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执行器有故障的多输入单输出系统的自适应输出反馈控制

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摘要: 对一类控制增益符号未知且执行器有故障的输出反馈多输入单输出非线性系统, 提出了一种后推容错控制方案. 该方案在系统状态不可量测的情况下, 利用Nussbaum函数处理符号未知的常数增益, 并通过构造K-滤波器来估计了系统不可量测的状态. 在容错控制器设计过程中, 引入变能量函数来处理利用虚拟控制律所无法抵消的部分. 与现有研究成果相比, 放宽了未知增益需要上下界均为已知的假设条件. 最后, 通过选取合适的李雅普诺夫函数, 证明了闭环系统所有信号半全局一致终结有界, 且跟踪误差收敛到原点的一个小邻域内. 仿真结果表明了所提控制方法的有效性.

关键词: 后推控制; 未建模动态; 输出反馈; 变能量函数

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Adaptive output feedback control for multi-input single-output systems with actuator failures

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Abstract: We proposed a fault-tolerant control scheme based on backstepping for a class of multi-input single-output (MISO) nonlinear output feedback systems with unknown control gain signs and actuator failures. Under the conditions that the states of the systems are unmeasured, we employ Nussbaum function and K-filters to deal with the constant gains with unknown signs and estimate the unavailable states, respectively. In designing the fault tolerant controller, the parts that the virtual control laws cannot counteract are disposed by introducing the changing supply function. Compared with the existing research results, the proposed approach relaxes the assumed restrictions that the upper and lower bounds of the unknown gains should be known. By choosing appropriate Lyapunov function, we show the closed-loop control system is semi-globally uniformly ultimately bounded, with the tracking error converging to a small neighborhood of the origin. Simulation results demonstrate the effectiveness of the proposed approach.

Key words: backstepping control; unmodeled dynamics; output feedback; changing supply function

1 引言(Introduction)

随着人类社会的发展与科学技术水平的进步, 控制理论与控制工程在工业生产、军工、航空航天、经济学与统计学等专业领域内获得了空前广泛的应用, 并且取得了丰硕的研究成果与令人瞩目的科技成就. 一方面, 精密复杂控制仪器的应用使得控制系统能够完成日趋复杂的控制目标, 但另一方面也使得系统发生故障的概率大大增加. 因此, 如何增强动态系统的安全性与可靠性来使得系统的稳定性能够不受故障的影响便成为了人们所急需解决的问题.

文献[1–3]对几类具有执行器故障的输出反馈非线性系统, 提出了若干种基于后推的容错控制方案, 并通过选取适当的李雅普洛夫函数证明了系统的稳定性. 然而, 文献[1]与文献[2–3]相比并不需要未知控制增益符号已知的假设条件, 并在基于后推的容错控制器设计过程中, 引入Nussbaum函数处理了符号未知的常数增益. 但文献[2]与文献[1,3]相比: 考虑了未知常数$k_{1,r}$的数值在故障发生的每个时间间隔$[t_k, t_{k+1})$内均有所不同, 因此, 需要在$[t_k, t_{k+1})$内对李雅普诺夫函数求导并逐段分析其稳定性. 而文献[1,3]将参数$k_{1,r}$的数值视为恒定不变. 文献[4–6]研究了若干类输出反馈系统的跟踪控制问题, 其利用K–滤波器估计了系统不可量测的状态. 文献[7–8]在文献[6]的基础上, 讨论了两类控制增益符号未知的输出反馈随机

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非线性系统的稳定性问题. 文献[9]对一类严格反馈非线性系统, 提出了一种基于后推的容错控制方案. 文献[10–11]针对具有执行器故障的分散控制系统, 提出了两种容错控制方案, 但文献[11]需要系统状态可量测. 文献[12]在文献[10–11]的基础上, 将后推控制策略拓展到具有执行器故障的随机分散系统中, 并拓展了文献[5–6]中关于未建模动态的假设条件. 文献[14–15]将文献[13]中的研究对象扩展为多输入多输出(multiple input multiple output, MIMO)系统, 但文献[15]状态方程中的未知函数需要满足局部利普希茨条件. 文献[16]研究了一类随机纯反馈系统的跟踪控制问题, 文献[17–18]通过引入变能量函数, 巧妙地设计了系统的控制器. 文献[19–20]相比于文献[1–3], 通过对神经网络未知理想权向量的模值进行估计来代替直接对权向量进行估计, 减少了在线调节参数的个数, 简化了系统设计. 文献[21–23]将小增益定理与后推控制方案结合起来, 得出了若干种控制领域的新成果.

本文在文献[1–3]的基础上, 对一类具有“常数值故障”与“衰减故障”的输出反馈非线性系统, 提出了一种基于后推的容错控制方案. 其主要贡献如下: 1) 在故障发生时刻与类型均为未知的情况下, 利用K-滤波器估计了系统不可量测的状态, 并在稳定性分析过程中, 考虑到未知参数$k_{1,r}$的数值在每两个故障发生的时间间隔内均有所不同, 从而采用对李雅普诺夫函数逐段求导与分析的方法完善了文献[1,3]的不足. 2) 利用非负且单调不减的变能量函数抵消了系统虚拟控制律所无法抵消的部分. 3) 将研究对象由文献[5]中的SISO系统拓展为MISO系统, 并利用中间控制变量v_0将系统模型简化为SISO系统进行研究, 降低了设计系统正常控制输入的难度.

2 问题描述及基本假设(Problem statement and basic assumptions)

考虑如下一类MISO非线性系统:

$$\begin{cases}\dot z = q(z,y),\\ \dot x_i = x_{i+1} + f_i(y) + \varDelta_i(z,y,t),\\ \qquad i=1,\cdots,\rho-1,\\ \dot x_\rho = x_{\rho+1} + f_\rho(y) + \sum\limits_{j=1}^{m} b_{\gamma j}\beta_j(y)u_j +\\ \qquad \varDelta_\rho(z,y,t),\\ \quad\vdots\\ \dot x_{n-1} = x_n + f_{n-1}(y) + \sum\limits_{j=1}^{m} b_{1j}\beta_j(y)u_j +\\ \qquad \varDelta_{n-1}(z,y,t),\\ \dot x_n = f_n(y) + \sum\limits_{j=1}^{m} b_{0j}\beta_j(y)u_j + \varDelta_n(z,y,t),\\ y = x_1\,,\end{cases} \tag{1}$$

其中: $\gamma = n-\rho$, $x=[x_1, x_2, \cdots, x_n]^{\mathrm T}\in\mathbb R^n$为系统不可量测的状态向量; $y\in\mathbb R$为系统可量测的输出; $f_i(y)(i=1,2,\cdots,n)$为未知光滑非线性函数; $\beta_j(y)\neq 0(j=1,2,\cdots,m)$为已知光滑非线性函数; $\varDelta_i(z,y,t)(i=1,2,\cdots,n)$为系统的外界扰动, 其为未知的光滑函数; b_{rj}, $r=0,1,\cdots,\gamma(j=1,2,\cdots,m)$为系统未知的常数控制增益, 且$b_{\gamma j}$的符号未知; $u_j(j=1,2,\cdots,m)$为系统可能发生故障的控制输入; $\dot z = q(z,y)$为系统的未建模动态.

本文所考虑的故障为“常数值故障”与“衰减故障”, 系统控制输入$u_j(j=1,2,\cdots,m)$既可能发生“常数值故障”, 也可能发生“衰减故障”. “常数值故障”的数学表达式为

$$u_j=\begin{cases}\bar u_j,\ t\geqslant t_j,\\ v_j,\ t<t_j,\end{cases} j\in\{j_1,\cdots,j_R\}\subset\{1,\cdots,m\}, \tag{2}$$

其中: 故障值$\bar u_j$, 故障发生的时刻t_j以及下标j未知, v_j为系统正常的控制输入, R为发生“常数值故障”的控制输入的个数.

“衰减故障”的数学表达式为

$$u_i=\begin{cases}\rho_i v_i,\ t\geqslant t_i,\\ v_i,\quad t<t_i,\end{cases} i\in\overline{\{j_1,\cdots,j_R\}}\cap\{1,\cdots,m\}, \tag{3}$$

其中: $\rho_i\in[\underline{\rho_i},1]$, $0<\underline{\rho_i}\leqslant 1$, ρ_i为“衰减故障”中的“衰减系数”, 其为未知正常数. $\underline{\rho_i}$为ρ_i的下界, 当$\rho_i=1$时, 等价于系统的控制输入u_i不发生故障. 故障发生的时刻t_i以及下标i未知, v_i为系统正常的控制输入.

故综合式(2)–(3), 系统输入$u_j(j=1,\cdots,m)$可写为如下形式:

$$u_j=\rho_j v_j+\sigma_j(\bar u_j-\rho_j v_j), \tag{4}$$

其中: 1) 当$\rho_j=1, \sigma_j=1$时, 系统控制输入u_j发生“常数值故障”; 2) 当$\rho_j\neq 1, \sigma_j=0$时, 系统控制输入u_j发生“衰减故障”; 3) 当$\rho_j=1$, $\sigma_j=0$时, 系统控制输入u_j不发生故障.

假设 1 系统(1)至多有$m-1$个执行器发生故障, 并且其余的执行器能够达到所期望的控制性能.

假设 2 $b_{\gamma j}, j=1,2,\cdots,m$为系统未知的常数增益, 且其符号也为未知.

假设 3 $B(s)$为Hurwitz多项式, 其中: $B(s)=k^*_{1,\gamma}s^\gamma+\cdots+k^*_{1,0}$. $k^*_{1,r}$, $r=0,\cdots,\gamma$稍后定义.

假设 4 参考信号$y_{\mathrm r}, \dot y_{\mathrm r}, \cdots, y_{\mathrm r}^{(\rho)}$都是有界且可量测的.

假设 5 外界动态扰动$\varDelta_i(z,y,t)(i=1,2,\cdots,n)$满足:

$$|\varDelta_i(z,y,t)|\leqslant p_i^*\varphi_{i1}(\|y\|)+p_i^*\varphi_{i2}(\|z\|), \tag{5}$$

其中: $\varphi_{i1}(\cdot)$为已知非负光滑函数, $\varphi_{i2}(\cdot)$为已知非负光滑增函数, 并且$\varphi_{i2}(0)=0$, p_i^*为未知常数.

假设 6[5] z称为指数输入状态实用稳定(exp-

ISpS), 即对于$\dot{z}=q(z,y)$, 若存在李雅普诺夫函数$V_0(z)$满足

$$\alpha_1(\|z\|)\leqslant V_0(z)\leqslant \alpha_2(\|z\|), \tag{6}$$

$$\frac{\partial V_0(z)}{\partial z}q(z,y)\leqslant -cV_0(z)+\gamma(\|y\|)+d, \tag{7}$$

其中: $\alpha_1(\cdot),\alpha_2(\cdot),\gamma(\cdot)$为$K_\infty$类函数且$\gamma(\cdot)$已知, c,d为已知正常数.

定义 1[1] 如果连续函数$N(k)$: $R\rightarrow\mathbb{R}$满足

$$\lim_{s\rightarrow+\infty}\sup\frac{1}{s}\int_0^s N(k)\mathrm{d}k=+\infty, \tag{8}$$

$$\lim_{s\rightarrow-\infty}\inf\frac{1}{s}\int_0^s N(k)\mathrm{d}k=-\infty, \tag{9}$$

则称为Nussbaum函数. 本文中, 取$N(k)=k^2\sin k$.

引理 1[5] 若V_0是指数输入状态实用稳定的(exp-ISpS)函数, 即式(6)–(7)成立, 则对于任意常数$\bar{c}\in(0,c)$, 任意初始时间$t_0>0$, 任意初始状态$z_0=z(t_0)$, $\upsilon_0>0$和任意$\bar{\gamma}(\|y\|)\geqslant\gamma(\|y\|)$, 存在有限时间:

$$T_0=\max\{0,\ln[V_0(z_0)/\upsilon_0]/(c-\bar{c})\}\geqslant 0,$$

对于非负函数$D(t_0,t)$, 定义动态信号: $\dot{\upsilon}=-\bar{c}\upsilon+\bar{\gamma}(\|y\|)+d$, $\upsilon(t_0)=\upsilon_0$, 当$t\geqslant t_0+T_0$时, 有$D(t_0,t)=0$, 使得: $V_0(z)\leqslant\upsilon(t)+D(t_0,t)$. 不失一般性, 取$\gamma(\|y\|)=\bar{\gamma}(\|y\|)$.

引理 2[1] 已知$V(\cdot),k(\cdot)$都是$[0,t_\mathrm{f})$上的光滑函数, 且$V(t)\geqslant 0$, $\forall t\in[0,t_\mathrm{f})$, $N(\cdot)$是Nussbaum函数, 如果下列不等式成立:

$$V(t)\leqslant c_0+\int_0^t(gN(k)+1)\dot{k}\mathrm{d}\tau,\ \forall t\in[0,t_\mathrm{f}), \tag{10}$$

其中: g是非零常数, c_0是常数, 那么$V(t)$, $k(t)$和$\int_0^t(gN(k)+1)\dot{k}\mathrm{d}\tau$一定在$[0,t_\mathrm{f})$上有界.

引理 3[1] 已知$V(\cdot),k(\cdot)$都是$[0,t_\mathrm{f})$上的光滑函数, 且$V(t)\geqslant 0,\forall t\in[0,t_\mathrm{f})$, $N(\cdot)$是Nussbaum函数, 如果下列不等式成立:

$$V(t)\leqslant c_0+\mathrm{e}^{-c_1t}\int_0^t g(x(\tau))N(k)\dot{k}\mathrm{e}^{c_1\tau}\mathrm{d}\tau+\mathrm{e}^{-c_1t}\int_0^t\dot{k}\mathrm{e}^{c_1\tau}\mathrm{d}\tau,\ \forall t\in[0,t_\mathrm{f}), \tag{11}$$

其中: 常数$c_1\geqslant 0$, $g(x(\tau))$是取值在闭区间$I=[l^{-1},l^{+1}]$, $0\notin I$上的时变参数, c_0是常数, 那么$V(t)$, $k(t)$和$\int_0^t(gN(k)+1)\dot{k}\mathrm{d}\tau$一定在$[0,t_\mathrm{f})$上有界.

引理 4[5] $h(Z)$为紧集$\Omega_Z\subset\mathbb{R}^n$内的任意连续函数, 则对于$\forall\varepsilon^*>0$存在RBF神经网络使得

$$\sup_{Z\in\Omega_Z}|h(Z)-W^{*\mathrm{T}}\xi(Z)|\leqslant\varepsilon^*, \tag{12}$$

式中: $h(Z)-W^{*\mathrm{T}}\xi(Z)$为神经网络的逼近误差, 最优权值向量$W^*$定义如下:

$$W^*=\arg\min_{W\in\mathbb{R}^l}\{\sup_{Z\in\Omega_Z}|h(Z)-W^{\mathrm{T}}\xi(Z)|\}. \tag{13}$$

假设系统(1)在$t_k(k=1,2,\cdots,q$, 且$t_0<t_1<t_2<\cdots<t_q<\infty)$时刻有$p_k$个执行器发生故障, 即对于系统(1), 当$t\in[t_k,t_{k+1})(t_{q+1}=\infty)$时, 共有$p=\sum_{i=1}^k p_i$个执行器发生故障. 其中: $u_j=\rho_jv_j+\sigma_j(\bar{u}_j-\rho_jv_j)$, $j=j_1,j_2,\cdots,j_p$, $0\leqslant p\leqslant m-1$为发生故障的控制输入. $u_j(t)=v_j(j\neq j_1,j_2,\cdots,j_p)$为正常的控制输入, 其为所需要设计的控制输入. 那么对于系统(1), 当$t\in[t_k,t_{k+1})$, 可写为如下形式:

$$\begin{cases}\dot{z}=q(z,y),\\ \dot{x}_i=x_{i+1}+f_i(y)+\varDelta_i(z,y,t),\ i=1,\cdots,\rho-1,\\ \dot{x}_\rho=x_{\rho+1}+f_\rho(y)+\sum\limits_{j=j_1,j_2,\cdots,j_p}b_{\gamma j}\beta_j(y)(1-\sigma_j)\rho_jv_j+\sum\limits_{j=j_1,j_2,\cdots,j_p}b_{\gamma j}\beta_j(y)\sigma_j\bar{u}_j+\\ \quad\sum\limits_{j\neq j_1,j_2,\cdots,j_p}b_{\gamma j}\beta_j(y)v_j+\varDelta_\rho(z,y,t),\\ \quad\vdots\\ \dot{x}_{n-1}=x_n+f_{n-1}(y)+\sum\limits_{j=j_1,j_2,\cdots,j_p}b_{1j}\beta_j(y)\times(1-\sigma_j)\rho_jv_j+\sum\limits_{j=j_1,j_2,\cdots,j_p}b_{1j}\beta_j(y)\sigma_j\bar{u}_j+\\ \quad\sum\limits_{j\neq j_1,j_2,\cdots,j_p}b_{1j}\beta_j(y)v_j+\varDelta_{n-1}(z,y,t),\\ \dot{x}_n=f_n(y)+\sum\limits_{j=j_1,j_2,\cdots,j_p}b_{0j}\beta_j(y)(1-\sigma_j)\rho_jv_j+\sum\limits_{j=j_1,j_2,\cdots,j_p}b_{0j}\beta_j(y)\sigma_j\bar{u}_j+\\ \quad\sum\limits_{j\neq j_1,j_2,\cdots,j_p}b_{0j}\beta_j(y)v_j+\varDelta_n(z,y,t),\\ y=x_1.\end{cases} \tag{14}$$

控制目标: 对于系统(1)在$t_k(k=1,2,\cdots,q)$时刻有$p\leqslant m-1$个执行器发生故障时, 设计系统正常的控制输入$v_j(j\in\{1,2,\cdots,m\})$, 使得系统输出$y$能够尽可能好的跟踪参考信号$y_\mathrm{r}$, 闭环系统半全局一致终结有界, 跟踪误差收敛到原点的一个小邻域内.

3 系统参数化与K–滤波器设计(System parameterization and K-filters design)

为了便于设计系统(1)正常的控制输入v_j, 参考文献[1]的设计方案, 由$\beta_j(y)$已知且$\beta_j(y)\neq 0$. 令

$$v_j=\frac{1}{\beta_j(y)}v_0,\ j=1,2,\cdots,m, \tag{15}$$

其中v_0为所需要设计的控制律, 其稍后定义.

同理, 为了便于系统(1)的参数化设计, 令

$$A=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix}\in\mathbb{R}^{n\times n},\ F(y)=\begin{bmatrix}f_1(y)\\f_2(y)\\\vdots\\f_n(y)\end{bmatrix},$$

$$\varDelta=\begin{bmatrix}\varDelta_1\\ \varDelta_2\\ \vdots\\ \varDelta_n\end{bmatrix},\ x=\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix},\ C=\begin{bmatrix}1\\ 0\\ \vdots\\ 0\end{bmatrix},$$

并且令

$$k_{1,rj}=\begin{cases}b_{rj}(1-\sigma_j)\rho_j,\\ \qquad r=0,\cdots,\gamma, j=j_1,\cdots,j_p,\\ b_{rj},\ r=0,\cdots,\gamma,\ j\neq j_1,\cdots,j_p,\end{cases}$$
$$k_{2,rj}=\begin{cases}b_{rj}\sigma_j\bar{u}_j,\\ \qquad r=0,\cdots,\gamma,\ j=j_1,\cdots,j_p,\\ 0,\ r=0,\cdots,\gamma, j\neq j_1,\cdots,j_p.\end{cases} \tag{16}$$

综合式(14)–(16), 令$k^*_{1,r}=\sum\limits_{j=1}^{m}k_{1,rj}(r=0,\cdots,\gamma)$, 则式(1)可写为如下向量形式:

$$\begin{cases}\dot{x}=Ax+F(y)+\sum\limits_{r=0}^{\gamma}e_{n-r}k^*_{1,r}v_0+\\ \quad\sum\limits_{r=0}^{\gamma}e_{n-r}\sum\limits_{j=1}^{m}k_{2,rj}\beta_j(y)+\varDelta,\\ y=C^{\mathrm T}x.\end{cases} \tag{17}$$

本文采用径向基神经网络$\hat{f}_i(y)=\theta_i^{\mathrm T}S_i(y)$在紧集$y\in\varOmega_y\subset\mathbb{R}$上逼近未知函数$f_i(y)$, 其中: $y\in\varOmega_y$表示神经网络的输入, $\theta_i=[\theta_{i1},\theta_{i2},\cdots,\theta_{iN_i}]^{\mathrm T}\in\mathbb{R}^{N_i}$表示神经网络的权向量, $N_i>1$表示神经元的节点个数, $S_i(y)=[S_{i1}(y),S_{i2}(y),\cdots,S_{iN_i}(y)]^{\mathrm T}\in\mathbb{R}^{N_i}$为第$i$个神经元网络的径向基函数向量, 一般取为高斯函数, 形式为

$$S_{ij}(y)=\exp(-\frac{(y-v_{ij})^2}{\sigma_{ij}^2}), \tag{18}$$

其中: v_{ij}为径向基函数的中心, $\sigma_{ij}>0$为高斯函数的宽度, $i=1,2,\cdots,n, j=1,2,\cdots,N_i$. 令未知理想权向量$\theta_i^*=\arg\min\limits_{\theta_i\in\mathbb{R}^{N_i}}[\sup\limits_{y\in\varOmega_y}|\theta_i^{\mathrm T}S_i(y)-f_i(y)|]$, 则

$$f_i(y)=\theta_i^{*\mathrm T}S_i(y)+\delta_i(y), \tag{19}$$

其中$\delta_i(\cdot)$为神经网络的逼近误差.

将式(19)代入式(17), 可写为如下形式:

$$\begin{cases}\dot{x}=Ax+S^{\mathrm T}(y)\theta_h+\delta(y)+\sum\limits_{r=0}^{\gamma}e_{n-r}k^*_{1,r}v_0+\\ \quad\sum\limits_{r=0}^{\gamma}e_{n-r}\sum\limits_{j=1}^{m}k_{2,rj}\beta_j(y)+\Delta,\\ y=C^{\mathrm T}x,\end{cases} \tag{20}$$

其中:

$$S^{\mathrm T}(y)=\mathrm{diag}\{S_1^{\mathrm T}(y),S_2^{\mathrm T}(y),\cdots,S_n^{\mathrm T}(y)\},$$
$$\theta_h=[\theta_1^{*\mathrm T},\cdots,\theta_n^{*\mathrm T}]^{\mathrm T},\ \delta(y)=[\delta_1(y),\cdots,\delta_n(y)]^{\mathrm T}.$$

为了便于设计系统(1)基于神经网络的滤波器, 令

$$\begin{cases}\varphi(y)=[e_{n-\gamma}\beta_1(y),\cdots,e_{n-\gamma}\beta_m(y),\cdots,\\ \qquad e_n\beta_m(y)],\\ K_1=[k^*_{1,\gamma},\cdots,k^*_{1,1},k^*_{1,0}]^{\mathrm T},\\ K_2=[k_{2,\gamma1},\cdots,k_{2,\gamma m},\cdots,k_{2,01},\cdots,k_{2,0m}]^{\mathrm T}.\end{cases} \tag{21}$$

故利用式(21), 式(20)最终可写为如下形式:

$$\begin{cases}\dot{x}=Ax+\delta(y)+\Delta+F^{\mathrm T}(y,v_0)\theta,\\ y=C^{\mathrm T}x,\end{cases} \tag{22}$$

其中:

$$F^{\mathrm T}(y,v_0)=\left[\begin{bmatrix}0_{(\rho-1)\times(\gamma+1)}\\ I_{\gamma+1}\end{bmatrix}v_0\ \ \varphi(y)\ \ S^{\mathrm T}(y)\right],$$
$$\theta=[K_1^{\mathrm T}\ \ K_2^{\mathrm T}\ \ \theta_h^{\mathrm T}]^{\mathrm T}.$$

由于系统的状态不可量测, 故利用如下的滤波器与观测器来重构系统状态. 滤波器设计为

$$\begin{cases}\dot{\xi}_0=A_0\xi_0+Ly,\xi_0\in\mathbb{R}^n,\\ \dot{\varOmega}^{\mathrm T}=A_0\varOmega^{\mathrm T}+F^{\mathrm T}(y,v_0),\end{cases} \tag{23}$$

其中: $A_0=A-LC^{\mathrm T},\ L=[l_1,l_2,\cdots,l_n]^{\mathrm T}, |sI-A_0|=s^n+l_1s^{n-1}+\cdots+l_{n-1}s+l_n$为Hurwitz多项式, $A_0^{\mathrm T}P+PA_0=-3I, P=P^{\mathrm T}>0$.

设矩阵$\varOmega^{\mathrm T}$的前$(\gamma+1)(m+1)$的列向量依次为: $\mu_\gamma,\mu_{\gamma-1},\cdots,\mu_0,\xi_{\gamma1},\xi_{\gamma2},\cdots,\xi_{\gamma m},\cdots,\xi_{01},\xi_{02},\cdots,\xi_{0m}$. 则: $\varOmega^{\mathrm T}=[\mu_\gamma,\mu_{\gamma-1},\cdots,\mu_0,\xi_{\gamma1},\xi_{\gamma2},\cdots,\xi_{\gamma m},\cdots,\xi_{01},\xi_{02},\cdots,\xi_{0m},\varXi]$. 故由式(23)可得, 列向量$\mu_\gamma,\mu_{\gamma-1},\cdots,\mu_0$满足下式:

$$\dot{\mu}_{\mathrm r}=A_0\mu_{\mathrm r}+e_{n-r}v_0,$$
$$r=0,1,\cdots,\gamma,\ \mu_{\mathrm r}\in\mathbb{R}^{n\times(\gamma+1)}. \tag{24}$$

由于$A_0^re_n=e_{n-r},\ r=0,1,\cdots,\gamma$. 令

$$\mu_{\mathrm r}=A_0^r\lambda,\ r=0,1,\cdots,\gamma, \tag{25}$$

故利用式(25), 式(24)可改写为如下形式:

$$\dot{\lambda}=A_0\lambda+e_nv_0. \tag{26}$$

令$\mu_{r,i}(r=0,1,\cdots,\gamma,\ i=1,2,\cdots,n)$为列向量$\mu_{\mathrm r}$的第$i$个分量. λ_k表示列向量λ的第k个分量. 根据文献[8]的讨论, 可知

$$\mu_{r,i}=[*,*,\cdots,1]\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_{r+i}\end{bmatrix},\ \lambda_k=0,\ k>n, \tag{27}$$

其中: $*$是关于$l_1,l_2,\cdots,l_n$的某一个多项式. 由于$l_1,l_2,\cdots,l_n$是设计参数, 所以$*$代表的多项式是有界的. 综合式(23)–(26), 可得到用于系统状态估计的K–滤波器:

$$\begin{cases}\dot{\xi}_0 = A_0\xi_0 + Ly, \xi_0 \in \mathbb{R}^n,\\ \dot{\lambda} = A_0\lambda + e_n v_0, \lambda \in \mathbb{R}^n,\\ \dot{\xi}_{rj} = A_0\xi_{rj} + e_{n-r}\beta_j(y),\\ j = 1,2,\cdots,m,\ r = 0,1,\cdots,\gamma, \xi_{rj} \in \mathbb{R}^n,\\ \dot{\varXi} = A_0\varXi + S^{\mathrm{T}}(y),\ \varXi \in \mathbb{R}^{n\times \sum\limits_{i=1}^{n} N_i},\end{cases} \tag{28}$$

定义系统(1)的状态估计$\hat{x} = \xi_0 + \varOmega^{\mathrm{T}}\theta$, 观测器误差$\varepsilon = x - \hat{x}$, 故$x = \varOmega^{\mathrm{T}}\theta + \varepsilon + \xi_0$. 则

$$x = \xi_0 + \sum_{r=0}^{\gamma} k_{1,r}^*\mu_{\mathrm{r}} + \sum_{r=0}^{\gamma}\sum_{j=1}^{m} k_{2,rj}\xi_{rj} + \varXi\theta_h + \varepsilon. \tag{29}$$

利用式(28)–(29), 可得系统的状态估计误差方程:

$$\dot{\varepsilon} = A_0\varepsilon + \varDelta + \delta(y). \tag{30}$$

4 自适应容错控制器设计(Adaptive fault-tolerant controller design)

本文采用后推控制技术设计系统的控制律v_0, 整个系统的控制器设计分为ρ步, 设计过程基于如下坐标变换: $z_1 = y - y_{\mathrm{r}},\ z_i = \mu_{\gamma,i} - \alpha_{i-1}(i = 2,3,\cdots,\rho)$.

第1步 定义系统(1)的跟踪误差为$z_1 = y - y_{\mathrm{r}}$, 对z_1求导并综合式(1)与式(29), 可得

$$\begin{aligned}\dot{z}_1 = {} & x_2 + f_1(y) + \varDelta_1(z,y,t) - \dot{y}_{\mathrm{r}} = \\ & \xi_{0,2} + k_{1,\gamma}^*\mu_{\gamma,2} + \bar{\omega}^{\mathrm{T}}\theta + \varepsilon_2 + \delta_1(y) + \\ & \varDelta_1(z,y,t) - \dot{y}_{\mathrm{r}},\end{aligned} \tag{31}$$

其中: $\bar{\omega}^{\mathrm{T}} = [0, \mu_{\gamma-1,2}, \cdots, \mu_{0,2}, \xi_{\gamma 1,2}, \xi_{\gamma 2,2}, \cdots, \xi_{\gamma m,2}, \cdots, \xi_{01,2}, \xi_{02,2}, \cdots, \xi_{0m,2}, \varXi_{(2)} + S_{(1)}^{\mathrm{T}}(y)]$, $\varXi_{(2)}$表示$\varXi$的第2行, $S_{(1)}^{\mathrm{T}}(y)$表示$S^{\mathrm{T}}(y)$的第1行.

考虑到$z_2 = \mu_{\gamma,2} - \alpha_1$, 将其代入式(31), 可得

$$\begin{aligned}\dot{z}_1 = {} & \xi_{0,2} + k_{1,\gamma}^*z_2 + k_{1,\gamma}^*\alpha_1 + \bar{\omega}^{\mathrm{T}}\theta + \varepsilon_2 + \\ & \delta_1(y) + \varDelta_1(z,y,t) - \dot{y}_{\mathrm{r}}.\end{aligned} \tag{32}$$

由假设2与式(16)可知, $k_{1,\gamma}^*$的符号未知, 故通过在虚拟控制律α_1中引入Nussbaum函数处理其符号未知的问题.

选取虚拟控制律α_1为

$$\begin{aligned}\alpha_1 = {} & N(k)[c_1z_1 + 2d_1z_1\hat{p} + \bar{\omega}^{\mathrm{T}}\hat{\theta} + \hat{p}z_1\varphi_{11}^2(\|y\|) + \\ & \frac{1}{4}\hat{p}z_1 - \dot{y}_{\mathrm{r}} + \xi_{0,2} + \frac{1}{4}z_1 - \hat{\theta}_0 + \frac{z_1^2}{\varepsilon^*}Q(y,\upsilon)].\end{aligned} \tag{33}$$

选取Nussbaum参数为

$$\begin{aligned}\dot{k} = {} & z_1[c_1z_1 + 2d_1z_1\hat{p} + \bar{\omega}^{\mathrm{T}}\hat{\theta} + \hat{p}z_1\varphi_{11}^2(\|y\|) + \\ & \frac{1}{4}\hat{p}z_1 - \dot{y}_{\mathrm{r}} + \xi_{0,2} + \frac{1}{4}z_1 - \hat{\theta}_0 + \frac{z_1^2}{\varepsilon^*}Q(y,\upsilon)],\end{aligned} \tag{34}$$

其中: $c_1 > 0,\ d_1 > 0,\ \varepsilon^* > 0$, 其为系统的设计常数. $\hat{\theta}$为θ的估计值, $\hat{p}$为p的估计值, $p = \max\{p_1^{*2}, p_2^{*2}, \cdots, p_n^{*2}\}$, 其为未知正常数, $\hat{\theta}_0$为θ_0的估计值, θ_0稍后定义, $Q(y,\upsilon)$为非负辅助函数, 形式如下所示:

$$Q(y,\upsilon) = \sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{d_i}\|P\|^2\varphi_{j1}^2(\|y\|) + \varrho(\upsilon)\gamma(\|y\|). \tag{35}$$

选取如下李雅普诺夫函数:

$$V_1 = \frac{1}{2}z_1^2. \tag{36}$$

对V_1求导, 并综合式(32), 可得

$$\begin{aligned}\dot{V}_1 = {} & z_1\xi_{0,2} + z_1k_{1,\gamma}^*\alpha_1 + z_1k_{1,\gamma}^*z_2 + z_1\bar{\omega}^{\mathrm{T}}\theta + \\ & z_1\varepsilon_2 + z_1\delta_1(y) + z_1\varDelta_1(z,y,t) - z_1\dot{y}_{\mathrm{r}}.\end{aligned} \tag{37}$$

综合假设5与式(37), 利用Young's不等式放缩, 可得

$$\begin{aligned}\dot{V}_1 \leqslant {} & z_1k_{1,\gamma}^*z_2 + k_{1,\gamma}^*N(k)\dot{k} + \dot{k} - c_1z_1^2 + \\ & z_1\bar{\omega}^{\mathrm{T}}\tilde{\theta} + z_1^2\tilde{p}\varphi_{11}^2(\|y\|) + \frac{1}{4}\tilde{p}z_1^2 + 2\tilde{p}d_1z_1^2 + \\ & \frac{1}{4d_1p}\varepsilon^{\mathrm{T}}\varepsilon + \frac{1}{4d_1p}\delta_1^2(y) + \frac{1}{4} - \frac{1}{4}z_1^2 + \\ & \varphi_{12}^2(\|z\|) + z_1\hat{\theta}_0 - \frac{z_1^2}{\varepsilon^*}Q(y,\upsilon).\end{aligned} \tag{38}$$

第2步 考虑到$z_2 = \mu_{\gamma,2} - \alpha_1$, 并将$z_3 = \mu_{\gamma,3} - \alpha_2$代入式(24), 对$z_2$求导可得

$$\dot{z}_2 = -l_2\mu_{\gamma,1} + z_3 + \alpha_2 - \dot{\alpha}_1. \tag{39}$$

选取虚拟控制律α_2为

$$\begin{aligned}\alpha_2 = {} & -\hat{k}_{1,\gamma}^*z_1 - c_2z_2 - 2d_2\hat{p}(\frac{\partial\alpha_1}{\partial y})^2z_2 + l_2\mu_{\gamma,1} + \\ & \frac{\partial\alpha_1}{\partial y}\xi_{0,2} + \frac{\partial\alpha_1}{\partial y}\omega^{\mathrm{T}}\hat{\theta} - \\ & \hat{p}\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_1}{\partial y})^2z_2 - \frac{1}{4}\hat{p}(\frac{\partial\alpha_1}{\partial y})^2z_2 + \\ & \frac{\partial\alpha_1}{\partial k}\dot{k} + \frac{\partial\alpha_1}{\partial\xi_0}(A_0\xi_0 + Ly) + \\ & \frac{\partial\alpha_1}{\partial\varXi}(A_0\varXi + S^{\mathrm{T}}(y)) + \frac{\partial\alpha_1}{\partial\hat{\theta}}\varGamma\tau_2 + \\ & \frac{\partial\alpha_1}{\partial\hat{p}}\eta_1\pi_2 + \sum_{j=1}^{\gamma+1}\frac{\partial\alpha_1}{\partial\lambda_j}(-l_j\lambda_1 + \lambda_{j+1}) + \\ & \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial y_{\mathrm{r}}^{(j-1)}}y_{\mathrm{r}}^{(j)} - \frac{\partial\alpha_1}{\partial\hat{\theta}}\varGamma\hat{\theta} - \eta_1\frac{\partial\alpha_1}{\partial\hat{p}}\hat{p} + \\ & \sum_{j=1}^{m}\sum_{r=0}^{\gamma}\frac{\partial\alpha_1}{\partial\xi_{rj}}(A_0\xi_{rj} + e_{n-r}\beta_j(y)) + \\ & \frac{\partial\alpha_1}{\partial\upsilon}(-\bar{c}\upsilon + \gamma(\|y\|) + d) + \frac{\partial\alpha_1}{\partial\hat{\theta}_0}\dot{\hat{\theta}}_0,\end{aligned} \tag{40}$$

其中: $c_2 > 0,\ d_2 > 0,\ \eta_1 > 0$为设计常数. τ_2, π_2为调节函数, 其稍后定义. $\hat{\theta}_0$为θ_0的估计, θ_0稍后定义.

$\omega^{\mathrm{T}}=[\mu_{\gamma,2},\bar{\omega}^{\mathrm{T}}]$, $\Gamma=\Gamma^{\mathrm{T}}>0$.

选取如下李雅普诺夫函数:

$$V_2=V_1+\frac{1}{2}z_2^2. \tag{41}$$

对V_2求导, 利用Young's不等式并综合式(39)与式(40), 可得

$$\begin{aligned}\dot{V}_2\leqslant{}&\dot{V}_1+z_2z_3-\hat{k}^*_{1,\gamma}z_1z_2-c_2z_2^2-z_2\frac{\partial\alpha_1}{\partial y}\omega^{\mathrm{T}}\tilde{\theta}+\\&z_2^2\tilde{p}\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_1}{\partial y})^2+\frac{1}{4}z_2^2\tilde{p}(\frac{\partial\alpha_1}{\partial y})^2+\\&z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}(\Gamma\tau_2-\dot{\hat{\theta}})+\frac{1}{4}+z_2\frac{\partial\alpha_1}{\partial\hat{p}}(\eta_1\pi_2-\\&\dot{\hat{p}})+\varphi_{12}^2(\|z\|)+\frac{1}{4d_2p}\varepsilon^{\mathrm{T}}\varepsilon+\frac{1}{4d_2p}\delta_1^2(y)-\\&z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\hat{\theta}-\eta_1z_2\frac{\partial\alpha_1}{\partial\hat{p}}\hat{p}+2d_2z_2^2\tilde{p}(\frac{\partial\alpha_1}{\partial y})^2.\end{aligned} \tag{42}$$

选取调节函数τ_2为

$$\tau_1=z_1\bar{\omega},\ \tau_2=\tau_1-\frac{\partial\alpha_1}{\partial y}\omega z_2+[z_1z_2,0,\cdots,0]^{\mathrm{T}}. \tag{43}$$

选取调节函数π_2为

$$\begin{aligned}\pi_2={}&z_1^2\varphi_{11}^2(\|y\|)+\frac{1}{4}z_1^2+z_2^2\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_1}{\partial y})^2+\\&\frac{1}{4}z_2^2(\frac{\partial\alpha_1}{\partial y})^2+2d_2z_1^2+2d_2z_2^2(\frac{\partial\alpha_1}{\partial y})^2.\end{aligned} \tag{44}$$

第i步$(3\leqslant i\leqslant\rho-1)$ 考虑到$z_i=\mu_{\gamma,i}-\alpha_{i-1}$, 并将$z_{i+1}=\mu_{\gamma,\,i+1}-\alpha_i$代入式(24), 对$z_i$求导可得

$$\dot{z}_i=-l_i\mu_{\gamma,\,1}+z_{i+1}+\alpha_i-\dot{\alpha}_{i-1}. \tag{45}$$

选取虚拟控制律α_i为

$$\begin{aligned}\alpha_i={}&-z_{i-1}-c_iz_i-2d_i\hat{p}(\frac{\partial\alpha_{i-1}}{\partial y})^2z_i+\\&l_i\mu_{\gamma,\,1}+\frac{\partial\alpha_{i-1}}{\partial y}\xi_{0,2}+\frac{\partial\alpha_{i-1}}{\partial y}\omega^{\mathrm{T}}\hat{\theta}-\\&\hat{p}\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{i-1}}{\partial y})^2z_i-\frac{1}{4}\hat{p}(\frac{\partial\alpha_{i-1}}{\partial y})^2z_i+\\&\frac{\partial\alpha_{i-1}}{\partial k}\dot{k}+\frac{\partial\alpha_{i-1}}{\partial\xi_0}(A_0\xi_0+Ly)+\\&\frac{\partial\alpha_{i-1}}{\partial\Xi}(A_0\Xi+S^{\mathrm{T}}(y))+\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\tau_i+\\&\eta_1\frac{\partial\alpha_{i-1}}{\partial\hat{p}}\pi_i+\sum_{j=1}^{\gamma+i-1}\frac{\partial\alpha_{i-1}}{\partial\lambda_j}(-l_j\lambda_1+\lambda_{j+1})+\\&\sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial y_{\mathrm{r}}^{(j-1)}}y_{\mathrm{r}}^{(j)}-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\hat{\theta}-\\&\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\Gamma\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\omega z_j+\\&\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{p}}\eta_1z_i\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{i-1}}{\partial y})^2z_j-\\&\eta_1\frac{\partial\alpha_{i-1}}{\partial\hat{p}}\hat{p}+\frac{1}{4}\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{p}}\eta_1z_i(\frac{\partial\alpha_{i-1}}{\partial y})^2z_j+\\&2\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{p}}\eta_1d_iz_i(\frac{\partial\alpha_{i-1}}{\partial y})^2z_j+\\&\sum_{j=1}^{m}\sum_{r=0}^{\gamma}\frac{\partial\alpha_{i-1}}{\partial\xi_{rj}}(A_0\xi_{rj}+e_{n-r}\beta_j(y))+\\&\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_0}\dot{\hat{\theta}}_0+\frac{\partial\alpha_{i-1}}{\partial\upsilon}(-\bar{c}\upsilon+\gamma(\|y\|)+d),\end{aligned} \tag{46}$$

其中: $c_i>0,d_i>0$为设计常数. τ_i,π_i为调节函数, 稍后定义.

选取如下李雅普诺夫函数:

$$V_i=V_{i-1}+\frac{1}{2}z_i^2. \tag{47}$$

对V_i求导, 由Young's不等式, 综合式(45)–(46), 可得

$$\begin{aligned}\dot{V}_i\leqslant{}&\dot{V}_{i-1}+z_iz_{i+1}-z_iz_{i-1}-c_iz_i^2-\\&z_i\frac{\partial\alpha_{i-1}}{\partial y}\omega^{\mathrm{T}}\tilde{\theta}+z_i^2\tilde{p}\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{i-1}}{\partial y})^2+\\&\frac{1}{4}z_i^2\tilde{p}(\frac{\partial\alpha_{i-1}}{\partial y})^2+z_2\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}(\Gamma\tau_i-\dot{\hat{\theta}})+\\&\frac{1}{4}+z_i\frac{\partial\alpha_{i-1}}{\partial\hat{p}}(\eta_1\pi_i-\dot{\hat{p}})+\varphi_{12}^2(\|z\|)+\\&\frac{1}{4d_ip}\varepsilon^{\mathrm{T}}\varepsilon+\frac{1}{4d_ip}\delta_1^2(y)-z_i\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\hat{\theta}-\\&\eta_1z_i\frac{\partial\alpha_{i-1}}{\partial\hat{p}}\hat{p}+2d_iz_i^2\tilde{p}(\frac{\partial\alpha_{i-1}}{\partial y})^2-\\&\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\Gamma\frac{\partial\alpha_{i-1}}{\partial y}\omega z_jz_i+\\&\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{p}}\eta_1z_i^2\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{i-1}}{\partial y})^2z_j+\\&\frac{1}{4}\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{p}}\eta_1z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2z_j+\\&2\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{p}}\eta_1d_iz_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2z_j.\end{aligned} \tag{48}$$

选取调节函数τ_i为

$$\tau_i=\tau_{i-1}-\frac{\partial\alpha_{i-1}}{\partial y}\omega z_i. \tag{49}$$

选取调节函数π_i为

$$\begin{aligned}\pi_i={}&\pi_{i-1}+z_i^2\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{i-1}}{\partial y})^2+\\&\frac{1}{4}z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2+2d_iz_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2.\end{aligned} \tag{50}$$

第ρ步 考虑到$z_\rho=\mu_{\gamma,\rho}-\alpha_{\rho-1}$, 对$z_\rho$求导可得

$$\dot{z}_\rho=-l_\rho\mu_{\gamma,\,1}+\mu_{\gamma,\rho+1}+v_0-\dot{\alpha}_{\rho-1}. \tag{51}$$

选取系统的控制律v_0为

$$v_0=\alpha_\rho-\mu_{\gamma,\rho+1}, \tag{52}$$

其中: 虚拟控制律α_ρ由式(46)确定, $i=\rho$, $c_\rho>0$,

$d_\rho > 0$为设计常数, τ_ρ, π_ρ为调节函数, 稍后定义.

选取如下李雅普诺夫函数:

$$V_\varepsilon = \frac{1}{p}\sum_{i=1}^{\rho}\frac{1}{2d_i}\varepsilon^{\mathrm T}P\varepsilon, \tag{53}$$

$$V_\rho = V_\varepsilon + V_{\rho-1} + \frac{1}{2}z_\rho^2 + \frac{1}{2}\tilde\theta^{\mathrm T}\Gamma^{-1}\tilde\theta + \frac{1}{2\eta_1}\tilde p^2 + \frac{1}{2\eta_2}\tilde\theta_0^2, \tag{54}$$

其中: $\eta_2 > 0$为设计常数; 参数估计误差为$\tilde\theta = \theta - \hat\theta$, $\tilde\theta_0 = \theta_0 - \hat\theta_0, \tilde p = p - \hat p$.

对V_ε求导, 综合假设5、式(30)与Young's不等式, 可得

$$\dot V_\varepsilon \leqslant -\frac{1}{p}\sum_{i=1}^{\rho}\frac{1}{2d_i}\varepsilon^{\mathrm T}\varepsilon + \sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{d_i}\|P\|^2[\varphi_{j1}^2(\|y\|) + \varphi_{j2}^2(\|z\|)] + \frac{1}{p}\|P\|^2\sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{2d_i}\delta_j^2(y). \tag{55}$$

由式(16)与式(21)可知: θ在时间段$[t_k, t_{k+1})$内为常数, 故对V_ρ在时间段$[t_k, t_{k+1})$内求导, 综合式(51)与式(52), 并利用Young's不等式放缩, 可得

$$\begin{aligned}\dot V_\rho \leqslant{}& \dot V_{\rho-1} - z_\rho z_{\rho-1} - c_\rho z_\rho^2 - z_\rho\frac{\partial\alpha_{\rho-1}}{\partial y}\omega^{\mathrm T}\tilde\theta + \\ & z_\rho^2\tilde p\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 + \frac{1}{4}z_\rho^2\tilde p(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 + \\ & z_\rho\frac{\partial\alpha_{\rho-1}}{\partial\hat\theta}(\Gamma\tau_\rho - \dot{\hat\theta}) + z_\rho\frac{\partial\alpha_{\rho-1}}{\partial\hat p}(\eta_1\pi_\rho - \dot{\hat p}) + \\ & \varphi_{12}^2(\|z\|) + \frac{1}{4d_\rho p}\varepsilon^{\mathrm T}\varepsilon + \frac{1}{4d_\rho p}\delta_1^2(y) - \\ & z_\rho\frac{\partial\alpha_{\rho-1}}{\partial\hat\theta}\Gamma\hat\theta - \eta_1 z_\rho\frac{\partial\alpha_{\rho-1}}{\partial\hat p}\hat p + 2d_\rho z_\rho^2\tilde p\times \\ & (\frac{\partial\alpha_{\rho-1}}{\partial y})^2 - \sum_{j=2}^{\rho-1}\frac{\partial\alpha_{j-1}}{\partial\hat\theta}\Gamma\frac{\partial\alpha_{\rho-1}}{\partial y}\omega z_j z_\rho + \\ & \sum_{j=2}^{\rho-1}\frac{\partial\alpha_{j-1}}{\partial\hat p}\eta_1 z_\rho^2\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 z_j + \\ & \frac{1}{4}\sum_{j=2}^{\rho-1}\frac{\partial\alpha_{j-1}}{\partial\hat p}\eta_1 z_\rho^2(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 z_j + \\ & 2\sum_{j=2}^{\rho-1}\frac{\partial\alpha_{j-1}}{\partial\hat p}\eta_1 d_\rho z_\rho^2(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 z_j - \\ & \frac{1}{p}\sum_{i=1}^{\rho}\frac{1}{2d_i}\varepsilon^{\mathrm T}\varepsilon + \frac{1}{4} + \\ & \sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{d_i}\|P\|^2\,[\varphi_{j1}^2(\|y\|) + \\ & \varphi_{j2}^2(\|z\|)] + \frac{1}{p}\|P\|^2\sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{2d_i}\delta_j^2(y) - \\ & \tilde\theta^{\mathrm T}\Gamma^{-1}\dot{\hat\theta} - \frac{1}{\eta_1}\tilde p\dot{\hat p} - \frac{1}{\eta_2}\tilde\theta_0\dot{\hat\theta}_0.\end{aligned} \tag{56}$$

选取调节函数τ_ρ为

$$\tau_\rho = \tau_{\rho-1} - \frac{\partial\alpha_{\rho-1}}{\partial y}\omega z_\rho. \tag{57}$$

选取调节函数π_ρ为

$$\pi_\rho = \pi_{\rho-1} + z_\rho^2\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 + \frac{1}{4}z_\rho^2(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 + 2d_\rho z_\rho^2(\frac{\partial\alpha_{\rho-1}}{\partial y})^2. \tag{58}$$

将式(37)(43)(48)代入式(56), 可得

$$\begin{aligned}\dot V_\rho \leqslant{}& z_1\hat k_{1,\gamma}^* z_2 + k_{1,\gamma}^* N(k)\dot k + \dot k - \\ & \sum_{i=1}^{\rho}c_i z_i^2 + z_1\bar\omega^{\mathrm T}\tilde\theta - \sum_{i=2}^{\rho}z_i(\frac{\partial\alpha_{i-1}}{\partial y})^2\omega^{\mathrm T}\tilde\theta + \\ & z_1^2\tilde p\varphi_{11}^2(\|y\|) + \sum_{i=2}^{\rho}z_i^2\tilde p\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{i-1}}{\partial y})^2 + \\ & \frac{1}{4}z_1^2\tilde p + \frac{1}{4}\sum_{i=2}^{\rho}z_i^2\tilde p(\frac{\partial\alpha_{i-1}}{\partial y})^2 + \\ & \sum_{i=2}^{\rho}\frac{\partial\alpha_{i-1}}{\partial\hat\theta}(\Gamma\tau_i - \dot{\hat\theta})z_i + \\ & \sum_{i=2}^{\rho}\frac{\partial\alpha_{i-1}}{\partial\hat p}(\eta_1\pi_i - \dot{\hat p})z_i + \frac{1}{4}\rho - \\ & \sum_{i=2}^{\rho}\frac{\partial\alpha_{i-1}}{\partial\hat\theta}\Gamma\hat\theta z_i + \rho\varphi_{12}^2(\|z\|) - \\ & \frac{1}{p}\sum_{i=1}^{\rho}\frac{1}{4d_i}\varepsilon^{\mathrm T}\varepsilon + \frac{1}{p}\sum_{i=1}^{\rho}\frac{1}{4d_i}\delta_1^2(y) - \\ & \sum_{i=3}^{\rho}\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat\theta}\Gamma\frac{\partial\alpha_{i-1}}{\partial y}\omega z_j z_i + \\ & \sum_{i=3}^{\rho}\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat p}\eta_1 z_i^2\varphi_{11}^2(\|y\|)(\frac{\partial\alpha_{i-1}}{\partial y})^2 z_j + \\ & \frac{1}{4}\sum_{i=3}^{\rho}\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat p}\eta_1 z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2 z_j - \\ & \sum_{i=2}^{\rho}\eta_1\frac{\partial\alpha_{i-1}}{\partial\hat p}\hat p z_i - \\ & \frac{1}{4}z_1^2 + z_1\hat\theta_0 + 2\tilde p d_1 z_1^2 + \sum_{i=2}^{\rho}2\tilde p d_i z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2 + \\ & 2\sum_{i=3}^{\rho}\sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat p}\eta_1 d_i z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2 z_j + \\ & \sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{d_i}\|P\|^2[\varphi_{j1}^2(\|y\|) + \\ & \varphi_{j2}^2(\|z\|)] + \frac{1}{p}\|P\|^2\sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{2d_i}\delta_j^2(y) - \\ & \frac{z_1^2}{\varepsilon^*}Q(y,\upsilon) - \tilde\theta^{\mathrm T}\Gamma^{-1}\dot{\hat\theta} - \frac{1}{\eta_1}\tilde p\dot{\hat p} - \frac{1}{\eta_2}\tilde\theta_0\dot{\hat\theta}_0.\end{aligned} \tag{59}$$

由引理4可知, 存在未知正常数δ_i^*, 满足:

$$|\delta_i(y)| \leqslant \delta_i^*(i = 1, 2, \cdots, n).$$

令未知正常数: $\delta^* = \max\{\delta_1^{*2}, \delta_2^{*2}, \cdots, \delta_n^{*2}\}$, 根据式(59), 令未知正常数$\theta_0$为

$$\theta_0 = \frac{1}{p}\sum_{i=1}^{\rho}\frac{1}{4d_i}\delta^* + \frac{1}{p}\|P\|^2\sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{2d_i}\delta^*. \tag{60}$$

选取参数自适应律:

$$\dot{\hat{\theta}} = \Gamma\tau_\rho - \Gamma\hat{\theta}, \tag{61}$$

$$\dot{\hat{\theta}}_0 = -\eta_2\hat{\theta}_0 - z_1\eta_2, \tag{62}$$

$$\dot{\hat{p}} = \eta_1\pi_\rho - \eta_1\hat{p}, \tag{63}$$

为了处理式(59)中利用虚拟控制律所无法抵消的部分, 引入非负且单调不减的变能量函数$\varrho(\cdot)$, 选取如下李雅普诺夫函数:

$$V_s = V_\rho + \int_0^\upsilon \varrho(s)\mathrm{d}s. \tag{64}$$

对V_s在时间段$[t_k, t_{k+1})$内求导并综合引理1可得

$$\dot{V}_s = \dot{V}_\rho + \varrho(\upsilon)(-\bar{c}\upsilon + \gamma(\|y\|) + d). \tag{65}$$

由式(59), 综合假设5、假设6和引理1, 可得

$$\begin{aligned}&\rho\varphi_{12}^2(\|z\|) + \sum_{i=1}^{\rho}\sum_{j=1}^{n}\|P\|^2\frac{1}{d_i}\varphi_{j2}^2(\|z\|) \leqslant \\&\rho\varphi_{12}^2[\alpha_1^{-1}(2\upsilon)] + \rho\varphi_{12}^2[\alpha_1^{-1}(2D(t,t_0))] + \\&\sum_{i=1}^{\rho}\sum_{j=1}^{n}\|P\|^2\frac{1}{d_i}\varphi_{j2}^2[\alpha_1^{-1}(2\upsilon)] + \\&\sum_{i=1}^{\rho}\sum_{j=1}^{n}\|P\|^2\frac{1}{d_i}\varphi_{j2}^2[\alpha_1^{-1}(2D(t,t_0))].\end{aligned} \tag{66}$$

由引理1可知, $D(t,t_0)$为有界函数, 故可设

$$\begin{aligned}&\rho\varphi_{12}^2[\alpha_1^{-1}(2D(t,t_0))] + \sum_{i=1}^{\rho}\sum_{j=1}^{n}\|P\|^2\frac{1}{d_i}\times \\&\varphi_{j2}^2[\alpha_1^{-1}(2D(t,t_0))] \leqslant \theta_0^*.\end{aligned} \tag{67}$$

令$\Pi(\upsilon) = \rho\varphi_{12}^2[\alpha_1^{-1}(2\upsilon)] + \sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{1}{d_i}\|P\|^2\varphi_{j2}^2[\alpha_1^{-1}(2\upsilon)]$, 由于$\Pi(\cdot)$为单调不减的非负光滑连续函数, 且$\Pi(0) = 0$, 故存在非负单调不减光滑函数$\Phi(\upsilon)$使得: $\Pi(\upsilon) = \upsilon\Phi(\upsilon)$.

综合式(66)–(67), 式(65)可写为如下形式:

$$\begin{aligned}\dot{V}_s \leqslant{}& \dot{V}_\rho - \frac{\bar{c}}{2}\varrho(\upsilon)\upsilon + \upsilon(-\frac{\bar{c}}{4}\varrho(\upsilon) + \Phi(\upsilon)) - \\&\frac{\bar{c}}{4}\varrho(\upsilon)\upsilon + \varrho(\upsilon)\gamma(\|y\|) + \varrho(\upsilon)d + \theta_0^*.\end{aligned} \tag{68}$$

由式(68), 选取变能量函数为

$$\varrho(\upsilon) = \frac{4}{\bar{c}}\Phi(\upsilon). \tag{69}$$

由于$\varrho(\cdot)$为非负单调不减函数, 由文献[17]可知:

$$\int_0^\upsilon \varrho(s)\mathrm{d}s \leqslant \varrho(\upsilon)\upsilon, \tag{70}$$

$$-\frac{\bar{c}}{4}\upsilon\varrho(\upsilon) + \varrho(\upsilon)d \leqslant \varrho(\frac{4d}{\bar{c}})d. \tag{71}$$

将式(69)–(71)代入式(68), 可得

$$\begin{aligned}\dot{V}_s \leqslant{}& \dot{V}_\rho - \frac{\bar{c}}{2}\int_0^\upsilon \varrho(s)\mathrm{d}s + \\&\varrho(\frac{4d}{\bar{c}})d + \varrho(\upsilon)\gamma(\|y\|) + \theta_0^*.\end{aligned} \tag{72}$$

5 稳定性分析(Stability analysis)

定理 1 考虑非线性系统(1)在时刻t_k发生执行器故障的情况下, 由其与控制律(52), 参数自适应律(61)–(63)以及变能量函数$\varrho(\upsilon)$所组成的系统, 若假设1–6成立, 则闭环系统是半全局一致终结有界的, 且跟踪误差收敛到原点的一个小邻域内.

证 定义李雅普诺夫函数

$$V = V_\rho + \int_0^\upsilon \varrho(s)\mathrm{d}s. \tag{73}$$

对V在$[t_k, t_{k+1})$内求导, 综合以上分析, 可得

$$\begin{aligned}\dot{V} \leqslant{}& k_{1,\gamma}^*N(k)\dot{k} + \dot{k} - \lambda_0 V + \\&C + (1 - \frac{z_1^2}{\varepsilon^{*2}})Q(y,\upsilon),\end{aligned} \tag{74}$$

其中:

$$\begin{aligned}\lambda_0 = \min\{&2c_1, 2c_2, \cdots, 2c_\rho, \frac{1}{\lambda_{\max}(\Gamma^{-1})}, \\&\eta_1, \eta_2, \frac{\bar{c}}{2}, \frac{1}{2\lambda_{\max}(P)}\},\end{aligned}$$

$$C = \varrho(\frac{4d}{\bar{c}})d + \frac{1}{2}\theta^{\mathrm{T}}\theta + \frac{3}{2}\theta_0^2 + \theta_0 + \theta_0^* + \frac{1}{2}p^2 + \frac{1}{4}\rho,$$

其为未知正常数. 根据式(74)可知, $(1 - \frac{z_1^2}{\varepsilon^{*2}})Q(y,\upsilon)$不确定, 故当$t \in [t_k, t_{k+1})$时, 对其分两种情况进行讨论:

1) $z_1 \in \Omega_{z1} = \{z_1 : |z_1| < \varepsilon^*\}$. 由$z_1 = y - y_{\mathrm{r}}$可知$z_1 \in L_\infty$. 由假设5可知$y_{\mathrm{r}} \in L_\infty$, 故$y \in L_\infty$. 由假设6可知$\bar{\gamma}(\|y\|) \in L_\infty$. 利用BIBO稳定性, 由引理1中的动态信号方程$\dot{\upsilon} = -\bar{c}\upsilon + \bar{\gamma}(\|y\|) + d$可知$\upsilon \in L_\infty$. 由于$\varphi_{i1}(\|y\|)$为非负光滑函数, 故$\varphi_{i1}(\|y\|) \in L_\infty$. 由于变能量函数$\varrho(\upsilon)$为光滑连续函数, 故$\varrho(\upsilon) \in L_\infty$. 则综上所述, 可得辅助函数$Q(y,\upsilon) \in L_\infty$, 令

$$|Q(y,\upsilon)| \leqslant \mu,\ \mu > 0, \tag{75}$$

则由式(74)可得

$$\dot{V} \leqslant k_{1,\gamma}^*N(k)\dot{k} + \dot{k} - \lambda_0 V + C + \mu. \tag{76}$$

将式(76)两边同时乘以$\mathrm{e}^{\lambda_0 t}$, 并对其在区间$[t_k, t]$上求定积分可得

$$\begin{aligned}0 \leqslant V \leqslant{}& \mathrm{e}^{-\lambda_0(t-t_k)}\int_{t_k}^{t}[k_{1,\gamma}^*N(k) + 1]\dot{k}\mathrm{e}^{\lambda_0\tau}\mathrm{d}\tau + \\&V(t_k) + \frac{C+\mu}{\lambda_0}.\end{aligned} \tag{77}$$

利用引理3, 可知$k \in L_\infty, V \in L_\infty$.

2) $z_1 \notin \Omega_{z_1}$. 当$z_1 \notin \Omega_{z_1}$时, 可得知$(1 - z_1^2/\varepsilon^{*2}) \times Q(y,\upsilon) \leqslant 0$, 故由式(74)可知

$$\dot{V} \leqslant k_{1,\gamma}^*N(k)\dot{k} + \dot{k} - \lambda_0 V + C. \tag{78}$$

与情况1)处理过程类似, 可得: $k \in L_\infty, V \in L_\infty$.

综合两种情况, 可得$z_1, z_2, \cdots, z_\rho, \tilde{\theta}, \tilde{\theta}_0, \tilde{p}, \varepsilon \in L_\infty$. 由假设4可知$y_{\mathrm{r}}, \dot{y}_{\mathrm{r}} \in L_\infty$, 故$y \in L_\infty$. 利用BIBO

稳定性, 由式(28)可知$\xi_0, \varXi, \xi_{rj} \in L_\infty$, 且由引理1可知$\upsilon \in L_\infty$. 利用式(26), 可得

$$\lambda_i = G(s)K(s)(y^{(n)} - \sum_{i=1}^{n} f_i^{(n-i)}(y) - \sum_{r=0}^{\gamma}\sum_{j=1}^{m} k_{2,rj}\beta_j^{(r)}(y) - \sum_{i=1}^{n} \varDelta_i^{(n-i)}(z,y,t)), \tag{79}$$

其中:

$$G(s) = \frac{1}{k_{1,\gamma}^* s^\gamma + \cdots + k_{1,1}^* s + k_{1,0}^*},$$
$$K(s) = \frac{s^{i-1} + l_1 s^{i-2} + \cdots + l_{i-1}}{s^n + l_1 s^{n-1} + \cdots + l_n}.$$

由假设3可得$G(s)$为稳定的多项式, 由于$y \in L_\infty$, 且$f_i(y)$与$\varDelta_i(z,y,t)$为光滑函数, 则由式(79)知: $\lambda_{\gamma+1}, \cdots, \lambda_2, \lambda_1 \in L_\infty$. 由于$\tilde\theta, \tilde\theta_0, \tilde p \in L_\infty$, 可得$\hat\theta, \hat\theta_0$, $\hat p \in L_\infty$. 考虑到

$$\mu_{\gamma,2} = z_2 + \alpha_1(y,\ k, \lambda_1,\ \lambda_2, \cdots, \lambda_{\gamma+1}, y_{\rm r}, \dot y_{\rm r}, \hat\theta, \hat\theta_0, \hat p, \xi_0, \xi_{rj}, \varXi, \upsilon), \tag{80}$$

$$\mu_{\gamma,i} = z_i + \alpha_{i-1}(y, k, \lambda_1, \lambda_2, \cdots, \lambda_{\gamma+i-1}, y_{\rm r}, \dot y_{\rm r}, \cdots, y_{\rm r}^{(i-1)}, \hat\theta, \hat\theta_0, \hat p, \xi_0, \xi_{rj}, \varXi, \upsilon), \tag{81}$$

其中$i = 3, 4, \cdots, \rho$.

由于$y, k, \lambda_1, \lambda_2, \cdots, \lambda_{\gamma+1}, y_{\rm r}, \dot y_{\rm r}, \hat\theta, \hat\theta_0, \hat p, \xi_0, \xi_{rj}, \varXi, \upsilon$, $z_2 \in L_\infty$, 故由式(80)可知$\mu_{\gamma,2} \in L_\infty$. 则由式(27)可得$\lambda_{\gamma+2} \in L_\infty$, 利用式(81)与假设4, 可得$\mu_{\gamma,3} \in L_\infty$. 重复利用式(27)与式(81), 可得$\lambda \in L_\infty$. 通过利用式(24)–(25), 可知$\mu_{\rm r} \in L_\infty, v_0 \in L_\infty$. 由于$\beta_j(y) \neq 0$, $j = 1, 2, \cdots, m$, 为光滑非线性函数, 故$\beta_j(y) \in L_\infty$. 由于$\beta_j(y) \in L_\infty$, 可得$v_j \in L_\infty$. 由于$\xi_0, \varXi, \xi_{rj}, \mu_{\rm r}, \varepsilon \in L_\infty$, 则由式(29)可知$x \in L_\infty$. 综上所述, 闭环系统半全局一致终结有界, 且跟踪误差能够收敛到原点的一个小邻域内.

6 仿真结果(Simulation results)

为了验证控制策略的有效性, 考虑如下2阶非线性系统:

$$\begin{cases} \dot z = -z + y^2 \sin y, \\ \dot x_1 = x_2 + f_1(y) + \varDelta_1, \\ \dot x_2 = f_2(y) + \beta_1(y)u_1 - 2\beta_2(y)u_2 - \\ \qquad 3\beta_3(y)u_3 - 2\beta_4(y)u_4 + \varDelta_2, \\ y = x_1, \end{cases} \tag{82}$$

其中:

$$f_1(y) = y^2 \sin t + y \cos y,$$
$$f_2(y) = \cos^2 y + y^2 \sin y^2,$$
$$\varDelta_1 = y^2 \cos y^2 + z^2 \sin z^2,$$
$$\varDelta_2 = y^2 \cos y^2 + z^2 \cos z^2.$$

跟踪信号: $y_{\rm r} = \sin(2t)\cos(2t) = 0.5 \times \sin(4t)$, 控制增益

$$b_{01} = 1,\ b_{02} = -2,\ b_{03} = -2,\ b_{04} = -3,$$
$$\beta_1(y) = 0.5 \sin^2 y \cos^2 y + 1,$$
$$\beta_2(y) = 0.5 \cos^2 y + 1,$$
$$\beta_3(y) = \beta_4(y) = 0.5 \sin^2 y + 1,$$

控制输入u_1, u_2, u_3发生故障, 形式如下所示:

$$u_1 = \begin{cases} v_1,\ 0 \leqslant t < 4, \\ -2,\ t \geqslant 4, \end{cases} \tag{83}$$

$$u_2 = \begin{cases} v_2, \quad 0 \leqslant t < 8, \\ 0.8v_2,\ t \geqslant 8, \end{cases} \tag{84}$$

$$u_3 = \begin{cases} v_3, \quad 0 \leqslant t < 12, \\ 0.7v_3,\ t \geqslant 12. \end{cases} \tag{85}$$

设计常数

$$l_1 = 3,\ l_2 = 2,\ \eta_1 = 0.08,\ \eta_2 = 0.05,$$
$$d_1 = 0.5,\ d_2 = 0.5,\ c_1 = 0.8,\ c_2 = 0.8,$$
$$\varepsilon^* = 20,\ \varGamma = 5I_{35},$$

初值

$$x(0) = [0.3\ \ -0.5]^{\rm T},\ z(0) = 0.1,$$
$$\hat\theta(0) = [0\ \ \cdots\ \ 0]^{\rm T} \in \mathbb{R}^{35},\ \hat\theta_0(0) = \hat p(0) = 0.3,$$

动态信号$\dot\upsilon = -10\upsilon + 0.125y^2 + 1$, $\upsilon(0) = 0.4$, $N(0) = 0.3$. 仿真结果如图1–5所示.

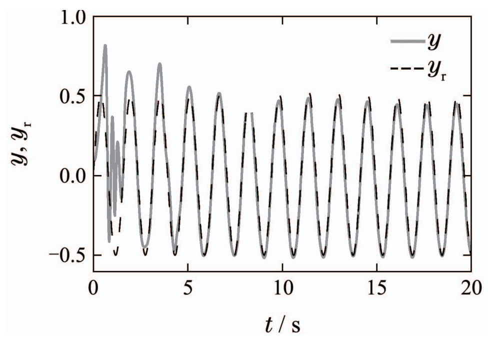

图 1 输出y和跟踪轨迹$y_{\rm r}$

Fig. 1 Output y and tracking trajectory $y_{\rm r}$

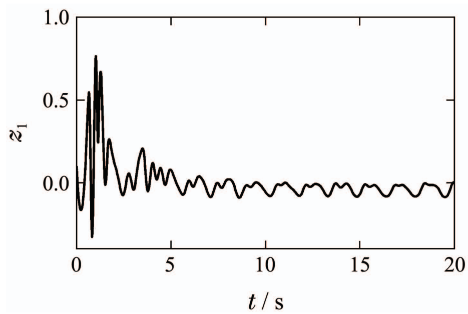

图 2 跟踪误差z_1

Fig. 2 Tracking error z_1

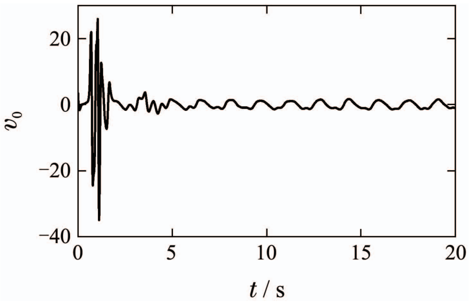

图 3 控制信号v_0

Fig. 3 Control signal v_0

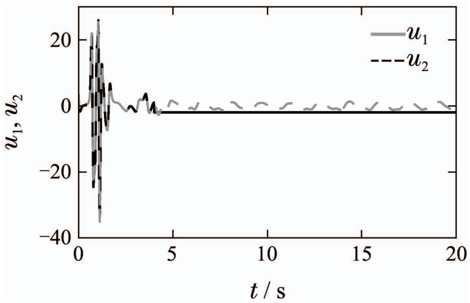

图 4 系统控制输入u_1和u_2

Fig. 4 System control input u_1 and u_2

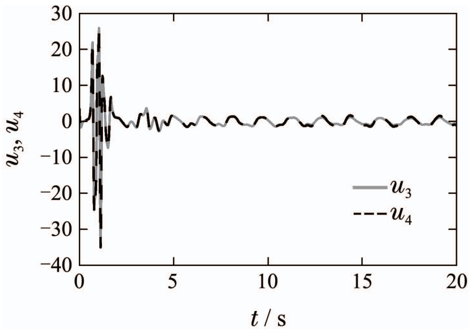

图 5 系统控制输入u_3和u_4

Fig. 5 System control input u_3 and u_4

7 结论(Conclusions)

本文对一类控制增益符号未知且具有执行器故障的输出反馈MISO系统, 提出了一种后推容错方案. 结合变能量函数与辅助函数处理了系统虚拟控制律所无法抵消的部分. 最终, 选取合适的李雅普诺夫函数证明了所设计的控制律能够在系统控制输入发生故障的情况下, 闭环系统半全局一致终结有界, 且跟踪误差能够收敛到原点的一个小邻域内.

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