

DOI: 10.7641/CTA.2015.40902

基于修正闭环子空间辨识-分段线性结构的 环管式丙烯聚合反应过程非线性模型预测控制

娄海川^{1,2}, 苏宏业^{1†}, 古勇¹, 侯卫锋², 谢磊¹, 荣冈¹

(1. 浙江大学工业控制技术国家重点实验室; 智能系统与控制研究所, 浙江 杭州 310027;

2. 浙江中控软件技术有限公司, 浙江 杭州 310053)

摘要: 针对环管式聚丙烯生产过程装置多变量、耦合和非线性等特性容易导致过程控制不稳定及质量指标波动问题, 本文提出了一种基于修正闭环子空间辨识-分段线性(MSSARX-PWL)维纳(Wiener)模型结构的非线性模型预测控制算法。利用修正的闭环子空间辨识方法(MSSARX)辨识对象在闭环工况下的线性状态空间模型, 并将该线性模型与多变量分段线性化(PWL)方法辨识得到的非线性稳态模型结合, 建立双环管丙烯聚合反应动态过程的非线性预测模型, 而后进一步将非线性模型转化为线性模型, 在线性预测控制算法框架下用二次线性规划方法(LQP)优化控制器, 无须用非线性规划方法(NLP)求解。从双环管丙烯聚合反应过程仿真例子表明, 该算法不仅能保证模型和控制精度, 而且能提高计算效率。

关键词: 非线性模型预测控制; MSSARX-PWL结构; 双环管丙烯聚合反应过程; 分段线性化

中图分类号: TP273 **文献标识码:** A

Nonlinear predictive control with modified closed-loop subspace identification-piecewise linear model for double-loop propylene polymerization process

LOU Hai-chuan^{1,2}, SU Hong-ye^{1†}, GU Yong¹, HOU Wei-feng², XIE Lei¹, RONG Gang¹

(1. State Laboratory of Industrial Control Technology; Institute of Cyber-Systems and Control,
Zhejiang University, Hangzhou Zhejiang 310027, China;

2. Zhejiang Supcon Software Co., Ltd., Hangzhou Zhejiang 310053, China)

Abstract: To solve the unstable and oscillatory problem of the double-loop propylene polymerization process with multivariable, coupling and nonlinearity, we propose a nonlinear model predictive control algorithm based on the Wiener-type model by modified closed-loop subspace identification (MSSARX). A linear state space model under closed-loop conditions and a nonlinear steady-state model are identified by using the modified closed-loop subspace identification method (MSSARX) and the multivariate piecewise linear (PWL) method, respectively. These two models are then combined into an MSSARX-PWL model structure, which is employed as the nonlinear predictive model of the process. To reduce the computational load, this nonlinear model is consequently linearized to ensure that linear quadratic programming (LQP) optimization controller can be applied instead of the nonlinear one. By applying the proposed algorithm, not only the accurate prediction and control are guaranteed, but the computational efficiency is also improved at the same time. The effectiveness of the proposed algorithm is demonstrated on the simulation process of double-loop propylene polymerization.

Key words: nonlinear model predictive control; MSSARX-PWL structure; double-loop propylene polymerization process; piece wise linear

1 Introduction

The industrial propylene polymerization reaction process, which is complex with serious nonlinear and coupling, is likely to cause the quality indicators (e.g., melt index) be fluctuated, as well as poor control accuracy of slurry density and hydrogen concentration. As it is difficult to be solved by the single-loop PID control or linear pre-

dictive control strategy, nonlinear modeling and predictive control methods are necessary.

Commonly, nonlinear modeling methods include rigorous mechanism methods, empirical (data-driven) methods, and combination of mechanism and empirical methods. Since propylene polymerization process involves complex chemical reactions, mechanism modeling method

Received 24 September 2014; accepted 18 May 2015.

† Corresponding Author. E-mail: hysu@ipc.zju.edu.cn; Tel.: +86 571-87952233-8209.

Supported by National Science Technology Pillar Program of China (2012BAF05B01) and National Natural Science Foundation of China (61374121, 61134007).

will be cost a lot of time and effort. Beside, a reliable and satisfaction model is difficult to establish if a certain assumptions be made that would limit their applications in the actual industrial process. On the contrary, empirical (data-driven) modeling methods widely used in the process control depend on data analysis in practice, is easier be obtained than mechanism based model.

There are many data-driven nonlinear modeling approaches, such as nonlinear autoregressive with exogenous inputs (NARX) models^[1–2] volterra series expansion^[3], artificial neural networks^[4], linear parameters time-varying (LPV)^[5–6] block oriented models^[7–9] and so on. And the block oriented (BO) models, combined of linear dynamic model and nonlinear static model, is relatively simple and easily to identify, can describe most nonlinear chemical process (such as, pH process, polymerization process, etc.). As the model can cover a wide of working area, there have been widely applied in the actual processes^[10]. Distinct from the connection order between dynamic linear model and nonlinear static model, the BO model can be divided into Hammerstein models^[7], Wiener models^[8–11] or combination of the both^[12]. And the Hammerstein models are used on the occasion that steady-state gain of process varied with the operating points, while the Wiener models are generally applied on the occasion when process dynamic changed with the operating points^[13]. Since the process dynamic of propylene polymerization is changed frequently with the operating points, the Wiener model is suitable for the process.

There are varieties of Wiener-type model identification methods, one is direct identification method that identifies linear dynamic and nonlinear static aspects together^[9]. While another approach identifies two parts of Wiener model respectively, in which linear part of model identified with step response model^[14], ARX^[15], orthonormal filter networks^[16]. Subspace^[17], etc. and the nonlinear static part uses with identification models such as polyno-

mia^[17], volterra kernels^[18], piecewise constant functions^[19], ANN^[20] and so on. Because the propylene polymerization process is open-loop unstable, it needs to be operated in closed loop case for safety. On the other hand, in order to avoid a large number of off-spec material loss during frequently testing of production process, and to maintain stable operate conditions, dynamic and static information for modeling are often extracted from the rich historical data or real-time database.

In this paper, a nonlinear model predictive control algorithm based on MSSARX–PWL (Wiener-type) model is proposed for double-loop polypropylene production process with multivariate, coupled and unstable nonlinear characteristics. The MSSARX–PWL model structure, in which linear state space model under the closed-loop conditions is identified by the improved closed-loop subspace identification method (MSSARX), combined with the nonlinear steady-state model identified by the multivariate PWL method, is established for the nonlinear predictive model of double loop propylene polymerization process. Furthermore, the non-linear model can be inverted to linear model that without non-linear programming methods (NLP) solver but only the linear quadratic programming (QP) optimization controller needed.

2 Problem description

In the industrial propylene polymerization process, various parameters (such as temperature, pressure, flow, level, etc.) underlying the process plant are controlled by PID controllers in control loops (see Figure 1). Most of these base control systems can play good control effect in actual that may provide a powerful guarantee for smooth operation of plants. However, due to the complex and couple of polypropylene production unit with serious nonlinear and frequent process parameters changed that can result in an unstable product quality, the conventional single-loop (or cascade) PID control and manual control based on the experience are powerless when dealing with these issues.

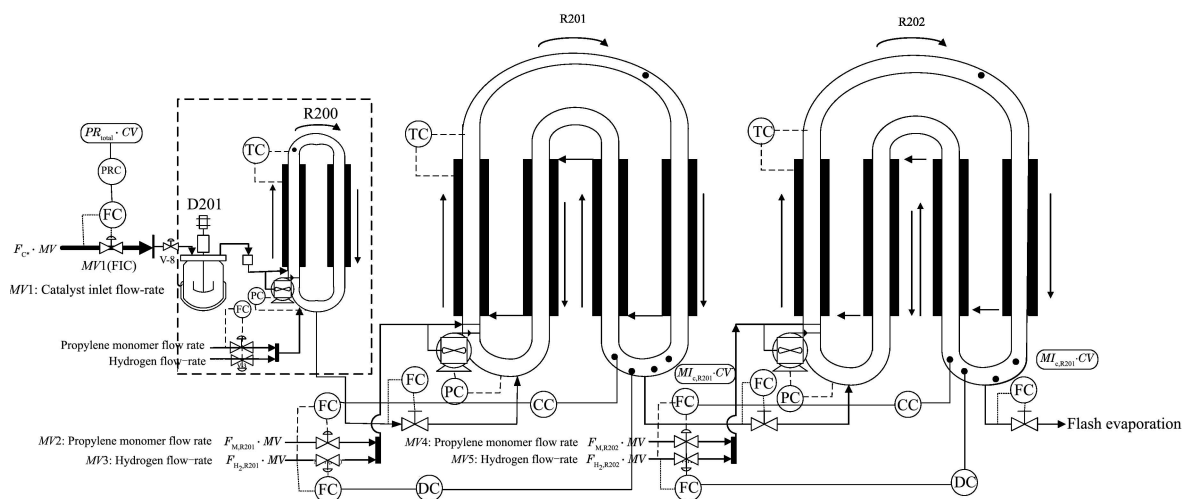


Fig. 1 Base control loops of propylene polymerization reaction process

Majority of engineering practices have already proved that multivariable advanced controller with model-based predictive control algorithm not only can increase the operation level of plant and ensure stable rates of key process parameters, but also raise the plant product quality, thus bring considerable economic benefits. In this paper, advanced control algorithms and strategies for double-loop propylene polymerization process are studied. The NMPC controller is to keep stability control for slurry density, hydrogen concentration and loop production rate, hence indirectly to realize the polypropylene product quality control.

As Figure 2 described, advanced process control diagram of double-loop propylene polymerization reaction process. In the reactor R201, the hydrogen inlet flow-rate $F_{H_2,R201}$, liquid propylene inlet rate $F_{M,R201}$, catalyst inlet flow-rate F_{C^*} are selected as the manipulated variables, and the slurry density $\rho_{s,R201}$, hydro-

gen concentration $C_{H_2,R201}$, loop production rate PR_1 are considered as controlled variables, and in the reactor R202, the hydrogen inlet flow-rate $F_{H_2,R202}$ and liquid propylene inlet flow-rate $F_{M,R202}$ are selected as the manipulated variables, while slurry density $\rho_{s,R202}$, hydrogen concentration $C_{H_2,R202}$, loop production rate PR_2 are considered as controlled variables. Besides, hydrogen inlet flow-rate, liquid propylene inlet flow-rate $F_{M,R201}$ of R201 are regarded as the main measured disturbance variables impacted on reactor R202.

From the mechanism analysis, the pre-polymerization reactor is easy to reach the steady state, the inlet flow-rate of hydrogen and liquid propylene can be controlled by conventional PID controllers, so multivariable nonlinear constrained predictive models and predictive controller for main reactors are only established. In addition, note that material flow of double-loop reactors are unidirectional.

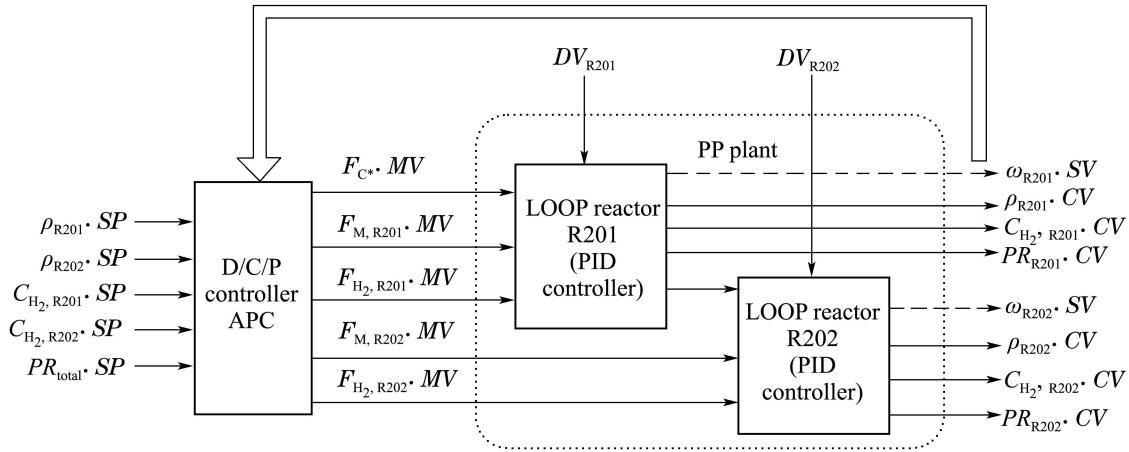


Fig. 2 Advanced process control diagram of double-loop propylene polymerization reaction process

3 MSSARX–PWL model identification

Since the propylene polymerization process shows strongly nonlinear dynamics with multiple operating points changed, Wiener model structure is selected to describe such process.

The Wiener model structure(see Figure 3), composed of linear dynamics part and nonlinear static part, can be expressed as

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + \xi, \\ v(k) = Cx(k) + Du(k) + \zeta, \\ y(k) = f(v(k)), \end{cases} \quad (1)$$

where $u(k) \in \mathbb{R}^{n_u}$, $x(k) \in \mathbb{R}^n$, $v(k) \in \mathbb{R}^{n_v}$, and $y(k) \in \mathbb{R}^{n_y}$ are input observation vector, state vector and output observation vector of system, respectively, n_u , n_v , n_x are orders of the input vector, output vector and state vector, ξ , ζ denote the zero mean white innovation process, respectively, that is

$$\begin{aligned} u(k) &= [u_1(k) \ u_2(k) \ \cdots \ u_{n_u}(k)]^T, \\ x(k) &= [x_1(k) \ x_2(k) \ \cdots \ x_n(k)]^T, \\ v(k) &= [v_1(k) \ v_2(k) \ \cdots \ v_{n_v}(k)]^T, \\ y(k) &= [y_1(k) \ y_2(k) \ \cdots \ y_{n_y}(k)]^T, \end{aligned}$$

where, coefficient matrixes

$$A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times n_u}, C \in \mathbb{R}^{n_v \times n}, D \in \mathbb{R}^{n_v \times n_u}.$$

In the Wiener model structure, dynamic linear part of model is identified by the closed-loop subspace algorithm (MSSARX) presented in this paper, while static nonlinear part is described by the multivariable piecewise linear (PWL) function.

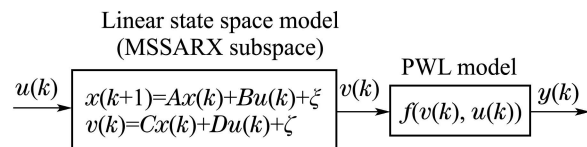


Fig. 3 MSSARX–PWL (Wiener-type) model structure

3.1 Closed-loop identification of linear model based on MSSARX

In the actual industrial field, continuous polypropylene production is carrying out under closed-loop and stable situation, identification and modeling data are generally obtained from the feedback loop for safety and economic considerations. Subspace identification methods based on state space model with simple and effective algorithm, which are especially for multivariable system identification, have been gained more and more attention for nearly twenty years. However, different from open-loop identification, the key problem of closed-loop identification lays in the correlation between input and unmeasured noise, that making some open-loop subspace identification algorithm used for closed-loop data identification lost in biased parameter estimates^[21].

In order to solve the above issue, many papers have proposed different methods to remove or circumvent the orthogonal projection calculations. Closed-loop subspace identification method, namely SSARX, which enables the input signal and noise separation^[22], is easily to implement in engineering. In this paper, modified-SSARX (MSSARX) is raised for simplifying the solving steps, which system states are estimated by the Kalman SVD method, and each parameter matrix of systems are computed, then linear part of the Wiener model is identified. Assuming the linear model of system is represented in the innovation form as

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ke(k), \\ v(k) &= Cx(k) + Du(k) + e(k). \end{aligned} \tag{2}$$

where $e(k) = v(k) - \hat{v}(k|k-1) \in \mathbb{R}^{n_v}$ defined as one step-ahead prediction error, is a zero-mean white noise process. Hence, system parameter matrix A, B, C, D, K of state-space model (2) are estimated online by input/output data acquired under open loop/closed loop conditions. The innovation form (2) is rewritten in the following Kalman predictor form

$$\begin{aligned} x(k+1) &= \tilde{A}x(k) + \tilde{B}u(k) + Kv(k), \\ v(k) &= Cx(k) + Du(k) + e(k), \end{aligned} \tag{3}$$

where

$$\begin{aligned} \tilde{A} &= A - KC, \\ \tilde{B} &= B - KD. \end{aligned}$$

Hankel matrixs, containing i block row and j block column, are constructed by the input and output data, defined as

$$\begin{cases} U_P = \begin{bmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j} \end{bmatrix}, \\ U_f = \begin{bmatrix} u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{bmatrix}. \end{cases} \tag{5}$$

$$\begin{cases} \mathcal{Y}_p = \begin{bmatrix} v_0 & v_1 & \cdots & v_{j-1} \\ v_1 & v_2 & \cdots & v_j \\ \vdots & \vdots & \ddots & \vdots \\ v_{i-1} & v_i & \cdots & v_{i+j-2} \end{bmatrix}, \\ \mathcal{Y}_f = \begin{bmatrix} v_i & v_{i+1} & \cdots & v_{i+j-1} \\ v_{i+1} & v_{i+2} & \cdots & v_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ v_{2i-1} & v_{2i} & \cdots & v_{2i+j-2} \end{bmatrix}, \end{cases} \tag{6}$$

wherein, the subscripts p and f represent the relative

time of past and future. $u_i = \begin{bmatrix} u_{i1} \\ u_{i2} \\ \vdots \\ u_{in_u} \end{bmatrix}, v_i = \begin{bmatrix} v_{i1} \\ v_{i2} \\ \vdots \\ v_{in_v} \end{bmatrix}$

denote the input and output column vectors, respectively. And i and j are user-defined parameters that reduce the sensitivity to noise, generally satisfied

$$i \geq n, j \gg \max(in_u, in_v).$$

Generalized observability matrix defined as

$$\tilde{\Gamma}_i = \begin{bmatrix} C \\ C\tilde{A} \\ \vdots \\ C\tilde{A}^{i-1} \end{bmatrix} \in \mathbb{R}^{in_v \times n}. \tag{7}$$

Generalized inverse observability matrix defined as

$$\tilde{C}_i^d = [\tilde{A}^{i-1}\tilde{B} \ \tilde{A}^{i-2}\tilde{B} \ \cdots \ \tilde{B}] \in \mathbb{R}^{n \times in_u}, \tag{8}$$

$$\tilde{C}_i^s = [\tilde{A}^{i-1}K \ \tilde{A}^{i-2}K \ \cdots \ K] \in \mathbb{R}^{n \times in_v}. \tag{9}$$

Low-dimensional triangular Toplitz matrix block $\tilde{H}_i^d, \tilde{H}_i^s$ defined as

$$\tilde{H}_i^d = \begin{bmatrix} D & 0 & \cdots & 0 \\ C\tilde{B} & D & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ C\tilde{A}^{i-2}\tilde{B} & \cdots & C\tilde{B} & D \end{bmatrix}, \tag{10}$$

$$\tilde{H}_i^s = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CK & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ C\tilde{A}^{i-2}K & \cdots & CK & 0 \end{bmatrix}. \tag{11}$$

And the future output prediction equation is obtained by (3) and (4),

$$\mathcal{Y}_f(k) = \tilde{\Gamma}_i x(k) + \tilde{H}_i^d U_f(k) + \tilde{H}_i^s \mathcal{Y}_f(k) + \varepsilon_f. \quad (12)$$

It can be seen the innovation noise term ε_f is related with the future Hankel matrix $U_f(k)$ and $\mathcal{Y}_f(k)$. In order to solve the correlation between the input signal and noise in the closed-loop identification, the both need to be separated. For closed-loop system, here let $D = 0$. Because the parameters of linear state-space model in the innovation form can be obtained from the higher order ARX model identification, and the input-output ARX model is launched by (4) as

$$\begin{aligned} x(k) = & \tilde{A}_1 v(k-1) + \dots + \tilde{A}_J v(k-J) + \\ & \tilde{B}_1 u(k-1) + \dots + \tilde{B}_J u(k-J) + \\ & \tilde{A}_J x(k-J) = \\ & \sum_{i=1}^J \tilde{A}_i v(k-(J+1-i)) + \\ & \sum_{i=1}^J \tilde{B}_i u(k-(J+1-i)) + \tilde{A}_1 x(k-J), \end{aligned} \quad (13)$$

where, Markov parameter coefficient matrix of Kalman filtering prediction model are

$$\begin{aligned} \tilde{A}_i &= \tilde{A}^{i-1} K \in \mathbb{R}^{n \times n}, \\ \tilde{B}_i &= \tilde{A}^i B \in \mathbb{R}^{n \times n_u}, \quad \forall i = 1, \dots, J. \end{aligned}$$

When the past time domain parameters $J \rightarrow \infty$, least-squares regression or ARX problem about (13) can be constructed as

$$\tilde{x}(k) = [\tilde{C}_i^d \quad \tilde{C}_i^s] \begin{bmatrix} U_P \\ V_P \end{bmatrix}, \quad (14)$$

where $\tilde{C}_i^d, \tilde{C}_i^s$ are matrix of unknown coefficients and U_P, V_P are vector containing delayed inputs and outputs J steps back.

Transposition (12) and defined as

$$\begin{aligned} \tilde{\mathcal{Y}}_f(k) = & \mathcal{Y}_f(k) - \tilde{H}_i^d U_f(k) - \tilde{H}_i^s \mathcal{Y}_f(k) \approx \\ & \tilde{\Gamma} \tilde{x}(k) + \varepsilon_f. \end{aligned} \quad (15)$$

Note Toplitz matrix is eliminated to separate the input signal and noise. Substitute (14) in the form as

$$\tilde{\mathcal{Y}}_f(k) = \tilde{\Gamma} [\tilde{C}_i^d \quad \tilde{C}_i^s] \begin{bmatrix} U_P \\ V_P \end{bmatrix} + \varepsilon_f.$$

Let

$$\tilde{\theta} = \tilde{\Gamma} [\tilde{C}_i^d \quad \tilde{C}_i^s], \quad (16)$$

then

$$\tilde{\mathcal{Y}}_f(k) = \tilde{\theta} W_P + \varepsilon_f \quad (17)$$

where

$$W_P = \begin{bmatrix} U_P \\ V_P \end{bmatrix}.$$

Obtained by the least squares identification

$$\hat{\tilde{\theta}} = \tilde{\mathcal{Y}}_f(k) W_P (W_P W_P^T)^{-1}. \quad (18)$$

Then SVD decomposition is carried out and obtained as

$$\theta = U S V^T \approx U_1 S_1 V_1^T, \quad (19)$$

where $U_1 = U(:, 1 : n)$, $S_1 = S(1 : n, 1 : n)$, $V_1 = V(:, 1 : n)$ is the order of system model. Compare (16) and (18), then give

$$\tilde{\Gamma} = U_1, \quad (20)$$

$$[\tilde{C}_i^d \quad \tilde{C}_i^s] = S_1 V_1^T. \quad (21)$$

State $\hat{x}(k)$ is identified as

$$\hat{x}(k) = [\tilde{C}_i^d \quad \tilde{C}_i^s] \begin{bmatrix} U_P \\ V_P \end{bmatrix} = S_1 V_1^T \begin{bmatrix} U_P \\ V_P \end{bmatrix}. \quad (22)$$

Finally, the output matrix is identified as

$$\hat{C} = v_f \hat{x}_f^T(k) (\hat{x}_f(k) \hat{x}_f^T(k))^{-1}. \quad (23)$$

Regression residuals information is got as

$$\varepsilon_f(k) = v_f(k) - v_f(k)/x_f(k). \quad (24)$$

Therefore, the parameters can be obtained by the least square method as

$$x_f(k+1) = [A \ B \ K] \begin{bmatrix} x(k) \\ U_f \\ \varepsilon_f \end{bmatrix}. \quad (25)$$

3.2 Modeling of nonlinear steady-state gain with piecewise linear method

Continuous PWL function is a powerful tool for nonlinear modeling and analysis, that any nonlinear function, through the division of input domain, can be expressed as^[23]

$$y(k) = f(v(k)) = \Phi^T \Psi(v(k)), \quad (26)$$

where

$$\begin{aligned} \Phi^T = & \\ & [\theta_{1,0}, \theta_{1,1}, \dots, \theta_{1,\sigma}, \dots, \theta_{n_y,0}, \dots, \theta_{n_y,\sigma}] \in \\ & \mathbb{R}^{n_y(\sigma+1)}. \end{aligned}$$

The base function is selected as

$$\Psi(v(k)) =$$

$$\begin{bmatrix} 1 \\ \frac{1}{2}(v(k) - \alpha_{n_y,0} + |v(k) - \alpha_{n_y,0}|) \\ \vdots \\ \frac{1}{2}(v(k) - \alpha_{n_y,\sigma-1} + |v(k) - \alpha_{n_y,\sigma-1}|) \end{bmatrix} \in \mathbb{R}^{n_y(\sigma+1) \times n_y},$$

$$\alpha_{n_y,0} \leq \alpha_{n_y,1} \leq \dots \leq \alpha_{n_y,\sigma-1},$$

where $[\theta_{1,0}, \theta_{1,1}, \dots, \theta_{1,\sigma}, \theta_{n_y,0}, \theta_{n_y,1}, \dots, \theta_{n_y,\sigma}]$ and $\alpha_{n_y,0}, \alpha_{n_y,1}, \dots, \alpha_{n_y,\sigma}$ are identified parameters, and solved from objective function (26) by QP optimization methods or intelligent optimization methods (such as genetic algorithms),

$$J = \min \sum (y(k) - \Phi^T \Psi(v(k)))^2$$

subject to

$$\alpha_{n_y,K-1} \leq \alpha_{n_y,K}, \quad K = 1, 2, \dots, \sigma - 1. \quad (27)$$

Compared with other nonlinear steady-state modeling methods (such as artificial neural networks, support vector machines, etc.), the PWL method not only can well describe the nonlinear processes^[23], but also avoid solving nonlinear optimization by linearization method to nonlinear controller, hence greatly reducing the burden of computation.

4 Multi-steps nonlinear prediction modeling based on MSSARX-PWL identification

In general, there are three methods for Wiener model identification (1): linear-nonlinear (L-N) methods^[24-25], nonlinear-linear (N-L) methods^[26-27], and combined with the both^[18]. Since L-N methods are relatively straightforward that can ensure better accuracy^[25], the method is adopted in this paper. Linear state-space model is identified based on MSSARX subspace methods with dynamic data from simulation model of propylene polymerization process, and nonlinear static gain is modeling with steady-state data of propylene polymerization process based on PWL identification.

In addition, various model parameters are identified off-line and nonlinear predictive model for application is built online. Steps of model parameter identification and prediction are listed as follows:

Step 1 State space model parameters A, B, C, D, K are identified by the dynamic input-output training data based on MSSARX algorithm.

Step 2 PWL parameters are training offline by steady input-output data $(u_{ss}(k), y_{ss}(k))$. When the

system is under steady state condition, then

$$x_{ss}(k+1) = Ax_{ss}(k) + Bu_{ss}(k) + \xi(k), \quad (28)$$

$$x_{ss}(k) = (I - A)^{-1}(Bu_{ss}(k) + Ke(k)). \quad (29)$$

As

$$v_{ss}(k) = Cx_{ss}(k),$$

$$y_{ss}(k) = f(v_{ss}(k)) = \Phi^T \Psi(v_{ss}(k)).$$

Parameters $\alpha_{n_y,0}, \alpha_{n_y,1}, \dots, \alpha_{n_y,\sigma}$ and $[\theta_{1,0}, \theta_{1,1}, \dots, \theta_{1,\sigma}, \theta_{n_y,0}, \theta_{n_y,1}, \dots, \theta_{n_y,\sigma}]$ can be obtained by the QP algorithm

Step 3 Nonlinear prediction model based on MSSARX-PWL is built online by using dynamic input-output testing data. Set model prediction horizon P , control horizon M , respectively, and the multi-step control and state variables are described as

$$\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+M) \\ \vdots \\ u(k+P) \end{bmatrix} = \begin{bmatrix} u_1(k) & u_2(k) & \dots & u_i(k) \\ u_1(k+1) & u_2(k+1) & \dots & u_i(k+1) \\ \vdots & \vdots & \vdots & \vdots \\ u_1(k+M) & u_2(k+M) & \dots & u_i(k+M) \\ \vdots & \vdots & \vdots & \vdots \\ u_1(k+P) & u_2(k+P) & \dots & u_i(k+P) \end{bmatrix}, \quad (30)$$

$$\begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+M) \\ \vdots \\ x(k+P) \end{bmatrix} = \begin{bmatrix} x_1(k+1) & x_2(k+1) & \dots & x_i(k+1) \\ x_1(k+2) & x_2(k+2) & \dots & x_i(k+2) \\ \vdots & \vdots & \vdots & \vdots \\ x_1(k+M) & x_2(k+M) & \dots & x_i(k+M) \\ \vdots & \vdots & \vdots & \vdots \\ x_1(k+P) & x_2(k+P) & \dots & x_i(k+P) \end{bmatrix}. \quad (31)$$

As $x(k+1) = Ax(k) + Bu(k) + \xi(k)$, equation (31) is transformed as

$$\begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+M) \\ \vdots \\ x(k+P) \end{bmatrix} = \begin{bmatrix} I & & & \\ A & I & & 0 \\ \vdots & \ddots & \ddots & \\ A^M & \dots & A & I \\ \vdots & & & \ddots & \ddots \\ A^{P-1} & A^{P-2} & \dots & A & I \end{bmatrix} \cdot B \cdot \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+M) \\ \vdots \\ u(k+P) \end{bmatrix} + \begin{bmatrix} I & & & \\ A & I & & 0 \\ \vdots & \ddots & \ddots & \\ A^M & \dots & A & I \\ \vdots & & & \ddots & \ddots \\ A^{P-1} & A^{P-2} & \dots & A & I \end{bmatrix} \cdot K \cdot e(k) + [A A^2 \dots A^M \dots A^{P-1}]^T x(k). \tag{32}$$

Similarly, as $v(k) = Cx(k) + Du(k) + \zeta(k)$, predicted output of linear model is got,

$$\begin{bmatrix} v(k+1) \\ v(k+2) \\ \vdots \\ v(k+M) \\ \vdots \\ v(k+P) \end{bmatrix} = \begin{bmatrix} C^T A \\ C^T A^2 \\ \vdots \\ C^T A^M \\ \vdots \\ C^T A^P \end{bmatrix} x(k) + \begin{bmatrix} \zeta(k) \\ \zeta(k+1) \\ \vdots \\ \zeta(k+M) \\ \vdots \\ \zeta(k+P) \end{bmatrix} + \begin{bmatrix} C^T B & D & & & 0 \\ C^T AB & C^T B & D & & \\ \vdots & \vdots & \vdots & \vdots & \\ C^T A^M B & C^T A^{M-1} B \dots & C^T B & D & \\ \vdots & \vdots & \vdots & \vdots & \\ C^T A^{P-1} B & C^T A^{P-2} B \dots & C^T AB & C^T B D & \end{bmatrix} \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+M) \\ \vdots \\ u(k+P) \end{bmatrix}. \tag{33}$$

Simply described as

$$\widehat{v}(k) = x(k) + U(k) + d(k), \tag{34}$$

where $x(k)$ is obtained by Kalman filter estimator when calculation online.

Finally, P -steps predicted model output can be obtained as

$$Y_m(k+1) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+M) \\ \vdots \\ y(k+P) \end{bmatrix} = \begin{bmatrix} y_1(k+1) & y_2(k+1) & \dots & y_k(k+1) \\ y_1(k+2) & y_2(k+2) & \dots & y_k(k+2) \\ \vdots & \vdots & \dots & \vdots \\ y_1(k+M) & y_2(k+M) & \dots & y_k(k+M) \\ \vdots & \vdots & \dots & \vdots \\ y_1(k+P) & y_2(k+P) & \dots & y_k(k+P) \end{bmatrix} = \begin{bmatrix} f(v(k+1)) \\ f(v(k+2)) \\ \vdots \\ f(v(k+M)) \\ \vdots \\ f(v(k+P)) \end{bmatrix} = \begin{bmatrix} \Phi^T \Psi(v(k+1)) \\ \Phi^T \Psi(v(k+2)) \\ \vdots \\ \Phi^T \Psi(v(k+M)) \\ \vdots \\ \Phi^T \Psi(v(k+P)) \end{bmatrix}. \tag{35}$$

5 Linearization of nonlinear model predictive controller

Nonlinear model predictive controller framework based on MSSARX-PWL(Wiener) model is shown as Figure 4.

In order to minimize the performance index, a series of control points $u(k+1), u(k+2), \dots, u(k+M)$

under various constraints are calculated by solving the optimization problem of nonlinear predictive controller. Construct nonlinear predictive controller optimization proposition as

$$\begin{aligned} \min_{u(k+1), u(k+2), \dots, u(k+M)} J &= \\ \min \{ \sum_{i=1}^P \|y_r(k+i) - y_P(k+i)\|_Q^2 + & \\ \sum_{j=1}^{M-1} \|\Delta u(k+j)\|_S^2 + \sum_{j=1}^M \|u(k+j)\|_R^2 \}, & \tag{36} \end{aligned}$$

subject to

$$\begin{cases} \widehat{v}(k) = Fx(k) + SU(k) + d(k), \\ e(k) = y(k) - y_m(k+1), \\ y_m(k+i) = f[v(k+i-1)], \\ y_P(k+i) = y_m(k+i) + e(k), \\ y_r(k+i) = y_{sp} \alpha^i + (1-\alpha)^i y(k), \\ \Delta u(k+j) = u(k+j) - u(k+j-1), \\ u(k+m) = u(k+M); m = M+1, \dots, P, \\ u_{\min} \leq u(k) \leq u_{\max}, \\ \Delta u_{\min} \leq \Delta u(k) \leq \Delta u_{\max}, \\ y_{\min} \leq y(k) \leq y_{\max}, \\ i = 1, \dots, P; j = 1, \dots, M-1, \end{cases} \tag{37}$$

where $e(k), y(k), y_m(k), y_P(k), y_r(k)$ denote prediction bias, actual system output, model predicted output, model calibration output and model reference input, respectively. And $u_{\min}, u_{\max}, \Delta u_{\min}, \Delta u_{\max}, y_{\min}, y_{\max}$ denote limits of control input, control increment and actual system output, respectively. Q, S, R denote weighted matrixes. It is a nonlinear constrained optimization problem for solving the above controller, commonly sequence quadratic programmer (SQP) or interior point optimization (IPOPT) algorithm can be used.

Nevertheless, in order to reduce the computation burden and ensure response time of the controller, the above nonlinear predictive controller is transformed into linear

predictive controller (see Figure 5). Because PWL is a reversible function that quadratic programming method can be used for solving to ensure real-time control.

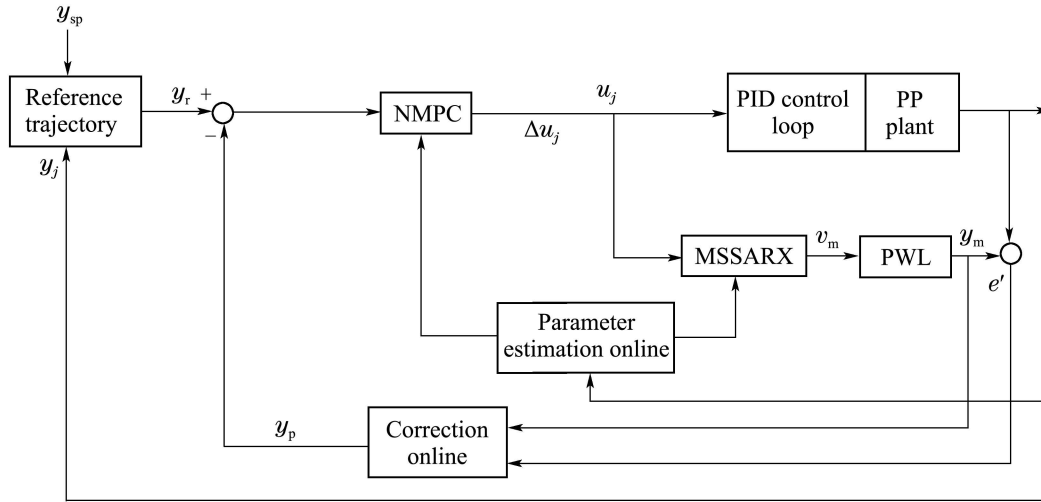


Fig. 4 Nonlinear model predictive control framework based on MSSARX-PWL

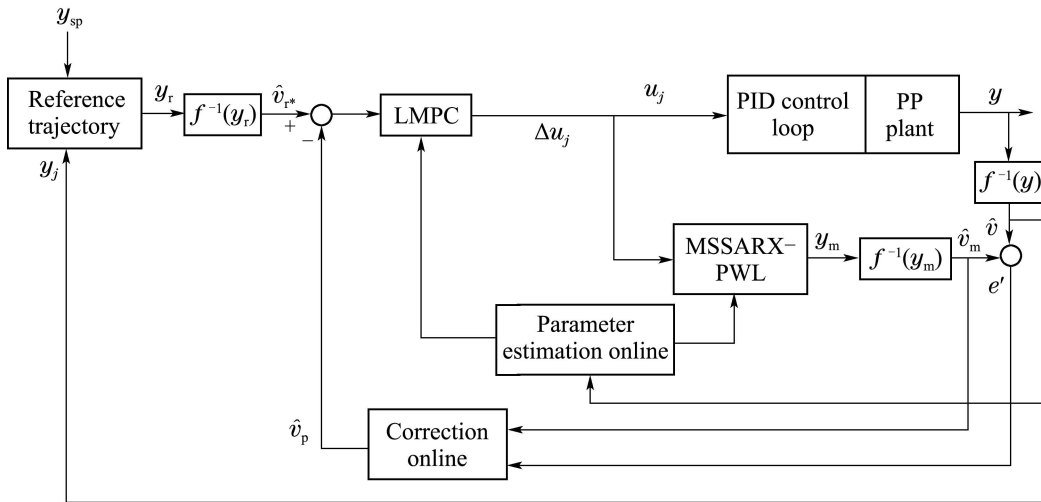


Fig. 5 Linearization of nonlinear model predictive control framework based on MSSARX-PWL

Since the PWL function $y(k) = f(v(k)) = \Phi^T \Psi(v(k))$ was assumed to be invertible^[28], it can be transformed to $v(k) = f^{-1}(y(k))$ ^[23]. Then the linear prediction controller is converted and reconstructed in the form of quadratic programming proposition under a series of constraints,

$$\begin{aligned} \min_{u(k+1), u(k+2), \dots, u(k+M)} J &= \\ \min \left\{ \sum_{i=1}^P \|\hat{v}_{r^*}(k+i) - \hat{v}_P(k+i)\|_Q^2 + \right. & \\ \left. \sum_{j=1}^{M-1} \|\Delta u(k+j)\|_S^2 + \sum_{j=1}^M \|u(k+j)\|_R^2 \right\} & \quad (38) \end{aligned}$$

subject to

$$\begin{cases} \hat{v}(k) = Fx(k) + SU(k) + d(k), \\ \dot{e}(k) = \hat{v}(k) - \hat{v}_m(k-1), \\ \hat{v}_P(k+i) = \hat{v}_m(k+i) + e(k), \\ \hat{v}_{r^*}(k+i) = f^{-1}(y_{sp}\alpha^i + (1-\alpha)^i y(k)), \\ \hat{v}_m(k+i) = f^{-1}[y_m(k+i-1)], \\ \Delta u(k+j) = u(k+j) - u(k+j-1), \\ u(k+m) = u(k+M); m = M+1, \dots, P, \\ u_{\min} \leq u(k) \leq u_{\max}, \\ \Delta u_{\min} \leq \Delta u(k) \leq \Delta u_{\max}, \\ \hat{v}_{\min} \leq \hat{v}(k) \leq \hat{v}_{\max}, \\ i = 1, \dots, P; j = 1, \dots, M-1, \end{cases} \quad (39)$$

where $\dot{e}(k)$, $\hat{v}(k)$, $\hat{v}_m(k)$, $\hat{v}_{r^*}(k)$ denote the prediction bias, linearization output of actual system, linear model

predicted output, linear model corrected output and linear model reference input. $\hat{v}_{\min}, \hat{v}_{\max}$ denote the upper and lower limits of linear model output.

This proposition can be directly solved by the second linear programming (QP) method.

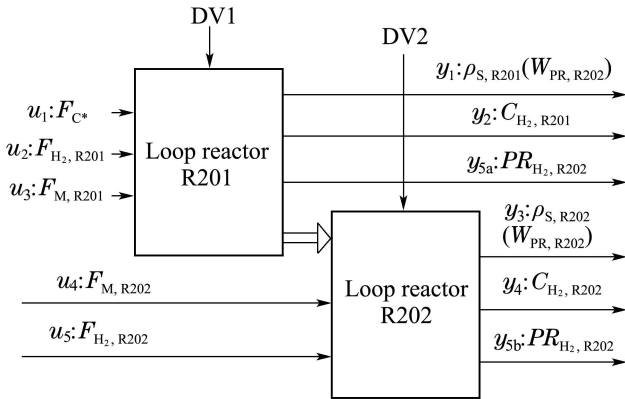


Fig. 6 Control relationship diagram of double-loop propylene polymerization process

6 Results and discussion

As problem description, propylene polymerization reaction is mainly carried out in the double-loop reactors, so nonlinear constrained and multivariable predictive models and controllers for both main reactors were established. According to the analysis of process mechanism, the hydrogen inlet flow-rate, liquid propylene inlet flow-rate and catalyst inlet flow-rate are regulated to keep stability control for loop density, inlet hydrogen concentration and process production rate, and thus polypropylene product qualities are indirectly controlled. (See Figure 6, control relationship diagram of double-loop propylene polymerization process).

6.1 Identification and modeling of the propylene polymerization process based on MSSARX-PWL model

1) Excitation signals and experimental test data.

In order to obtain an accurate estimate of nonlinear process systems, generalized multi-valued noise (GMN) easily realized in actual is selected as the dynamic test signals in closed-loop, while the stair signals is used for testing process steady-state gain. In addition, Gaussian noise signal is selected as the measurement noise. As Figure 7 shown, test signal and output response of manipulated variables based on generalized multi-valued noise (GMN).

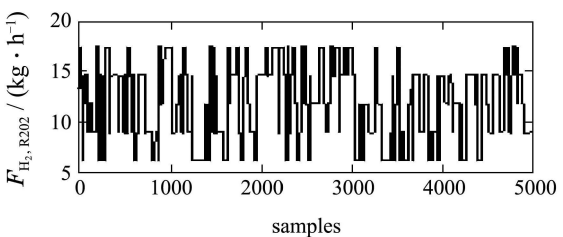
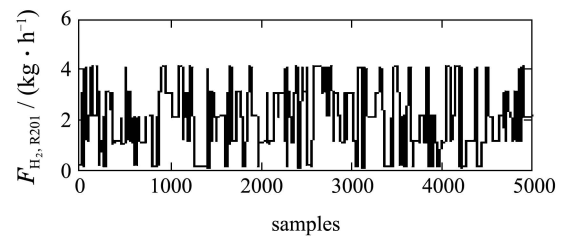
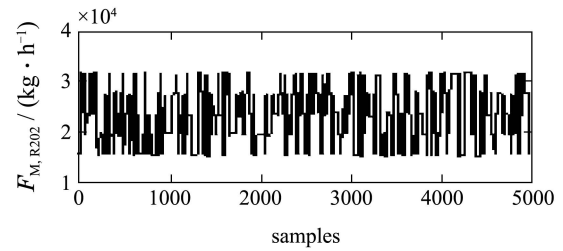
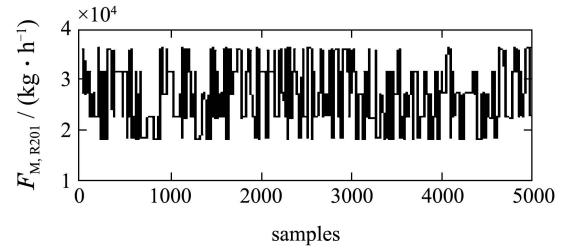
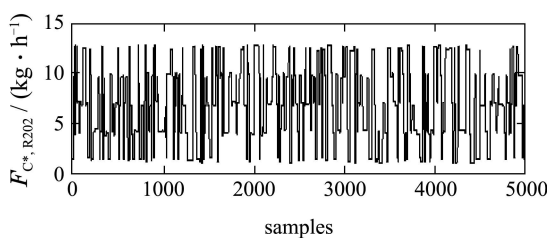
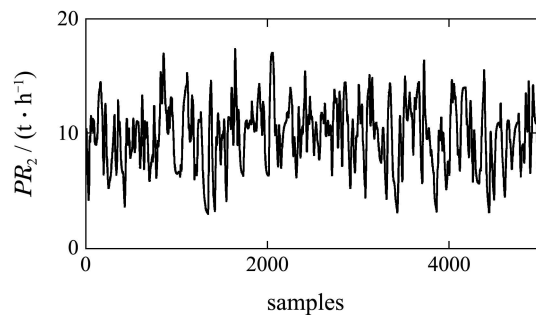
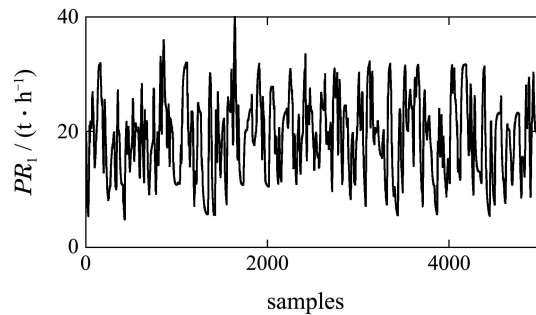


Fig. 7(a) Test signal of manipulated variables based on generalized multi-valued noise (GMN)



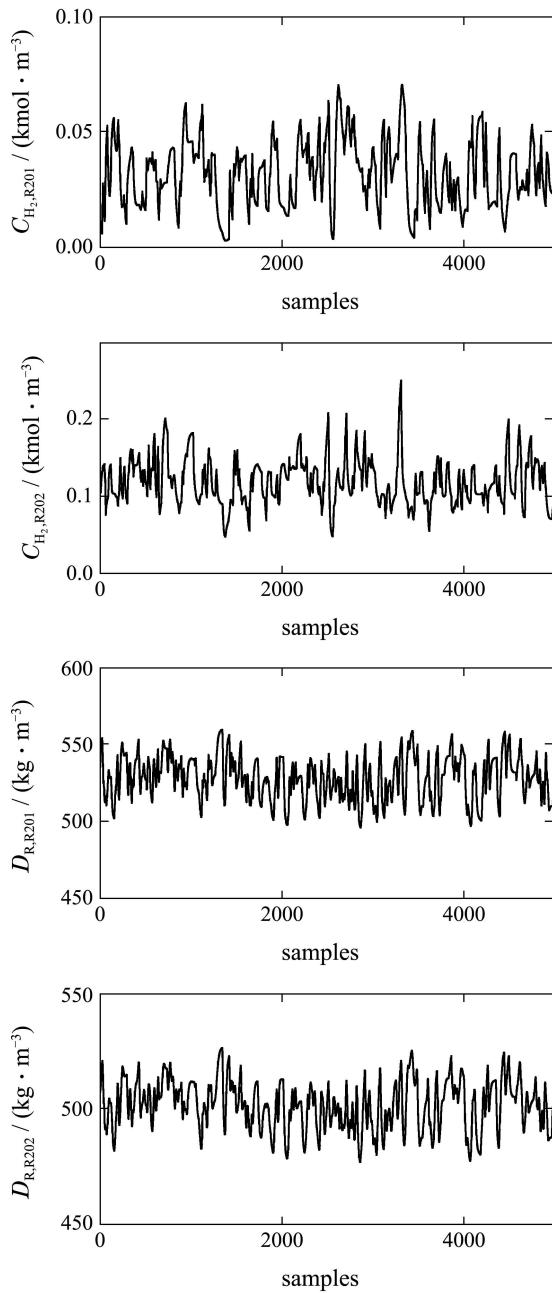


Fig. 7(b) Output response of manipulated variables based on generalized multi-valued noise (GMN)

2) Model identification and prediction with MSSARX-PWL.

Double-loop propylene polymerization process model is built on the Wiener based predictive model, in which parameters of linear part are identified in closed-loop identification based on MSSARX subspace algorithms with 5000 group dynamic testing data of input and output generated by polypropylene dynamic mathematical model, and on the other hand, the steady-state data is generated for identification of nonlinear steady-state gain with multivariable PWL approach. According to the detail steps of model parameter identification and predictive models (see part 4), the model identification results are derived as shown in Figure 8 (of which data the first 3000 group is for modeled while the rest of 2000 group is for validity data).

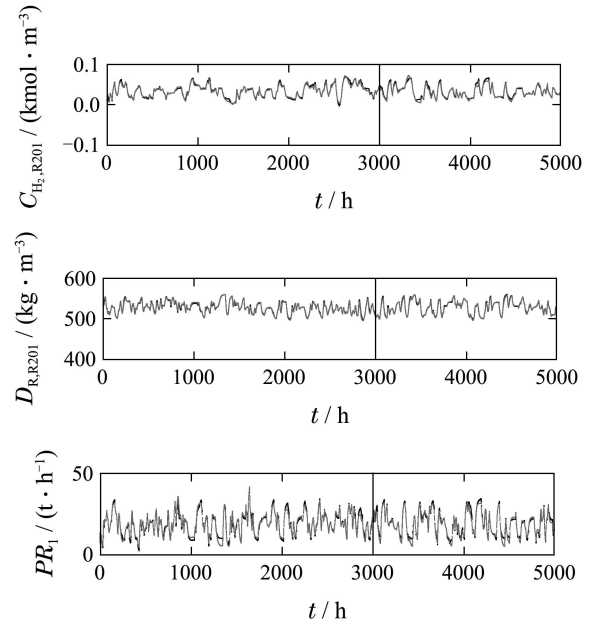


Fig. 8(a) Identification results of controlled variable in R201 reactor

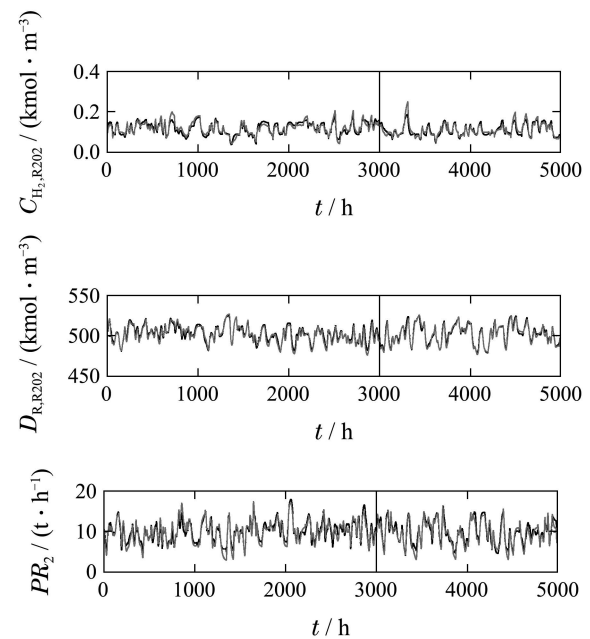


Fig. 8(b) Identification results of controlled variable in R202 reactor

6.2 Predictive control strategy of double-loop propylene polymerization process

The predictive controller based on MSSARX-PWL (Wiener-type) model for double-loop propylene polymerization process is designed. Parameters of R201 and R202 controller are setting as Table 1.

Table 1 Controller parameters of R201 and R202

Reactor	Control horizon M	Prediction horizon P	Weight matrix		
			Q	S	R
R201	2	10	1.3	0.08	0.08
R202	3	10	1.5	0.07	0.05

Meanwhile, constraint ranges of manipulated variables and controlled variables are defined, as shown in Table 2.

Table 2 Constraints range of manipulated variable and controlled variable

Manipulated variable	Constraint range	Controlled variable	Constraint range
F_{C^*}	[1,15]	$C_{H_2,R201}$	[0.01,0.1]
$F_{H_2,R201}$	[0.1,5]	$\rho_{s,R201}$	[500,560]
$F_{M,R201}$	[20000,40000]	PR_1	[10,25]
$F_{H_2,R202}$	[6,20]	$C_{H_2,R202}$	[0.05,0.5]
$F_{M,R202}$	[15000,35000]	$\rho_{s,R202}$	[500,560]
		PR_2	[10,25]

Then performance testing, including set-points tracking and disturbance rejection (where set-point are randomly generated, and the step disturbance triggered at $t = 80$ h), are performed on the MSSARX–PWL based model prediction controller, and the simulation results are given. Seen from Figure 9, nonlinear model predictive controller based on MSSARX–PWL model can well track set-points of propylene polymerization process with nonlinear constrained. In addition, it can quickly suppress the external disturbance and achieved good control effect.

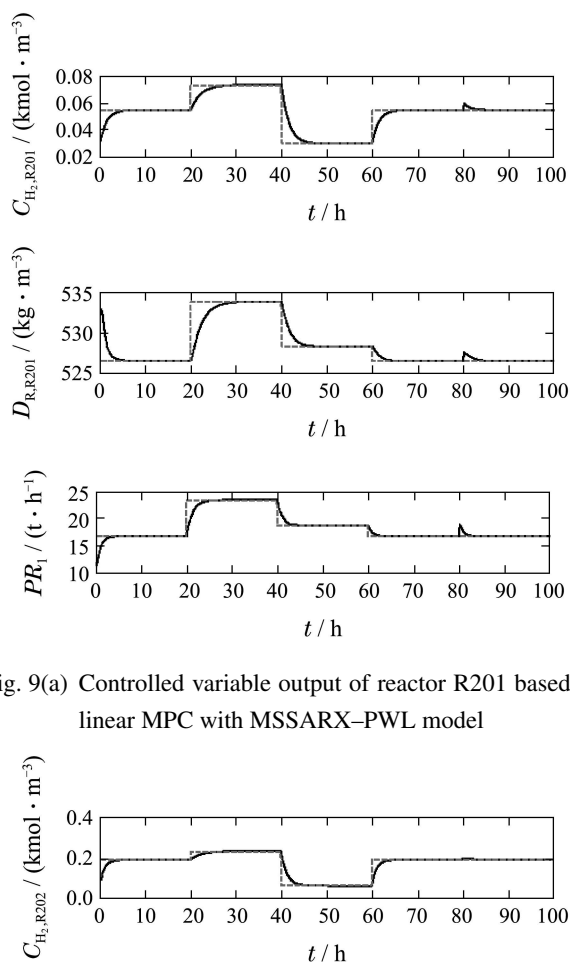


Fig. 9(a) Controlled variable output of reactor R201 based on linear MPC with MSSARX–PWL model

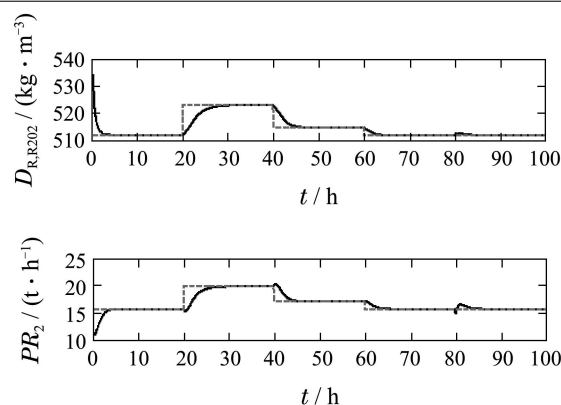


Fig. 9(b) Controlled variable output of reactor R202 based on linear MPC with MSSARX–PWL model

7 Conclusion

Double-loop propylene polymerization process, which is multi-variable, strongly coupling and nonlinear, can easily lost in instability and fluctuations in quality indicators when uncorrected modeling and process control. In this paper, a nonlinear model predictive control algorithm based on MSSARX–PWL (Wiener-type) model is proposed to overcome this problem. The MSSARX–PWL model structure, in which linear state space model under the closed-loop conditions is identified by the modified closed-loop subspace identification method (MSSARX), combined with the nonlinear steady-state model identified by the multivariate PWL method, is established for the nonlinear predictive model. Then the nonlinear model can be inverted to linear model that without nonlinear programming methods (NLP) solver but only the linear quadratic programming (QP) optimization controller needed. The controllers are adopted for multiple-variable control to the hydrogen concentration, slurry density and loop production rate in the loop reactor R201 and R202. From the simulation results, the algorithm proposed not only can guarantee model and control accuracy, as well as the computational efficiency, but also can better reject disturbance.

References:

- RAHROOH A, SHEPARD S. Identification of nonlinear systems using NARMAX model [J]. *Nonlinear Analysis*, 2009, 71(12): 1198 – 1202.
- KISHOR N. Nonlinear predictive control to track deviated power of an identified NNARX model of a hydro plant [J]. *Expert Systems with Applications*, 2008, 35(4): 1741 – 1751.
- GRUBER J K, GUZMAN J L, RODRIGUEZ F, et al. Nonlinear MPC based on a Volterra series model for green house temperature control using natural ventilation [J]. *Control Engineering Practice*, 2011, 19(4): 354 – 366.
- ALVAREZ L A, ODLOAK D. Robust integration of real time optimization with linear model predictive control [J]. *Computers Chemical Engineering*, 2010, 34(12): 1937 – 1944.
- BAMIEH B, GIARRE L. Identification for linear parameter varying models [J]. *International Journal of Robust and Nonlinear Control*, 2002, 12(3): 841 – 853.
- XU Z, ZHAO J, QIAN J, et al. Nonlinear MPC using an identified LPV model [J]. *Industrial Engineering Chemistry Research*, 2009, 48(7): 3043 – 3051.
- DING F, LIU X P, LIU G. Identification methods for Hammerstein nonlinear systems [J]. *Digital Signal Processing*, 2011, 21(9): 215 – 238.

- [8] NORQUAY S J, PALAZOGLU A, ROMAGOLI J A. Model predictive control based on Wiener models [J]. *Chemical Engineering Science*, 1998, 53(1): 75 – 84.
- [9] DESHPANDE S, PATWARDHAN S C, METHEKAR R, et al. Development of a closed form nonlinear predictive control law based on a class of Wiener models [J]. *Industrial Engineering Chemistry Research*, 2009, 49(8): 148 – 165.
- [10] SAHA P, KRISHNAN S, RAO V, et al. Identification method for nonlinear dynamic system using Wiener neural network [J]. *Chemical Engineering Communications*, 2004, 191(3): 1083 – 1119.
- [11] WU D H. Identification method for nonlinear dynamic system using Wiener neural network [J]. *Control Theory & Applications*, 2009, 26(11): 1192 – 1196.
- [12] DING B, PING X. Dynamic output feedback model predictive control for nonlinear systems represented by Hammerstein-Wiener model [J]. *Journal of Process Control*, 2012, 22(9): 1773 – 1784.
- [13] AGURIRRE L, COELHO M, CORRÉA M. On the interpretation and practice of dynamical differences between Hammerstein and Wiener models [J]. *IEEE Proceeding of D: Control Theory and Applications*, 2005, 152(4): 349 – 356.
- [14] TANG Y, QIAO L, GUAN X. Identification of Wiener model using step signals and particle swarm optimization [J]. *Expert Systems with Applications*, 2010, 37(4): 3398 – 3404.
- [15] HSU Y L, WANG J S. A Wiener-type recurrent neural network and its control strategy for nonlinear dynamic applications [J]. *Journal of Process Control*, 2009, 19(6): 942 – 953.
- [16] FIGUEROA J, BIAGIOLA S, AGAMENNONI O. An approach for identification of uncertain Wiener systems [J]. *Mathematical and Computer Modelling*, 2008, 48(1): 305 – 315.
- [17] NA S S, RHEE H K. An experimental study for property control in a continuous styrene polymerization reactor using a polynomial AR-MA model [J]. *Chemical Engineering Science*, 2002, 57(7): 1165 – 1173.
- [18] MANER B R, DOYLE F J. Polymerization reactor control using autoregressive-plus Volterra-based MPC [J]. *AIChE Journal*, 1997, 43(7): 1763 – 1784.
- [19] CERVANTES A L, AGAMENNONI O E, FIGUEROA J L. A nonlinear model predictive control system based on Wiener piecewise linear models [J]. *Journal of Process Control*, 2003, 13(7): 655 – 666.
- [20] ARTO V, HANNU P, HALME A. Modeling of chromatographic separation process with Wiener-MLP representation [J]. *Journal of Process Control*, 2001, 11(5): 443 – 458.
- [21] QIN S J. An overview of subspace identification [J]. *Computers Chemical Engineering*, 2006, 30(10): 1502 – 1513.
- [22] JANSSON M. A new subspace identification method for open and closed loop data [C] // *Proceedings of the 16th IFAC World Congress*, New York: IEEE, 2005: 372 – 378.
- [23] JULIAN P. A high level canonical piecewise linear representation: theory and applications [D]. Argentina: Universidad Nacional del Sur, Bahia Blanca, 1999.
- [24] GERKSIC S, JURICIC D, STRMCNIK S, MATKO D. Wiener model based nonlinear predictive control [J]. *International Journal of Systems Science*, 2000, 31(2): 189 – 202.
- [25] WIGREN T. Recursive prediction error identification using the nonlinear Wiener model [J]. *Automatica*, 1993, 29(4): 1011 – 1025.
- [26] HOSEN M A, HUSSAIN M A, MJALLI F S. Control of polystyrene batch reactors using neural network based model predictive control (NNMPC): An experimental investigation [J]. *Control Engineering Practice*, 2011, 19(5): 709 – 726.
- [27] SHAFIEE G, AREFI M, JAHED M M, et al. Nonlinear predictive control of a polymerization reactor based on piecewise linear Wiener model [J]. *Chemical Engineering Journal*, 2008, 143(3): 282 – 292.
- [28] FIGUEROA J L, DESAGES A C. Use of piecewise linear approximations for steady state back off analysis [J]. *Optimal Control Applications and Methods*, 1998, 19(3): 93 – 110.

作者简介:

娄海川 (1981–), 男, 博士, 做博士后研究工作, 研究方向为复杂过程系统建模、优化与控制, E-mail: lhczju@126.com;

苏宏业 (1969–), 男, 教授, 博士生导师, 研究方向为鲁棒控制、先进控制、过程系统优化等, E-mail: hysu@iipc.zju.edu.cn;

古勇 (1972–), 男, 副教授, 硕士生导师, 研究方向为过程控制与优化, E-mail: sup_guy@supcon.com;

谢磊 (1979–), 男, 副教授, 博士生导师, 研究方向为故障诊断、过程优化等, E-mail: leix@csc.zju.edu.cn;

侯卫锋 (1978–), 男, 高级工程师, 博士, 研究方向为流程模拟, E-mail: houweifeng@supcon.com;

荣冈 (1963–), 男, 教授, 博士生导师, 研究方向为数据挖掘、智能工厂等, E-mail: grong@iipc.zju.edu.cn.