DOI: 10.7641/CTA.2014.30035

广义输出误差模型的两阶段最小二乘递推辨识

贾 杰[1], 陈 晨[1], 曹 姣[1], 罗小娜[1†], 丁 锋[2]

(1. 南昌航空大学 信息工程学院, 江西 南昌 330063; 2. 江南大学 物联网工程学院, 江苏 无锡 214122)

摘要: 在有色噪声干扰系统中有一类系统, 它具有广义输出误差模型(OEARMA), 本文提出一类广义输出误差模型的两阶段递推最小二乘参数估计算法. 该算法基本思想是结合辅助模型辨识思想和分解技术, 将系统分解成两个子系统, 每个子系统包含一个参数向量. 借助基于辅助模型和递推最小二乘理论, 用辅助模型的输出代替辨识模型信息向量中未知中间变量, 用估计残差代替信息向量中不可测噪声项, 从而可以运用递推辨识思想来估计系统所有参数. 该算法具有较高的计算效率, 仿真例子说明提出算法的有效性.

关键词: 随机系统; 最小二乘; 两阶段递推; 辅助模型

中图分类号: TP273 **文献标识码**: A

Two-stage least squares recursive identification for generalized output error models

JIA Jie[1], CHEN Chen[1], CAO Jiao[1], LUO Xiao-na[1†], DING Feng[2]

(1. School of Information Engineering, Nanchang Hangkong University, Nanchang Jiangxi 330063, China;
2. School of Internet of Things Engineering, Jiangnan University, Wuxi Jiangsu 214122, China)

Abstract: A class of the colored noise interference systems is with the generalized output error model (OEARMA). This paper presents a two-stage recursive least squares algorithm for their parameter identifications. The basic idea is to combine the auxiliary model identification idea and the decomposition technique to decompose a system into two subsystems, each of which contains one parameter vector. When applying the auxiliary model-based recursive extended least squares theory, we employ the auxiliary model output to replace the unknown intermediate variables in the identified model information vector, and use the estimated residuals to replace the immeasurable noise terms in the information vector. This makes it possible to apply the recursive identification idea to estimate all the parameters of the system with a high computational efficiency. The simulation examples validate the effectiveness of the proposed algorithm.

Key words: stochastic system; least squares; two-stage recursion; auxiliary model

1 引言(Introduction)

有色噪声干扰的系统辨识一直都是国内外学者关心的研究领域[1–2]. 对于有色噪声干扰的随机系统, 常规的最小二乘参数估计是有偏的[2]. 针对最小二乘算法辨识有色噪声系统存在偏差的问题, 许多学者做了大量工作, 也提出了不少有效的方法. 例如, 偏差补偿最小二乘算法(BCLS)[3]、递推广义最小二乘(RGLS)算法[4]、递推广义增广最小二乘(RGELS)算法[4]、两阶段递推最小二乘参数估计算法[5]以及1991年由丁锋提出的辅助模型辨识思想[6]、递阶辨识原理[7]、多新息辨识理论[8]和参数估计误差界理论[9–12]等等. 这些方法不仅能给出系统模型的参数估计, 而且后两种能产生噪声模型参数估计.

然而, 理论分析表明偏差补偿最小二乘算法针对广义输出误差(OEARMA)模型很难做到无偏估计, 且要求输入是平稳的(stationary)各态遍历的(ergodic). 丁锋等提出了改进型的偏差补偿最小二乘算法克服了上述缺点[13], 文献[14]中利用滤波器对输入数据进行滤波, 势必加大计算量. RGLS算法[15]在过程的输出信噪比比较大时或模型参数比较多时, 这种数据白色化处理的可靠性就会下降. 此时可能出现多个局部收敛点, 辨识精度也低, 这样最终辨识结果也是有偏的. RGELS算法是否收敛以及在什么条件下收敛的理论证明是极具挑战性的课题, 文献中仅给出一个近似分析. 丁锋等人[16]提出用迭代辨识方法辨识非线性受控自回归模型(CARAR)模型虽然能获得满意的精度要求, 但由于实际问题复杂性, 不易在线辨识, 白化处理时模型的选取比较困难, 同时算法也比较复杂, 在

收稿日期: 2013−01−11; 录用日期: 2013−09−17.
†通信作者. E-mail: nhluoxiaona@163.com.
基金项目: 中国国家自然基金资助项目(61263012, 61263040); 中国博士后基金资助项目(2012M510593); 航空科学基金资助项目(20120156001).

什么条件下收敛也没有很好解决. 然而, 理论分析表明递推增广最小二乘算法(RELS)的收敛性要求噪声模型是严格正实传递函数[17–19].

递推最小二乘算法解决了ARX模型的辨识问题[20], 递推增广最小二乘算法解决了ARMAX模型的辨识问题[20], 辅助模型辨识方法和偏差补偿方法解决了输出误差模型的辨识问题[21–25]. 尽管辅助变量最小二乘算法可以用于辨识系统, 但不能给出噪声模型的参数估计[26]. 对于输出误差模型, 除了上述提到的方法外, 文献[27]采用有理分式等价方法, 进一步利用相关技术, 提出了有限脉冲响应模型阶次递增的参数估计算法, 文献[28]利用有理分式等价方法研究了多输入单输出系统的辨识问题; 文献[29]利用有理分式等价方法来简化有色噪声干扰随机系统, 使用获得的近似简化模型可以用增广最小二乘算法来估计其参数, 然后确定原系统的参数.

上述提到的方法只解决了OEARMA中某一两个多项式为1的特殊模型的辨识问题. 对于一般形式的随机系统, 本文提出一类广义输出误差模型的两阶段递推最小二乘参数估计算法. 该算法基本思想是结合辅助模型辨识思想和分解技术, 将系统分解成两个子系统, 每个子系统包含一个参数向量. 借助基于辅助模型和递推最小二乘理论, 用辅助模型的输出代替辨识模型信息向量中未知中间变量, 用估计残差代替信息向量中不可测噪声项, 从而可以运用递推辨识思想来估计系统所有参数.

2 模型描述(Model description)

定义符号, “$A := X$”或“$X := A$”表示A等于X, 符号I表示适当维数$n\times n$的单位阵, 上标T代表矩阵或向量的转置, I_n表示n维单位列向量.

考虑下列有色噪声干扰的OEARMA系统, 见图1描述.

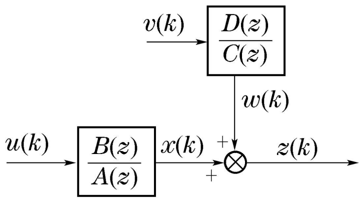

图 1 随机系统的结构框图

Fig. 1 Stochastic system structure diagram

$$z(k)=\frac{B(z)}{A(z)}u(k)+\frac{D(z)}{C(z)}v(k), \tag{1}$$

其中: $z(k)$为系统k时刻输出, $u(k)$为系统k时刻输入, $v(k)$为零均值, 不相关随机白噪声, $A(z),B(z),C(z)$和$D(z)$均为单位后移算子z^{-1}的多项式$[z^{-1}]z(k)=z(k-1)$, 且

$$A(z):=1+a_1z^{-1}+a_2z^{-2}+\cdots+a_{n_\mathrm{a}}z^{-n_\mathrm{a}},$$
$$B(z):=b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_\mathrm{b}}z^{-n_\mathrm{b}},$$
$$C(z):=1+c_1z^{-1}+c_2z^{-2}+\cdots+c_{n_\mathrm{c}}z^{-n_\mathrm{c}},$$
$$D(z):=1+d_1z^{-1}+d_2z^{-2}+\cdots+d_{n_\mathrm{d}}z^{-n_\mathrm{d}}.$$

不妨设$k\leqslant 0$时, $u(k)=0, z(k)=0, v(k)=0$, 且阶次$n_\mathrm{a},n_\mathrm{b},n_\mathrm{c},n_\mathrm{d}$已知. 本文目标是利用基于分离技术的二阶段辨识算法, 将原辨识系统转化为两个具有较小阶次的子问题. 定义参数向量:

$$\theta:=\begin{bmatrix}\theta_s\\ \theta_n\end{bmatrix}\in\mathbb{R}^n,\ n=n_\mathrm{a}+n_\mathrm{b}+n_\mathrm{c}+n_\mathrm{d},$$
$$\theta_s:=(a_1,a_2,\cdots,a_{n_\mathrm{a}},b_1,b_2,\cdots,b_{n_\mathrm{b}})\in\mathbb{R}^{(n_\mathrm{a}+n_\mathrm{b})},$$
$$\theta_n:=(c_1,c_2,\cdots,c_{n_\mathrm{c}},d_1,d_2,\cdots,d_{n_\mathrm{d}})\in\mathbb{R}^{(n_\mathrm{c}+n_\mathrm{d})}.$$

定义信息向量:

$$\varphi(k):=\begin{bmatrix}\varphi_s(k)\\ \varphi_n(k)\end{bmatrix}\in\mathbb{R}^n,\ n=n_\mathrm{a}+n_\mathrm{b}+n_\mathrm{c}+n_\mathrm{d},$$
$$\varphi_s(k):=(-z_{k-1},-z_{k-2},\cdots,-z_{k-n_\mathrm{a}},u_{k-1},u_{k-2},\cdots,u_{k-n_\mathrm{b}})\in\mathbb{R}^{(n_\mathrm{a}+n_\mathrm{b})},$$
$$\varphi_n(k):=(-w_{k-1},-w_{k-2},\cdots,-w_{k-n_\mathrm{c}},v_{k-1},v_{k-2},\cdots,v_{k-n_\mathrm{d}})\in\mathbb{R}^{(n_\mathrm{c}+n_\mathrm{d})}.$$

定义中间变量$x(k)$和$w(k)$如下:

$$x(k):=\frac{A(z)}{B(z)}u(k), \tag{2}$$

或

$$\begin{aligned}&x(k)=[1-A(z)]x(k)+B(z)u(k)=\\&(-a_1z^{-1}-a_2z^{-2}-\cdots-a_{n_\mathrm{a}}z^{-n_\mathrm{a}})x(k)+\\&(b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_\mathrm{b}}z^{-n_\mathrm{b}})u(k)=\\&-a_1x(k-1)-a_2x(k-2)-\cdots-\\&a_{n_\mathrm{a}}x(k-n_\mathrm{a})+b_1u(k-1)+b_2u(k-2)+\\&\cdots+b_{n_\mathrm{b}}u(k-n_\mathrm{b})=\\&\varphi_s^\mathrm{T}(k)\theta_s,\end{aligned}$$

$$w(k):=\frac{D(z)}{C(z)}v(k), \tag{3}$$

或

$$\begin{aligned}&w(k)=[1-C(z)]w(k)+D(z)v(k)=\\&(-c_1z^{-1}-c_2z^{-2}-\cdots-c_{n_\mathrm{c}}z^{-n_\mathrm{c}})w(k)+\\&(1+d_1z^{-1}+d_2z^{-2}+\cdots+d_{n_\mathrm{d}}z^{-n_\mathrm{d}})v(k)=\\&-c_1w(k-1)-c_2w(k-2)-\cdots-\\&c_{n_\mathrm{c}}w(k-n_\mathrm{c})+v(k)+d_1v(k-1)+\\&d_2v(k-2)+\cdots+d_{n_\mathrm{d}}v(k-n_\mathrm{d})=\\&\varphi_n^\mathrm{T}(k)\theta_n+v(k).\end{aligned}$$

利用式(2)–(3), 式(1)可写为

$$\begin{aligned}z(k) &= x(k)+w(k) = \\ &\varphi_s^{\rm T}(k)\theta_s+\varphi_n^{\rm T}(k)\theta_n+v(k) = \\ &\varphi^{\rm T}(k)\theta+v(k).\end{aligned} \tag{4}$$

3 二阶段最小二乘递推辨识算法 (Two-stage least squares recursive identification algorithm)

二阶段最小二乘递推辨识算法的基本思想是将系统转化为两个子系统, 将参数和信息向量分别转化为两个参数子向量和两个信息子向量, 然后利用辅助模型思想辨识每个子系统参数.

定义两个中间变量:

$$z_1(k) := z(k)-\varphi_n^{\rm T}(k)\theta_n, \tag{5}$$

$$z_2(k) := z(k)-\varphi_s^{\rm T}(k)\theta_s. \tag{6}$$

系统(4)可以转化为下列两个虚拟辨识子系统:

$$z_1(k) = \varphi_s^{\rm T}(k)\theta_s+v(k),$$

$$z_2(k) = \varphi_n^{\rm T}(k)\theta_n+v(k).$$

这两个子系统分别包含参数向量θ_s和θ_n, 定义两个准则函数:

$$J_1(\theta_s) := \sum_{j=1}^{k}[z_1(k)-\varphi_s^{\rm T}(k)\theta_s]^2,$$

$$J_2(\theta_n) := \sum_{j=1}^{k}[z_2(k)-\varphi_n^{\rm T}(k)\theta_n]^2.$$

令$J_1(\theta_s)$和$J_2(\theta_n)$分别对θ_s和θ_n的偏导数为零,

$$\frac{\partial J_1(\theta_s)}{\partial\theta_s} = -2\varphi_s(j)\sum_{j=1}^{k}[z_1(j)-\varphi_s^{\rm T}(j)\theta_s]=0,$$

$$\frac{\partial J_2(\theta_n)}{\partial\theta_n} = -2\varphi_n(j)\sum_{j=1}^{k}[z_2(j)-\varphi_n^{\rm T}(j)\theta_n]=0.$$

令 $\hat{\theta}(t) := \begin{bmatrix}\theta_s(k)\\ \theta_n(k)\end{bmatrix}\in\mathbb{R}^n$是$\theta=\begin{bmatrix}\theta_s\\ \theta_n\end{bmatrix}\in\mathbb{R}^n$在$k$时刻的估计, 最小化准则函数, 可以得到递推最小二乘RLS算法,

$$\hat{\theta}_s(k)=\hat{\theta}_s(k-1)+K_s(k)[z_1(k)-\varphi_s^{\rm T}(k)\hat{\theta}_s(k-1)], \tag{7}$$

$$K_s(k)=P_s(k-1)\varphi_s(k)[1+\varphi_s^{\rm T}P_s(k-1)\varphi_s(k)]^{-1}, \tag{8}$$

$$P_s(k)=[I-K_s(k)\varphi_s^{\rm T}(k)]P_s(k-1),\ P_s(0)=P_0I, \tag{9}$$

$$\hat{\theta}_n(k)=\hat{\theta}_n(k-1)+K_n(k)[z_2(k)-\varphi_n^{\rm T}(k)\hat{\theta}_n(k-1)], \tag{10}$$

$$K_n(k)=P_n(k-1)\varphi_n(k)[1+\varphi_n^{\rm T}P_n(k-1)\varphi_n(k)]^{-1}, \tag{11}$$

$$P_n(k)=[I-K_n(k)\varphi_n^{\rm T}(k)]P_n(k-1),\ P_n(0)=P_0I. \tag{12}$$

将式(5)和式(6)代入式(7)和式(10)得

$$\begin{aligned}\hat{\theta}_s(k) = \hat{\theta}_s(k-1)+K_s(k)[z(k) - \\ \varphi_n^{\rm T}(k)\theta_n-\varphi_s^{\rm T}(k)\hat{\theta}_s(k-1)],\end{aligned} \tag{13}$$

$$\begin{aligned}\hat{\theta}_n(k) = \hat{\theta}_n(k-1)+K_n(k)[z(k) - \\ \varphi_s^{\rm T}(k)\theta_s-\varphi_n^{\rm T}(k)\hat{\theta}_n(k-1)].\end{aligned} \tag{14}$$

分别用估计$\hat{\theta}_s(k-1)$和$\hat{\theta}_n(k-1)$来代替式(13)和式(14)右边存在的未知参数向量θ_s, θ_n得

$$\begin{aligned}\hat{\theta}_s(k)=\hat{\theta}_s(k-1)+K_s(k)[z(k)- \\ \varphi_s^{\rm T}(k)\hat{\theta}_s(k-1)-\varphi_n^{\rm T}(k)\hat{\theta}_n(k-1)],\end{aligned} \tag{15}$$

$$\begin{aligned}\hat{\theta}_n(k)=\hat{\theta}_n(k-1)+K_n(k)[z(k)- \\ \varphi_s^{\rm T}(k)\hat{\theta}_s(k-1)-\varphi_n^{\rm T}(k)\hat{\theta}_n(k-1)].\end{aligned} \tag{16}$$

分别用估计$\hat{\varphi}_s(k)$和$\hat{\varphi}_n(k)$来代替式(8)(11)(15)–(16)右边的未知信息向量$\varphi_s(k)$和$\varphi_n(k)$, 最终得

$$\begin{aligned}\hat{\varphi}_s(k) :=(&-\hat{x}(k-1), -\hat{x}(k-2),\cdots,-\hat{x}(k-n_{\rm a}), \\ &u(k-1), u(k-2),\cdots,u(k-n_{\rm b})),\end{aligned} \tag{17}$$

$$\begin{aligned}\hat{\varphi}_n(k) :=(&-\hat{w}(k-1), -\hat{w}(k-2),\cdots,-\hat{w}(k-n_{\rm c}), \\ &\hat{v}(k-1), \hat{v}(k-2),\cdots,\hat{v}(k-n_{\rm d})).\end{aligned} \tag{18}$$

定义

$$\hat{\varphi}(k) := \begin{bmatrix}\hat{\varphi}_s(k)\\ \hat{\varphi}_n(k)\end{bmatrix}\in\mathbb{R}^n,$$

利用$\hat{\varphi}_s(k)$, $\hat{\varphi}_n(k)$, $\hat{\theta}_s(k)$, $\hat{\theta}_n(k)$代替式(2)–(4)中的$\varphi_s(k)$, $\varphi_n(k)$, $\theta_s(k)$, $\theta_n(k)$得

$$\begin{cases}\hat{x}(k) = \hat{\varphi}_s^{\rm T}(k)\hat{\theta}_s, \\ \hat{w}(k) = z(k)-\hat{\varphi}_s^{\rm T}(k)\hat{\theta}_s, \\ \hat{v}(k) = z(k)-\hat{\varphi}^{\rm T}(k)\hat{\theta},\end{cases} \tag{19}$$

得出广义输出误差模型的参数向量$\theta_s(k)$, $\theta_n(k)$的二阶段最小二乘递推辨识算法如下:

$$\begin{aligned}\hat{\theta}_s(k) = \hat{\theta}_s(k-1)+K_s(k)[z(k)-\hat{\varphi}_n^{\rm T}(k)\hat{\theta}_n(k- \\ 1)-\hat{\varphi}_s^{\rm T}(k)\hat{\theta}_s(k-1)],\end{aligned} \tag{20}$$

$$K_s(k) = \frac{P_s(k-1)\hat{\varphi}_s^{\rm T}(k)}{[1+\hat{\varphi}_s^{\rm T}(k)P_s(k-1)\hat{\varphi}_s(k)]^{-1}}, \tag{21}$$

$$\begin{cases}P_s(k) = [I-K_s(k)\hat{\varphi}_s^{\rm T}(k)]P_s(k-1), \\ P_s(0) = P_0I,\end{cases} \tag{22}$$

$$\begin{aligned}\hat{\varphi}_s(k) = (&-\hat{x}(k-1), -\hat{x}(k-2),\cdots,-\hat{x}(k-n_{\rm a}), \\ &u(k-1), u(k-2),\cdots,u(k-n_{\rm b})),\end{aligned} \tag{23}$$

$$\begin{aligned}\hat{\theta}_n(k) = \hat{\theta}_n(k-1)+K_n(k)[z(k)-\hat{\varphi}_s^{\rm T}(k)\hat{\theta}_s - \\ \hat{\varphi}_n^{\rm T}(k)\hat{\theta}_n(k-1)]^{-1},\end{aligned} \tag{24}$$

$$K_n(k) = \frac{P_n(k-1)\hat{\varphi}_n(k)}{[1+\hat{\varphi}_n^{\mathrm{T}}(k)P_n(k-1)\hat{\varphi}_n(k)]^{-1}}, \tag{25}$$

$$\begin{cases} P_n(k) = [I - K_n(k)\hat{\varphi}_n^{\mathrm{T}}(k)]P_n(k-1), \\ P_n(0) = P_0 I, \end{cases} \tag{26}$$

$$\begin{aligned} \hat{\varphi}_n(k) = (&-\hat{w}(k-1), -\hat{w}(k-2), \cdots, -\hat{w}(k-n_{\mathrm{c}}), \\ &\hat{v}(k-1), \hat{v}(k-2), \cdots, \hat{v}(k-n_{\mathrm{d}})), \end{aligned} \tag{27}$$

$$\begin{cases} \hat{x}(k) = \hat{\varphi}_s^{\mathrm{T}}(k)\hat{\theta}_s, \\ \hat{w}(k) = z(k) - \hat{\varphi}_s^{\mathrm{T}}(k)\hat{\theta}_s, \\ \hat{v}(k) = z(k) - \hat{\varphi}^{\mathrm{T}}(k)\hat{\theta}, \end{cases} \tag{28}$$

$K_s(k)$和$K_n(k)$为增益向量, $P_s(k)$和$P_n(k)$为协方差矩阵.

计算$\hat{\theta}_s(k)$, $\hat{\theta}_n(k)$的二阶段最小二乘递推辨识算法步骤列举如图2所示.

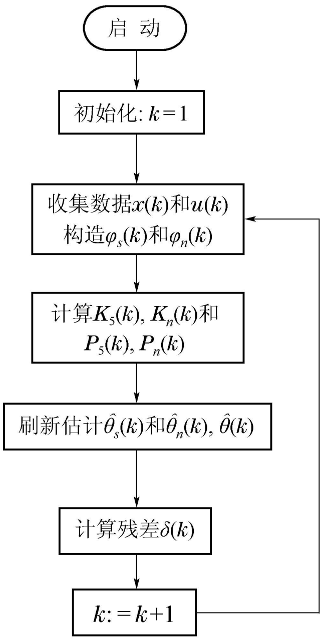

图 2 计算RLS参数估计$\hat{\theta}(t)$的流程图

Fig. 2 The flow chart of computing RLS parameter estimates $\hat{\theta}(t)$

为比较该算法, 该部分介绍递推增广最小二乘算法. 递推增广最小二乘算法是通过增加参数向量和信息向量的维数, 来处理受控自回归滑动平均模型(CARMA)中有色噪声的一种辨识方法, 即在信息向量中加入噪声回归项, 在参数向量中加入噪声模型的参数:

$$\begin{aligned} \theta := (&a_1, a_2, \cdots, a_{n_{\mathrm{a}}}, b_1, b_2, \cdots, b_{n_{\mathrm{b}}}, \\ &c_1, c_2, \cdots, c_{n_{\mathrm{c}}}, d_1, d_2, \cdots, d_{n_{\mathrm{d}}})^{\mathrm{T}} \end{aligned}$$

$$\begin{aligned} \varphi(k) :=& \\ &(-z(k-1), -z(k-2), \cdots, -z(k-n_{\mathrm{a}}), \\ &u(k-1), u(k-2), \cdots, u(k-n_{\mathrm{b}}), \\ &-w(k-1), -w(k-2), \cdots, -w(k-n_{\mathrm{c}}), \\ &v(k-1), v(k-2), \cdots, v(k-n_{\mathrm{d}})), \end{aligned}$$

$$J(\theta) := \sum_{j=1}^{k}[z(k) - \varphi^{\mathrm{T}}(k)\theta]^2,$$

$$\begin{aligned} \hat{\theta}(k) = &\hat{\theta}(k-1) + P(k)\hat{\varphi}(k)[z(k) - \\ &\hat{\varphi}^{\mathrm{T}}(k)\hat{\theta}(k-1)], \end{aligned}$$

$$P(k) = P(k-1) - \frac{P(k-1)\hat{\varphi}^{\mathrm{T}}(k)P(k-1)}{1+\hat{\varphi}^{\mathrm{T}}P(k-1)\hat{\varphi}(k)},$$

$$P(0) = p_0 I,$$

$$\hat{v}(k) = z(k) - \hat{\varphi}^{\mathrm{T}}(k)\hat{\theta}(k-1),$$

$$\begin{aligned} \hat{\varphi}(k) =& \\ &(-z(k-1), -z(k-2), \cdots, -z(k-n_{\mathrm{a}}), \\ &u(k-1), u(k-2), \cdots, u(k-n_{\mathrm{b}}), \\ &-\hat{w}(k-1), -\hat{w}(k-2), \cdots, -\hat{w}(k-n_{\mathrm{c}}), \\ &\hat{v}(k-1), \hat{v}(k-2), \cdots, \hat{v}(k-n_{\mathrm{d}})), \end{aligned}$$

$$\begin{aligned} \hat{\theta} = (&\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_{n_{\mathrm{a}}}, \hat{b}_1, \hat{b}_2, \cdots, \hat{b}_{n_{\mathrm{b}}}, \\ &\hat{c}_1, \hat{c}_2, \cdots, \hat{c}_{n_{\mathrm{c}}}, \hat{d}_1, \hat{d}_2, \cdots, \hat{d}_{n_{\mathrm{d}}})^{\mathrm{T}}. \end{aligned}$$

4 仿真例子(Simulation example)

考虑OEARMA模型描述的仿真对象如下:

$$z(k) = \frac{B(z)}{A(z)}u(k) + \frac{D(z)}{C(z)}v(k),$$

$$w(k) = \frac{D(z)}{C(z)}v(k),$$

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} = 1 + 1.60z^{-1} + 0.8z^{-2},$$

$$B(z) = b_1 z^{-1} + b_2 z^{-2} = 0.412z^{-1} + 0.309z^{-2},$$

$$C(z) = 1 + c_1 z^{-1} = 1 + 0.8z^{-1},$$

$$D(z) = 1 + d_1 z^{-1} = 1 - 0.64z^{-1}.$$

模型参数

$$\theta^{\mathrm{T}} = (1.60, 0.8, 0.412, 0.309, 0.8, -0.64).$$

仿真时, 输入$\{u(k)\}$采用零均值单位方差不相关可测随机信号序列, $\{v(k)\}$采用零均值方差为σ^2的白噪声序列.

在不同噪声方差参数σ^2的RLS估计误差随时间k的变化曲线如图3所示.

该RLS算法与递推增广最小二乘算法RELS在噪声方差为$\sigma^2 = 0.4^2$和噪信比为$\delta_{ns} = 144.94\%$下的残差仿真对比如图4所示.

数据对比如表1和表2所示, 计算量对比如表3所示.

1) 随着噪声均方差σ^2的减小, 参数估计的精度逐渐提高, 可参考图3来对照;

2) 随着数据长度的增加, 参数估计误差越来越小, 并且在图4中可以直观的看到RLS算法明显优于RELS算法;

3) 参考表3可知RLS算法的计算量相对于RELS算法得到大大的简化, 其中$n = n_{\mathrm{a}} + n_{\mathrm{b}} + n_{\mathrm{c}} + n_{\mathrm{d}}$.

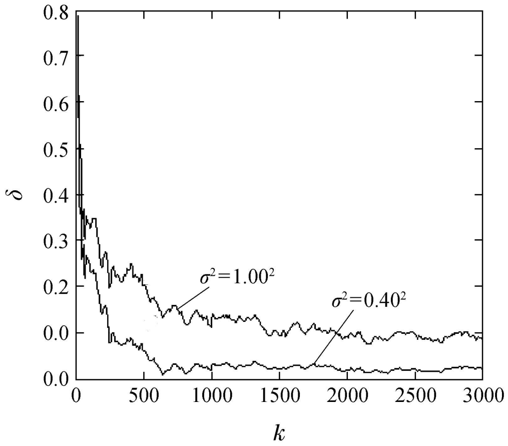

图 3 参数估计误差随时间k的变化($\sigma^2=1.00^2$, $\sigma^2=0.40^2$)

Fig. 3 The RLS paramater estimation errors σ versus k ($\sigma^2=1.00^2$ and $\sigma^2=0.40^2$)

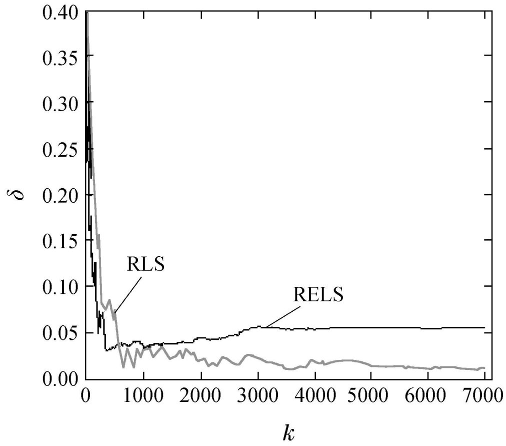

图 4 RLS和RELS在相同条件下的误差($\sigma^2=0.40^2$)对比图

Fig. 4 The parameter estimation errors versus k for the algorithms ($\sigma^2=0.40^2$)

表 1 RLS算法的参数估计和残差

Table 1 The RLS estimates and errors

k	a_1	a_2	b_1	b_2	c_1	d_1	δ/%
100	1.20260	0.50691	0.44384	0.10396	0.79078	– 0.61422	25.24060
200	1.38256	0.62701	0.43945	0.20664	0.80789	– 0.73193	14.65398
500	1.50781	0.73472	0.41297	0.25746	0.82262	– 0.63651	5.94194
1000	1.56554	0.77144	0.42204	0.29850	0.79953	– 0.61600	2.48625
2000	1.56720	0.77511	0.40969	0.30610	0.79876	– 0.63711	1.95130
3000	1.57315	0.77594	0.41251	0.30417	0.79270	– 0.67137	2.28639
4000	1.61920	0.81367	0.40720	0.32474	0.79790	– 0.66544	1.80940
5000	1.60369	0.79932	0.40738	0.31820	0.79974	– 0.66888	1.45374
6000	1.61022	0.80409	0.40868	0.32003	0.80252	– 0.65743	1.11775
7000	1.60706	0.80172	0.41220	0.31841	0.80221	– 0.66047	1.11907
真值	1.60	0.80	0.412	0.309	0.80	– 0.64	0

表 2 RELS算法的参数估计和残差

Table 2 The RELS estimates and errors

k	a_1	a_2	b_1	b_2	c_1	d_1	δ/%
100	1.64648	0.77137	0.42953	0.30980	0.54707	– 0.60255	12.33223
200	1.63711	0.78087	0.41288	0.32958	0.68643	– 0.68990	6.23565
500	1.59082	0.74141	0.40771	0.28768	0.77035	– 0.65435	3.35251
1000	1.58187	0.73913	0.42097	0.30178	0.76132	– 0.65371	3.60007
2000	1.56228	0.72573	0.40866	0.30378	0.78236	– 0.68412	4.52285
3000	1.55633	0.72314	0.41106	0.29813	0.78266	– 0.72008	5.69570
4000	1.54884	0.72009	0.40550	0.29869	0.80517	– 0.70123	5.35111
5000	1.54388	0.71793	0.40621	0.29756	0.81598	– 0.70100	5.57394
6000	1.54133	0.71753	0.40815	0.29671	0.82358	– 0.68668	5.39588
7000	1.53665	0.71462	0.41102	0.29458	0.82989	– 0.68782	5.70455
真值	1.60	0.80	0.412	0.309	0.80	– 0.64	0

表 3 RLS算法与RELS算法的计算量对比

Table 3 Comparison of the computational efficiency of the RLS and RELS algorithms

算法	乘法次数	加法次数	总次数
RLS	$2(n_\mathrm{a}+n_\mathrm{b})^2+2(n_\mathrm{c}+n_\mathrm{d})^2+4n$	$2(n_\mathrm{a}+n_\mathrm{b})^2+2(n_\mathrm{c}+n_\mathrm{d})^2+2n$	$4(n_\mathrm{a}+n_\mathrm{b})^2+4(n_\mathrm{c}+n_\mathrm{d})^2+6n$
RELS	$2n^2+4n$	$2n^2+2n$	$4n^2+6n$

5 总结(Conclusion)

本文借助辅助辨识模型思想和分解技术, 推导出了一类广义输出误差模型的两阶段递推最小二乘参数估计算法. 该算法的推导过程简单、计算量少、精度高. 理论分析和仿真结果验证了这些结论.

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作者简介:

贾 杰 (1972–), 男, 博士, 副教授, 主要研究方向为飞行器制导与控制、非线性系统建模与故障检测等, E-mail: jiajie757@126.com;

陈 晨 (1988–), 女, 硕士研究生, 主要研究方向为成像目标自动检测与分类, E-mail: 15879123815@139.com;

曹 姣 (1988–), 女, 硕士研究生, 主要研究方向为多变量系统辨识, E-mail: caojiao0121@126.com;

罗小娜 (1988–), 女, 硕士研究生, 主要研究方向为空间绳网捕获, E-mail: nhluoxiaona@163.com;

丁 锋 (1963–), 男, 博士生导师, 主要研究方向为系统辨识理论与方法、复杂工业过建模理论、多率系统辨识与控制等, E-mail: fding@jiangnan.edu.cn.