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具有动态不确定性互联大系统的分散自适应控制

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摘要: 对一类具有未建模动态结构相似形的严格反馈非线性互联大系统, 提出一种基于神经网络的分散自适应动态面控制方案. 该方案引入Lyapunov函数来约束未建模动态, 利用神经网络逼近理论分析中所产生的未知非线性连续函数. 通过Young's不等式和三重求和项的分解, 有效地处理了耦合作用项, 并利用动态面控制技术, 实现了系统的分散控制. 与现有研究结果相比, 所设计的分散控制律中不含有控制增益下界常数. 通过构造的方法, 利用动态面控制设计中引入的紧集有效地处理了未建模动态和分析中产生的不确定连续函数. 理论分析证明了闭环控制系统中所有信号半全局一致终结有界, 且跟踪误差收敛到原点的一个小邻域内. 两个数值算例的仿真结果表明所提控制方案的有效性.

关键词: 未建模动态; 动态面控制; 严格反馈; 互联大系统; 分散自适应控制
中图分类号: TP13　　**文献标识码**: A

Decentralized adaptive control for large-scale interconnected systems with dynamic uncertainties

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Abstract: We present a decentralized adaptive control scheme based on neural networks and dynamic surface control for a class of interconnected nonlinear large-scale systems in strict-feedback form with similar structure and unmodeled dynamics. In the designed scheme, unmodeled dynamics is described by using the Lyapunov function method, and neural networks are used to approximate the unknown nonlinear continuous functions which are produced in theoretical analysis. The interconnected terms are effectively dealt with by using Young's inequality and decomposition of the threefold summation term, and the decentralized control is realized by utilizing dynamic surface control technique. Compared with the existing results, the designed decentralized control laws do not contain the lower bound of control gain. By the constructing method and the compact set introduced in dynamic surface control design, the unmodeled dynamics and uncertain continuous functions generated in the recursive design are effectively handled. By theoretical analysis, the closed-loop control system is shown to be semi-globally uniformly ultimately bounded, with the tracking error converging to a small neighborhood of the origin. Simulation results of two numerical examples show the effectiveness of the proposed scheme.

Key words: unmodeled dynamics; dynamic surface control; strict-feedback; interconnected system; decentralized adaptive control

1 引言(Introduction)

自从后推技术和动态面设计方法在文献[1–2]中被提出后, 这两种方法已经成为非线性控制系统设计的重要手段, 并在各种非线性系统自适应控制器设计中得到了广泛应用. 文献[3]利用后推技术, 对一类严格反馈非线性系统, 设计了自适应神经网络控制器. 由于后推设计需要对虚拟控制器反复求导, 从而导致控制器设计越来越复杂. 针对上述不足, 文献[2]通过在递推设计中引入一阶滤波器, 实现了代数运算代替微分运算, 简化了控制器的设计. 基于动态面控制, 文献[4–9]对严格反馈单输入单输出非线性系统提出了多种控制方案. 文献[4]对具有未知控制增益的严格反馈系统, 结合最小学习参数、输入状态稳定和小增益方法, 提出了鲁棒自适应神经网络控制方法. 文献[3,5,9]分别讨论了具有参数化形式的外界扰动, 具有周期性外界扰动和具有状态时滞情况下的控制系统信号的有界性和稳定性问题. 文献[10]利用积分型Lyapunov函数, 对具有未知死区和不确定扰动的纯反馈非线性系统, 提出了基于神经网络的动态面自适应控制.

在实际控制系统中, 人们很难建立被控对象的精确模型, 因而存在建模误差, 系统模型简化和参数测

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量误差也会导致系统具有不确定性, 这些不确定性统称为未建模动态. 未建模动态和系统的外部扰动都可能对控制系统的稳定性造成不利影响. 文献[11–12]中提出引入动态信号和利用Lyapunov函数两种用来处理未建模动态的方法. 文献[13]通过Lyapunov函数约束未建模动态, 对一类具有不确定性的严格反馈非线性系统, 设计了神经网络自适应动态面控制器. 文献[14]借助Nussbaum函数, 对一类带有未知增益和未建模动态的纯反馈非线性系统, 引入动态信号并利用后推技术设计了神经网络自适应控制器. 文献[15]对一类具有未建模动态非仿射纯反馈非线性系统, 提出了神经网络输出反馈控制方案.

文献[16–17]将文献[11]中提出的动态信号的方法推广应用到多输入多输出非线性系统. 文献[16]针对含有未知参数、不确定非线性、外界扰动和未建模动态的多种不确定性多输入多输出系统, 设计了具有自适应非线性阻尼项的观测器. 文献[17]提出模糊输出反馈形式的动态面控制方案. 正如所知道的, 在耦合大系统中, 由于各子系统的状态之间不可避免地存在耦合作用, 因此如何有效地处理耦合作用项, 实现大系统的分散控制是研究耦合大系统控制器设计的关键技术. 文献[18]对具有耦合作用项和未建模动态的耦合大系统, 提出分散L_1自适应控制器设计方案. 文献[19]引入修正的动态信号处理未建模动态, 利用自适应非线性阻尼项来抵消高阶耦合项的影响. 文献[20]针对多机组电力系统的模型实例设计的自适应模糊控制方案中每个子系统能够自适应补偿未知的扰动和互联项. 文献[21]基于动态面设计方法, 对具有严格反馈形式的耦合非线性系统, 设计了神经分散自适应控制器, 其中关于耦合项的假设无需满足匹配条件. 文献[22–23]对系统状态不可测且具有外部扰动和未知控制增益符号或未知输入死区的MIMO非线性系统提出了输出反馈模糊跟踪控制方案. 文献[24]采用滑模控制, 利用Nussbaum函数, 对含有未知死区和未知控制增益方向的MIMO非线性系统, 设计了自适应神经跟踪控制器. 文献[25]利用控制系数矩阵的谱半径放宽了控制系数矩阵非奇异的假设条件, 对具有非对称输入约束的MIMO系统提出了自适应跟踪控制方案. 文献[22, 24–25]中, 第i个子系统的控制器中需要使用前i个子系统的状态或输入信息, 因此是集中控制. 文献[20–25]的控制方案中都未考虑未建模动态对系统稳定性的影响. 文献[26]讨论了一类耦合大系统的分散控制问题.

本文在文献[13, 26]基础上, 对一类具有未建模动态的结构相似形严格反馈非线性耦合大系统, 提出一种分散自适应神经网络动态面控制策略. 与文献[26]相比较, 主要贡献与区别如下: 1) 本文引入Lyapunov函数来约束未建模动态, 避免了动态信号的使用[26]. 所设计的虚拟控制律和控制律中避免了使用控制增益下界常数[26], 并通过重新选取适当的Lyapunov函数, 证明了该控制方案的有效性. 2) 通过Young's不等式和三重求和项的分解, 有效地处理了耦合作用项和动态不确定性, 并利用动态面控制技术, 实现了系统的分散控制; 减少了处理耦合项所产生的需要自适应的未知参数个数, 从而减少了控制策略中的估计参数数目. 而文献[26]利用多重求和项下标交换规则处理耦合作用项的方法是值得商榷的. 3) 自适应律和控制律中包含了神经网络径向基函数向量, 充分地利用了子系统的信息调节参数, 并且避免控制律中使用符号函数, 有效地抑制了控制律的抖动.

2 问题描述及基本假设(Problem statement and basic assumptions)

考虑一类由N个互联不确定非线性子系统组成的大系统, 其第i个子系统描述如下:

$$\begin{cases} \dot{z}_i = q_i(t, z_i, x_{i1}), \\ \dot{x}_{ij} = f_{ij}(\bar{x}_{ij}) + g_{ij}(\bar{x}_{ij})x_{i,j+1} + \Delta_{ij}(t, z_i, \bar{x}_{in_i}), \\ \qquad 1 \leqslant j \leqslant n_i - 1, \\ \dot{x}_{in_i} = f_{in_i}(\bar{x}_{in_i}) + g_{in_i}(\bar{x}_{in_i})u_i + \\ \qquad \Delta_{in_i}(t, z_i, \bar{x}_{in_i}) + d_i(x, t), \\ y_i = x_{i1},\ i = 1, \cdots, N, \end{cases} \tag{1}$$

其中: $\bar{x}_{ij} = (x_{i1}, x_{i2}, \cdots, x_{ij})^{\mathrm{T}} \in \mathbb{R}^j$, $j = 1, 2, \cdots, n_i$, $i = 1, 2, \cdots, N$; $x = (\bar{x}_{1n_1}^{\mathrm{T}}, \cdots, \bar{x}_{Nn_N}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{\bar{N}}$为包含$N$个子系统全部状态的向量, $\bar{N} = \sum\limits_{i=1}^{N} n_i$; $u_i \in \mathbb{R}$, $y_i \in \mathbb{R}$分别为第i个子系统的输入和输出; $g_{ij}(\bar{x}_{ij})$, $f_{ij}(\bar{x}_{ij})$为未知光滑函数, 且满足$f_{ij}(0, \cdots, 0) = 0$; $z_i \in \mathbb{R}^{n_{i0}}$为不可测状态部分, 又称为未建模动态; $\Delta_{ij}(t, z_i, \bar{x}_{in_i})$为动态扰动, 且$\Delta_{ij}(t, z_i, \bar{x}_{in_i})$和$q_i(t, z_i, x_{i1})$是满足Lipschitz条件的连续函数, $d_i(x, t)$是耦合作用项, $j = 1, 2, \cdots, n_i$, $i = 1, 2, \cdots, N$.

控制目标: 对第i个子系统设计分散自适应控制器u_i, 使系统输出y_i跟踪一个给定的期望轨迹$y_{i\mathrm{d}}$, 闭环系统全局一致终结有界, 跟踪误差收敛到一个小的残差集内.

假设 1 光滑非线性函数$g_{ij}(\bar{x}_{ij})$符号已知且满足$0 < g_{ij0} \leqslant |g_{ij}(\bar{x}_{ij})| \leqslant g_{ij1}$, g_{ij0}和g_{ij1}为未知正常数. 不失一般性, 假设$g_{ij}(\bar{x}_{ij}) > 0$.

假设 2 参考输入$x_{i\mathrm{d}} = (y_{i\mathrm{d}}, \dot{y}_{i\mathrm{d}}, \ddot{y}_{i\mathrm{d}})^{\mathrm{T}} \in \Omega_{i\mathrm{d}}$光滑可测, 且$\Omega_{i\mathrm{d}} = \{x_{i\mathrm{d}} : y_{i\mathrm{d}}^2 + \dot{y}_{i\mathrm{d}}^2 + \ddot{y}_{i\mathrm{d}}^2 \leqslant B_{i0}\}$, B_{i0}为已知正常数.

假设 3 $|\Delta_{ij}(t, z_i, \bar{x}_{in_i})| \leqslant \rho_{ij2}(\bar{x}_{ij})\|z_i\| + \rho_{ij1}(\|\bar{x}_{ij}\|)$, $j = 1, \cdots, n_i$, $i = 1, \cdots, N$, $\forall(\bar{x}_{ij}, t) \in \mathbb{R}^j \times \mathbb{R}_+$, $\rho_{ij1}(\|\bar{x}_{ij}\|)$, $\rho_{ij2}(\bar{x}_{ij})$为未知非负连续函数.

假设 4 系统$\dot{z}_i = q_i(t, z_i, 0) - q_i(t, 0, 0)$在$z_i = 0$时是全局指数稳定的, 即存在一个Lyapunov函数W_i

满足下列不等式:

$$\begin{cases} c_{i1}\|z_i\|^2 \leqslant W_i(t,z_i) \leqslant c_{i2}\|z_i\|^2, \\ \dfrac{\partial W_i}{\partial z_i}(t,z_i)(q_i(t,z_i,0)-q_i(t,0,0))+ \\ \dfrac{\partial W_i}{\partial t}(t,z_i) \leqslant -c_{i3}\|z_i\|^2, \\ \Big|\dfrac{\partial W_i}{\partial z_i}(t,z_i)\Big| \leqslant c_{i4}\,\|z_i\|, \end{cases} \tag{2}$$

其中: $c_{i1},c_{i2},c_{i3},c_{i4}$均为正常数, 并存在$c_{i5}\geqslant 0$使得$\|q_i(t,0,0)\| \leqslant c_{i5}, \forall t\geqslant 0$.

假设 5　存在未知函数$\psi_{i0}\in\mathbb{C}^1$且$\psi_{i0}(0)=0$满足下式:

$$\|q_i(t,z_i,x_{i1})-q_i(t,z_i,0)\| \leqslant \psi_{i0}(|x_{i1}|).$$

假设 6　存在未知非负系数ς_{ij}^k, 使得系统耦合作用项满足

$$|d_i(x,t)| \leqslant \sum_{j=1}^{N}\sum_{k=0}^{P}\varsigma_{ij}^k\|\bar{x}_{jn_j}\|^k, \tag{3}$$

其中P是一个已知的非负整数.

引理 1[27]　对于任意实值连续函数$f(x,y)$, 存在光滑的纯量函数$\phi(x)\geqslant 0, \vartheta(y)\geqslant 0$, 使以下不等式成立:

$$|f(x,y)| \leqslant \phi(x)+\vartheta(y), \tag{4}$$

其中: $x\in\mathbb{R}^m$, $y\in\mathbb{R}^n$.

注 1　本文研究对象的每一个子系统均含有未建模动态和动态不确定项, 而文献[21]不含未建模动态和动态不确定项. 与文献[17]相比, 本文研究了一类具有未建模动态和耦合作用项的严格反馈非线性大系统, 而文献[17]中的研究对象是输出反馈形式. 此外, 本文通过引入假设4–5有效地处理了未建模动态, 避免了动态信号的引入, 从而减少了用于逼近未知连续函数的神经网络的输入变量个数, 而文献[17]通过引入一种动态信号来消除未建模动态的影响.

3　分散神经网络动态面控制器设计(Design of decentralized neural dynamic surface controller)

在本节中, 利用动态面控制方法, 对式(1)所描述的严格反馈非线性互联大系统设计分散自适应控制方案. 对大系统中第i个子系统定义动态面: $s_{i1}=x_{i1}-y_{1\rm d}$, $\omega_{i1}=y_{1\rm d}$, $s_{ij}=x_{ij}-\omega_{ij}$, $j=2,\cdots,n_i$, 其中ω_{ij}是以α_{ij}为输入的一阶滤波器的输出, α_{ij}是递推设计过程第$j-1$步设计的虚拟控制器, 整个设计过程需要n_i步, 最后在第n_i步, 设计每个子系统的分散控制律u_i. 定义给定的紧集$\Omega_{Z_{ij}}\subset\mathbb{R}^{j+2}$, 用径向基函数神经网络${W_{ij}^*}^{\rm T}S_{ij}(Z_{ij})$在紧集$\Omega_{Z_{ij}}$上对未知连续函数$h_{ij}(Z_{ij})$进行逼近[28–29],

$$h_{ij}(Z_{ij})={W_{ij}^*}^{\rm T}S_{ij}(Z_{ij})+\varepsilon_{ij}(Z_{ij}), \tag{5}$$

其中未知连续函数$h_{ij}(Z_{ij})$在控制器设计的第j步给出.

$$Z_{i1}=(x_{i1},s_{i1},\dot{\omega}_{i1})^{\rm T},\ Z_{ij}=(\bar{x}_{ij},s_{ij},\dot{\omega}_{ij})^{\rm T}, \tag{6}$$

基向量$S_{ij}(Z_{ij})=(s_{ij1}(Z_{ij}),\cdots,s_{ijl_{ij}}(Z_{ij}))^{\rm T}\in\mathbb{R}^{l_{ij}}$, $s_{ijk}(Z_{ij})$采用普遍使用的高斯函数

$$s_{ijk}(Z_{ij})=\exp[-\frac{(Z_{ij}-\mu_{ijk})^{\rm T}(Z_{ij}-\mu_{ijk})}{\phi_{ijk}^2}], \tag{7}$$

$k=1,\cdots,l_{ij}$, $j=1,\cdots,n_i$, $\mu_{ijk}=(\mu_{ijk1},\mu_{ijk2},\cdots,\mu_{ijkq_{ijk}})^{\rm T}$, $q_{ijk}=j+2$, 高斯函数的中心; ϕ_{ijk}是高斯函数的宽度; 理想权向量是W_{ij}^*定义为

$$W_{ij}^* = \arg\min_{W_{ij}\in\mathbb{R}^{l_{ij}}}[\sup_{Z_{ij}\in\Omega_{Z_{ij}}}|W_{ij}^{\rm T}S_{ij}(Z_{ij})-h_{ij}(Z_{ij})|], \tag{8}$$

$\varepsilon_{ij}(Z_{ij})$为逼近误差. 为了减少自适应调节参数的数目, 定义未知常数$\lambda_i^*=\max\{|W_{ij}^*|^2,\ j=1,2,\cdots,n_i\}$. 进一步, 令

$$\lambda_{i\rm g}^*=\frac{\lambda_i^*}{g_{i,\min}}, \tag{9}$$

其中$g_{i,\min}=\min\{g_{i10},\cdots,g_{in_i0}\}$. 本文直接估计$\lambda_{i\rm g}^*$ $(i=1,2,\cdots,N)$来取代估计理想权向量W_{ij}^*, 定义$\hat{\lambda}_{i\rm g}$是$\lambda_{i\rm g}^*$的估计, 估计误差$\tilde{\lambda}_{i\rm g}=\lambda_{i\rm g}^*-\hat{\lambda}_{i\rm g}$.

定义如下记号:

$$\bar{s}_{ij}=(s_{i1},\cdots,s_{ij})^{\rm T}, \tag{10}$$

$$V_{is_{ij}}=\frac{1}{2}s_{ij}^2, \tag{11}$$

$$\bar{y}_{ij}=(y_{i2},\cdots,y_{ij})^{\rm T}, \tag{12}$$

其中: $y_{ij}=\omega_{ij}-\alpha_{ij}$, $j=2,\cdots,n_i$, $i=1,\cdots,N$.

Step 1　考虑第i个子系统

$$\dot{x}_{i1}=f_{i1}(x_{i1})+g_{i1}(x_{i1})x_{i2}+\varDelta_{i1}(t,z_i,\bar{x}_{in_1}). \tag{13}$$

令$V_{is\rm W}=W_i/\lambda_{i0}+V_{is_{i1}}$, 其中$\lambda_{i0}>0$是一个正常数, W_i在假设4中给出. 将$V_{is\rm W}$关于时间t求导得

$$\dot{V}_{is\rm W}=\frac{1}{\lambda_{i0}}\cdot\frac{{\rm d}W_i}{{\rm d}t}+\dot{V}_{is_{i1}}. \tag{14}$$

根据假设3和Young's不等式,

$$|s_{i1}\varDelta_{i1}| \leqslant |s_{i1}|[\rho_{i11}(\|x_{i1}\|)+\|z_i\|\rho_{i12}(x_{i1})] \leqslant s_{i1}^2\rho_{i11}^2(\|x_{i1}\|)+\frac{1}{4}+\frac{c_{i3}}{4\lambda_{i0}}\|z_i\|^2+\frac{\lambda_{i0}}{c_{i3}}s_{i1}^2{\rho_{i12}}^2(x_{i1}). \tag{15}$$

利用假设4–5, 可得

$$\dot{V}_{is\rm W} \leqslant \frac{1}{\lambda_{i0}}[-c_{i3}\|z_i\|^2+c_{i4}c_{i5}\|z_i\|+c_{i4}\|z_i\|\psi_{i0}(|x_{i1}|)]+ g_{i1}(x_{i1})s_{i1}x_{i2}+s_{i1}h_{i1}(Z_{i1})+$$

$$\frac{1}{4}+\frac{c_{i3}}{4\lambda_{i0}}\|z_i\|^2+\frac{\lambda_{i0}^2}{4c_{i3}^2}\leqslant$$
$$-\frac{c_{i3}}{2\lambda_{i0}}\|z_i\|^2+\psi_{i0}^4(|x_{i1}|)+$$
$$g_{i1}(x_{i1})s_{i1}x_{i2}+\frac{g_{i,\min}}{2a_{i1}^2}\lambda_{ig}^*s_{i1}^2\|S_{i1}(Z_{i1})\|^2+$$
$$s_{i1}\varepsilon_{i1}(Z_{i1})+D_{i1},\tag{16}$$

其中$a_{i1}>0$是一个设计常数,

$$h_{i1}(Z_{i1})=$$
$$f_{i1}(x_{i1})+s_{i1}\rho_{i11}^2(\|x_{i1}\|)+s_{i1}^3\rho_{i12}^4(x_{i1})-\dot{\omega}_{i1},$$
$$D_{i1}=\frac{2c_{i4}^2c_{i5}^2}{\lambda_{i0}c_{i3}}+\frac{c_{i4}^4}{c_{i3}^2\lambda_{i0}^2}+\frac{\lambda_{i0}^2}{4c_{i3}^2}+\frac{a_{i1}^2}{2}+\frac{1}{4}.$$

选取虚拟控制律

$$\alpha_{i2}=-k_{i1}s_{i1}-\frac{1}{2a_{i1}^2}\hat{\lambda}_{ig}s_{i1}\|S_{i1}(Z_{i1})\|^2.\tag{17}$$

引入一阶滤波器、虚拟控制α_{i2}和变量ω_{i2}为一阶滤波器的输入和输出, τ_{i2}为时间常数. 即

$$\tau_{i2}\dot{\omega}_{i2}+\omega_{i2}=\alpha_{i2},\ \omega_{i2}(0)=\alpha_{i2}(0).\tag{18}$$

由上式可得$\dot{\omega}_{i2}=-y_{i2}/\tau_{i2}$. 由于

$$x_{i2}=s_{i2}+y_{i2}+\alpha_{i2}=s_{i2}+y_{i2}-k_{i1}s_{i1}-\frac{s_{i1}\hat{\lambda}_{ig}\|S_{i1}(Z_{i1})\|^2}{2a_{i1}^2},$$

因此, 利用Young's不等式, 式(16)可进一步化为

$$\dot{V}_{is\mathrm{W}}\leqslant$$
$$-\frac{c_{i3}}{2\lambda_{i0}}\|z_i\|^2+\psi_{i0}^4(|x_{i1}|)+g_{i1}(x_{i1})s_{i1}(s_{i2}+y_{i2})-$$
$$g_{i10}k_{i1}s_{i1}^2-\frac{g_{i,\min}\hat{\lambda}_{ig}s_{i1}^2\|S_{i1}(Z_{i1})\|^2}{2a_{i1}^2}+$$
$$\frac{g_{i,\min}\lambda_{ig}^*s_{i1}^2\|S_{i1}(Z_{i1})\|^2}{2a_{i1}^2}+s_{i1}\varepsilon_{i1}(Z_{i1})+D_{i1}\leqslant$$
$$-\frac{c_{i3}}{2\lambda_{i0}}\|z_i\|^2+(-k_{i1}g_{i10}+2)s_{i1}^2+\frac{g_{i11}^2}{4}s_{i2}^2+$$
$$\frac{g_{i11}^2}{4}y_{i2}^2+\frac{g_{i,\min}\tilde{\lambda}_{ig}s_{i1}^2\|S_{i1}(Z_{i1})\|^2}{2a_{i1}^2}+$$
$$D_{i1}+\eta_{i1}(s_{i1},y_{\mathrm{id}},\dot{y}_{\mathrm{id}}),\tag{19}$$

其中$\eta_{i1}(s_{i1},y_{\mathrm{id}},\dot{y}_{\mathrm{id}})$是一个连续非负函数, 满足

$$|\psi_{i0}^4(|x_{i1}|)+s_{i1}\varepsilon_{i1}(Z_{i1})|\leqslant\eta_{i1}(s_{i1},y_{\mathrm{id}},\dot{y}_{\mathrm{id}}).\tag{20}$$

$$\dot{y}_{i2}=-\frac{y_{i2}}{\tau_{i2}}+\Big[k_{i1}\dot{s}_{i1}+\frac{\dot{s}_{i1}\hat{\lambda}_{ig}\|S_{i1}(Z_{i1})\|^2}{2a_{i1}^2}+\frac{s_{i1}\dot{\hat{\lambda}}_{ig}\|S_{i1}(Z_{i1})\|^2}{2a_{i1}^2}+\frac{s_{i1}\hat{\lambda}_{ig}}{2a_{i1}^2}\frac{\mathrm{d}\|S_{i1}(Z_{i1})\|^2}{\mathrm{d}t}\Big],\tag{21}$$

$$|\dot{y}_{i2}+\frac{y_{i2}}{\tau_{i2}}|\leqslant\xi_{i2}(\bar{s}_{i2},\bar{y}_{i2},\hat{\lambda}_{ig},\|z_i\|,y_{\mathrm{id}},\dot{y}_{\mathrm{id}},\ddot{y}_{\mathrm{id}}),\tag{22}$$

其中$\xi_{i2}(\bar{s}_{i2},\bar{y}_{i2},\hat{\lambda}_{ig},\|z_i\|,y_{\mathrm{id}},\dot{y}_{\mathrm{id}},\ddot{y}_{\mathrm{id}})$是一个连续非负函数. 由式(21)–(22)及Young's不等式, 得

$$y_{i2}\dot{y}_{i2}\leqslant$$
$$-\frac{y_{i2}^2}{\tau_{i2}}+|y_{i2}|\xi_{i2}(\bar{s}_{i2},\bar{y}_{i2},\hat{\lambda}_{ig},\|z_i\|,y_{\mathrm{id}},\dot{y}_{\mathrm{id}},\ddot{y}_{\mathrm{id}})\leqslant$$
$$-\frac{y_{i2}^2}{\tau_{i2}}+y_{i2}^2+\frac{1}{4}\xi_{i2}^2.\tag{23}$$

Step $\boldsymbol{j}$$(2\leqslant j\leqslant n_i-1)$ 由式(1)可知, s_{ij}的导数为

$$\dot{s}_{ij}=f_{ij}(\bar{x}_{ij})+g_{ij}(\bar{x}_{ij})x_{i,j+1}+\Delta_{ij}(t,z,\bar{x}_{in_i})-\dot{\omega}_{ij}.\tag{24}$$

$V_{is_{ij}}$关于时间t的导数为

$$\dot{V}_{is_{ij}}=s_{ij}\dot{s}_{ij}.\tag{25}$$

根据假设3, 并利用Young's不等式, 可得

$$|s_{ij}\Delta_{ij}|\leqslant s_{ij}^2\rho_{ij1}^2(\|\bar{x}_{ij}\|)+\frac{c_{i3}}{2^{j+1}\lambda_{i0}}\|z_i\|^2+s_{ij}^4{\rho_{ij2}}^4(\bar{x}_{ij})+\frac{1}{4}+\frac{4^{j-2}\lambda_{i0}^2}{c_{i3}^2}.\tag{26}$$

将式(24)和式(26)代入式(25), 整理得

$$\dot{V}_{is_{ij}}\leqslant g_{ij}(\bar{x}_{ij})s_{ij}x_{i,j+1}+s_{ij}h_{ij}(Z_{ij})+\frac{c_{i3}}{2^{j+1}\lambda_{i0}}\|z_i\|^2+\frac{1}{4}+\frac{4^{j-2}\lambda_{i0}^2}{c_{i3}^2},\tag{27}$$

其中

$$h_{ij}(Z_{ij})=$$
$$f_{ij}(\bar{x}_{ij})+s_{ij}\rho_{ij1}^2(\|\bar{x}_{ij}\|)+s_{ij}^3\rho_{ij2}^4(\bar{x}_{ij})-\dot{\omega}_{ij}.$$

选取虚拟控制律$\alpha_{i,j+1}$为

$$\alpha_{i,j+1}=-k_{ij}s_{ij}-\frac{1}{2a_{ij}^2}s_{ij}\hat{\lambda}_{ig}\|S_{ij}(Z_{ij})\|^2,\tag{28}$$

其中$k_{ij}>0$和$a_{ij}>0$是两个设计常数.

引入一阶滤波器

$$\tau_{i,j+1}\dot{\omega}_{i,j+1}+\omega_{i,j+1}=\alpha_{i,j+1},\tag{29}$$

其中: $\omega_{i,j+1}$是一阶滤波器的输出, $\omega_{i,j+1}(0)=\alpha_{i,j+1}(0)$, $\tau_{i,j+1}>0$是一个设计常数. 由式(29)可得$\dot{\omega}_{i,j+1}=-y_{i,j+1}/\tau_{i,j+1}$. 由于

$$x_{i,j+1}=s_{i,j+1}+y_{i,j+1}+\alpha_{i,j+1}=$$
$$s_{i,j+1}+y_{i,j+1}-k_{ij}s_{ij}-\frac{1}{2a_{ij}^2}s_{ij}\hat{\lambda}_{ig}\|S_{ij}(Z_{ij})\|^2,$$

利用式(27)和Young's不等式, 易得

$$\dot{V}_{is_{ij}}\leqslant(-k_{ij}g_{ij0}+2)s_{ij}^2+\frac{g_{ij1}^2}{4}s_{i,j+1}^2+$$
$$\frac{g_{ij1}^2}{4}y_{i,j+1}^2+\frac{g_{i,\min}\tilde{\lambda}_{ig}s_{ij}^2\|S_{ij}(Z_{ij})\|^2}{2a_{ij}^2}+$$
$$\eta_{ij}(\bar{s}_{ij},\bar{y}_{ij},\hat{\lambda}_{ig},\|z_i\|,y_{\mathrm{id}},\dot{y}_{\mathrm{id}})+$$

$$\frac{c_{i3}}{2^{j+1}\lambda_{i0}}\|z_i\|^2+D_{ij}, \tag{30}$$

其中

$$D_{ij}=\frac{1}{4}+\frac{4^{j-2}\lambda_{i0}^2}{c_{i3}^2}+\frac{a_{ij}^2}{2}.$$

连续非负函数$\eta_{ij}(\bar{s}_{ij},\bar{y}_{ij},\hat{\lambda}_{ig},\|z_i\|,y_{id},\dot{y}_{id})$满足

$$|s_{ij}\varepsilon_{ij}(Z_{ij})|\leqslant\eta_{ij}(\bar{s}_{ij},\bar{y}_{ij},\hat{\lambda}_{ig},\|z_i\|,y_{id},\dot{y}_{id}), \tag{31}$$

$$\begin{aligned}&\dot{y}_{i,j+1}=\\&-\frac{y_{i,j+1}}{\tau_{i,j+1}}+\Big[k_{ij}\dot{s}_{ij}+\frac{\dot{s}_{ij}\hat{\lambda}_{ig}\|S_{ij}(Z_{ij})\|^2}{2a_{ij}^2}+\\&\frac{s_{ij}\dot{\hat{\lambda}}_{ig}\|S_{ij}(Z_{ij})\|^2}{2a_{ij}^2}+\frac{s_{ij}\hat{\lambda}_{ig}}{2a_{ij}^2}\frac{\mathrm{d}\|S_{ij}(Z_{ij})\|^2}{\mathrm{d}t}\Big],\end{aligned} \tag{32}$$

$$\begin{aligned}&|\dot{y}_{i,j+1}+\frac{y_{i,j+1}}{\tau_{i,j+1}}|\leqslant\\&\xi_{i,j+1}(\bar{s}_{i,j+1},\bar{y}_{i,j+1},\hat{\lambda}_{ig},\|z_i\|,y_{id},\dot{y}_{id},\ddot{y}_{id}),\end{aligned} \tag{33}$$

其中$\xi_{i,j+1}(\bar{s}_{i,j+1},\bar{y}_{i,j+1},\hat{\lambda}_{ig},\|z_i\|,y_{id},\dot{y}_{id},\ddot{y}_{id})$是一个非负连续函数.

由式(32)−(33), 可得

$$y_{i,j+1}\dot{y}_{i,j+1}\leqslant-\frac{y_{i,j+1}^2}{\tau_{i,j+1}}+y_{i,j+1}^2+\frac{1}{4}\xi_{i,j+1}^2. \tag{34}$$

Step n_i 这一步将确定最终的控制律. 由于$s_{in_i}=x_{in_i}-\omega_{in_i}$, s_{in_i}的导数为

$$\begin{aligned}\dot{s}_{in_i}=&f_{in_i}(\bar{x}_{in_i})+g_{in_i}(\bar{x}_{in_i})u_i-\dot{\omega}_{in_i}+\\&\Delta_{in_i}(t,z_i,\bar{x}_{in_i})+d_i(x,t).\end{aligned} \tag{35}$$

由假设6, 可得

$$\begin{aligned}&|s_{in_i}d_i(x,t)|\leqslant\\&\sum_{j=1}^N\varsigma_{ij}^0|s_{in_i}|+\sum_{j=1}^N\sum_{k=1}^P\varsigma_{ij}^k\|\bar{x}_{jn_j}\|^k|s_{in_i}|.\end{aligned} \tag{36}$$

令$\beta_i^*=[\sum_{j=1}^N\varsigma_{ij}^0]^2$, 则根据Young's不等式, 式(36)可化为

$$\begin{aligned}|s_{in_i}d_i(x,t)|\leqslant&\frac{1}{2}s_{in_i}^2\beta_i^*+\frac{1}{2}+\frac{NP}{2}+\\&\frac{1}{2}\sum_{j=1}^N\sum_{k=1}^P(\varsigma_{ij}^k)^2\|\bar{x}_{jn_j}\|^{2k}s_{in_i}^2,\end{aligned}$$

令未知常数

$$d_{ik}^*=\sum_{j=1}^N(\varsigma_{ij}^k)^2, \tag{37}$$

$$d_{ig}^*=\frac{\max\{d_{i1}^*,\cdots,d_{iP}^*,\beta_i^*\}}{g_{i,\min}}, \tag{38}$$

将三重求和项分解, 可得

$$\begin{aligned}&\sum_{i=1}^N\sum_{j=1}^N\sum_{k=1}^P(\varsigma_{ij}^k)^2\left\|\bar{x}_{jn_j}\right\|^{2k}s_{in_i}^2\leqslant\\&g_{i,\min}\sum_{i=1}^N\sum_{j=1}^N\sum_{k=1}^P d_{ig}^*\left\|\bar{x}_{jn_j}\right\|^{2k}s_{in_i}^2\leqslant\\&g_{i,\min}\sum_{i=1}^N d_{ig}^*\sum_{k=1}^P\left\|\bar{x}_{in_i}\right\|^{2k}s_{in_i}^2+\Delta(\cdot),\end{aligned}$$

其中$\Delta(\cdot)$是变量$\bar{x}_{1n_1},\cdots,\bar{x}_{Nn_N},s_{1n_1},\cdots,s_{Nn_N}$的连续函数. 因此

$$\begin{aligned}&\sum_{i=1}^N|s_{in_i}d_i(x,t)|\leqslant\\&\frac{(NP+1)N}{2}+\frac{g_{i,\min}}{2}\sum_{i=1}^N d_{ig}^*\sum_{k=0}^P\|\bar{x}_{in_i}\|^{2k}s_{in_i}^2+\frac{\Delta(\cdot)}{2}.\end{aligned} \tag{39}$$

类似于第i步的讨论, 易得

$$\begin{aligned}&\sum_{i=1}^N\dot{V}_{is_{in_i}}\leqslant\\&\sum_{i=1}^N\Big[g_{in_i}(\bar{x}_{in_i})s_{in_i}u_i+s_{in_i}h_{in_i}(Z_{in_i})+\\&\frac{c_{i3}}{2^{n_i+1}\lambda_{i0}}\|z_i\|^2+\frac{1}{4}+\frac{4^{n_i-2}\lambda_{i0}^2}{c_{i3}^2}\Big]+\frac{(NP+1)N}{2}+\\&\frac{\Delta(\cdot)}{2}+\frac{g_{i,\min}}{2}\sum_{i=1}^N d_{ig}^*\sum_{k=0}^P\|\bar{x}_{in_i}\|^{2k}s_{in_i}^2,\end{aligned} \tag{40}$$

其中

$$\begin{aligned}h_{in_i}(Z_{in_i})=&f_{in_i}(\bar{x}_{in_i})+s_{in_i}\rho_{in_i1}^2(\|\bar{x}_{in_i}\|)+\\&s_{in_i}^3\rho_{in_i2}^4(\bar{x}_{in_i})-\dot{\omega}_{in_i}.\end{aligned} \tag{41}$$

设计控制律为

$$u_i(t)=u_{id}(t)+u_{ic}(t), \tag{42}$$

$$u_{id}(t)=-k_{in_i}s_{in_i}-\frac{1}{2a_{in_i}^2}s_{in_i}\hat{\lambda}_{ig}\|S_{in_i}(Z_{in_i})\|^2, \tag{43}$$

$$u_{ic}(t)=-\frac{1}{2}\hat{d}_{ig}\sum_{k=0}^P\|\bar{x}_{in_i}\|^{2k}s_{in_i}, \tag{44}$$

其中: $k_{in_i}\geqslant0$和$a_{in_i}\geqslant0$是两个设计常数, u_{ic}是为了补偿系统互联及外部干扰项. $\hat{d}_{ig}$是d_{ig}^*的估计, $\tilde{d}_{ig}=d_{ig}^*-\hat{d}_{ig}$. 利用式(42)−(44), 式(40)可转化为

$$\begin{aligned}\sum_{i=1}^N\dot{V}_{is_{in_i}}\leqslant&\sum_{i=1}^N\Big[(-k_{in_i}g_{in_i0})s_{in_i}^2+\\&\frac{g_{i,\min}\tilde{\lambda}_{ig}s_{in_i}^2\|S_{in_i}(Z_{in_i})\|^2}{2a_{in_i}^2}+\\&\eta_{in_i}(\bar{s}_{in_i},\bar{y}_{in_i},\hat{\lambda}_{ig},\|z_i\|,y_{id},\dot{y}_{id})+\\&\frac{c_{i3}}{2^{n_i+1}\lambda_{i0}}\|z_i\|^2+\frac{\Delta(\cdot)}{2}+\\&\frac{g_{i,\min}}{2}\tilde{d}_{ig}\sum_{k=0}^P\|\bar{x}_{in_i}\|^{2k}s_{in_i}^2\Big],\end{aligned} \tag{45}$$

其中

$$D_{in_i}=\frac{1}{4}+\frac{4^{n_i-2}\lambda_{i0}^2}{c_{i3}^2}+\frac{a_{in_i}^2}{2}+\frac{(NP+1)N}{2},$$

连续函数$\eta_{in_i}(\bar{s}_{in_i},\bar{y}_{in_i},\hat{\lambda}_{ig},\|z_i\|,y_{id},\dot{y}_{id})$满足

$$|s_{in_i}\varepsilon_{n_i}(Z_{n_i})|\leqslant\eta_{in_i}(\bar{s}_{in_i},\bar{y}_{in_i},\hat{\lambda}_{ig},\|z_i\|,y_{id},\dot{y}_{id}).$$

采用如下自适应律:

$$\dot{\hat{\lambda}}_{ig}=\gamma_{i1}\Big(\sum_{j=1}^{n_i}\frac{1}{2a_{ij}^2}\|S_{ij}(Z_{ij})\|^2s_{ij}^2-\sigma_{i1}\hat{\lambda}_{ig}\Big),\tag{46}$$

$$\dot{\hat{d}}_{ig}=\gamma_{i2}\Big(\frac{1}{2}\sum_{k=0}^{P}\|\bar{x}_{in_i}\|^{2k}s_{in_i}^2-\sigma_{i2}\hat{d}_{ig}\Big),\tag{47}$$

其中: $\gamma_{i1},\gamma_{i2},\sigma_{i1},\sigma_{i2}(i=1,\cdots,N)$均为正的设计常数.

4 稳定性分析(Stability analysis)

令

$$V=\sum_{i=1}^{N}\Big(\sum_{j=1}^{n_i}s_{ij}^2+\sum_{j=2}^{n_i}y_{ij}^2+\frac{g_{i,\min}}{\gamma_{i1}}\tilde{\lambda}_{ig}^2+\frac{c_{i3}}{\lambda_{i0}}\|z_i\|^2+\frac{g_{i,\min}}{\gamma_{i2}}\tilde{d}_{ig}^2\Big).$$

定义有界闭集

$$\varOmega_N=\{(\bar{s}_{1n_1},\cdots,\bar{s}_{Nn_N},\bar{y}_{1n_1},\cdots,\bar{y}_{Nn_N},\|z_1\|,\cdots,\|z_N\|,\hat{\lambda}_{1g},\cdots,\hat{\lambda}_{Ng},\hat{d}_{1g},\cdots,\hat{d}_{Ng})^{\mathrm T}:V\leqslant p\}\subset\mathbb{R}^{p_N},$$

其中: $p_N=2\bar{N}+3N$, p为与任意给定的正常数c相关的一个常数. 令连续函数η_{ij}在有界闭集$\varOmega_{\rm d}\times\varOmega_N$上的最大值为$M_{ij}$, $j=1,\cdots,n_i$; $i=1,\cdots,N$. 连续函数$\xi_{i,j+1}$在有界闭集$\varOmega_{\rm d}\times\varOmega_N$上的最大值为$N_{i,j+1}$, $j=1,\cdots,n_i-1$, $i=1,\cdots,N$.

定理 1 考虑由系统(1), 控制律(42)及自适应律(46)和式(47)组成的闭环系统, 若假设1−6成立, 则对于任意给定的正常数c及初始条件满足$\bar{V}(0)\leqslant c$, 满足式(48)的正常数$k_{ij},\sigma_{ij},\tau_{i,j+1}$, 使得闭环系统半全局一致终结有界, 且跟踪误差收敛到一个小的残差内, 而$k_{ij},\tau_{i,j+1}$满足

$$\begin{cases}k_{ij}\geqslant g_{ij0}^{-1}[2+\dfrac{g_{i,j-1,1}^2}{4}+\dfrac{\alpha_0}{2}],\\ j=1,\cdots,n_i,\ i=1,\cdots,N,\\ \dfrac{1}{\tau_{i,j+1}}\geqslant 1+\dfrac{g_{ij1}^2}{4}+\dfrac{\alpha_0}{2},\\ j=1,\cdots,n_i-1,\ i=1,\cdots,N,\\ \alpha_0=\\ \min\limits_{1\leqslant i\leqslant N}\{\gamma_{i1}\sigma_{i1},\gamma_{i2}\sigma_{i2},(\dfrac{1}{4}+\dfrac{1}{2^{n_{\max}+1}})\dfrac{c_{3\min}}{c_{2\max}}\},\\ c_{2\max}=\max\{c_{12},\cdots,c_{N2}\},\\ c_{3\min}=\min\{c_{13},\cdots,c_{N3}\},\\ n_{\max}=\max\{n_1,\cdots,n_N\}.\end{cases}\tag{48}$$

证 考虑如下Lyapunov函数:

$$\bar{V}=\sum_{i=1}^{N}\Big(V_{is\rm W}+\sum_{j=2}^{n_i}V_{is_{ij}}+\sum_{j=2}^{n_i}\frac{1}{2}y_{ij}^2+\frac{g_{i,\min}}{2\gamma_{i1}}\tilde{\lambda}_{ig}^2+\frac{g_{i,\min}}{2\gamma_{i2}}\tilde{d}_{ig}^2\Big).\tag{49}$$

对于任意给定的正常数c, 若$\bar{V}=c$, 则$\sum\limits_{i=1}^{N}V_{isW}=\sum\limits_{i=1}^{N}(W_i/\lambda_{i0}+s_{i1}^2/2)\leqslant\bar{V}$, 又由假设4可知$\sum\limits_{i=1}^{N}(c_{i1}\|z_i\|^2/\lambda_{i0})\leqslant\sum\limits_{i=1}^{N}W_i/\lambda_{i0}\leqslant c$, 令$c_{1\min}=\min\{c_{11},\cdots,c_{N1}\}$, $c_{3\max}=\max\{c_{13},\cdots,c_{N3}\}$, 所以

$$\sum_{i=1}^{N}\frac{1}{\lambda_{i0}}\|z_i\|^2\leqslant\frac{c}{c_{1\min}}.\tag{50}$$

式(49)可改写为

$$\bar{V}=\sum_{i=1}^{N}\frac{1}{\lambda_{i0}}W_i+\frac{1}{2}V-\sum_{i=1}^{N}\frac{c_{i3}}{2\lambda_{i0}}\|z_i\|^2,\tag{51}$$

所以

$$V\leqslant 2c+\frac{c_{3\max}}{c_{1\min}}c.\tag{52}$$

由上式可知, 取$p=(2+c_{3\max}/c_{1\min})c$, 当$\bar{V}=c$时, $V\leqslant p$, 则在紧集$\varOmega_N$上, $\eta_{ij}\leqslant M_{ij}$, $\xi_{i,j+1}^2\leqslant N_{i,j+1}^2$, 且$\sum\limits_{i=1}^{N}c_{i3}\|z_i\|^2/\lambda_{i0}\leqslant p$. 将$\bar{V}$关于时间$t$求导, 得

$$\dot{\bar{V}}=\sum_{i=1}^{N}\Big(\dot{V}_{is\rm W}+\sum_{j=2}^{n_i}\dot{V}_{is_{ij}}+\sum_{j=2}^{n_i}y_{ij}\dot{y}_{ij}-\frac{g_{i,\min}}{\gamma_{i1}}\tilde{\lambda}_{ig}\dot{\hat{\lambda}}_{ig}-\frac{g_{i,\min}}{\gamma_{i2}}\tilde{d}_{ig}\dot{\hat{d}}_{ig}\Big).\tag{53}$$

将式(19)(30)(34)和式(45)代入式(53), 并利用式(46)−(47)及

$$\sigma_{i1}\tilde{\lambda}_{ig}\hat{\lambda}_{ig}\leqslant\sigma_{i1}\Big(-\frac{\tilde{\lambda}_{ig}^2}{2}+\frac{\lambda_{ig}^{*2}}{2}\Big),\tag{54}$$

则式(53)可化简得

$$\begin{aligned}&\dot{\bar{V}}\leqslant\\&\sum_{i=1}^{N}\Big[\sum_{j=1}^{n_i}\Big(-k_{ij}g_{ij0}+2+\frac{g_{i,j-1,1}^2}{4}\Big)s_{ij}^2+\\&\sum_{j=1}^{n_i-1}\Big(-\frac{1}{\tau_{i,j+1}}+1+\frac{g_{ij1}^2}{4}\Big)y_{i,j+1}^2-g_{i,\min}\frac{\sigma_{i1}\tilde{\lambda}_{ig}^2}{2}-\\&\Big(\frac{1}{4}+\frac{1}{2^{n_i+1}}\Big)\frac{c_{i3}}{\lambda_{i0}}\|z_i\|^2+\frac{1}{4}\sum_{j=1}^{n_i-1}N_{i,j+1}^2+\sum_{j=1}^{n_i}M_{ij}+\\&\sum_{j=1}^{n_i}D_{ij}+g_{i,\min}\frac{\sigma_{i1}\lambda_{ig}^{*2}}{2}-g_{i,\min}\frac{\sigma_{i2}\tilde{d}_{ig}^2}{2}+\\&g_{i,\min}\frac{\sigma_{i2}d_{ig}^{*2}}{2}\Big]+\frac{\varDelta(\cdot)}{2},\end{aligned}\tag{55}$$

其中$g_{i01}=0$. 当$\bar{V}\leqslant c$时, 易知$s_{i1},\cdots,s_{in_i},y_{i2},\cdots,y_{in_i}$, $i=1,\cdots,N$, $\hat{\lambda}_{1g},\cdots,\hat{\lambda}_{Ng}$, $\hat{d}_{1g},\cdots,\hat{d}_{Ng}$, $y_1,\cdots,y_N$, $z_1,\cdots,z_N$是有界的. 由式(17)−(18)可知$\alpha_{i2},\omega_{i2},x_{i2}\in L_\infty$, 进一步由式(28)−(29)可知$\alpha_{ij}$,

$\omega_{ij}, x_{ij} \in L_\infty, j=3,\cdots,n_i$. 因此可得$x \in L_\infty$. 又因为$\Delta(\cdot)$是连续函数, 所以存在正常数$C$, 使得$|\Delta(\cdot)/2| \leqslant C$. 令

$$\mu = \sum_{i=1}^{N}\Big[\sum_{j=1}^{n_i} D_{ij} + \sum_{j=1}^{n_i} M_{ij} + \frac{1}{4}\sum_{j=1}^{n_i-1} N_{i+1}^2 + g_{i,\min}\frac{\sigma_{i1}\lambda_{ig}^{*2}}{2} + g_{i,\min}\frac{\sigma_{i2}d_{ig}^{*2}}{2}\Big] + C, \tag{56}$$

于是, 将式(48)和式(56)代入式(55)可得

$$\dot{\bar{V}} \leqslant -\alpha_0\bar{V} + \mu. \tag{57}$$

当$\bar{V}=c, \alpha_0 > \mu/c$, 则有$\dot{\bar{V}} \leqslant 0$, 由此可知当初始条件$\bar{V}(0) \leqslant c$时, $\bar{V}(t) \leqslant c, \forall t \geqslant 0$. 将式(57)两边同乘$\mathrm{e}^{\alpha_0 t}$可得

$$\frac{\mathrm{d}}{\mathrm{d}t}(\bar{V}(t)\mathrm{e}^{\alpha_0 t}) \leqslant \mathrm{e}^{\alpha_0 t}\mu. \tag{58}$$

将上式两边在$[0,t]$上积分得

$$0 \leqslant \bar{V}(t) \leqslant \frac{\mu}{\alpha_0} + [\bar{V}(0) - \frac{\mu}{\alpha_0}]\mathrm{e}^{-\alpha_0 t}. \tag{59}$$

因此, 闭环控制系统中的所有信号$s_{ij}, y_{i,j+1}, \tilde{\lambda}_{ig}, \tilde{d}_{ig}$, $\|z_i\|$都是一致终结有界的, 从而$x_{ij}, \alpha_{i,j+1}, \omega_{ij}, u_i$也都是一致终结有界的.

5 仿真结果(Simulation results)

在本节中, 为了验证所提控制算法的有效性, 两个数值仿真例子被演示.

例 1 考虑具有未建模动态的小车上双倒立摆系统[18,22,30–31], 其动态方程描述如下:

$$\begin{cases}\dot{z}_1 = -2z_1 + 1.5y_1^2\sin(y_1 t),\\ \dot{x}_{11} = x_{12} + 0.5z_1,\\ \dot{x}_{12} = g_1x_{11} + g_3u_1 + g_2x_{21} - (\beta_1x_{12}^2 + g_4) + 2z_1^2,\\ y_1 = x_{11},\\ \dot{z}_2 = -2z_2 + 0.5y_2^2\sin(y_2 t)\\ \dot{x}_{21} = x_{22} + 2z_2,\\ \dot{x}_{22} = g_1x_{21} + g_3u_2 + g_2x_{11} - (\beta_2x_{22}^2 - g_4) + 3z_2^2,\\ y_2 = x_{21},\end{cases} \tag{60}$$

其中:

$$x_{11} = \theta_1,\ x_{12} = \dot{\theta}_1,\ x_{21} = \theta_2,$$
$$x_{22} = \dot{\theta}_2,\ g_1 = \frac{g}{c\times l} - g_2,\ g_3 = \frac{1}{c\times m\times l^2},$$
$$g_2 = k\times a(t)\times(a(t) - c\times l)\times g_3,$$
$$g_4 = k\times(a(t) - c\times l)\times(y_1 - y_2)\times g_3,$$
$$\beta_1 = m\times\frac{\sin x_{11}}{M},\ \beta_2 = m\times\frac{\sin x_{21}}{M},$$
$$c = \frac{m}{m+M}.$$

令$x_1 = (x_{11}, x_{12})^{\mathrm{T}},\ x_2 = (x_{21}, x_{22})^{\mathrm{T}},\ x = (x_1^{\mathrm{T}}, x_2^{\mathrm{T}})^{\mathrm{T}}$. 耦合作用项为

$$d_1(x,t) = \frac{1}{c\times m}[a(t)(a(t) - c\times l)(-x_{11} + x_{21}) - (a(t) - 0.5)(y_1 - y_2)],$$
$$d_2(x,t) = \frac{1}{c\times m}[a(t)(a(t) - c\times l)(x_{11} - x_{21}) - (a(t) - c\times l)(y_2 - y_1)].$$

取

$$a(t) = \sin(5t),\ y_1 = \sin(2t),$$
$$y_2 = L + \sin(3t),\ k = 1,$$
$$M = m = 10,\ l = 1,$$
$$L = 2,\ g = 1,\ c = 0.5.$$

因此

$$|d_i(x,t)| \leqslant \frac{12}{m} + \frac{3}{m}\|x_1\| + \frac{3}{m}\|x_2\|,\ i = 1,2, \tag{61}$$

$$f_{11}(x_{11}) = 0,\ f_{12}(\bar{x}_{12}) = 2x_{11} - \beta_1x_{12}^2,$$
$$g_{11}(x_{11}) = 1,\ g_{12}(\bar{x}_{12}) = g_3,\ \Delta_{11} = 0.5z_1,$$
$$\Delta_{12} = 2z_1^2,\ f_{21}(x_{21}) = 0,$$
$$f_{22}(\bar{x}_{22}) = 2x_{21} - \beta_2x_{22}^2,\ g_{21}(x_{21}) = 1,$$
$$g_{22}(\bar{x}_{22}) = g_3,\ \Delta_{21} = 2z_2,\ \Delta_{22} = 3z_2^2,$$

跟踪的期望轨迹$(y_{1\mathrm{d}}, y_{2\mathrm{d}})^{\mathrm{T}} = (0,0)^{\mathrm{T}}$.

自适应律和控制律设计如下:

$$\dot{\hat{\lambda}}_{ig} = \gamma_{i1}\Big[\sum_{j=1}^{2}\frac{1}{2a_{ij}^2}\|S_{ij}(Z_{ij})\|^2 s_{ij}^2 - \sigma_{i1}\hat{\lambda}_{ig}\Big], \tag{62}$$

$$\dot{\hat{d}}_{ig} = \gamma_{i2}\Big[\frac{1}{2}\sum_{k=0}^{1}\|\bar{x}_{i2}\|^{2k} s_{i2}^2 - \sigma_{i2}\hat{d}_{ig}\Big], \tag{63}$$

$$\alpha_{i2} = -k_{i1}s_{i1} - \frac{1}{2a_{i1}^2}\hat{\lambda}_{ig}s_{i1}\|S_{i1}(Z_{i1})\|^2, \tag{64}$$

$$u_i(t) = -k_{i2}s_{i2} - \frac{1}{2a_{i2}^2}s_{i2}\hat{\lambda}_{ig}\|S_{i2}(Z_{i2})\|^2 - \frac{1}{2}\hat{d}_{ig}\sum_{k=0}^{1}[\|\bar{x}_{i2}\|^{2k}s_{i2}],\ i = 1,2. \tag{65}$$

仿真中, 取

$$k_{11} = k_{21} = k_{12} = k_{22} = 15,$$
$$\gamma_{11} = \gamma_{12} = 15,\ \gamma_{21} = \gamma_{22} = 15,$$
$$\sigma_{11} = \sigma_{12} = 0.001,\ \sigma_{21} = \sigma_{22} = 0.001,$$
$$\tau_{12} = 0.01,\ \tau_{22} = 0.02,\ a_{11} = a_{21} = 1.5,$$
$$a_{12} = a_{22} = 2.25.$$

初始状态取为

$$(x_{11}(0), x_{12}(0))^{\mathrm{T}} = (0.3, 0)^{\mathrm{T}},$$
$$(x_{21}(0), x_{22}(0))^{\mathrm{T}} = (-0.3, 0)^{\mathrm{T}},$$
$$z_1(0) = z_2(0) = 0,\ \hat{d}_{1\mathrm{g}}(0) = \hat{d}_{2\mathrm{g}}(0) = 0.2,$$

$\hat{\lambda}_1(0)=\hat{\lambda}_2(0)=0.1,\ \omega_{12}(0)=0,\ \omega_{22}(0)=0.$

仿真结果如图1–4所示. 由图1–2可以看出, 本文提出的自适应控制算法具有较强的鲁棒性和良好的跟踪性能.

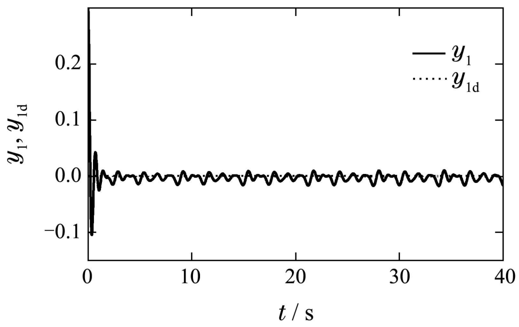

图 1 输出y_1和跟踪轨迹y_{1d}

Fig. 1 Output y_1 and tracking trajectory y_{1d}

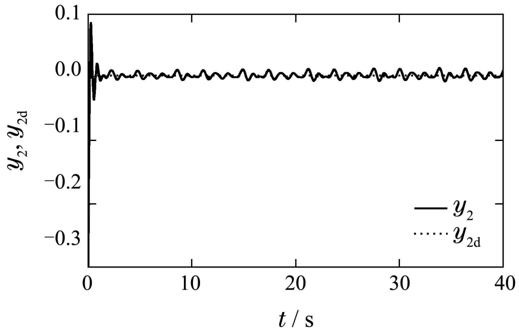

图 2 输出y_2和跟踪轨迹y_{2d}

Fig. 2 Output y_2 and tracking trajectory y_{2d}

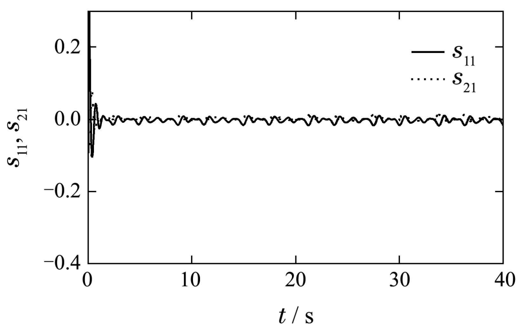

图 3 跟踪误差s_{11}和s_{21}

Fig. 3 Tracking errors s_{11} and s_{21}

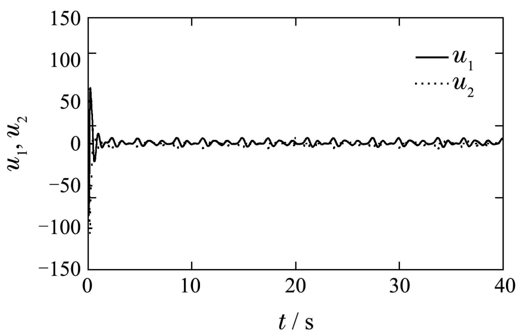

图 4 控制信号u_1和u_2

Fig. 4 Control signals u_1 and u_2

例 2 考虑具有未建模动态的3个倒立摆组成的耦合系统[17], 其动态方程描述如下:

子系统1:

$$\begin{cases}\dot{z}_1=q_1(z_1,y_1),\\ \dot{x}_{11}=x_{12},\\ \dot{x}_{12}=\dfrac{\mathrm{g}}{l}\sin y_1+u_1+\\ \qquad \varDelta_{12}(z_1,y_1)+d_1(y_1,y_2,y_3),\\ y_1=x_{11}.\end{cases}\tag{66}$$

子系统2:

$$\begin{cases}\dot{z}_2=q_2(z_2,y_2),\\ \dot{x}_{21}=x_{22},\\ \dot{x}_{22}=\dfrac{\mathrm{g}}{l}\sin y_2+0.7u_2+\\ \qquad \varDelta_{22}(z_2,y_2)+d_2(y_1,y_2,y_3),\\ y_2=x_{21}.\end{cases}\tag{67}$$

子系统3:

$$\begin{cases}\dot{z}_3=q_3(z_3,y_3),\\ \dot{x}_{31}=x_{32},\\ \dot{x}_{32}=\dfrac{\mathrm{g}}{l}\sin y_3+1.2u_3+\\ \qquad \varDelta_{32}(z_3,y_3)+d_3(y_1,y_2,y_3),\\ y_3=x_{31},\end{cases}\tag{68}$$

其中: x_{i1}是第i个摆的角度, g是重力加速度, M_i是第i个杆的质量, l是每一个杆的长度, a是支点到杆的重力中心的距离, k_1和k_2是弹簧的弹性系数, u_i表示第i个子系统的输入. 仿真中, 类似于文献[17]的讨论, 取未建模动态为$q_i(z_i,y_i)=-z_i+y_i^2$, 动态干扰为$\varDelta_{12}(z_1,y_1,t)=z_1^2,\ \varDelta_{22}(z_2,y_2,t)=z_2^2\sin t,\ \varDelta_{32}(z_3,y_3,t)=z_3^2\cos t$, 耦合作用项为

$$\begin{aligned}&d_1(y_1,y_2,y_3)=\\ &\frac{k_1a^2}{M_1l^2}(\sin y_2\cos y_2-\sin y_1\cos y_1),\\ &d_2(y_1,y_2,y_3)=\\ &\frac{k_1a^2}{M_2l^2}(\sin y_1\cos y_1-\sin y_2\cos y_2)+\\ &\frac{k_2a^2}{M_2l^2}(\sin y_3\cos y_3-\sin y_2\cos y_2),\\ &d_3(y_1,y_2,y_3)=\\ &\frac{k_2a^2}{M_3l^2}(\sin y_2\cos y_2-\sin y_3\cos y_3),\end{aligned}$$

期望的跟踪轨迹为$y_{id}=0,\ i=1,2,3.$

自适应律和控制律设计如下:

$$\dot{\hat{\lambda}}_{ig}=\gamma_{i1}\Big[\sum_{j=1}^{2}\frac{1}{2a_{ij}^2}\|S_{ij}(Z_{ij})\|^2s_{ij}^2-\sigma_{i1}\hat{\lambda}_{ig}\Big],\tag{69}$$

$$\dot{\hat{d}}_{ig}=\gamma_{i2}\Big[\frac{1}{2}s_{i2}^2-\sigma_{i2}\hat{d}_{ig}\Big],\tag{70}$$

$$\alpha_{i2} = -k_{i1}s_{i1} - \frac{1}{2a_{i1}^2}\hat{\lambda}_{ig}s_{i1}\|S_{i1}(Z_{i1})\|^2, \quad (71)$$

$$u_i(t) = -k_{i2}s_{i2} - \frac{1}{2a_{i2}^2}s_{i2}\hat{\lambda}_{ig}\|S_{i2}(Z_{i2})\|^2 - \frac{1}{2}\hat{d}_{ig}s_{i2},\ i=1,2,3. \quad (72)$$

仿真中:

$$M_i = 5,\ g=1,\ k_i=1,\ l=1,\ a=0.25,$$
$$s_{i1} = y_i,\ k_{i1}=k_{i2}=5,\ \tau_{i2}=0.01,$$
$$\gamma_{i1}=\gamma_{i2}=15,\ \sigma_{i1}=\sigma_{i2}=0.001,$$
$$a_{i1}=1.5,\ a_{i2}=2.25,\ i=1,2,3,$$

初始状态取为

$$x(0) = (1,0,-1,0,1.4,0,0.1,0.1,0.1)^{\mathrm{T}},$$
$$\hat{\lambda}_{ig}(0)=0.1,\ \hat{d}_{ig}(0)=0.5,$$
$$\omega_{i2}(0)=0.5,\ i=1,2,3.$$

仿真结果如图5−6所示.

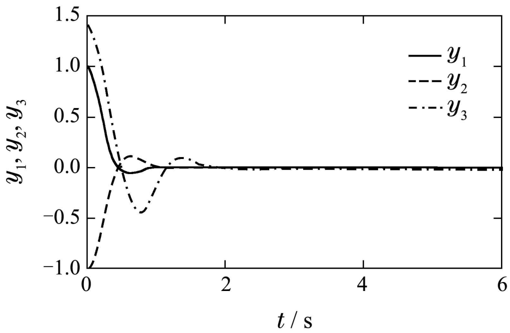

图 5 输出y_1, y_2和y_3

Fig. 5 Outputs y_1, y_2 and y_3

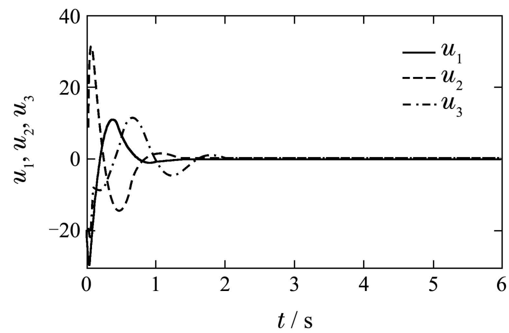

图 6 控制信号u_1, u_2和u_3

Fig. 6 Control signals u_1, u_2 and u_3

注 2　例1中耦合作用项上界多项式的阶次$P=1$, 而例2中耦合作用项上界多项式的阶次$P=0$. 仿真结果表明本文所提控制算法对文献[17]中的3倒立摆系统是有效的. 从图5和文献[17]中的图5可知, 本文获得的输出进入稳态的时间比文献[17]短.

6　结论(Conclusions)

本文研究了一类具有未建模动态和动态扰动的严格反馈非线性耦合大系统的分散自适应控制问题. 采用Lyapunov函数来描述未建模动态的特性, 利用Young's不等式和三重求和项的分解, 有效地处理了耦合作用项, 进一步, 借助于动态面控制设计方法的特点, 设计了分散自适应神经网络控制器. 与现有结果相比, 所设计的分散控制律中不含有控制增益下界常数, 自适应律和控制律中包含了神经网络径向基函数向量, 充分地利用了子系统的信息调节参数, 并且避免控制律中使用符号函数, 有效地抑制了控制信号的抖动. 通过理论分析, 证明了闭环控制系统中的所有信号一致终结有界, 且跟踪误差收敛到原点的一个小邻域内.

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