epsilon_i$一致有界. 由$S_{i,1}, y_{\mathrm{r}i}$有界可知, $y_i = x_{i,1}$有界. 由于y_i和$\varPhi_i(y_i)$是有界的, 根据式(9)–(10)可推得$\xi_{i,0}$, $\varXi_i$有界. 由式(8)可知

$$v_{i,q} = (sI - k_i A_{i,0})^{-1} e_{n_i, n_i - q} k_i^{1+q-n_i} u_i,$$
$$q = 0, 1, \cdots, n_i - \rho_i, \tag{62}$$

则

$$v_{i,(n_i-\rho_i,1)} = G_{\rho_i}(s) u_i,\ v_{i,(n_i-\rho_i,\rho_i+1)} = G_{\rho_i+1}(s) u_i, \tag{63}$$

其中$G_{\rho_i}(s)$和$G_{\rho_i+1}(s)$分别为相对阶为ρ_i和ρ_i+1的稳定的传递函数. 将y_i分别通过$s^{n_i} G_{\rho_i}(s)/N_i(s)$和$s^{n_i} G_{\rho_i+1}(s)/N_i(s)$可得,

$$\frac{s^{n_i} G_{\rho_i}(s)}{N_i(s)} y_i = \sum_{j=1}^{n_i} \frac{s^{n_i-j} G_{\rho_i}(s)}{N_i(s)} (\varPhi_{i,j}(y_i)\theta_{i,j} + \sum_{q=1}^{N} f_{iq,j}(t, y_j) + d_{i,j}(t)) + G_{\rho_i}(s) u_i + \frac{G_{\rho_i} F[u_i](t)}{\lambda_i},$$

$$\frac{s^{n_i} G_{\rho_i}(s)}{N_i(s)} y_i = \sum_{j=1}^{n_i} \frac{s^{n_i-j} G_{\rho_i}(s)}{N_i(s)} (\varPhi_{i,j}(y_i)\theta_{i,j} + \sum_{q=1}^{N} f_{iq,j}(t, y_j) + d_{i,j}(t)) + G_{\rho_i+1}(s) u_i + \frac{G_{\rho_i+1} F[u_i](t)}{\lambda_i}.$$

由于$y_i, \varPhi_{i,j}(y_i), F[u_i](t)/\lambda_i$是有界的, 且$G_{\rho_i}(s)u_i$和$G_{\rho_i+1}(s)u_i$有界. 根据式(62)可以得出$v_{i,(n_i-\rho_i,1)}$和$v_{i,(n_i-\rho_i,\rho_i+1)}$是有界的. 而$\bar{y}_{\rho_i}$的有界性保证了$\dot{z}_{\rho_i} = -\bar{y}_{\rho_i}/\tau_{\rho_i}$的有界性, 由式(30)得出$u_i$有界; 因此, 式(8)中的$v_{i,q}, i = 1, \cdots, N, q = 1, \cdots, \rho_i$有界. 再由式(12)–(13)可得$x_i$有界. 从而, 闭环系统所有信号一致有界. 又根据式(59)得

$$V_i(t) \leqslant \frac{C_i}{2\zeta_i} + (V_i(0) - \frac{C_i}{2\zeta_i}) \mathrm{e}^{-2\zeta_i t}, \tag{64}$$

从而

$$\lim_{t\to\infty} V_i(t) \leqslant \frac{C_i}{2\zeta_i}, \tag{65}$$

$$\lim_{t\to\infty} |S_{i,1}| \leqslant \sqrt{\frac{C_i}{\zeta_i}}. \tag{66}$$

需说明的是通过选定η_i和γ_{ν_i}使得$\eta_i \geqslant 2\zeta_i/\gamma_{\nu_i}$. 当$\zeta_i$充分大时, 各个子系统跟踪误差可以任意小.

另外, 令$y_{\mathrm{r}i}(0) = y_i(0)$, 则由式(15)可得

$$S_{i,1}(0) = 0. \tag{67}$$

根据定理1的假设条件及式(20), $\bar{v}_{i,(n_i-\rho_i,2)}(0) = 0$; 由式(23), $z_{i,2}(0) = 0$, $\dot{z}_{i,2}(0) = 0$. 则由式(24)(26)可得, $S_{i,2}(0) = 0$, $\bar{v}_{i,(n_i-\rho_i,3)}(0) = 0$. 同理, 可得

$$S_{i,q}(0) = 0,\ q = 3, \cdots, \rho_i. \tag{68}$$

根据式(31)可得

$$\bar{y}_{i,q}(0) = 0,\ q = 2, \cdots, \rho_i. \tag{69}$$

由式(59)(65)(69)和$\eta_i \geqslant 2\zeta_i/\gamma_{\nu_i}$, 可得

$$V_i(0) = \frac{1}{2}(\sum_{q=1}^{\rho_i} S_{i,q}^2(0) + \sum_{q=1}^{\rho_i} \bar{y}_{i,q+1}^2(0) + \frac{\beta_{i,n_i-\rho_i} \tilde{\nu}_i^2(0)}{2\gamma_{\nu_i}} + V_{\epsilon_i}(0)) = \frac{\beta_{i,n_i-\rho_i} \tilde{\nu}_i^2(0)}{2\gamma_{\nu_i}} + V_{\epsilon_i}(0) \leqslant \frac{\beta_{i,n_i-\rho_i} \eta_i}{4\zeta_i} \nu_i^{*2} + V_{\epsilon_i}(0) \leqslant \frac{C_i}{2\zeta_i} + V_{\epsilon_i}(0). \tag{70}$$

由式(12), 若$v_{i,q}, q = 0, \cdots, n_i - \rho_i, \varXi_i(0) = 0, i = 1, \cdots, N$, 则$\xi_{i,(0,1)}(0) = \hat{x}_{i,1}(0)$. 因此, 由式(13), 若$\xi_{i,(0,1)}(0) = y_i(0) = x_{i,1}(0)$, 则

$$\epsilon_{i,1}(0) = 0, \tag{71}$$

同时也说明$\varPsi_i^{-1}\epsilon_i(0) = \mathrm{diag}\{1, 1/k_i, \cdots, 1/k_i^{n_i-1}\} \cdot \epsilon_i(0)$. 由于$k_i \geqslant 1$, 则

$$\|\varPsi_i^{-1}\epsilon_i(0)\| \leqslant \|\epsilon_i(0)\|. \tag{72}$$

因此, $V_i(t)$的上界为

$$V_i(t) \leqslant \frac{C_i}{2\zeta_i} + \frac{\lambda_{\max}(\bar{P}_i)\|\epsilon_i(0)\|}{k_i^2}. \tag{73}$$

证毕.

5 仿真算例(Simulation results)

考虑如下由4个子系统构成的内联系统:

$$\begin{cases} \dot{x}_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x_1 + \begin{pmatrix} 2y_1 & y_1^2 \\ 0 & y_1 \end{pmatrix} \theta_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_1 + \sum_{j=1}^{4} f_{1j}(t, y_j) + d_1(t), \\ y_1 = x_{1,1}, \end{cases} \tag{74}$$

$$\begin{cases}\dot{x}_2=\begin{pmatrix}0&1\\0&0\end{pmatrix}x_2+\begin{pmatrix}0&0\\0.5y_2&1+y_2\end{pmatrix}\theta_2+\\ \qquad\begin{bmatrix}0\\1\end{bmatrix}w_2+\sum_{j=1}^{4}f_{2j}(t,y_j)+d_2(t),\\ y_2=x_{2,1},\end{cases}\tag{75}$$

$$\begin{cases}\dot{x}_3=\begin{pmatrix}0&1\\0&0\end{pmatrix}x_3+\begin{pmatrix}0&0\\y_3&1+y_3\end{pmatrix}\theta_3+\\ \qquad\begin{bmatrix}0\\1\end{bmatrix}w_2+\sum_{j=1}^{4}f_{3j}(t,y_j)+d_3(t),\\ y_3=x_{3,1},\end{cases}\tag{76}$$

$$\begin{cases}\dot{x}_4=\begin{pmatrix}0&1\\0&0\end{pmatrix}x_4+\begin{pmatrix}0&0\\y_4&1+y_4\end{pmatrix}\theta_4+\\ \qquad\begin{bmatrix}0\\1\end{bmatrix}w_4+\sum_{j=1}^{4}f_{4j}(t,y_j)+d_4(t),\\ y_4=x_{4,1},\end{cases}\tag{77}$$

其中: $\theta_1=[1\ \ 1]^{\mathrm{T}}$, $\theta_2=\theta_3=\theta_4=[0.5\ \ 1]^{\mathrm{T}}$, 磁滞环节为未知类间隙磁滞模型(backlash-like), 由本文中的式(2)给出, 且 $\lambda_i=0.9$, $\alpha_i=1$, $\psi_i=0.3$, $i=1,\cdots,4$. 各个子系统间的相互作用关系为:

$$f_{11}(t,y_1)=\begin{bmatrix}0\\\sin\ y_1\end{bmatrix},\ f_{12}(t,y_2)=\begin{bmatrix}y_2^2\\0\end{bmatrix},$$

$$f_{13}(t,y_3)=\begin{bmatrix}0.1y_3^2\\0\end{bmatrix},\ f_{14}(t,y_4)=\begin{bmatrix}0.1y_4\\0\end{bmatrix},$$

$$f_{21}(t,y_1)=\begin{bmatrix}0.2y_1^2\\0\end{bmatrix},\ f_{22}(t,y_2)=\begin{bmatrix}\sin\ y_2\\0\end{bmatrix},$$

$$f_{23}(t,y_3)=\begin{bmatrix}0.2y_3^2\\0\end{bmatrix},\ f_{24}(t,y_4)=\begin{bmatrix}0.2y_4^2\\0\end{bmatrix},$$

$$f_{31}(t,y_1)=\begin{bmatrix}0\\0.3y_1^2\end{bmatrix},\ f_{32}(t,y_2)=\begin{bmatrix}0.1\cos\ y_2\\0\end{bmatrix},$$

$$f_{33}(t,y_3)=\begin{bmatrix}0\\\sin\ y_3\end{bmatrix},\ f_{34}(t,y_2)=\begin{bmatrix}0.1y_4\\0\end{bmatrix},$$

$$f_{41}(t,y_1)=\begin{bmatrix}0.2y_1^2\\0\end{bmatrix},\ f_{42}(t,y_2)=\begin{bmatrix}\sin\ y_2\\0\end{bmatrix},$$

$$f_{43}(t,y_1)=\begin{bmatrix}0.2y_3^2\\0\end{bmatrix},\ f_{44}(t,y_2)=\begin{bmatrix}0.2y_4^2\\0\end{bmatrix}.$$

系统的扰动为:

$$d_1(t)=\begin{bmatrix}0.5\cos t\\0.2\sin t\end{bmatrix},\ d_2(t)=\begin{bmatrix}0\\0.2\sin(2t)\end{bmatrix},$$

$$d_3(t)=\begin{bmatrix}0\\0.1\cos(2t)\end{bmatrix},\ d_4(t)=\begin{bmatrix}0.1\sin(2t)\\0.2\cos(2t)\end{bmatrix}.$$

控制目标是使输出 y_1,y_2,y_3,y_4 跟踪对应的参考信号 $y_{\mathrm{r}1}=2\sin(1.5t)$, $y_{\mathrm{r}2}=2\sin(0.5t)$, $y_{\mathrm{r}3}=\sin(0.5t)$ 和 $y_{\mathrm{r}4}=\sin(0.5t)$. 基于论文中式(8)–(10), 高增益 K–观测器为如下形式:

$$\dot{v}_{i,0}=k_iA_{i,0}v_{i,0}+\Psi_i^{-1}e_{2,0}u_i,\ v_{i,0}=0,$$
$$\dot{\xi}_{i,0}=k_iA_{i,0}\xi_{i,0}+k_iq_iy_i,\ \xi_{i,0}=0,$$
$$\dot{\Xi}_i=k_iA_{i,0}\Xi_i+\Psi_i^{-1}\Phi_i(y_i),\ \Xi_i(0)=0,$$

其中:

$$A_{i,0}=\begin{pmatrix}-q_{i,1}&1\\-q_{i,2}&0\end{pmatrix},\ q_i=\begin{bmatrix}q_{i,1}\\q_{i,2}\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix},$$

$$k_i=2,\ \Psi_i=\mathrm{diag}\{1,k_i\}=1,2,3,4,$$

$$\Phi_1(y_1)=\begin{pmatrix}2y_1&y_1^2\\0&y_1\end{pmatrix},\ \Phi_2(y_2)=\begin{pmatrix}0&0\\0.5y_2&1+y_2\end{pmatrix},$$

$$\Phi_3(y_3)=\begin{pmatrix}0&0\\y_3&1+y_3\end{pmatrix},\ \Phi_4(y_4)=\begin{pmatrix}0&0\\y_4&1+y_4\end{pmatrix}.$$

其他参数的选择分别为:

$$l_{1,1}=10,\ l_{2,1}=l_{3,1}=l_{4,1}=2,$$
$$l_{1,2}=10,\ l_{2,2}=l_{3,2}=l_{4,2}=2,$$
$$l_1^*=1.25,\ l_2^*=0.3,\ l_3^*=l_4^*=0.5,$$
$$\psi_1(y_1)=1+y_1^2,\ \psi_2(y_2)=2.1+y_2^2,$$
$$\psi_3(y_3)=1+0.5y_3^2,\ \psi_4(y_4)=0.6y_4^2,$$
$$\gamma_{\nu_i}=2,\ \eta_i=0.1,\ \tau_{1,2}=0.01,$$
$$\tau_{2,2}=\tau_{3,2}=\tau_{4,2}=0.02.$$

仿真中系统状态的初始条件为 $x_{1,1}(0)=x_{1,2}(0)=x_{2,1}(0)=x_{2,2}(0)=x_{3,1}(0)=x_{3,2}(0)=x_{4,1}(0)=x_{4,2}(0)=0.1$; 在调参律中: $\hat{\nu}_i(0)=0$, $i=1,2,3,4$. 图1(a)–1(d)分别给出了4个子系统的跟踪效果; 图2–4分别给出4个子系统的跟踪误差、控制信号以及磁滞输出仿真结果. 图1表明第系统尽管存在磁滞现象、外界扰动及系统间的相互作用, 但在笔者所提出的控制策略下, 通过合理选择设计参数后, 具有理想的跟踪效果.

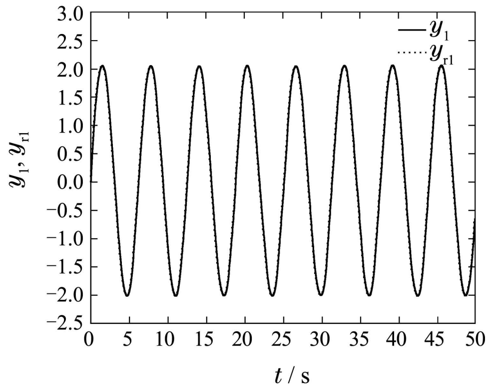

图 1(a) 第1个子系统的跟踪效果

Fig. 1(a) Tracking performance of subsystem 1

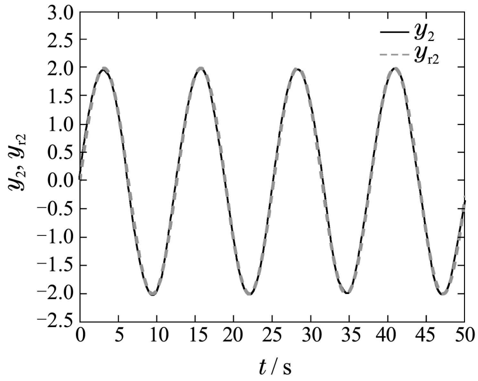

图 1(b) 第2个子系统的跟踪效果

Fig. 1(b) Tracking performance of subsystem 2

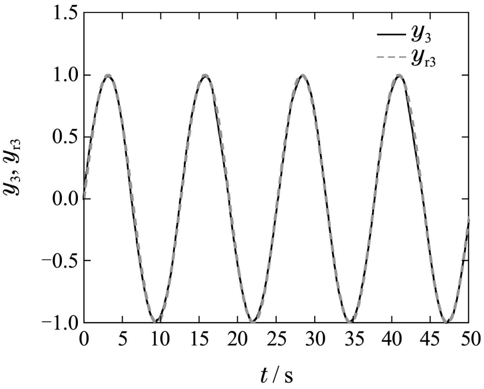

图 1(c) 第3个子系统的跟踪效果

Fig. 1(c) Tracking performance of subsystem 3

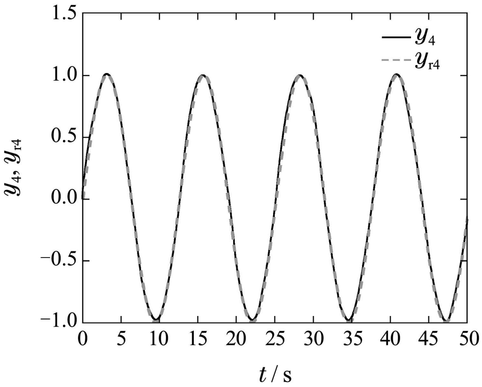

图 1(d) 第4个子系统的跟踪效果

Fig. 1(d) Tracking performance of subsystem 4

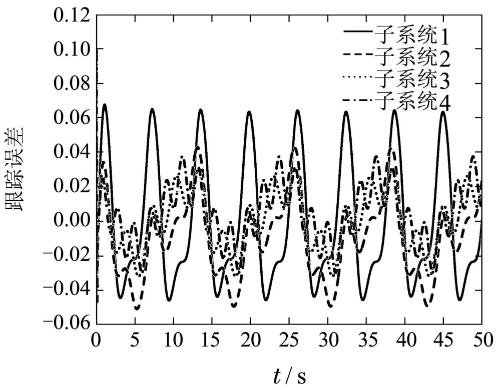

图 2 4个子系统的跟踪误差

Fig. 2 Tracking errors of the whole system

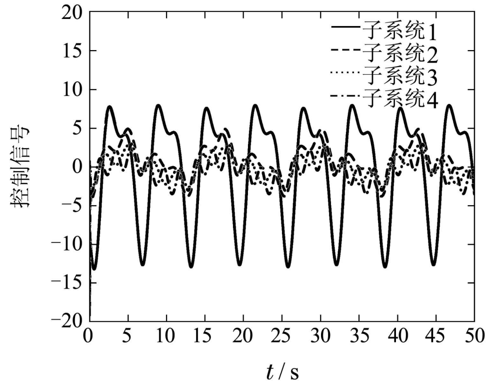

图 3 4个子系统的控制信号

Fig. 3 Control signals of the whole system

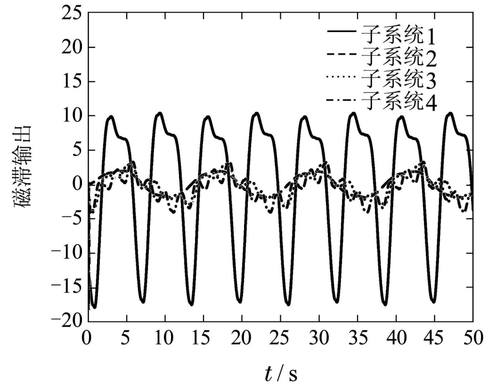

图 4 4个子系统的磁滞输出

Fig. 4 Hysteresis outputs of the whole system

注 4 在本文中笔者将磁滞环节分成线性和有界扰动两部分来处理(见式(2)–(3)), 因此, 磁滞环节的引入相当于引入一个外部有界扰动, 必然使得控制系统的跟踪误差精度变坏(仿真结果见图5), 但由于本文设计的控制器具有鲁棒性, 因此可通过提高反馈增益$l_{1,1}, h_{2,1}, h_{3,1}, h_{4,1}$来提高跟踪误差精度, 如当$l_{1,1} = 10$时, 在有、无磁滞输入两种情况下第1个子系统的跟踪误差如图5所示. 从图5可以看出无磁滞输入是系统的跟踪误差精度比有磁滞输入时要好, 然而通过增大反馈增益系数$l_{1,1} = 30$后, 如图6所示, 可提高跟踪误差精度.

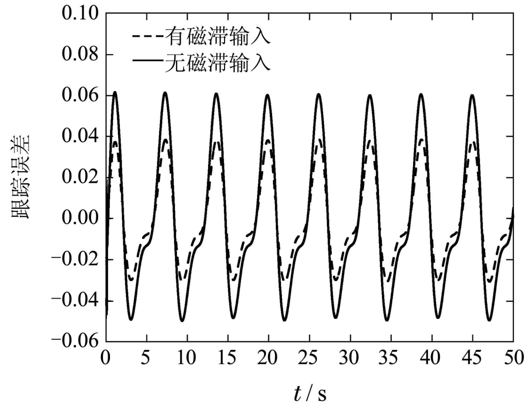

图 5 有、无磁滞输入两种情况下第1个子系统的跟踪误差

Fig. 5 Tracking errors of subsystem 1 with and without hysteresis inputs

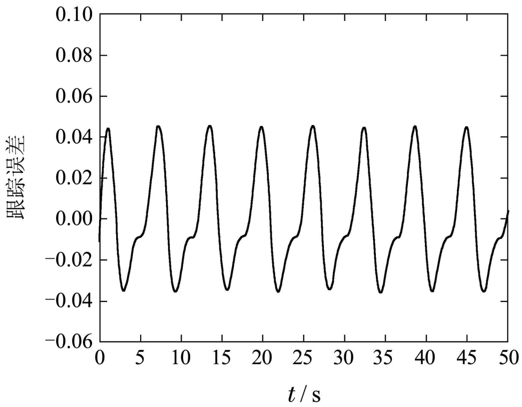

图 6 增大反馈增益系数后, 第1个子系统的跟踪误差

Fig. 6 Tracking errors of subsystem 1 when the feedback gains are increased

6 结论(Conclusions)

本文针对具有类间隙磁滞输入的互联系统, 给出了分布自适应输出反馈动态面控制方案, 并设计了高增益K–观测器. 所提出的方案简化了控制器的复杂度, 保证了互联系统中各个子系统跟踪性能指标; 引入了一种高增益观测器, 保证了互联系统中各个子系统跟踪误差可以收敛到任意小. 此外, 由于采用估计未知参数向量的范数来代替估计未知参数地向量, 极大地减轻了系统的计算负担.

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