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随机马尔科夫跳跃系统有限时间控制

张维海[1†], 刘鹤鸣[2]

(1. 山东科技大学 电气与自动化工程学院, 山东 青岛 266590; 2. 山东科技大学 数学与系统科学学院, 山东 青岛 266590)

摘要: 研究了转移概率部分信息未知的随机马尔科夫跳跃系统的有限时间控制问题. 首先, 介绍了有限时间随机稳定性与随机镇定性的概念; 然后, 利用矩阵变换、数学期望以及Gronwall不等式等方法, 给出了系统为有限时间随机稳定的判定准则. 利用上述结果, 得出了系统状态输出反馈随机镇定的充分条件; 进一步, 考虑到实际工程中系统状态的不完全可测性, 给出了保证系统有限时间随机镇定的动态输出反馈控制器设计方式, 并求得了此控制器存在的判定条件. 最后, 用一个数值算例说明了控制器设计方法的有效性.

关键词: 有限时间稳定性; 马尔科夫跳跃; 转移概率部分未知

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Finite-time control of stochastic Markovian jump systems

ZHANG Wei-hai[1†], LIU He-ming[2]

(1. College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao Shandong 266590, China;
2. College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao Shandong 266590, China)

Abstract: This paper considers the finite-time control problem for stochastic Markovian jump system subject to partial information on transition probabilities. Firstly, the concepts of finite-time stochastic stability and stabilization for stochastic Markovian jump systems are defined. Secondly, with the aid of matrix transformations, mathematical expectation and Gronwall inequality, a practical criterion, under which the stochastic systems are finite-time stochastically stable, is given. After that, a sufficient condition for stochastic stabilization via state feedback is obtained. Considering that the state of the system in practical engineering is often with partial information, we develop a dynamic output feedback controller which ensures the stochastic system to be finite-time stochastically stabilizable. The testing condition for the existence of such a controller is proved. Finally, a numerical example is given to show the validity of the designed method.

Key words: finite-time stability; Markovian jumps; partly unknown transition probabilities

1 引言(Introduction)

近年来, 有限时间控制正受到学者越来越多的关注, 已成为控制理论界热门的研究方向之一. 所谓有限时间稳定性, 粗略地讲, 是指在给定的时间区间上, 系统轨道应满足预定的要求[1–2]. 文献[3–4]重新给出了有限时间稳定性的定义, 并给出了状态反馈控制器与动态输出反馈控制器存在的条件. 随后, 有限时间稳定性被进一步研究, 文献[5]研究了随机系统的有限时间控制问题并给出了状态反馈控制器存在的条件, 文献[6–7]分别研究了基于有限时间的随机系统H_∞控制与滤波问题, 文献[8]研究了切换系统的有限稳定与镇定, 并给出了状态反馈控制器存在的充分条件. 文献[9–10]分别研究了马尔科夫跳变系统的有限时间稳定性与镇定问题. 然而, 上述有限时间稳定性[1–10]仅仅考虑了系统轨道的上界, 没有考虑其下界. 在许多实际问题中, 系统轨道的下界也有明显的物理意义, 基于这些实际问题, 文献[11]对随机系统给出了新的有限时间稳定性概念, 其涵义是在给定的时间区间上, 系统的轨道被限制在一个环域上. 随后, 文献[12]研究了伊藤型随机奇异系统的有限时间稳定性与镇定问题, 并给出了伊藤型随机奇异系统解的存在唯一性条件.

本文主要讨论了转移概率部分未知的随机马尔科夫跳跃系统的有限时间稳定性与镇定问题. 第2节介绍了稳定性与镇定性的新定义和一些预备知识. 在第3节中, 利用矩阵不等式方法, 给出了系统有限时间随机稳定性与镇定性的充分条件. 第4节考虑了系统的状态不能观测情形, 设计了动态输出反馈控制器,

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†通信作者. E-mail: w_hzhang@163.com; Tel.: +86 532-80691786.
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并给出了控制器存在的条件. 在第5节中, 用数值算例来说明文中方法的有效性. 最后对本文进行了总结.

为了方便, 文中采用以下记号: $\mathbb{R}^n$: n维的欧氏空间; $I_{n\times n}$: n阶单位矩阵; A^{T}: 矩阵或向量A的转置; $A>0(A\geqslant 0)$: A是正定(半正定)对称矩阵; $\mathrm{E}[\cdot]$: 数学期望算子; $\lambda_{\max}(A)(\lambda_{\min}(A))$: 矩阵$A$的最大(最小)特征值; $(\varOmega,\mathcal{F},\mathcal{P})$: 完备概率空间; $C^2(\mathbb{R}^n\times D;\mathbb{R}^+)$: 关于向量$\boldsymbol{x}$为二次可微的非负函数$f(\boldsymbol{x}(t),i)$的集合.

2 预备知识(Preliminaries)

考虑概率空间$(\varOmega,\mathcal{F},\mathcal{P})$上的马尔科夫跳跃随机系统

$$\begin{aligned}\mathrm{d}\boldsymbol{x}(t)=&[A_{r(t)}\boldsymbol{x}(t)+B_{r(t)}\boldsymbol{u}(t)]\mathrm{d}t+\\&[C_{r(t)}\boldsymbol{x}(t)+D_{r(t)}\boldsymbol{u}(t)]\mathrm{d}w(t),\end{aligned}\tag{1}$$

其中: $\boldsymbol{x}(t)\in\mathbb{R}^n, \boldsymbol{u}(t)\in\mathbb{R}^m, r(t)$分别是系统状态、控制输入、系统模态. 不失一般性, 这里假设$w(t)$是一维标准维纳过程. $\{r(t),t\geqslant 0\}$是一个时间连续的齐次马尔科夫过程, 且在集合D={1, 2, $\cdots$, N}中取值. 此外, 模态$r(t)$也满足

$$\mathrm{P}\{r(t+\varDelta)=\frac{j}{r(t)}=i\}=\begin{cases}q_{ij}\varDelta+o(\varDelta), & \text{如果}j\neq i,\\ 1+q_{ii}\varDelta+o(\varDelta), & \text{如果}j=i,\end{cases}$$

其中: $\varDelta>0$, $o(\varDelta)$为$\varDelta$的高阶无穷小量, 对于模态$r(t)$, 当$j{\neq}i$有$q_{ij}\geqslant 0$, $q_{ii}=-\sum\limits_{j\neq i}q_{ij}$. 取

$$\varLambda=\begin{bmatrix}q_{11} & q_{12} & \cdots & q_{1N}\\ q_{21} & q_{22} & \cdots & q_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ q_{N1} & q_{N2} & \cdots & q_{NN}\end{bmatrix}$$

为系统的转移概率矩阵, 并假设$N=4$, 则有

$$\varLambda=\begin{bmatrix}q_{11} & \widehat{q}_{12} & \widehat{q}_{13} & q_{14}\\ \widehat{q}_{21} & \widehat{q}_{22} & q_{23} & \widehat{q}_{24}\\ q_{31} & q_{32} & q_{33} & q_{34}\\ \widehat{q}_{41} & q_{42} & \widehat{q}_{43} & q_{44}\end{bmatrix},$$

其中$\widehat{q}_{ij}$表示该元素为未知元素. 参考文献[16], 引入下列记号:

$$q_k^i=\sum_{j\in D_k^i}q_{ij},\ P_k^i=\sum_{j\in D_k^i}q_{ij}P_j,\ D^i=D_k^i+D_{uk}^i,$$

其中: P_i为正定矩阵,

$$D_k^i\triangleq\{j:q_{ij}\text{为已知的}\},$$
$$D_{uk}^i\triangleq\{j:\widehat{q}_{ij}\text{为未知的}\}.$$

另外, 若D_k^i非空, 则它可以表示为

$$D_k^i=(\kappa_1^i,\kappa_2^i,\cdots,\kappa_m^i),\ 1\leqslant m\leqslant N,$$

其中κ_m^i代表转移概率矩阵$\varLambda$中第i行已知元素. 如果q_{ii}未知, 即$i\in D_{uk}^i$, 假设q_{d}^i为$\widehat{q}_{ii}$的下界, 则有$q_{\mathrm{d}}^i\leqslant\widehat{q}_{ii}<-q_k^i$.

注 1 系统(1)是转移概率部分未知的随机马尔科夫跳变系统, 这类系统在实际中经常遇到, 如网络通信中的数据包信息漏失等问题, 文献[13–16]针对此类问题做了研究.

定义 1[11] 给定对称矩阵$R>0$以及正数T, $0<c_4<c_3<c_1<c_2$, 系统

$$\mathrm{d}\boldsymbol{x}(t)=A_{r(t)}\boldsymbol{x}(t)\mathrm{d}t+C_{r(t)}\boldsymbol{x}(t)\mathrm{d}w\tag{2}$$

是有限时间随机稳定的, 如果对于$t\in[0,T]$, 都有

$$\begin{aligned}&c_3\leqslant\boldsymbol{x}^{\mathrm{T}}(0)R\boldsymbol{x}(0)\leqslant c_1\Rightarrow\\&c_4<\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)]<c_2.\end{aligned}$$

定义 2[11] 系统(1)是有限时间随机镇定的, 如果存在一个反馈控制器$\boldsymbol{u}(t)=K_{r(t)}\boldsymbol{x}(t)$, 使得系统

$$\begin{aligned}\mathrm{d}\boldsymbol{x}(t)=&[A_{r(t)}+B_{r(t)}K_{r(t)}]\boldsymbol{x}(t)\mathrm{d}t+\\&[C_{r(t)}+D_{r(t)}K_{r(t)}]\boldsymbol{x}(t)\mathrm{d}w(t)\end{aligned}$$

是有限时间随机稳定的.

引理 1[17] 对系统(2)给定$V(\boldsymbol{x}(t),i)\in C^2(\mathbb{R}^n\times D;\mathbb{R}^+)$, 定义线性算子$\mathcal{L}V$, 即

$$\begin{aligned}&\mathcal{L}V(\boldsymbol{x}(t),i)=\\&V_{\mathrm{t}}(\boldsymbol{x}(t),i)+V_{\mathrm{x}}(\boldsymbol{x}(t),i)A_i\boldsymbol{x}(t)+\\&2^{-1}\mathrm{tr}[(C_i\boldsymbol{x}(t))^{\mathrm{T}}V_{\mathrm{xx}}(\boldsymbol{x}(t),i)C_i\boldsymbol{x}(t)]+\\&\sum_{j=1}^{N}q_{ij}V(\boldsymbol{x}(t),j),\end{aligned}$$

其中:

$$\begin{aligned}&V_{\mathrm{t}}(\boldsymbol{x}(t),i)=\frac{\partial V(\boldsymbol{x}(t),i)}{\partial t},\\&V_{\mathrm{x}}(\boldsymbol{x}(t),i)=(\frac{\partial V(\boldsymbol{x}(t),i)}{\partial x_1},\cdots,\frac{\partial V(\boldsymbol{x}(t),i)}{\partial x_n}),\\&V_{\mathrm{xx}}(\boldsymbol{x}(t),i)=(\frac{\partial^2V(\boldsymbol{x}(t),i)}{\partial x_i\partial x_j})_{n\times n},\ i\in D.\end{aligned}$$

引理 2(Gronwall不等式) 给定一些标量$a>0$, $b>0$以及非负的函数$\theta(t)$, 如果

$$\theta(t)\leqslant a+b\int_0^t\theta(s)\mathrm{d}s,\ 0\leqslant t\leqslant T,$$

则

$$\theta(t)\leqslant a\mathrm{e}^{bt},\ 0\leqslant t\leqslant T.$$

引理 3[11] 给定一些标量$a>0$, $b>0$以及非负的函数$\theta(t)$, 如果

$$\theta(t)\geqslant a+b\int_0^t\theta(s)\mathrm{d}s,\ 0\leqslant t\leqslant T,$$

则有

$$\theta(t)\geqslant a\mathrm{e}^{bt},\ 0\leqslant t\leqslant T.$$

3 状态反馈镇定(Stabilization via state feedback)

本节的主要目的是寻求如下状态反馈控制器:

$$\boldsymbol{u}(t)=K_i\boldsymbol{x}(t), \tag{3}$$

用以镇定系统(1). 为此, 先给出系统(2)有限时间随机稳定的充分条件.

定理 1 给定两个正数$\alpha\geqslant 0$, $\beta\geqslant 0$. 系统(2)是关于(c_1,c_2,c_3,c_4,T,R)有限时间随机稳定的, 如果存在两个正数λ_1,λ_2以及正定的对称矩阵X_i满足下列不等式:

$$\begin{bmatrix}\varOmega_{11}+q_{ii}\tilde X_i & * & * & *\\ C_i\tilde X_i & -\tilde X_i & * & *\\ \varOmega_{31} & 0 & -\varOmega_{33} & *\\ \sqrt{-q_k^i}\tilde X_i & 0 & 0 & -\tilde X_j\end{bmatrix}<0, \tag{4}$$

$\forall i\in D_k^i,\ j\in D_{uk}^i$,

$$\begin{bmatrix}\varPi_{11}-q_{ii}\tilde X_i & * & * & *\\ C_i\tilde X_i & -\tilde X_i & * & *\\ \varOmega_{31} & 0 & -\varOmega_{33} & *\\ \sqrt{-q_k^i}\tilde X_i & 0 & 0 & -\tilde X_j\end{bmatrix}<0, \tag{5}$$

$\forall i\in D_k^i,\ j\in D_{uk}^i$,

$$\begin{bmatrix}\varOmega_{11}+q_{\rm d}^i\tilde X_i & * & * & *\\ C_i\tilde X_i & -\tilde X_i & * & *\\ \varOmega_{31} & 0 & -\varOmega_{33} & *\\ \sqrt{-q_{\rm d}^i-q_k^i}\tilde X_i & 0 & 0 & -\tilde X_j\end{bmatrix}<0, \tag{6}$$

$\forall i\in D_{uk}^i,\ j\in D_{uk}^i$,

$$\begin{bmatrix}\varPi_{11}-q_{\rm d}^i\tilde X_i & * & * & *\\ C_i\tilde X_i & -\tilde X_i & * & *\\ \varOmega_{31} & 0 & -\varOmega_{33} & *\\ \sqrt{-q_{\rm d}^i-q_k^i}\tilde X_i & 0 & 0 & -\tilde X_j\end{bmatrix}<0, \tag{7}$$

$\forall i\in D_{uk}^i,\ j\in D_{uk}^i$,

$$c_1\lambda_2-c_2\lambda_1{\rm e}^{-\alpha T}<0, \tag{8}$$

$$\lambda_2c_4-\lambda_1c_3<0, \tag{9}$$

$$\lambda_1I<X_i<\lambda_2I, \tag{10}$$

其中:

$$\varOmega_{11}=\tilde X_iA_i^{\rm T}+A_i\tilde X_i-\alpha\tilde X_i,$$

$$\varPi_{11}=\beta\tilde X_i-\tilde X_iA_i^{\rm T}-A_i\tilde X_i,$$

$$\varOmega_{31}=[\sqrt{q_{i\kappa_1^i}}\tilde X_i\ \ \cdots\ \ \sqrt{q_{i\kappa_m^i}}\tilde X_i]^{\rm T},\ \kappa_m^i\neq i,$$

$$\varOmega_{33}={\rm diag}\{\tilde X_{\kappa_1^i},\cdots,\tilde X_{\kappa_m^i}\},\ \kappa_m^i\neq i,$$

$$\tilde X_i=R^{-1/2}X_iR^{-1/2}.$$

证 首先证明

$$\boldsymbol{x}^{\rm T}(0)R\boldsymbol{x}(0)\leqslant c_1\Rightarrow {\rm E}[\boldsymbol{x}^{\rm T}(t)R\boldsymbol{x}(t)]<c_2.$$

取二次函数

$$V(\boldsymbol{x}(t),i)=\boldsymbol{x}^{\rm T}(t)P_i\boldsymbol{x}(t),$$

其中$P_i=\tilde X_i^{-1}$.

根据引理1, 得到

$$\mathcal{L}V(\boldsymbol{x}(t),i)-\alpha V(\boldsymbol{x}(t),i)=\boldsymbol{x}^{\rm T}(t)Q\boldsymbol{x}(t),$$

其中

$$Q=A_i^{\rm T}P_i+P_iA_i+C_i^{\rm T}P_iC_i+\sum_{j\in D_k^i}q_{ij}P_j+\sum_{j\in D_{uk}^i}\widehat q_{ij}P_j-\alpha P_i.$$

当$i\in D_k^i$, 根据模态性质有

$$0\leqslant\frac{\widehat q_{ij}}{-q_k^i}\leqslant 1,\ \sum_{j\in D_{uk}^i}\frac{\widehat q_{ij}}{-q_k^i}=1.$$

因此, 得到

$$Q=\sum_{j\in D_{uk}^i}\frac{\widehat q_{ij}}{-q_k^i}\{A_i^{\rm T}P_i+P_iA_i+C_i^{\rm T}P_iC_i-\alpha P_i+P_k^i-q_k^iP_j\}.$$

所以$Q<0$等价于

$$A_i^{\rm T}P_i+P_iA_i+C_i^{\rm T}P_iC_i-\alpha P_i+P_k^i-q_k^iP_j<0.$$

对上式左右两边同乘以$\tilde X_i$利用Schur补, 可知上式等价于(4).

当$i\in D_{uk}^i$时, $\widehat q_{ii}$是未知的. 类似地, 根据模态性质, 有

$$0\leqslant\frac{\widehat q_{ij}}{-\widehat q_{ii}-q_k^i}\leqslant 1,\ \sum_{j\in D_{uk}^i,i\neq j}\frac{\widehat q_{ij}}{-\widehat q_{ii}-q_k^i}=1.$$

因此有

$$Q=\sum_{j\in D_{uk}^i,i\neq j}\frac{\widehat q_{ij}}{-\widehat q_{ii}-q_k^i}\{A_i^{\rm T}P_i+P_iA_i+C_i^{\rm T}P_iC_i-\alpha P_i+P_k^i+\widehat q_{ii}P_i-\widehat q_{ii}P_j-q_k^iP_j\}.$$

所以$Q<0$等价于

$$A_i^{\rm T}P_i+P_iA_i+C_i^{\rm T}P_iC_i-\alpha P_i+P_k^i+\widehat q_{ii}P_i-\widehat q_{ii}P_j-q_k^iP_j<0.$$

又$q_{\rm d}^i$为$\widehat q_{ii}$的一个下界, 这意味着对于无限小$\varepsilon<0$, $\widehat q_{ii}$可在区间$[q_{\rm d}^i,-q_k^i+\varepsilon]$中取值. 这时可以把$\widehat q_{ii}$写作一个凸组合

$$\widehat q_{ii}=-\theta q_k^i+\theta\varepsilon+(1-\theta)q_{\rm d}^i,$$

其中θ可以在区间$[0,1]$任意取值. 因此, $Q<0$对$\forall j\in D_{uk}^i, i\neq j$应该同时满足

$$A_i^{\rm T}P_i+P_iA_i+C_i^{\rm T}P_iC_i-\alpha P_i+P_k^i-q_k^iP_i+\varepsilon(P_i-P_j)+q_k^iP_j-q_k^iP_j<0,$$

$$A_i^{\rm T}P_i+P_iA_i+C_i^{\rm T}P_iC_i-\alpha P_i+q_{\rm d}^iP_i+P_k^i-q_{\rm d}^iP_j-q_k^iP_j<0.$$

因为ε可以无穷小, 因此上面第1式等价于

$$A_i^{\mathrm{T}}P_i + P_iA_i + C_i^{\mathrm{T}}P_iC_i - \alpha P_i + P_k^i - q_k^iP_i < 0,$$

当$i=j$时, 又等价于上面第2式. 对上面第2式左右两边都乘以$\tilde{X}_i$并利用Schur补, 得到上式等价于式(6).

综上所述, 式(4)和式(6)保证了$Q<0$, 也就是

$$\mathcal{L}V(\boldsymbol{x}(t),i) < \alpha V(\boldsymbol{x}(t),i). \tag{11}$$

先对式(11)两边同时从0到t进行积分, 再取数学期望, 得出

$$\mathrm{E}V(\boldsymbol{x}(t),i) < V(\boldsymbol{x}(0),i) + \alpha\int_0^t \mathrm{E}V(\boldsymbol{x}(s),i)\mathrm{d}s.$$

利用引理2, 得到

$$\mathrm{E}V(\boldsymbol{x}(t),i) < V(\boldsymbol{x}(0),i)\mathrm{e}^{\alpha t}. \tag{12}$$

又根据本定理中的一些已知条件, 可以得出

$$\begin{aligned}&V(\boldsymbol{x}(0),i)\mathrm{e}^{\alpha t} \leqslant \\ &\lambda_{\max}(X_i^{-1})\boldsymbol{x}^{\mathrm{T}}(0)R\boldsymbol{x}(0)\mathrm{e}^{\alpha t} < c_1\mathrm{e}^{\alpha t}\lambda_1,\end{aligned} \tag{13}$$

$$\begin{aligned}&\mathrm{E}V(\boldsymbol{x}(t),i) = \\ &\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)\tilde{X}_i^{-1}\boldsymbol{x}(t)] > \frac{\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)]}{\lambda_2}.\end{aligned} \tag{14}$$

根据式(12)−(14), 有

$$\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)] < \frac{\lambda_2c_1\mathrm{e}^{\alpha T}}{\lambda_1}, \tag{15}$$

从式(15)和式(8), 得到$\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)] < c_2$.

其次证明

$$c_3 \leqslant \boldsymbol{x}^{\mathrm{T}}(0)R\boldsymbol{x}(0) \Rightarrow c_4 < \mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)].$$

类似地, 可得

$$\mathcal{L}V(\boldsymbol{x}(t),i) - \beta V(\boldsymbol{x}(t),i) = \boldsymbol{x}^{\mathrm{T}}(t)H\boldsymbol{x}(t),$$

其中

$$\begin{aligned}H =\,&\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - C_i^{\mathrm{T}}P_iC_i - \\ &\sum_{j\in D_k^i} q_{ij}P_j - \sum_{j\in D_{uk}^i}\widehat{q}_{ij}P_j.\end{aligned}$$

当$i\in D_k^i$, 同理可以得到

$$\begin{aligned}H = &\sum_{j\in D_{uk}^i}\frac{\widehat{q}_{ij}}{-q_k^i}\{\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - \\ &C_i^{\mathrm{T}}P_iC_i - P_k^i + q_k^iP_j\}.\end{aligned}$$

所以$H<0$等价于

$$\begin{aligned}&\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - C_i^{\mathrm{T}}P_iC_i - P_k^i + \\ &q_k^iP_j < 0.\end{aligned} \tag{16}$$

又下面不等式:

$$\begin{aligned}&\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i + C_i^{\mathrm{T}}P_iC_i - \\ &q_{ii}P_i + (P_k^i - q_{ii}P_i) - q_k^iP_j < 0,\end{aligned} \tag{17}$$

意味着式(16)成立. 对式(17)左右两边都乘以$\tilde{X}_i$并利用Schur补, 得到上式等价于条件(5).

当$i\in D_{uk}^i$时$\widehat{q}_{ii}$是未知的, 同理有

$$\begin{aligned}H = &\sum_{j\in D_{uk}^i, i\neq j}\frac{\widehat{q}_{ij}}{-\widehat{q}_{ii}-q_k^i}\{\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - \\ &C_i^{\mathrm{T}}P_iC_i - P_k^i - \widehat{q}_{ii}P_i + \widehat{q}_{ii}P_j + q_k^iP_j\}.\end{aligned}$$

所以$H<0$等价于

$$\begin{aligned}&\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - C_i^{\mathrm{T}}P_iC_i - P_k^i - \\ &\widehat{q}_{ii}P_i + \widehat{q}_{ii}P_j + q_k^iP_j < 0.\end{aligned}$$

又q_{d}^i为$\widehat{q}_{ii}$的一个下界, 这意味着对于一些$\varepsilon<0$, $\widehat{q}_{ii}$可在区间$[q_{\mathrm{d}}^i, -q_k^i+\varepsilon]$中取值. 这时可以把$\widehat{q}_{ii}$写作一个凸组合

$$\widehat{q}_{ii} = -\theta q_k^i + \theta\varepsilon + (1-\theta)q_{\mathrm{d}}^i,$$

其中θ可以在区间$[0,1]$任意取值. 因此, $H<0$对$\forall j\in D_{uk}^i, i\neq j$应该同时满足

$$\begin{aligned}&\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - C_i^{\mathrm{T}}P_iC_i - P_k^i + \\ &q_k^iP_i - \varepsilon(P_i - P_j) - q_k^iP_j + q_k^iP_j < 0,\end{aligned} \tag{18}$$

$$\begin{aligned}&\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - C_i^{\mathrm{T}}P_iC_i - q_{\mathrm{d}}^iP_i - \\ &P_k^i + q_{\mathrm{d}}^iP_j + q_k^iP_j < 0.\end{aligned} \tag{19}$$

因为ε可以无穷小, 因此式(18)等价于

$$\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i - C_i^{\mathrm{T}}P_iC_i - P_k^i + q_k^iP_i < 0,$$

当$i=j$时, 又等价于式(19). 由下面不等式

$$\begin{aligned}&\beta P_i - A_i^{\mathrm{T}}P_i - P_iA_i + C_i^{\mathrm{T}}P_iC_i - q_{\mathrm{d}}^iP_i + P_k^i - \\ &q_{\mathrm{d}}^iP_j - q_k^iP_j < 0,\end{aligned} \tag{20}$$

可知式(19)成立. 对式(20)左右两边都乘以$\tilde{X}_i$并利用Schur补, 得到上式等价于式(7). 综上所述, 式(5)和式(7)保证了$H<0$, 从而

$$\mathcal{L}V(\boldsymbol{x}(t),i) > \beta V(\boldsymbol{x}(t),i).$$

先对上式的两边同时求从0到t的积分, 再取数学期望, 有

$$\mathrm{E}V(\boldsymbol{x}(t),i) > V(\boldsymbol{x}(0),i) + \beta\int_0^t \mathrm{E}V(\boldsymbol{x}(s),i)\mathrm{d}s.$$

利用引理3, 得到

$$\mathrm{E}V(\boldsymbol{x}(t),i) > V(\boldsymbol{x}(0),i)\mathrm{e}^{\beta t}. \tag{21}$$

又根据本定理中的一些已知条件, 可以得出

$$\begin{aligned}&V(\boldsymbol{x}(0),i)\mathrm{e}^{\beta t} \geqslant \\ &\lambda_{\min}(X_i^{-1})\boldsymbol{x}^{\mathrm{T}}(0)R\boldsymbol{x}(0)\mathrm{e}^{\beta t} > \frac{c_3}{\lambda_2},\end{aligned} \tag{22}$$

$$\begin{aligned}&\mathrm{E}V(\boldsymbol{x}(t),i) = \\ &\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)\tilde{X}_i^{-1}\boldsymbol{x}(t)] \leqslant \frac{\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)]}{\lambda_1}.\end{aligned} \tag{23}$$

根据式(21)−(23), 有

$$\mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)] > \frac{\lambda_1c_3}{\lambda_2}. \tag{24}$$

从式(24)和式(9)得到$c_4 < \mathrm{E}[\boldsymbol{x}^{\mathrm{T}}(t)R\boldsymbol{x}(t)]$.

应用上面这个结论, 可以得到系统(1)的有限时间随机镇定的充分条件, 也就是下面这个定理.

定理 2 给定标量$\alpha \geqslant 0$, $\beta \geqslant 0$, 系统(1)是关于$(c_1, c_2, c_3, c_4, T, R)$有限时间随机镇定的, 如果存在两个正标量$\lambda_1, \lambda_2$, 正定的对称矩阵$X_i$以及矩阵$T_i$满足下列不等式:

$$\begin{bmatrix} \varOmega_{11}+q_{ii}\tilde{X}_i & * & * & * \\ C_i\tilde{X}_i+D_iT_i & -\tilde{X}_i & * & * \\ \varOmega_{31} & 0 & -\varOmega_{33} & * \\ \sqrt{-q_k^i}\tilde{X}_i & 0 & 0 & -\tilde{X}_j \end{bmatrix} < 0, \tag{25}$$

$\forall i \in D_k^i,\ j \in D_{uk}^i$,

$$\begin{bmatrix} \varPi_{11}-q_{ii}\tilde{X}_i & * & * & * \\ C_i\tilde{X}_i+D_iT_i & -\tilde{X}_i & * & * \\ \varOmega_{31} & 0 & -\varOmega_{33} & * \\ \sqrt{-q_k^i}\tilde{X}_i & 0 & 0 & -\tilde{X}_j \end{bmatrix} < 0, \tag{26}$$

$\forall i \in D_k^i,\ j \in D_{uk}^i$,

$$\begin{bmatrix} \varOmega_{11}+q_{\mathrm{d}}^i\tilde{X}_i & * & * & * \\ C_i\tilde{X}_i+D_iT_i & -\tilde{X}_i & * & * \\ \varOmega_{31} & 0 & -\varOmega_{33} & * \\ \sqrt{-q_{\mathrm{d}}^i-q_k^i}\tilde{X}_i & 0 & 0 & -\tilde{X}_j \end{bmatrix} < 0, \tag{27}$$

$\forall i \in D_{uk}^i,\ j \in D_{uk}^i$,

$$\begin{bmatrix} \varPi_{11}-q_{\mathrm{d}}^i\tilde{X}_i & * & * & * \\ C_i\tilde{X}_i+D_iT_i & -\tilde{X}_i & * & * \\ \varOmega_{31} & 0 & -\varOmega_{33} & * \\ \sqrt{-q_{\mathrm{d}}^i-q_k^i}\tilde{X}_i & 0 & 0 & -\tilde{X}_j \end{bmatrix} < 0, \tag{28}$$

$\forall i \in D_{uk}^i,\ j \in D_{uk}^i$,

$$c_1\lambda_2 - c_2\lambda_1\mathrm{e}^{-\alpha T} < 0, \tag{29}$$

$$\lambda_2 c_4 - \lambda_1 c_3 < 0, \tag{30}$$

$$\lambda_1 I < X_i < \lambda_2 I, \tag{31}$$

其中:

$$\varOmega_{11} = \tilde{X}_i A_i^{\mathrm{T}} + A_i\tilde{X}_i + B_iT_i + T_i^{\mathrm{T}}B_i^{\mathrm{T}} - \alpha\tilde{X}_i,$$
$$\varPi_{11} = \beta\tilde{X}_i - \tilde{X}_iA_i^{\mathrm{T}} - A_i\tilde{X}_i - B_iT_i - T_i^{\mathrm{T}}B_i^{\mathrm{T}},$$
$$\varOmega_{31} = [\sqrt{q_{i\kappa_1^i}}\tilde{X}_i, \cdots, \sqrt{q_{i\kappa_m^i}}\tilde{X}_i]^{\mathrm{T}},\ \kappa_m^i \neq i,$$
$$\varOmega_{33} = \mathrm{diag}\{\tilde{X}_{\kappa_1^i}, \cdots, \tilde{X}_{\kappa_m^i}\},\ \kappa_m^i \neq i,$$
$$\tilde{X}_i = R^{-1/2}X_iR^{-1/2},\ K_i = T_i\tilde{X}_i^{-1}.$$

4 输出反馈镇定(Stabilization via output feedback)

本节设计了动态输出反馈控制器. 不失一般意义, 首先给出如下假设和可测输出$\boldsymbol{y}(t) = J_i\boldsymbol{x}(t)$.

假设 1 存在$\boldsymbol{u}(t) = K_i\boldsymbol{x}(t)$使得系统(1)满足定理2.

给出如下形式的动态输出反馈控制器:

$$\begin{cases} \mathrm{d}\hat{\boldsymbol{x}}(t) = \hat{A}_i\mathrm{d}t + \hat{C}_i\mathrm{d}w(t), \\ \boldsymbol{u}(t) = K_i\hat{\boldsymbol{x}}(t), \end{cases} \tag{32}$$

其中:

$$\hat{A}_i = A_i\boldsymbol{x}(t) + B_i\boldsymbol{u}(t) + L_i(\boldsymbol{y}(t) - J_i\hat{\boldsymbol{x}}(t)),$$
$$\hat{C}_i = C_i\boldsymbol{x}(t) + D_i\boldsymbol{u}(t) + L_i(\boldsymbol{y}(t) - J_i\hat{\boldsymbol{x}}(t)),$$

$\hat{\boldsymbol{x}}(t)$是系统的预测轨迹, L_i是适当维数的反馈增益矩阵. 令$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$, 则可得误差系统

$$\begin{aligned} \mathrm{d}\boldsymbol{e}(t) = {} & (A_i - L_iJ_i)\boldsymbol{e}(t)\mathrm{d}t + \\ & (C_i - L_iJ_i)\boldsymbol{e}(t)\mathrm{d}w(t). \end{aligned} \tag{33}$$

不失一般意义, 假定误差系统满足$\mathrm{E}[\boldsymbol{e}^{\mathrm{T}}(t)R\boldsymbol{e}(t)] < 1, t \in [0, T]$. 令

$$\boldsymbol{z}(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{e}(t) \end{bmatrix},$$

根据式(33), 得到以下系统:

$$\begin{cases} \mathrm{d}\boldsymbol{z}(t) = \tilde{A}_i\boldsymbol{z}(t)\mathrm{d}t + \tilde{C}_i\boldsymbol{z}(t)\mathrm{d}w(t), \\ \boldsymbol{z}(0) \in \mathbb{R}^{2n}, \end{cases} \tag{34}$$

其中:

$$\tilde{A}_i = \begin{bmatrix} A_i + B_iK_i & -B_iK_i \\ 0 & A_i - L_iJ_i \end{bmatrix},$$
$$\tilde{C}_i = \begin{bmatrix} C_i + D_iK_i & -D_iK_i \\ 0 & C_i - L_iJ_i \end{bmatrix}.$$

根据假设1, 下面这个定理给出了L_i存在的条件.

定理 3 给定一些标量$\alpha \geqslant 0$, $\beta \geqslant 0$, 如果存在3个正标量$\lambda_1, \lambda_2, \lambda_3$, 矩阵$T_i$以及正定的对称矩阵$X_i, Y_i$, 满足下列不等式:

$$\begin{bmatrix} \tilde{\varOmega}_{11} - \alpha\tilde{X}_i & * & * \\ \tilde{\varOmega}_{21} & \tilde{\varOmega}_{22} - \alpha\tilde{Y}_i & * \\ 0 & \tilde{Y}_iC_i - T_iJ_i & -\tilde{Y}_i \end{bmatrix} < 0, \tag{35}$$

$\forall i \in D_k^i,\ j \in D_{uk}^i$,

$$\begin{bmatrix} \tilde{\varPi}_{11} - \alpha\tilde{X}_i & * & * \\ \tilde{\varOmega}_{21} & \tilde{\varPi}_{22} - \alpha\tilde{Y}_i & * \\ 0 & \tilde{Y}_iC_i - T_iJ_i & -\tilde{Y}_i \end{bmatrix} < 0, \tag{36}$$

$\forall i \in D_{uk}^i,\ j \in D_{uk}^i$,

$$\begin{bmatrix} \beta\tilde{X}_i - \tilde{\varOmega}_{11} & * \\ -\tilde{\varOmega}_{21} & \beta\tilde{Y}_i - \tilde{\varOmega}_{22} \end{bmatrix} < 0, \tag{37}$$

$\forall i \in D_k^i,\ j \in D_{uk}^i$,

$$\begin{bmatrix} \beta\tilde{X}_i - \tilde{\varPi}_{11} & * \\ -\tilde{\varOmega}_{21} & \beta\tilde{Y}_i - \tilde{\varPi}_{22} \end{bmatrix} < 0, \tag{38}$$

$\forall i \in D_{uk}^i,\ j \in D_{uk}^i$,

$$(\lambda_2 c_1 + \lambda_3)\mathrm{e}^{\alpha T} - \lambda_1 c_2 < 0, \tag{39}$$

$$\lambda_2 c_4 - \lambda_1 c_3 + \lambda_3 < 0, \tag{40}$$

$$\lambda_1 I < X_i < \lambda_2 I, \tag{41}$$

$$0 < Y_i < \lambda_3 I, \tag{42}$$

则存在一个动态输出反馈控制器(32)使得系统(1)关于$(c_1, c_2, c_3, c_4, T, R)$有限时间随机镇定, 其中:

$$\begin{aligned}
\tilde{\Omega}_{11} = {} & (A_i + B_i K_i)^{\mathrm{T}} \tilde{X}_i + \tilde{X}_i (A_i + B_i K_i) + \\
& (C_i + D_i K_i)^{\mathrm{T}} \tilde{X}_i (C_i + D_i K_i) + \\
& \sum_{k \in D_k^i} q_{ik} \tilde{X}_k - q_k^i \tilde{X}_j,
\end{aligned}$$

$$\begin{aligned}
\tilde{\Omega}_{22} = {} & (D_i K_i)^{\mathrm{T}} \tilde{X}_i (D_i K_i) + A_i^{\mathrm{T}} \tilde{Y}_i - J_i^{\mathrm{T}} T_i^{\mathrm{T}} + \\
& \tilde{Y}_i A_i - T_i J_i + \sum_{k \in D_k^i} q_{ik} \tilde{Y}_k - q_k^i \tilde{Y}_j,
\end{aligned}$$

$$\tilde{\Omega}_{21} = (B_i K_i)^{\mathrm{T}} \tilde{X}_i + (D_i K_i)^{\mathrm{T}} \tilde{X}_i (C_i + D_i K_i),$$

$$\tilde{X}_i = R^{1/2} X_i R^{1/2},\ \tilde{Y}_i = R^{1/2} Y_i R^{1/2},\ T_i = \tilde{Y}_i L_i,$$

$$\tilde{\Pi}_{11} = \tilde{\Omega}_{11} + q_{\mathrm{d}}^i \tilde{X}_i - q_{\mathrm{d}}^i \tilde{X}_j,$$

$$\tilde{\Pi}_{22} = \tilde{\Omega}_{22} + q_{\mathrm{d}}^i \tilde{Y}_i - q_{\mathrm{d}}^i \tilde{Y}_j.$$

证　只需令

$$P_i = \begin{bmatrix} \tilde{X}_i & 0 \\ 0 & \tilde{Y}_i \end{bmatrix},$$

然后, 利用证明定理1的方法, 即可得到该定理.

5　例子(Examples)

给出$\{r(t)\}$表示集合$D = \{1, 2, 3, 4\}$上的马尔科夫链和转移概率矩阵

$$\Lambda = \begin{bmatrix} -0.9 & ? & ? & 0.3 \\ ? & ? & 0.5 & ? \\ 0.1 & 0.6 & -1.1 & 0.4 \\ ? & 0.5 & ? & -1.2 \end{bmatrix}.$$

式中“?”表示转移信息未知的元素.

给系统(1)以下系数:

$$A_1 = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix},\ A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\ A_3 = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 3 & 0 \\ 0 & 2.5 \end{bmatrix},\ B_1 = \begin{bmatrix} 1.1 \\ 1.0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.9 \\ 1.2 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 1.1 \\ 0.9 \end{bmatrix},\ B_4 = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix},\ C_1 = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 1.0 \end{bmatrix},\ C_3 = \begin{bmatrix} 1.0 & 0 \\ 0 & 0.9 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix},\ D_1 = \begin{bmatrix} 1.1 \\ 1.0 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.9 \\ 1.2 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 1.1 \\ 0.9 \end{bmatrix},\ D_4 = \begin{bmatrix} 1.6 \\ 0.7 \end{bmatrix},\ J_1 = [1\ \ 2],$$

$$J_2 = [2\ \ 1],\ J_3 = [1\ \ 1],\ J_4 = [2\ \ 2].$$

另外取

$$c_1 = 5,\ c_2 = 100,\ c_3 = 4,\ c_4 = 1,$$

$$T = 0.1,\ R = I,\ \alpha = 15,\ \beta = 1.$$

为了方便, 特给出下界$q_{\mathrm{d}}^i = -1$来说明文中理论. 通过解不等式(25)−(31)得到

$$X_1 = \begin{bmatrix} 0.63 & 0 \\ 0 & 0.55 \end{bmatrix}, T_1 = \begin{bmatrix} -0.11 \\ -0.03 \end{bmatrix}^{\mathrm{T}},$$

$$X_2 = \begin{bmatrix} 0.61 & 0 \\ 0 & 0.60 \end{bmatrix}, T_2 = \begin{bmatrix} 0.03 \\ -0.01 \end{bmatrix}^{\mathrm{T}},$$

$$X_3 = \begin{bmatrix} 0.80 & 0 \\ 0 & 0.89 \end{bmatrix}, T_3 = \begin{bmatrix} -0.05 \\ 0 \end{bmatrix}^{\mathrm{T}},$$

$$X_4 = \begin{bmatrix} 0.66 & 0 \\ 0 & 0.63 \end{bmatrix}, T_4 = \begin{bmatrix} -0.06 \\ -0.09 \end{bmatrix}^{\mathrm{T}},$$

$$\lambda_1 = 0.35, \lambda_2 = 1.36.$$

因此

$$K_1 = [-0.18\ \ -0.06],\ K_2 = [0.04\ \ -0.02],$$

$$K_3 = [-0.06\ \ 0],\ K_4 = [-0.10\ \ -0.15].$$

由式(3)和定理2, 可见系统(1)是状态反馈有限时间随机镇定的. 通过解不等式(35)−(42), 得到

$$X_1 = \begin{bmatrix} 0.78 & -0.01 \\ -0.01 & 0.77 \end{bmatrix},\ X_2 = \begin{bmatrix} 0.69 & 0.01 \\ 0.01 & 0.69 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 0.93 & 0.01 \\ 0.01 & 0.94 \end{bmatrix},\ X_4 = \begin{bmatrix} 0.77 & -0.01 \\ -0.01 & 0.78 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 0.68 & -0.01 \\ -0.01 & 0.67 \end{bmatrix}, T_1 = \begin{bmatrix} 0.05 \\ -0.09 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} 0.69 & 0.01 \\ 0.01 & 0.68 \end{bmatrix},\ T_2 = \begin{bmatrix} -0.11 \\ -0.04 \end{bmatrix},$$

$$Y_3 = \begin{bmatrix} 0.68 & -0.01 \\ -0.01 & 0.70 \end{bmatrix},\ T_3 = \begin{bmatrix} -0.02 \\ -0.05 \end{bmatrix},$$

$$Y_4 = \begin{bmatrix} 0.68 & -0.01 \\ -0.01 & 0.68 \end{bmatrix},\ T_4 = \begin{bmatrix} 0.03 \\ -0.01 \end{bmatrix},$$

$$\lambda_1 = 0.65, \lambda_2 = 1.38, \lambda_3 = 1.17.$$

因此, 反馈增益矩阵为

$$L_1 = \begin{bmatrix} 0.07 \\ -0.13 \end{bmatrix},\ L_2 = \begin{bmatrix} -0.17 \\ -0.06 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} -0.03 \\ -0.07 \end{bmatrix},\ L_4 = \begin{bmatrix} 0.04 \\ -0.01 \end{bmatrix}.$$

式(32)和定理3表明存在动态输出反馈控制器使系

统(1)是有限时间随机镇定的.

6 结论(Conclusions)

本文给出了转移概率矩阵部分信息未知情况下随机马尔科夫跳跃系统的有限时间随机稳定性与镇定的充分条件, 并且利用线性矩阵不等式的方法, 设计了状态与动态输出反馈控制器.

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作者简介:

张维海 (1965–), 男, 博士, 教授, 博士生导师, 山东省“泰山学者”特聘教授, Asian Journal of Control编委, 研究方向为随机控制、鲁棒控制等, E-mail: w_hzhang@163.com;

刘鹤鸣 (1990–), 男, 硕士研究生, 研究方向为系统与控制理论, E-mail: hliuheming@163.com.