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考虑随机量测时滞和同步相关噪声的改进高斯滤波算法

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摘要: 经典高斯滤波算法存在量测信息实时获取, 以及过程噪声和量测噪声相互独立的假设条件. 然而, 在工程实际应用中该假设条件有时难以满足. 本文针对一类具有随机量测时滞和同步相关噪声的高斯系统的状态估计问题, 设计了一种高斯滤波框架形式的最优估计算法, 并给出了所设计算法的三阶球径容积法则的次优实现形式–考虑随机量测时滞和同步相关噪声的容积卡尔曼滤波器(CKF–RDSCN). 其借助Bernoulli随机序列, 来描述系统中可能存在的量测时滞现象, 并利用高斯条件分布性质来解决噪声相关问题, 在此基础上构建所提出的最优估计算法. 仿真结果表明, 相比于扩展卡尔曼滤波(EKF), 无迹卡尔曼滤波(UKF)以及容积卡尔曼滤波(CKF), 在含有随机量测时滞和噪声同步相关的状态估计问题中, CKF–RDSCN具有更高的精度和更好的数值稳定性.

关键词: 高斯滤波; 容积卡尔曼滤波; 随机时滞; 同步相关噪声

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An improved Gaussian filter with randomly delayed measurements and synchronously correlated noises

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Abstract: The classical Gaussian filters are based on the assumption that measurements are acquired in time and noises of process and measurement are independent of each other. However, this assumption is sometimes hard to satisfy in practical applications. In this paper, an optimal estimation algorithm in the form of Gaussian filter framework is designed to solve the problem of states estimation of a Gaussian system with randomly delayed measurements and synchronously correlated noises, and the rule of third-degree spherical-radial cubature is employed to deduced the suboptimal estimation implementation of the proposed algorithms which is named cubature Kalman filter with randomly delayed measurements and synchronously correlated noises(CKF–RDSCN). It takes random sequence of Bernoulli to describe the possible situation with respect to random delay in observation measurement and the property of Gaussian conditional distribution is utilized to solve the problem of noises correlation. Simulation results demonstrate that CKF–RDSCN is more accurate and stability than the extended Kalman filter(EKF), unscented Kalman filter(UKF) and CKF in the states estimation problem involved with randomly delayed measurements and synchronously correlated noises.

Key words: Gaussian filter; cubature Kalman filter; random delay; synchronously correlated noises

1 引言(Introduction)

在许多工程实际问题中, 如目标跟踪 [1]、无人机导航 [2]、图像信息处理 [3]以及化工生产 [4]等问题, 要求在线的获取系统状态信息, 以达到实时监控和控制系统的目的. 因此, 对系统状态的最优估计问题, 由于其重要的应用价值与广泛的应用前景, 自提出以来就引起了研究人员极大的兴趣, 并取得了丰富的研究成果. 其中, 贝叶斯估计方法给出了状态估计问题的最

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优解, 它要求基于时间量测序列$\boldsymbol{Z}_k=[\boldsymbol{z}_1^{\mathrm{T}}\ \boldsymbol{z}_2^{\mathrm{T}}\cdots\boldsymbol{z}_k^{\mathrm{T}}]^{\mathrm{T}}$, 计算关于系统当前状态$\boldsymbol{x}_k$的后验概率密度函数$p(\boldsymbol{x}_k|\boldsymbol{Z}_k)$, 以此来达到对状态$\boldsymbol{x}_k$的估计[5]. 在以线性高斯系统为研究对象的状态估计问题中, 对后验概率密度函数$p(\boldsymbol{x}_k|\boldsymbol{Z}_k)$的计算是容易实现的, 其典型代表就是著名的卡尔曼滤波器(KF). 然而, 对于非线性高斯系统而言, 由于其自身的非线性特性, 使得对于上述函数的计算变得异常困难. 因此, 研究人员只能采用近似计算的方式, 以达到对非线性系统状态的次优估计.

对于非线性系统的状态估计问题, 研究人员相继展开了一系列的研究. Ito等[6]首次给出了关于高斯离散系统的一般形式的最优滤波框架, 并将概率密度计算问题转化为高斯加权积分计算问题. 该框架虽然仅具有理论层面上的意义, 但其重要性在于, 为一类非线性高斯系统的状态估计, 设计了一个具有普遍意义的框架, 在此基础上, 研究人员通过采用不同的高斯加权积分近似计算方法, 设计相应的非线性高斯滤波算法. 例如, 基于高斯厄尔米特正交法则的高斯厄尔米特滤波算法(GHQF)[6], 基于无迹变换的无迹卡尔曼滤波算法(UKF)[7], 基于斯特林多项式插值的差分滤波算法(DDF)[8], 以及近年来提出的基于球径容积法则的容积卡尔曼滤波算法(CKF)[9]. 在上述的高斯滤波算法中, GHQF具有最高的状态估计精度, 然而巨大的运算量限制了其在工程中的实际应用, 所以常常作为参考标准, 用来衡量其他滤波算法的性能高低. 此外, UKF, DDF以及CKF属于同一类滤波算法, 即均采用确定性的加权点来近似状态的后验概率估计. 值得注意的是, 虽然CKF滤波算法仅是UKF滤波算法的实现特例, 但前者的数值稳定性却要远好于后者, 随着系统状态维数的增多, 这种差异性的表现越来越明显[9].

然而, 为了方便滤波算法的设计, 高斯滤波算法存在如下假设条件, 即系统过程噪声和量测噪声彼此独立, 且实时获取量测信息. 但在工程实际应用中, 上述假设条件有时难以满足. 尤其是在多传感器所构成的量测值输出反馈网络系统中, 由于网络拥堵、传输机制随机失效或某段时间内数据的不可获取等原因, 使得该系统的量测值具有随机时滞特性. 与此同时, 由于量测值作为输出反馈传递给网络系统, 从而使得系统的过程噪声和量测噪声具有相关特性. 因此, 该系统模型具有随机量测时滞和噪声相关的特征[10–12], 上述干扰因素破坏了高斯滤波算法的假设条件, 所以UKF、DDF和CKF等高斯滤波算法, 在对具有噪声相关和量测时滞特征的系统进行状态估计时, 会出现滤波算法性能下降, 甚至估计结果严重发散的情形. 针对噪声相关问题, 其解决方法主要有两类, 一类是对卡尔曼类型的滤波器进行改进, 采用解耦方法使噪声相关问题转化为标准的状态估计问题[13–15]; 一类是从设计通用算法的角度出发, 构建适用于噪声相关情况下的通用算法框架[16–17]. 其中, 基于噪声解耦的研究成果较多, 其核心思想是通过构建伪状态方程的方式, 迫使过程噪声和量测噪声彼此独立. Yaakov[13]和Chang等[14]分别给出了基于EKF和UKF的解决方法, 徐小良等[15]将其推广到系统噪声任意相关的情况, 并提出了基于容积粒子滤波下的估计算法. 王小旭等[16]基于最小均方误差估计准则, 提出了噪声相关条件下的滤波递推公式. 随后其又以两步预测代替原高斯滤波算法中一步预测的方式, 即以对概率密度函数$p(\boldsymbol{x}_k|\boldsymbol{Z}_{k-2})$的计算来取代对$p(\boldsymbol{x}_k|\boldsymbol{Z}_{k-1})$计算的方式, 从而达到噪声解耦目的, 并实现对状态$\boldsymbol{x}_k$的估计, 将噪声相关问题的解决统一到高斯滤波算法框架下[17]. 此外, 考虑到条件概率密度比原概率密度含有更多信息的情况, Chang利用高斯条件分布, 来解决噪声相关问题, 然而其仅给出了适用于线性系统的, 基于KF的估计算法[18].

针对量测时滞问题, 其主要研究方法同样可分为两类, 一类是假设量测时滞为确定性延迟[19–20], 一类是假设量测时滞为随机延迟[12, 21–22]. 其中, 在确定性延迟问题中, 一般利用延迟信息确定时滞状态, 再选取量测信息对其更新. Prasad等[19]提出了基于多速率EKF的解决方法, Asadi等[20]通过扩展随机克隆方法, 以延迟状态为状态增量的方式, 解决时滞问题. 考虑到数据通信信道带宽的限制, 使得量测时滞常常具有随机特性. Wang等[12]在文献[17]的基础上, 考虑随机时滞问题, 将结果拓展到适用于随机时滞和噪声相关的估计问题中. Hermoso等[21]以Bernoulli分布的方式来描述系统中所存在的随机时滞现象, 以量测噪声为状态增量, 给出了基于EKF和UKF的时滞滤波器. 随后, 其又针对两步随机时滞问题, 提出了UKF下的解决方法[22]. 此外, 在鲁棒滤波算法框架下, 对于噪声相关干扰情况下的状态估计问题, Qian等[23–26]借助于线性矩阵不等式和Riccati方程等方法, 对同步相关、两步相关以及有限步相关等情况进行了深入的研究, 并取得了丰富的研究成果; 对于时滞干扰情况下的状态估计问题, Yang等[27–29]分别给出了考虑随机时滞丢包、随机饱和丢包以及随机量化丢包等情况下的估计算法, 设计了相应的H_∞滤波器.

虽然, 针对随机时滞和噪声相关条件下状态估计问题的研究, 已经取得了上述丰富的研究成果, 然而同时考虑两类干扰因素共同作用下的最优估计方面的理论研究还不够完善. 尤其是针对上述干扰, 给出形式为高斯滤波框架的结果还很少见. 受文献[12, 18]启发, 本文研究了一类具有随机量测时滞和同步相关噪声的高斯系统的状态估计问题, 提出了高斯框架形式的最优估计算法, 并采用三阶球径容积法则来近似

计算所提框架中的高斯加权积分, 以此提出考虑随机量测时滞和同步相关噪声的改进容积卡尔曼滤波器(CKF–RDSCN).

2 问题描述(Problem formulation)

考虑具有随机量测时滞和同步相关噪声的非线性离散系统如下:

$$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}) + \boldsymbol{v}_{k-1}, \tag{1}$$

$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{n}_k, \tag{2}$$

$$\boldsymbol{y}_k = (1-\varepsilon_k)\boldsymbol{z}_k + \varepsilon_k \boldsymbol{z}_{k-1}, k > 1; \boldsymbol{y}_1 = \boldsymbol{z}_1, \tag{3}$$

式中: $\boldsymbol{x}_k \in \mathbb{R}^n$表示系统在$k$时刻的状态; $\boldsymbol{u}_k \in \mathbb{R}^{n_u}$表示系统在$k$时刻的外部输入; $\boldsymbol{z}_k \in \mathbb{R}^m$表示系统在$k$时刻的理想观测值; $\boldsymbol{y}_k \in \mathbb{R}^m$表示系统在$k$时刻的实际观测值; $\boldsymbol{f}(\cdot)$表示系统的状态方程; $\boldsymbol{h}(\cdot)$表示系统的观测方程; $\boldsymbol{v}_k$和$\boldsymbol{n}_k$分别表示系统的过程噪声和量测噪声, 且均为零均值的高斯白噪声. 不失一般性, 对上述非线性离散系统作如下假设.

假设 1　过程噪声$\boldsymbol{v}_k$和量测噪声$\boldsymbol{n}_k$为同步相关高斯白噪声, 且满足

$$\mathrm{E}\{\begin{bmatrix}\boldsymbol{v}_k \\ \boldsymbol{n}_k\end{bmatrix}[\boldsymbol{v}_l^{\mathrm{T}}\ \boldsymbol{n}_l^{\mathrm{T}}]\} = \begin{bmatrix}\boldsymbol{Q}_k & \boldsymbol{S}_k \\ \boldsymbol{S}_k^{\mathrm{T}} & \boldsymbol{R}_k\end{bmatrix}\delta_{kl}, \tag{4}$$

式中: $\boldsymbol{Q}_k$和$\boldsymbol{R}_k$分别表示过程噪声和量测噪声在k时刻的协方差矩阵; $\boldsymbol{S}_k$表示上述噪声在k时刻的互协方差阵, 且$\boldsymbol{S}_k \neq 0$; δ_{kl}表示克罗内克函数.

假设 2　设$\varepsilon_k(k>1)$表示取值为0或1的满足Bernoulli分布的互不相关随机序列, 其统计特性满足

$$p(\varepsilon_k = 1) = \mathrm{E}\{\varepsilon_k\} = p_k. \tag{5}$$

在假设2条件下, 容易获得:

$$p(\varepsilon_k = 0) = 1 - \mathrm{E}\{\varepsilon_k\} = 1 - p_k, \tag{6}$$

$$\mathrm{E}\{\varepsilon_k \varepsilon_k^{\mathrm{T}}\} = p_k, \tag{7}$$

$$\mathrm{E}\{(1-\varepsilon_k)(1-\varepsilon_k)^{\mathrm{T}}\} = (1-p_k), \tag{8}$$

$$\mathrm{E}\{(1-\varepsilon_k)\varepsilon_k^{\mathrm{T}}\} = \mathrm{E}\{\varepsilon_k(1-\varepsilon_k)^{\mathrm{T}}\} = 0, \tag{9}$$

$$\mathrm{E}\{\varepsilon_k(p_k-\varepsilon_k)^{\mathrm{T}}\} = \mathrm{E}\{(p_k-\varepsilon_k)\varepsilon_k^{\mathrm{T}}\} = p_k(p_k-1), \tag{10}$$

$$\mathrm{E}\{(p_k-\varepsilon_k)(p_k-\varepsilon_k)^{\mathrm{T}}\} = -p_k(p_k-1), \tag{11}$$

$$\mathrm{E}\{(1-\varepsilon_k)(p_k-\varepsilon_k)^{\mathrm{T}}\} = \mathrm{E}\{(p_k-\varepsilon_k)(1-\varepsilon_k)^{\mathrm{T}}\} = -p_k(p_k-1). \tag{12}$$

注 1　在一般形式的高斯滤波框架中, 要求假设1中的$\boldsymbol{S}_k \equiv 0$以及假设2中的$p_k \equiv 1$, 即仅考虑了噪声互不相关和量测无时滞情况下的状态估计问题. 一方面基于假设1可知, 由于相关系数$\boldsymbol{S}_k \neq 0$, 使得过程噪声和量测噪声为相关噪声, 且由于克罗内克函数δ_{kl}的存在, 使得仅在$k=l$时, 过程噪声$\boldsymbol{v}_k$和量测噪声$\boldsymbol{n}_k$才具有相关性, 即$\boldsymbol{v}_k$和$\boldsymbol{n}_k$为同步相关噪声. 另一方面, 假设2中对随机序列ε_k的引入, 使系统以概率值p_k获取观测值, 当$\varepsilon_k=1$时, 系统观测值为$\boldsymbol{z}_k$, 当$\varepsilon_k=0$时, 系统观测值为$\boldsymbol{z}_{k-1}$. 综上所述, 由于假设1和假设2的引入, 使得式(1)–(3)中所考虑的系统模型, 既具有噪声相关特性, 又具有随机量测时滞特性.

3 改进高斯滤波算法(Improved Gaussian filter)

本节中, 提出一种高斯滤波框架形式的最优估计算法, 用以解决随机量测时滞和噪声同步相关情况下的状态估计问题. 与一般形式的高斯滤波框架不同的是, 由于系统观测值中存在一步随机时滞问题, 因此在所提出的改进框架中, 以对后验概率密度函数$p(\boldsymbol{x}_k|\boldsymbol{Y}_k)$的估计来取代对$p(\boldsymbol{x}_k|\boldsymbol{Z}_k)$的估计. 其中, $\boldsymbol{Y}_k$表示由初始时刻到k时刻的系统观测序列, 其取值为$\boldsymbol{Y}_k = [\boldsymbol{y}_1^{\mathrm{T}}\ \boldsymbol{y}_2^{\mathrm{T}} \cdots \boldsymbol{y}_k^{\mathrm{T}}]^{\mathrm{T}}$. 此外, 由下述的分析可知, 在所设计的滤波算法中, 其$k-1$时刻对观测值$\boldsymbol{y}_k$的一步预测值$\hat{\boldsymbol{y}}_{k|k-1}$中含有估计项$\hat{\boldsymbol{z}}_{k-1|k-1}$, 而$\hat{\boldsymbol{z}}_{k-1|k-1}$的计算中含有对$k-1$时刻噪声项$\hat{\boldsymbol{n}}_{k-1|k-1}$的估计. 因而, 在本节所提出的最优估计算法中, 除含有对系统状态估计值$\hat{\boldsymbol{x}}_{k|k}$的迭代更新外, 还需实现对噪声估计值$\hat{\boldsymbol{n}}_{k|k}$的迭代更新. 故重新定义系统状态$\boldsymbol{x}_k^a$为

$$\boldsymbol{x}_k^a = [\boldsymbol{x}_k^{\mathrm{T}}\ \ \boldsymbol{n}_k^{\mathrm{T}}]^{\mathrm{T}}, \tag{13}$$

其状态估计与协方差矩阵分别为

$$\hat{\boldsymbol{x}}_{k|k}^a = \begin{bmatrix}\hat{\boldsymbol{x}}_{k|k} \\ \hat{\boldsymbol{n}}_{k|k}\end{bmatrix}, \boldsymbol{P}_{k|k}^a = \begin{bmatrix}\boldsymbol{P}_{k|k} & \boldsymbol{P}_{\tilde{x}\tilde{n},k|k} \\ \boldsymbol{P}_{\tilde{x}\tilde{n},k|k}^{\mathrm{T}} & \boldsymbol{P}_{\tilde{n}\tilde{n},k|k}\end{bmatrix}, \tag{14}$$

式中:

$$\boldsymbol{P}_{k|k} = \mathrm{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})^{\mathrm{T}}|\boldsymbol{Y}_k\},$$

$$\boldsymbol{P}_{\tilde{x}\tilde{n},k|k} = \mathrm{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})(\boldsymbol{n}_k - \hat{\boldsymbol{n}}_{k|k})^{\mathrm{T}}|\boldsymbol{Y}_k\},$$

$$\boldsymbol{P}_{\tilde{n}\tilde{n},k|k} = \mathrm{E}\{(\boldsymbol{n}_k - \hat{\boldsymbol{n}}_{k|k})(\boldsymbol{n}_k - \hat{\boldsymbol{n}}_{k|k})^{\mathrm{T}}|\boldsymbol{Y}_k\}.$$

此外, 为方便算法设计, 定义如下变量:

$$\hat{\boldsymbol{x}}_{k|k-1} = \mathrm{E}\{\boldsymbol{x}_k|\boldsymbol{Y}_{k-1}\}, \hat{\boldsymbol{n}}_{k|k-1} = \mathrm{E}\{\boldsymbol{n}_k|\boldsymbol{Y}_{k-1}\},$$

$$\hat{\boldsymbol{z}}_{k|k-1} = \mathrm{E}\{\boldsymbol{z}_k|\boldsymbol{Y}_{k-1}\}, \hat{\boldsymbol{z}}_{k-1|k-1} = \mathrm{E}\{\boldsymbol{z}_{k-1}|\boldsymbol{Y}_{k-1}\},$$

$$\hat{\boldsymbol{y}}_{k|k-1} = \mathrm{E}\{\boldsymbol{y}_k|\boldsymbol{Y}_{k-1}\}, \hat{\boldsymbol{y}}_{k-1|k-1} = \mathrm{E}\{\boldsymbol{y}_{k-1}|\boldsymbol{Y}_{k-1}\},$$

$$\boldsymbol{P}_{k|k-1} = \mathrm{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$

$$\boldsymbol{P}_{\tilde{n}\tilde{n},k|k-1} = \mathrm{E}\{(\boldsymbol{n}_k - \hat{\boldsymbol{n}}_{k|k-1})(\boldsymbol{n}_k - \hat{\boldsymbol{n}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$

$$\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1} = \mathrm{E}\{(\boldsymbol{n}_k - \hat{\boldsymbol{n}}_{k|k-1})(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$

$$\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1} = \mathrm{E}\{(\boldsymbol{v}_k - \hat{\boldsymbol{v}}_{k-1|k-1})(\boldsymbol{y}_{k-1} - \hat{\boldsymbol{y}}_{k-1|k-1})^{\mathrm{T}}\},$$

$$\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1} = \mathrm{E}\{(\boldsymbol{y}_{k-1} - \hat{\boldsymbol{y}}_{k-1|k-1})(\boldsymbol{y}_{k-1} - \hat{\boldsymbol{y}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$

$$\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1} = \mathrm{E}\{(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_{k-1|k-1})(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$

$$\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1}=$$
$$\mathrm{E}\{(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$
$$\boldsymbol{P}_{\tilde{z}\tilde{z},k-1|k-1}=$$
$$\mathrm{E}\{(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$
$$\boldsymbol{P}_{\tilde{x}\tilde{y},k|k-1}=$$
$$\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$
$$\boldsymbol{P}_{\tilde{x}\tilde{z},k|k-1}=$$
$$\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},$$
$$\boldsymbol{P}_{\tilde{x}\tilde{z},k-1|k-1}=$$
$$\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}.$$

所设计的最优估计算法由时间预测和量测更新两部分构成, 具体内容如下.

3.1 时间预测(Time predication)

引理 1[13] 设$\boldsymbol{X}$和$\boldsymbol{Y}$为满足高斯分布的随机向量, 其联合高斯分布如下:

$$\begin{pmatrix}\boldsymbol{X}\\ \boldsymbol{Y}\end{pmatrix}\sim\mathcal{N}\left(\begin{pmatrix}\boldsymbol{u}_{\mathrm{x}}\\ \boldsymbol{u}_{\mathrm{y}}\end{pmatrix},\begin{bmatrix}\boldsymbol{P}_{\mathrm{xx}} & \boldsymbol{P}_{\mathrm{xy}}\\ \boldsymbol{P}_{\mathrm{xy}}^{\mathrm{T}} & \boldsymbol{P}_{\mathrm{yy}}\end{bmatrix}\right),\tag{15}$$

则在给定观测值$\boldsymbol{Y}=\boldsymbol{y}$下, 关于随机向量$\boldsymbol{X}$的条件概率密度为

$$(\boldsymbol{X}|\boldsymbol{Y}=\boldsymbol{y})\sim$$
$$\mathcal{N}(\boldsymbol{u}_{\mathrm{x}}+\boldsymbol{P}_{\mathrm{xy}}\boldsymbol{P}_{\mathrm{yy}}^{-1}(\boldsymbol{y}-\boldsymbol{u}_{\mathrm{y}}),\boldsymbol{P}_{\mathrm{xx}}-\boldsymbol{P}_{\mathrm{xy}}\boldsymbol{P}_{\mathrm{yy}}^{-1}\boldsymbol{P}_{\mathrm{xy}}^{\mathrm{T}}),\tag{16}$$

式中: $\boldsymbol{u}_{\mathrm{x}}$和$\boldsymbol{u}_{\mathrm{y}}$分别表示随机向量$\boldsymbol{X}$和随机向量$\boldsymbol{Y}$的均值; $\boldsymbol{P}_{\mathrm{xx}}$和$\boldsymbol{P}_{\mathrm{yy}}$分别表示两向量的协方差矩阵; $\boldsymbol{P}_{\mathrm{xy}}$表示两向量间的互协方差阵; $\mathcal{N}$表示高斯分布.

定理 1 在假设1和假设2条件下, 给定滤波器在$k-1$时刻状态量$\boldsymbol{x}_{k-1}^a$的估计值$\hat{\boldsymbol{x}}_{k-1|k-1}^a$和协方差矩阵$\boldsymbol{P}_{k-1|k-1}^a$, 以及$k-2$时刻的估计值$\hat{\boldsymbol{x}}_{k-2|k-2}$和协方差阵$\boldsymbol{P}_{k-2|k-2}$, 则关于$k$时刻状态量$\boldsymbol{x}_k$的一步预测值为

$$\begin{aligned}\hat{\boldsymbol{x}}_{k|k-1}=&\int\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1}+\\&\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}^{-1}\cdot\\&(\boldsymbol{y}_{k-1}-(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})-\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})),\end{aligned}\tag{17}$$

$$\begin{aligned}\boldsymbol{P}_{k|k-1}=&\int\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1}-\\&\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{x}}_{k|k-1}^{\mathrm{T}}+\boldsymbol{Q}_{k-1}-\\&\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}^{-1}\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}^{\mathrm{T}},\end{aligned}\tag{18}$$

式中:

$$\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}=(1-p_{k-1})\boldsymbol{S}_{k-1}^{\mathrm{T}},\tag{19}$$

$$\begin{aligned}\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}=&\\&(1-p_{k-1})\boldsymbol{M}_1+p_{k-1}\boldsymbol{M}_2+\boldsymbol{M}_3+\boldsymbol{M}_3^{\mathrm{T}}+\\&\boldsymbol{M}_4+\boldsymbol{M}_4^{\mathrm{T}}+p_{k-1}\boldsymbol{R}_{k-2}+p_{k-1}(1-p_{k-1})\boldsymbol{M}_5^{\mathrm{T}}+\\&p_{k-1}(1-p_{k-1})\boldsymbol{M}_5+(1-p_{k-1})\boldsymbol{R}_{k-1}+\\&p_{k-1}^2\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})+\\&(1-p_{k-1})^2\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1}),\end{aligned}\tag{20}$$

其中:

$$\begin{aligned}\boldsymbol{M}_1=&\int\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1},\end{aligned}\tag{21}$$

$$\begin{aligned}\boldsymbol{M}_2=&\int\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-2};\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{P}_{k-2|k-2})\mathrm{d}\boldsymbol{x}_{k-2},\end{aligned}\tag{22}$$

$$\begin{aligned}\boldsymbol{M}_3=&\int\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\\&\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1}\cdot\\&(-(1-p_{k-1})^2\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})-\\&p_{k-1}(1-p_{k-1})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})),\end{aligned}\tag{23}$$

$$\begin{aligned}\boldsymbol{M}_4=&\int\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-2};\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{P}_{k-2|k-2})\mathrm{d}\boldsymbol{x}_{k-2}\cdot\\&(-p_{k-1}(1-p_{k-1})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})-\\&p_{k-1}^2\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})),\end{aligned}\tag{24}$$

$$\boldsymbol{M}_5=\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2}).\tag{25}$$

证 首先, 将式(1)代入到$\hat{\boldsymbol{x}}_{k|k-1}$的定义中, 易得

$$\begin{aligned}&\hat{\boldsymbol{x}}_{k|k-1}=\\&\mathrm{E}\{(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{v}_{k-1})|\boldsymbol{Y}_{k-1}\}=\\&\mathrm{E}\{\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})|\boldsymbol{Y}_{k-1}\}+\mathrm{E}\{\boldsymbol{v}_{k-1}|\boldsymbol{Y}_{k-1}\}=\\&\int\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1}+\\&\mathrm{E}\{\boldsymbol{v}_{k-1}|\boldsymbol{Y}_{k-1}\}.\end{aligned}\tag{26}$$

由于过程噪声$\boldsymbol{v}_{k-1}$和量测噪声$\boldsymbol{n}_{k-1}$为同步相关噪声, 故$\mathrm{E}\{\boldsymbol{v}_{k-1}|\boldsymbol{Y}_{k-1}\}\neq 0$. 为确定$\mathrm{E}\{\boldsymbol{v}_{k-1}|\boldsymbol{Y}_{k-1}\}$的取值, 将式(3) 代入$\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}$ 定义中, 易得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}=\\&\mathrm{E}\{\boldsymbol{v}_{k-1}(\boldsymbol{y}_{k-1}-((1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})))^{\mathrm{T}}\}=\\&\mathrm{E}\{\boldsymbol{v}_{k-1}((1-\varepsilon_{k-1})\boldsymbol{z}_{k-1}+\varepsilon_{k-1}\boldsymbol{z}_{k-2}-\\&((1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})))^{\mathrm{T}}\}=\\&\mathrm{E}\{(1-\varepsilon_{k-1})\boldsymbol{v}_{k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}+\\&\varepsilon_{k-1}\boldsymbol{v}_{k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})+\boldsymbol{n}_{k-2})^{\mathrm{T}}-\\&\boldsymbol{v}_{k-1}((1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+\end{aligned}$$

$$p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2}))^{\mathrm{T}}\}. \tag{27}$$

由于过程噪声$\boldsymbol{v}_{k-1}$与$k-2$时刻量测噪声$\boldsymbol{n}_{k-2}$不相关, 且噪声$\boldsymbol{v}_{k-1}$为高斯白噪声故, 故其与状态$\boldsymbol{x}_{k-1}$中含有的噪声项$\boldsymbol{v}_{k-2}$不相关, 所以

$$\mathrm{E}\{\boldsymbol{v}_{k-1}\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\}=0, \tag{28}$$

$$\mathrm{E}\{\varepsilon_{k-1}\boldsymbol{v}_{k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})+\boldsymbol{n}_{k-2})^{\mathrm{T}}\}=0, \tag{29}$$

$$\mathrm{E}\{\boldsymbol{v}_{k-1}((1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+ p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2}))^{\mathrm{T}}\}=0. \tag{30}$$

将式(28)–(30)代入式(27)中, 得式(19).

其次, 将式(2)–(3)代入$\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}$定义中, 易得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}=\\&\mathrm{E}\{((1-\varepsilon_{k-1})(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})+\\&\varepsilon_{k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})+\boldsymbol{n}_{k-2})-\\&((1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})))\cdot\\&((1-\varepsilon_{k-1})(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})+\\&\varepsilon_{k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})+\boldsymbol{n}_{k-2})-\\&((1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})))^{\mathrm{T}}\}.\end{aligned} \tag{31}$$

由于量测噪声$\boldsymbol{n}_{k-1}$为高斯白噪声, 且与状态量$\boldsymbol{x}_{k-1}$不相关, 则根据式(9), 对式(31)展开整理可得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}=\\&\mathrm{E}\{(1-\varepsilon_{k-1})\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\\&(1-\varepsilon_{k-1})-(1-\varepsilon_{k-1})\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})(1-p_{k-1})-\\&(1-\varepsilon_{k-1})\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})p_{k-1}+\\&(1-\varepsilon_{k-1})\boldsymbol{n}_{k-1}\boldsymbol{n}_{k-1}^{\mathrm{T}}(1-\varepsilon_{k-1})+\\&\varepsilon_{k-1}\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\varepsilon_{k-1}-\\&\varepsilon_{k-1}\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})(1-p_{k-1})-\\&\varepsilon_{k-1}\boldsymbol{h}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})p_{k-1}+\\&\varepsilon_{k-1}\boldsymbol{n}_{k-2}\boldsymbol{n}_{k-2}^{\mathrm{T}}\varepsilon_{k-1}-(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\\&\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})(1-\varepsilon_{k-1})-\\&(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\varepsilon_{k-1}+\\&(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})(1-p_{k-1})+\\&(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})p_{k-1}-p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})(1-\varepsilon_{k-1})-\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-2},\boldsymbol{u}_{k-2})\varepsilon_{k-1}+\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})(1-p_{k-1})+\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})\cdot\\&\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})p_{k-1}\}.\end{aligned} \tag{32}$$

将式(7)–(8)代入式(32)中, 得式(20).

根据引理1, 取$\boldsymbol{X}=\boldsymbol{v}_{k-1}|\hat{\boldsymbol{x}}_{k-1|k-1},\hat{\boldsymbol{x}}_{k-2|k-2}$以及$\boldsymbol{Y}=\boldsymbol{y}_{k-1}|\hat{\boldsymbol{x}}_{k-1|k-1},\hat{\boldsymbol{x}}_{k-2|k-2}$, 可知$\boldsymbol{X}$和$\boldsymbol{Y}$的联合高斯分布如下:

$$\begin{aligned}&\begin{pmatrix}\boldsymbol{v}_{k-1}|\hat{\boldsymbol{x}}_{k-1|k-1},\hat{\boldsymbol{x}}_{k-2|k-2}\\ \boldsymbol{y}_{k-1}|\hat{\boldsymbol{x}}_{k-1|k-1},\hat{\boldsymbol{x}}_{k-2|k-2}\end{pmatrix}\sim\\&\mathcal{N}\left(\begin{pmatrix}0\\(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})\end{pmatrix},\begin{bmatrix}\boldsymbol{Q}_{k-1}&\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\\ \boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}^{\mathrm{T}}&\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}\end{bmatrix}\right).\end{aligned} \tag{33}$$

那么噪声项$\boldsymbol{v}_{k-1}$在观测值$\boldsymbol{y}_{k-1}$下的条件概率密度为

$$\begin{aligned}&p(\boldsymbol{v}_{k-1}|\hat{\boldsymbol{x}}_{k-1|k-1},\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{y}_{k-1})=\\&\mathcal{N}(\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}^{-1}\cdot\\&(\boldsymbol{y}_{k-1}-(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})-\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})),\\&\boldsymbol{Q}_{k-1}-\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}^{-1}\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}^{\mathrm{T}}).\end{aligned} \tag{34}$$

由于过程噪声$\boldsymbol{v}_{k-1}$独立于观测序列$\boldsymbol{Y}_{k-2}$, 易得

$$\begin{aligned}&p(\boldsymbol{v}_{k-1}|\hat{\boldsymbol{x}}_{k-1|k-1},\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{y}_{k-1})=\\&p(\boldsymbol{v}_{k-1}|\hat{\boldsymbol{x}}_{k-1|k-1},\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{Y}_{k-1}),\end{aligned} \tag{35}$$

因而

$$\begin{aligned}&\mathrm{E}\{\boldsymbol{v}_{k-1}|\boldsymbol{Y}_{k-1}\}=\\&\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}^{-1}\cdot\\&(\boldsymbol{y}_{k-1}-(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})-\\&p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})),\end{aligned} \tag{36}$$

$$\begin{aligned}&\mathrm{E}\{\boldsymbol{v}_{k-1}\boldsymbol{v}_{k-1}^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&\boldsymbol{Q}_{k-1}-\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}^{-1}\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}^{\mathrm{T}}.\end{aligned} \tag{37}$$

将式(36)代入式(26)中, 得式(17).

最后, 根据$\boldsymbol{P}_{k|k-1}$定义, 易得

$$\begin{aligned}&\boldsymbol{P}_{k|k-1}=\\&\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&\mathrm{E}\{(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{v}_{k-1}-\hat{\boldsymbol{x}}_{k|k-1})\cdot\\&(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{v}_{k-1}-\hat{\boldsymbol{x}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&\mathrm{E}\{(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\\&\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{v}_{k-1}^{\mathrm{T}}+\boldsymbol{v}_{k-1}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})-\\&(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{v}_{k-1})\hat{\boldsymbol{x}}_{k|k-1}^{\mathrm{T}}+\boldsymbol{v}_{k-1}\boldsymbol{v}_{k-1}^{\mathrm{T}}-\\&\hat{\boldsymbol{x}}_{k|k-1}(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{v}_{k-1})^{\mathrm{T}}+\\&\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{x}}_{k|k-1}^{\mathrm{T}})|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{38}$$

由于$\boldsymbol{v}_{k-1}$为高斯白噪声, 所以

$$\begin{aligned}&\mathrm{E}\{\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{v}_{k-1}^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&\mathrm{E}\{\boldsymbol{v}_{k-1}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})|\boldsymbol{Y}_{k-1}\}=0.\end{aligned}\tag{39}$$

将式(37)(39)代入式(38)中, 得式(18).

综上所述, 定理1得证.

3.2 量测更新(Measurement update)

定理 2 在假设1和假设2条件下, 给定滤波器在$k-1$时刻状态量$\boldsymbol{x}_{k-1}^a$的估计值$\hat{\boldsymbol{x}}_{k-1|k-1}^a$和协方差矩阵$\boldsymbol{P}_{k-1|k-1}^a$, 以及关于状态量$\boldsymbol{x}_k$的一步预测值$\hat{\boldsymbol{x}}_{k|k-1}$和$\boldsymbol{P}_{k|k-1}$, 则关于$k$时刻状态量$\boldsymbol{x}_k^a$的估计值为

$$\hat{\boldsymbol{x}}_{k|k}=\hat{\boldsymbol{x}}_{k|k-1}+\boldsymbol{K}_k(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1}),\tag{40}$$

$$\boldsymbol{P}_{k|k}=\boldsymbol{P}_{k|k-1}-\boldsymbol{K}_k\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}\boldsymbol{K}_k^{\mathrm{T}},\tag{41}$$

$$\boldsymbol{K}_k=\boldsymbol{P}_{\tilde{x}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1},\tag{42}$$

$$\hat{\boldsymbol{n}}_{k|k}=\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1}(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1}),\tag{43}$$

$$\boldsymbol{P}_{\tilde{n}\tilde{n},k|k}=\boldsymbol{R}_k-\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1}\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}^{\mathrm{T}},\tag{44}$$

$$\boldsymbol{P}_{\tilde{x}\tilde{n},k|k}=-\boldsymbol{K}_k(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1})^{\mathrm{T}},\tag{45}$$

式中:

$$\hat{\boldsymbol{y}}_{k|k-1}=(1-p_k)\hat{\boldsymbol{z}}_{k|k-1}+p_k\hat{\boldsymbol{z}}_{k-1|k-1},\tag{46}$$

$$\begin{aligned}&\hat{\boldsymbol{z}}_{k|k-1}=\\&\int\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)\mathcal{N}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k-1},\boldsymbol{P}_{k|k-1})\mathrm{d}\boldsymbol{x}_k,\end{aligned}\tag{47}$$

$$\begin{aligned}&\hat{\boldsymbol{z}}_{k-1|k-1}=\\&\int(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})\\&\mathcal{N}(\boldsymbol{x}_{k-1}^a;\hat{\boldsymbol{x}}_{k-1|k-1}^a,\boldsymbol{P}_{k-1|k-1}^a)\mathrm{d}\boldsymbol{x}_{k-1}^a,\end{aligned}\tag{48}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}=\\&(1-p_k)\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1}+p_k\boldsymbol{P}_{\tilde{z}\tilde{z},k-1|k-1}+\\&(1-p_k)p_k(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})\cdot\\&(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}},\end{aligned}\tag{49}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1}=\\&\int\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)\cdot\\&\mathcal{N}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k-1},\boldsymbol{P}_{k|k-1})\mathrm{d}\boldsymbol{x}_k-\\&\hat{\boldsymbol{z}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}+\boldsymbol{R}_k,\end{aligned}\tag{50}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{z}\tilde{z},k-1|k-1}=\\&\int(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})\cdot\\&(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1}^a;\hat{\boldsymbol{x}}_{k-1|k-1}^a,\boldsymbol{P}_{k-1|k-1}^a)\mathrm{d}\boldsymbol{x}_{k-1}^a-\\&\hat{\boldsymbol{z}}_{k-1|k-1}\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}},\end{aligned}\tag{51}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{x}\tilde{y},k|k-1}=\\&(1-p_k)\boldsymbol{P}_{\tilde{x}\tilde{z},k|k-1}+p_k\boldsymbol{P}_{\tilde{x}\tilde{z},k-1|k-1},\end{aligned}\tag{52}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{x}\tilde{z},k|k-1}=\\&\int\boldsymbol{x}_k\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)\mathcal{N}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k-1},\boldsymbol{P}_{k|k-1})\mathrm{d}\boldsymbol{x}_k-\\&\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}},\end{aligned}\tag{53}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{x}\tilde{z},k-1|k-1}=\\&\int\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1}^a;\hat{\boldsymbol{x}}_{k-1|k-1}^a,\boldsymbol{P}_{k-1|k-1}^a)\mathrm{d}\boldsymbol{x}_{k-1}^a+\\&\boldsymbol{S}_{k-1}-\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}},\end{aligned}\tag{54}$$

$$\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}=(1-p_k)\boldsymbol{R}_k.\tag{55}$$

证 首先, 将式(2)–(3)代入$\hat{\boldsymbol{y}}_{k|k-1}$, $\hat{\boldsymbol{z}}_{k|k-1}$以及$\hat{\boldsymbol{z}}_{k-1|k-1}$的定义中, 易得

$$\begin{aligned}&\hat{\boldsymbol{y}}_{k|k-1}=\\&\mathrm{E}\{((1-\varepsilon_k)\boldsymbol{z}_k+\varepsilon_k\boldsymbol{z}_{k-1})|\boldsymbol{Y}_{k-1}\}=\\&(1-p_k)\hat{\boldsymbol{z}}_{k|k-1}+p_k\hat{\boldsymbol{z}}_{k-1|k-1},\end{aligned}\tag{56}$$

$$\begin{aligned}&\hat{\boldsymbol{z}}_{k|k-1}=\\&\mathrm{E}\{(\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)+\boldsymbol{n}_k)|\boldsymbol{Y}_{k-1}\}=\\&\int\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)\mathcal{N}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k-1},\boldsymbol{P}_{k|k-1})\mathrm{d}\boldsymbol{x}_k,\end{aligned}\tag{57}$$

$$\begin{aligned}&\hat{\boldsymbol{z}}_{k-1|k-1}=\\&\mathrm{E}\{(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})|\boldsymbol{Y}_{k-1}\}=\\&\int(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1}^a;\hat{\boldsymbol{x}}_{k-1|k-1}^a,\boldsymbol{P}_{k-1|k-1}^a)\mathrm{d}\boldsymbol{x}_{k-1}^a,\end{aligned}\tag{58}$$

式(46)–(48)得证. 其次, 将式(3)(56)代入$\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}$定义中, 得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}=\\&\mathrm{E}\{((1-\varepsilon_k)\boldsymbol{z}_k+\varepsilon_k\boldsymbol{z}_{k-1}-\\&((1-p_k)\hat{\boldsymbol{z}}_{k|k-1}+p_k\hat{\boldsymbol{z}}_{k-1|k-1}))\cdot\\&((1-\varepsilon_k)\boldsymbol{z}_k+\varepsilon_k\boldsymbol{z}_{k-1}-\\&((1-p_k)\hat{\boldsymbol{z}}_{k|k-1}+p_k\hat{\boldsymbol{z}}_{k-1|k-1}))^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{59}$$

对式 (59) 中的乘积因子加 $(1-\varepsilon_k)\hat{\boldsymbol{z}}_{k|k-1}$, 再减去 $(1-\varepsilon_k)\hat{\boldsymbol{z}}_{k|k-1}$, 则整理可得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}=\\&\mathrm{E}\{((1-\varepsilon_k)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})+\\&\varepsilon_k(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})+\\&(p_k-\varepsilon_k)(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1}))\cdot\\&((1-\varepsilon_k)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})+\varepsilon_k(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})+\\&(p_k-\varepsilon_k)(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1}))^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{60}$$

对式(60)右侧展开, 整理后可得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}=\\&\mathrm{E}\{((1-\varepsilon_k)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})\cdot\\&(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}(1-\varepsilon_k)^{\mathrm{T}}+(1-\varepsilon_k)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})\cdot\\&(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}\varepsilon_k^{\mathrm{T}}+(1-\varepsilon_k)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})\cdot\\&(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}(p_k-\varepsilon_k)^{\mathrm{T}}+\\&\varepsilon_k(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}(1-\varepsilon_k)^{\mathrm{T}}+\\&\varepsilon_k(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}\varepsilon_k^{\mathrm{T}}+\\&\varepsilon_k(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})\cdot\\&(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}(p_k-\varepsilon_k)^{\mathrm{T}}+\\&(p_k-\varepsilon_k)(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})\cdot\\&(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}(1-\varepsilon_k)^{\mathrm{T}}+\\&(p_k-\varepsilon_k)(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})\cdot\\&(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}\varepsilon_k^{\mathrm{T}}+\\&(p_k-\varepsilon_k)(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})\cdot\\&(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}(p_k-\varepsilon_k)^{\mathrm{T}})|\boldsymbol{Y}_{k-1}\}.\end{aligned}\tag{61}$$

定义

$$\begin{aligned}&\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1}=\\&\mathrm{E}\{(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{62}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{z}\tilde{z},k-1|k-1}=\\&\mathrm{E}\{(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{63}$$

将式(7)–(12), 以及式(62)–(63)代入式(61) 中, 得式(49). 根据$\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1}$定义, 将式(2)(57)代入式(62)中, 易得

$$\begin{aligned}\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1}=&\mathrm{E}\{(\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)+\\&\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)\boldsymbol{n}_k^{\mathrm{T}}-\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}+\\&\boldsymbol{n}_k\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)+\boldsymbol{n}_k\boldsymbol{n}_k^{\mathrm{T}}-\boldsymbol{n}_k\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}-\\&\hat{\boldsymbol{z}}_{k|k-1}\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)-\hat{\boldsymbol{z}}_{k|k-1}\boldsymbol{n}_k^{\mathrm{T}}+\\&\hat{\boldsymbol{z}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}})|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{64}$$

由于$\boldsymbol{x}_k$中含有的过程噪声项$\boldsymbol{v}_{k-1}$与$\boldsymbol{n}_k$不相关, 故

$$\begin{aligned}&\mathrm{E}\{\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)\boldsymbol{n}_k^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&\mathrm{E}\{\boldsymbol{n}_k\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)|\boldsymbol{Y}_{k-1}\}=0,\end{aligned}\tag{65}$$

将式 (65) 代入式 (64)中, 得式 (50). 类似地, 将$\boldsymbol{z}_{k-1}$对应的表达式代入$\boldsymbol{P}_{\tilde{z}\tilde{z},k-1|k-1}$定义中, 易得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{z}\tilde{z},k-1|k-1}=\\&\mathrm{E}\{(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})\cdot\\&(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&\mathrm{E}\{((\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})\cdot\\&(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}+\\&(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}}-\\&\hat{\boldsymbol{z}}_{k-1|k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}+\\&\hat{\boldsymbol{z}}_{k-1|k-1}\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}})|\boldsymbol{Y}_{k-1}\}=\\&\int(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})\cdot\\&(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}\cdot\\&\mathcal{N}(\boldsymbol{x}_{k-1}^a;\hat{\boldsymbol{x}}_{k-1|k-1}^a,\boldsymbol{P}_{k-1|k-1}^a)\mathrm{d}\boldsymbol{x}_{k-1}^a-\\&\hat{\boldsymbol{z}}_{k-1|k-1}\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}},\end{aligned}\tag{66}$$

式(51)得证.

将式(3)(56)代入$\boldsymbol{P}_{\tilde{x}\tilde{y},k|k-1}$定义中, 得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{x}\tilde{y},k|k-1}=\\&\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})((1-\varepsilon_k)(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})+\\&\varepsilon_k(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})+(p_k-\varepsilon_k)\cdot\\&(\hat{\boldsymbol{z}}_{k|k-1}-\hat{\boldsymbol{z}}_{k-1|k-1}))^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&(1-p_k)\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})\cdot\\&(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}+p_k\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})\cdot\\&(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}.\end{aligned}\tag{67}$$

定义

$$\begin{aligned}&\boldsymbol{P}_{\tilde{x}\tilde{z},k|k-1}=\\&\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1}(\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{68}$$

$$\begin{aligned}&\boldsymbol{P}_{\tilde{x}\tilde{z},k-1|k-1}=\\&\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{z}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\},\end{aligned}\tag{69}$$

将式(68)–(69)代入式(67)中, 得式(52). 将式(2)代入式(68)中, 得

$$\begin{aligned}&\boldsymbol{P}_{\tilde{x}\tilde{z},k|k-1}=\\&\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})(\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)+\\&\boldsymbol{n}_k-\hat{\boldsymbol{z}}_{k|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\\&\mathrm{E}\{(\boldsymbol{x}_k\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)+\boldsymbol{x}_k\boldsymbol{n}_k^{\mathrm{T}}-\boldsymbol{x}_k\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}-\end{aligned}$$

$$\hat{\boldsymbol{x}}_{k|k-1}\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)-\hat{\boldsymbol{x}}_{k|k-1}\boldsymbol{n}_k^{\mathrm{T}}+\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}})|\boldsymbol{Y}_{k-1}\}=\int\boldsymbol{x}_k\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)\mathcal{N}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k-1},\boldsymbol{P}_{k|k-1})\mathrm{d}\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}-\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}+\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}=\int\boldsymbol{x}_k\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_k,\boldsymbol{u}_k)\mathcal{N}(\boldsymbol{x}_k;\hat{\boldsymbol{x}}_{k|k-1},\boldsymbol{P}_{k|k-1})\mathrm{d}\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}},\tag{70}$$

式(53)得证. 类似地, 将式(1)和$\boldsymbol{z}_{k-1}$对应的表达式代入$\boldsymbol{P}_{\tilde{x}\tilde{z},k-1|k-1}$定义中, 易得

$$\boldsymbol{P}_{\tilde{x}\tilde{z},k-1|k-1}=\mathrm{E}\{(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{v}_{k-1}-\hat{\boldsymbol{x}}_{k|k-1})\cdot(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1}-\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\mathrm{E}\{(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}+\boldsymbol{v}_{k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}-(\boldsymbol{f}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{v}_{k-1})\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}}-\hat{\boldsymbol{x}}_{k|k-1}(\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})+\boldsymbol{n}_{k-1})^{\mathrm{T}}+\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}})|\boldsymbol{Y}_{k-1}\},\tag{71}$$

由于$\boldsymbol{v}_{k-1}$与$\boldsymbol{n}_{k-1}$为相关噪声, 并将式(28)代入式(71)中, 得式(54).

将式(3)(56)以及式(2)代入$\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}$定义中, 易得

$$\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}=\mathrm{E}\{\boldsymbol{n}_k((1-\varepsilon_k)(\boldsymbol{h}(\boldsymbol{x}_k,\boldsymbol{u}_k)+\boldsymbol{n}_k)+\varepsilon_k\boldsymbol{z}_{k-1}-(1-p_k)\hat{\boldsymbol{z}}_{k|k-1}-p_k\hat{\boldsymbol{z}}_{k-1|k-1})^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}.\tag{72}$$

由于$\boldsymbol{n}_k$为高斯白噪声, 所以

$$\mathrm{E}\{\boldsymbol{n}_k\boldsymbol{z}_{k-1}^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\mathrm{E}\{\boldsymbol{n}_k\hat{\boldsymbol{z}}_{k|k-1}^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=\mathrm{E}\{\boldsymbol{n}_k\hat{\boldsymbol{z}}_{k-1|k-1}^{\mathrm{T}}|\boldsymbol{Y}_{k-1}\}=0,\tag{73}$$

将式(65)(73)代入式(72)中, 得式(55).

此外, 根据引理1, 取$\boldsymbol{X}=\boldsymbol{n}_k|\boldsymbol{Y}_{k-1}$以及$\boldsymbol{Y}=\boldsymbol{y}_k|\boldsymbol{Y}_{k-1}$, 可知$\boldsymbol{X}$和$\boldsymbol{Y}$的联合高斯分布如下:

$$\begin{pmatrix}\boldsymbol{n}_k|\boldsymbol{Y}_{k-1}\\ \boldsymbol{y}_k|\boldsymbol{Y}_{k-1}\end{pmatrix}\sim\mathcal{N}\left(\begin{pmatrix}0\\ \hat{\boldsymbol{y}}_{k|k-1}\end{pmatrix},\begin{bmatrix}\boldsymbol{R}_k & \boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\\ \boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}^{\mathrm{T}} & \boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}\end{bmatrix}\right),\tag{74}$$

那么噪声$\boldsymbol{n}_k$在观测值$\boldsymbol{y}_k$下的条件概率密度为

$$p(\boldsymbol{n}_k|\boldsymbol{y}_k,\boldsymbol{Y}_{k-1})=p(\boldsymbol{n}_k|\boldsymbol{Y}_k)=\mathcal{N}(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1}(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1}),\boldsymbol{R}_k-\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1}\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}^{\mathrm{T}}).\tag{75}$$

因而, 式(43)–(44)得证.

最后, 将式(40)(43)代入$\boldsymbol{P}_{\tilde{x}\tilde{n},k|k}$定义中, 得

$$\boldsymbol{P}_{\tilde{x}\tilde{n},k|k}=\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1}-\boldsymbol{K}_k(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1}))\cdot(\boldsymbol{n}_k-\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1}(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1}))^{\mathrm{T}}|\boldsymbol{Y}_k\}=\mathrm{E}\{((\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})\boldsymbol{n}_k^{\mathrm{T}}-(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})\cdot(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1})^{\mathrm{T}}(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1})^{\mathrm{T}}-\boldsymbol{K}_k(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1})\boldsymbol{n}_k^{\mathrm{T}}+\boldsymbol{K}_k(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1})\cdot(\boldsymbol{y}_k-\hat{\boldsymbol{y}}_{k|k-1})^{\mathrm{T}}(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1})^{\mathrm{T}})|\boldsymbol{Y}_k\}.\tag{76}$$

由于

$$\boldsymbol{P}_{\tilde{x}\tilde{y},k|k-1}(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1})^{\mathrm{T}}=\boldsymbol{K}_k(\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1})^{\mathrm{T}}(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}^{-1})^{\mathrm{T}}=\boldsymbol{K}_k(\boldsymbol{P}_{n\tilde{y},k|k-1})^{\mathrm{T}},\tag{77}$$

且$\boldsymbol{n}_k$为高斯白噪声, 因此式(76)等价于

$$\boldsymbol{P}_{\tilde{x}\tilde{n},k|k}=\mathrm{E}\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k-1})\boldsymbol{n}_k^{\mathrm{T}}|\boldsymbol{Y}_k\}-\boldsymbol{K}_k(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1})^{\mathrm{T}}=-\boldsymbol{K}_k(\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1})^{\mathrm{T}},\tag{78}$$

式(45)得证.

综上所述, 定理2得证.

注 2 分析式(19)(20)(46)(49)(52)(55)可知, 当$p_k=0$, $\boldsymbol{S}_k=0$时, 上述各式分别退化为

$$\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}=0,$$

$$\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}=\int\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1}-\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\int\boldsymbol{h}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1}-\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\int\boldsymbol{h}^{\mathrm{T}}(\boldsymbol{x}_{k-1},\boldsymbol{u}_{k-1})\cdot\mathcal{N}(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{P}_{k-1|k-1})\mathrm{d}\boldsymbol{x}_{k-1}+\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})\boldsymbol{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})+\boldsymbol{R}_{k-1},$$

$$\hat{\boldsymbol{y}}_{k|k-1}=\hat{\boldsymbol{z}}_{k|k-1},$$

$$\boldsymbol{P}_{\tilde{y}\tilde{y},k|k-1}=\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1},$$

$$\boldsymbol{P}_{\tilde{x}\tilde{y},k|k-1}=\boldsymbol{P}_{\tilde{x}\tilde{z},k|k-1},$$

$$\boldsymbol{P}_{\tilde{n}\tilde{y},k|k-1}=0.$$

可知当系统中不存在随机量测时滞和噪声同步相关的情况时, 本文所提出的改进框架等价于一般形式的高斯滤波框架. 因此, 后者是前者在无时滞、无同步相关噪声情况下的实

现特例. 相比于一般形式的高斯滤波框架, 该改进框架具有更广阔的适用范围, 更符合实际状态估计问题的需求.

注 3 　关于随机量测时滞和噪声相关情况下高斯滤波器的已有结果文献[12], 是在文献[17]的基础上发展而来的, 其继承了后者对于噪声相关情况的处理思路, 并采用视量测噪声为状态增量的方式来处理随机量测时滞干扰因素. 同文献[12]相比, 由定理3.1和定理3.2所构成的最优估计算法, 其相同之处在于均以状态扩维的方式来处理随机量测时滞干扰, 不同之处在于对于噪声相关情况的处理. 如前文所述, 文献[12]采用二步高斯预测的噪声解耦方法, 而本文所提出的算法仍沿用原高斯滤波算法中一步预测的方式, 因此, 较后者而言, 当处理具有高动态背景的状态估计问题时, 前者存在精度较低的先天劣势. 此外, 本文所提出的算法利用高斯条件分布性质, 来深入挖掘噪声相关信息, 相比而言可以获得更高精度的估计值.

4 CKF–RDSCN滤波算法(Algorithm of the CKF–RDSCN)

由定理1和定理2可知, 在对于系统状态估计值$\hat{\boldsymbol{x}}_{k|k}$的迭代更新过程中, 涉及到较多的多维高斯加权积分计算问题. 对于这一问题, 可以通过相应的数值积分方法来近似求解, 较为常用的方法有斯特林多项式插值、无迹变换以及球径容积法则等. 其中, 球径容积法则具有精度高、数值稳定性强的优点, 因而在本文中给出基于三阶球径容积法则的实现形式——CKF–RDSCN滤波算法.

4.1 三阶球径容积法则(Third-degree spherical-radical rule)

考虑如下形式的多维高斯加权积分问题:

$$\boldsymbol{I}_{\mathcal{N}}(\boldsymbol{f})=\int \boldsymbol{f}(\boldsymbol{x})\mathcal{N}(\boldsymbol{x};\hat{\boldsymbol{x}},\boldsymbol{P})\mathrm{d}\boldsymbol{x}, \tag{79}$$

式中: $\boldsymbol{x}$为服从均值为$\hat{\boldsymbol{x}}$、方差为$\boldsymbol{P}$的n维高斯变量. 根据三阶球径容积法则, 式(79)可近似为

$$\boldsymbol{I}_{\mathcal{N}}(\boldsymbol{f})=\int \boldsymbol{f}(\boldsymbol{U}\boldsymbol{\xi}_d+\hat{\boldsymbol{x}})\mathcal{N}(\boldsymbol{x};\ \boldsymbol{0},\ \boldsymbol{I})\mathrm{d}\boldsymbol{x}\approx \sum_{d=1}^{2n} w\boldsymbol{f}(\boldsymbol{U}\boldsymbol{\xi}_d+\hat{\boldsymbol{x}}), \tag{80}$$

式中: w表示取值为$\dfrac{1}{2n}$的权重系数; $\boldsymbol{I}$表示单位矩阵; $\boldsymbol{U}$表示方差阵$\boldsymbol{P}$的平方根矩阵; $\boldsymbol{\xi}_d$表示具有$2n$个容积点集合$\{\boldsymbol{\xi}_d\}$的第d列向量, $\{\boldsymbol{\xi}_d\}$定义如式(81):

$$\{\boldsymbol{\xi}_d\}=\sqrt{n}\left\{\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}\begin{bmatrix}0\\1\\\vdots\\0\end{bmatrix}\cdots\begin{bmatrix}0\\0\\\vdots\\1\end{bmatrix}\begin{bmatrix}-1\\0\\\vdots\\0\end{bmatrix}\begin{bmatrix}0\\-1\\\vdots\\0\end{bmatrix}\cdots\begin{bmatrix}0\\0\\\vdots\\-1\end{bmatrix}\right\}. \tag{81}$$

4.2 CKF–RDSCN 滤波算法(Algorithm of the CKF–RDSCN)

在本节中, 依据三阶球径容积法则对式(17)–(18)(21)–(24)(47)–(48)(50)–(51)以及(53)–(54)中的高斯加权积分部分近似计算, 并根据定理1和定理2给出CKF–RDSCN滤波算法的具体形式. 易知该算法由时间预测和量测更新两部分构成, 其内容如下:

a) 时间预测:

1) 分别对矩阵$\boldsymbol{P}^a_{k-1|k-1}$和$\boldsymbol{P}_{k-2|k-2}$进行Cholesky分解.

$$\boldsymbol{P}^a_{k-1|k-1}=\boldsymbol{U}^a_{k-1|k-1}(\boldsymbol{U}^a_{k-1|k-1})^{\mathrm{T}}, \tag{82}$$

$$\boldsymbol{P}_{k-2|k-2}=\boldsymbol{U}_{k-2|k-2}\boldsymbol{U}^{\mathrm{T}}_{k-2|k-2}. \tag{83}$$

2) 容积点计算.

$$\boldsymbol{X}^a_{i,k-1|k-1}=[(\boldsymbol{X}^x_{i,k-1|k-1})^{\mathrm{T}}\ (\boldsymbol{X}^n_{i,k-1|k-1})^{\mathrm{T}}]^{\mathrm{T}}=\boldsymbol{U}^a_{k-1|k-1}\boldsymbol{\xi}_{i,L}+\hat{\boldsymbol{x}}^a_{k-1|k-1}, i=1,\cdots,2L, \tag{84}$$

$$\boldsymbol{X}_{i,k-2|k-2}=\boldsymbol{U}_{k-2|k-2}\boldsymbol{\xi}_{i,n}+\hat{\boldsymbol{x}}_{k-2|k-2}, i=1,\cdots,2n, \tag{85}$$

其中: $\boldsymbol{\xi}_{i,L}$和$\boldsymbol{\xi}_{i,n}$分别表示$L\times 2L$维以及$n\times 2n$维容积点集合的第i列向量; $L=n+m$.

3) 容积点扩散.

$$\boldsymbol{X}^{*x}_{i,k|k-1}=\boldsymbol{f}(\boldsymbol{X}^x_{i,k-1|k-1},\boldsymbol{u}_{k-1}), i=1,\cdots,2L, \tag{86}$$

$$\boldsymbol{Z}_{i,k-1|k-1}=\boldsymbol{h}(\boldsymbol{X}^x_{i,k-1|k-1},\boldsymbol{u}_{k-1}), i=1,\cdots,2L, \tag{87}$$

$$\boldsymbol{Z}_{i,k-2|k-2}=\boldsymbol{h}(\boldsymbol{X}_{i,k-2|k-2},\boldsymbol{u}_{k-2}), i=1,\cdots,2n. \tag{88}$$

4) 计算式(21)–(24).

$$\boldsymbol{M}_1=\frac{1}{2L}\sum_{i=1}^{2L}\boldsymbol{Z}_{i,k-1|k-1}(\boldsymbol{Z}_{i,k-1|k-1})^{\mathrm{T}}, \tag{89}$$

$$\boldsymbol{M}_2=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{Z}_{i,k-2|k-2}(\boldsymbol{Z}_{i,k-2|k-2})^{\mathrm{T}}, \tag{90}$$

$$\boldsymbol{M}_3=\frac{1}{2L}\sum_{i=1}^{2L}\boldsymbol{Z}_{i,k-1|k-1}(-(1-p_{k-1})^2\boldsymbol{h}^{\mathrm{T}}\cdot(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})-p_{k-1}(1-p_{k-1})\boldsymbol{h}^{\mathrm{T}}\cdot(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})), \tag{91}$$

$$\boldsymbol{M}_4=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{Z}_{i,k-2|k-2}(-p_{k-1}(1-p_{k-1})\boldsymbol{h}^{\mathrm{T}}\cdot(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})-p^2_{k-1}\boldsymbol{h}^{\mathrm{T}}\cdot(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})), \tag{92}$$

将式(89)–(92)和式(25)代入式(20)中, 求取$\boldsymbol{P}_{\tilde{y}\tilde{y},k-1|k-1}$.

5) 估计一步预测值$\hat{\boldsymbol{x}}_{k|k-1}$和$\boldsymbol{P}_{k|k-1}$.

$$\hat{\boldsymbol{x}}_{k|k-1}= \frac{1}{2L}\sum_{i=1}^{2L}\boldsymbol{X}^{*x}_{i,k|k-1}+\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}^{-1}_{\tilde{y}\tilde{y},k-1|k-1}\cdot (\boldsymbol{y}_{k-1}-(1-p_{k-1})\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-1|k-1},\boldsymbol{u}_{k-1})- p_{k-1}\boldsymbol{h}(\hat{\boldsymbol{x}}_{k-2|k-2},\boldsymbol{u}_{k-2})), \tag{93}$$

$$\boldsymbol{P}_{k|k-1}= \frac{1}{2L}\sum_{i=1}^{2L}\boldsymbol{X}^{*x}_{i,k|k-1}(\boldsymbol{X}^{*x}_{i,k|k-1})^{\mathrm{T}}-\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{x}}^{\mathrm{T}}_{k|k-1}+ \boldsymbol{Q}_{k-1}-\boldsymbol{P}_{\tilde{v}\tilde{y},k-1|k-1}\boldsymbol{P}^{-1}_{\tilde{y}\tilde{y},k-1|k-1}\boldsymbol{P}^{\mathrm{T}}_{\tilde{v}\tilde{y},k-1|k-1}. \tag{94}$$

b) 量测更新:

1) 对矩阵$\boldsymbol{P}_{k|k-1}$进行Cholesky分解.

$$\boldsymbol{P}_{k|k-1}=\boldsymbol{U}_{k|k-1}\boldsymbol{U}^{\mathrm{T}}_{k|k-1}. \tag{95}$$

2) 容积点计算.

$$\boldsymbol{X}_{i,k|k-1}= \boldsymbol{U}_{k|k-1}\boldsymbol{\xi}_{i,n}+\hat{\boldsymbol{x}}_{k|k-1}, i=1,\cdots,2n. \tag{96}$$

3) 容积点扩散.

$$\boldsymbol{Z}_{i,k|k-1}= \boldsymbol{h}(\boldsymbol{X}_{i,k|k-1},\boldsymbol{u}_k), i=1,\cdots,2n. \tag{97}$$

4) 计算式(47)–(48)(50)–(51)(53)–(54).

$$\hat{\boldsymbol{z}}_{k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{Z}_{i,k|k-1}, \tag{98}$$

$$\hat{\boldsymbol{z}}_{k-1|k-1}=\frac{1}{2L}\sum_{i=1}^{2L}(\boldsymbol{Z}_{i,k-1|k-1}+\boldsymbol{X}^{n}_{i,k-1|k-1}), \tag{99}$$

$$\boldsymbol{P}_{\tilde{z}\tilde{z},k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{Z}_{i,k|k-1}(\boldsymbol{Z}_{i,k|k-1})^{\mathrm{T}}- \hat{\boldsymbol{z}}_{k|k-1}\hat{\boldsymbol{z}}^{\mathrm{T}}_{k|k-1}+\boldsymbol{R}_k, \tag{100}$$

$$\boldsymbol{P}_{\tilde{z}\tilde{z},k-1|k-1}=\frac{1}{2L}\sum_{i=1}^{2L}(\boldsymbol{Z}_{i,k-1|k-1}+\boldsymbol{X}^{n}_{i,k-1|k-1})\cdot (\boldsymbol{Z}_{i,k-1|k-1}+\boldsymbol{X}^{n}_{i,k-1|k-1})^{\mathrm{T}}- \hat{\boldsymbol{z}}_{k-1|k-1}\hat{\boldsymbol{z}}^{\mathrm{T}}_{k-1|k-1}, \tag{101}$$

$$\boldsymbol{P}_{\tilde{x}\tilde{z},k|k-1}= \frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}_{i,k|k-1}(\boldsymbol{Z}_{i,k|k-1})^{\mathrm{T}}-\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}^{\mathrm{T}}_{k|k-1}, \tag{102}$$

$$\boldsymbol{P}_{\tilde{x}\tilde{z},k-1|k-1}= \frac{1}{2L}\sum_{i=1}^{2L}\boldsymbol{X}^{*x}_{i,k|k-1}(\boldsymbol{Z}_{i,k-1|k-1}+\boldsymbol{X}^{n}_{i,k-1|k-1})^{\mathrm{T}}+ \boldsymbol{S}_{k-1}-\hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{z}}^{\mathrm{T}}_{k-1|k-1}, \tag{103}$$

将式(98)–(103)代入定理2中对应各项, 求取k时刻下系统状态的估计值$\hat{\boldsymbol{x}}_{k|k}$以及对应的协方差矩阵$\boldsymbol{P}_{k|k}$.

注 4 分析可知, 当参数$\boldsymbol{S}_k=0, p_k=0$时, 即系统无噪声相关和量测时滞的情况, 则CKF–RDSCN算法退化为标准的CKF滤波算法, 因而标准CKF算法为本文所提出的CKF–RDSCN算法, 在系统模型无噪声相关和量测时滞情况下的特殊实现. 此外, 本文所给出的最优估计算法, 是噪声相关和量测时滞条件下, 针对状态估计问题具有通用性质的一般框架, 因而其实现形式不局限于CKF–RDSCN算法, 研究人员可依据不同的数值近似方法, 给出相应的实现形式.

5 仿真分析(Simulations)

在本节中, 对所提出的CKF–RDSCN算法进行验证分析, 并与经典的EKF, UKF以及CKF算法进行性能比较. 一方面测试所提出的CKF–RDSCN算法同上述经典滤波算法, 对随机量测时滞和同步相关噪声等两类干扰因素的处理能力, 另一方面测试前者对非线性模型的处理能力. 因此, 仿真采用非线性滤波领域中常用的测试模型—单变量非静态增长模型(UNGM)[30–31]. 该模型含有三角函数、分数以及平方函数等形式的非线性项, 具有较强的非线性特性, 其具体形式如下:

$$x_k=0.5x_{k-1}+25\frac{x_{k-1}}{1+x^2_{k-1}}+ 8\cos(1.2(k-1))+v_{k-1},$$

$$z_k=\frac{x^2_k}{20}+n_k,$$

$$y_k=(1-\varepsilon_k)z_k+\varepsilon_k z_{k-1},\ k>1;\ y_1=z_1,$$

式中: 过程噪声v_{k-1}和量测噪声n_k分别表示方差为$Q_{k-1}=2$和$R_k=10$的相关高斯白噪声, 其相关系数S_k的取值为$0.1\sim0.7$; $\varepsilon_k(k>1)$表示满足Bernoulli分布的互不相关随机序列, 其一步随机时滞概率p_k的取值为$0.1\sim0.9$. 设定仿真参数的初始值为$x_0=-0.3$, $P_0=1$, $T=200$. 仿真中采用状态估计结果的均方根误差(RMSE), 作为定量评价各滤波算法的性能指标, 其定义如下:

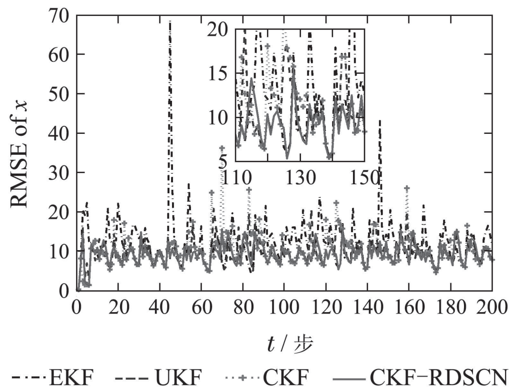
$$\mathrm{RMSE}(k)=\sqrt{\frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{x}^{(n)}_k-\hat{\boldsymbol{x}}^{(n)}_{k|k})^2}, 1\leqslant k\leqslant 200,$$

式中: N表示蒙特卡洛仿真次数, 仿真中设定$N=100$; $\boldsymbol{x}^{(n)}_k$和$\hat{\boldsymbol{x}}^{(n)}_{k|k}$分别表示第$n$次蒙特卡洛仿真下$k$时刻的状态真值和滤波算法的估计值.

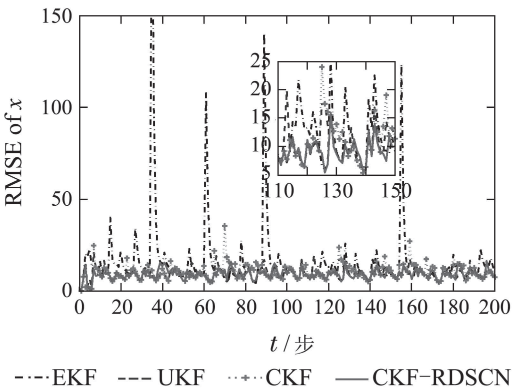
图1–3以噪声相关和量测时滞不同参数的情况为研究对象, 对比了经典的扩展卡尔曼滤波(EKF), UKF以及CKF算法同本文所提出的CKF–RDSCN算法的性能差异. 为了分析噪声相关因素对滤波算法性能的影响, 在图1中给出了时滞参数相同, 而相关噪声系数不同情况下, 各滤波算法对RMSE的计算结果. 由图1可知, CKF–RDSCN算法的估计结果要明显好于其他3种算法, 且噪声相关系数增大时, 对其估计结果的影响很小. 这说明 CKF–RDSCN算法对噪声相关因素的影响具有很好的鲁棒性, 且可以给出较高精度的估计结果. 对比估计结果可知, EKF算法的性能最差, 结果波动性最大, 其在图1(a)中$k=50$和$k=150$附近时, 估计结果出现了幅值较大的尖峰. 造成该现象的原因, 一方面是由于测试模型本身的强非线性, 导致了EKF的估计结果较差, 另一方面是由于噪声相关以及量测时滞等因素的引入, 破坏了算法的稳定性. 在图1(b)中, 随着噪声相关系数的增大, EKF估计结果出现尖峰的次数增多, 进一步证实了EKF无法满足噪声相关条件下的状态估计要求. 此外, 由于测试模型的维数较低, 因而UKF和CKF 的估计结果较为接近, 当噪声相关系数增大时, 二者具有相似的数值稳定性, 从侧面验证了CKF是UKF算法特例的已有结论.

图2展示了噪声相关系数相同, 而时滞参数不同情况下的状态估计结果. 与图1中所呈现的结果类似, 在时滞参数$p_k=0.3$和$p_k=0.7$时, CKF–RDSCN的算法性能均优于其他算法, 且时滞参数的增大对其数值稳定性的影响很小, 说明该算法对时滞因素的鲁棒性较强, 且可以给出较高精度的估计结果. 对比图1和图2中EKF算法的估计结果可知, 虽然每幅图中均出现了尖峰, 但图1中尖峰的幅值和出现的次数都要大于图2, 这说明噪声相关因素对估计结果的影响要大于时滞因素. 此外, 虽然图2的子图部分显示, UKF和CKF–RDSCN的估计结果接近, 但分析整个估计过程可知, 后者的性能仍然要优于前者.

图3中对比了不同噪声相关系数和不同量测时滞参数, 共同作用下的滤波算法结果分布. 对比分析可知, 在由 $p_k=0.1, 0.2, \cdots, 0.9$ 和 $S_k=0.1, 0.2, \cdots, 0.7$所构成的测试区间中, 同其他三种算法相比, CKF–RDSCN算法的估计精度最高, 且估计结果仅在$9\sim9.5$的区间内波动, 具有较好的数值稳定性. 而EKF算法的性能最差, 但与CKF和UKF算法不同的是, 其RMSE取值最大点并不在噪声相关系数和量测时滞参数取值同为最大的时刻, 而出现$S_k=0.7$和$p_k=0.5$ 的时刻, 进一步验证了, 模型非线性特性对估计结果的影响要大于噪声相关和量测时滞等因素. 由于UKF算法和CKF算法对低维模型非线性的处理能力相似, 因而其RMSE最大值的取值, 均发生在噪声相关系数和量测时滞参数同为最大值的附近. 从而说明, 在滤波算法的设计过程中, 对非线性特性的解决是应考虑的主要因素, 而噪声相关和测量时滞等干扰因素的处理, 对于提高估计结果精度具有重要意义.



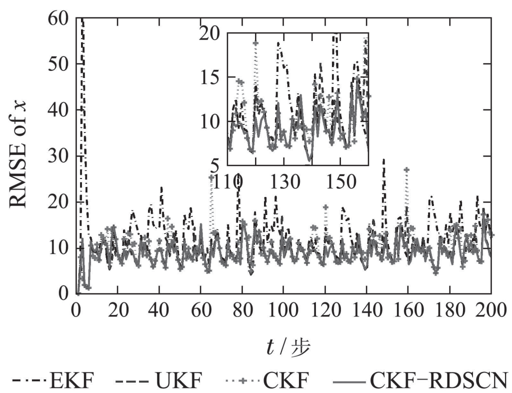
(a) $S_k=0.1$和$p_k=0.5$的RMSE计算结果



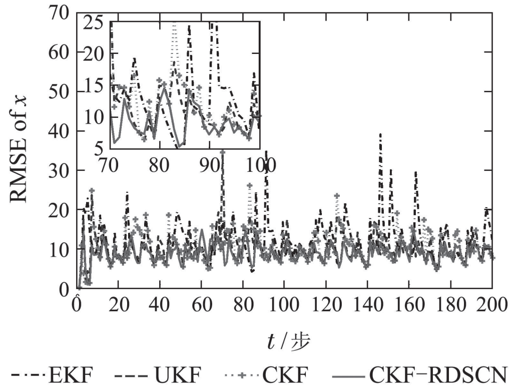
(b) $S_k=0.7$和$p_k=0.5$的RMSE计算结果

图 1 $S_k=0.1$和$S_k=0.7$情况下的EKF, UKF, CKF以及CKF–RDSCN性能比较

Fig. 1 Comparison with the EKF, UKF, CKF and CKF–RDSCN in$S_k=0.1$ and $S_k=0.7$



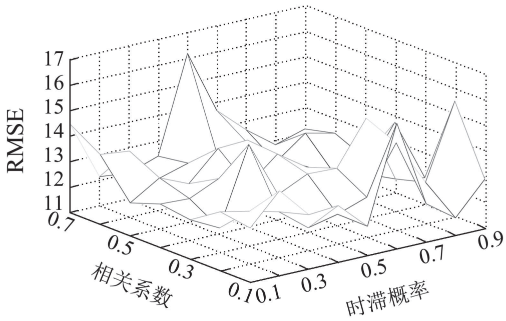
(a) $S_k=0.6$和$p_k=0.3$的RMSE计算结果



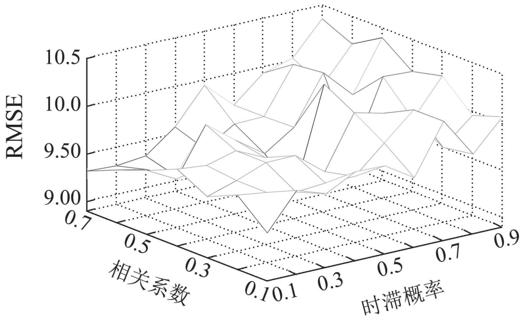
(b) $S_k = 0.6$和$p_k = 0.7$的RMSE计算结果

图 2 $p_k = 0.3$和$p_k = 0.7$情况下的EKF, UKF, CKF以及CKF–RDSCN性能比较

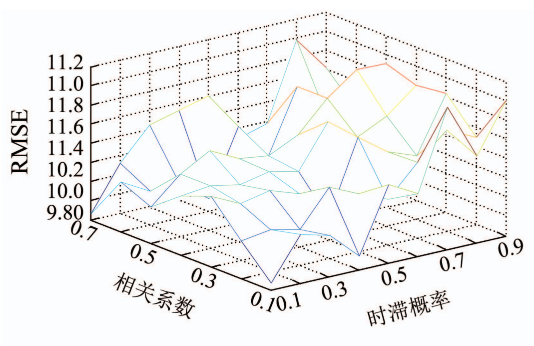
Fig. 2 Comparison with the EKF, UKF, CKF and CKF–RDSCN in $p_k = 0.3$ and $p_k = 0.7$



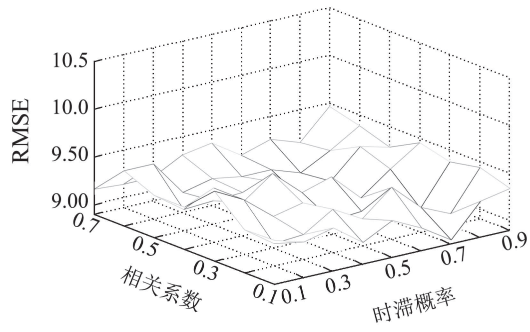
(a) EKF算法的RMSE计算结果



(b) UKF算法的RMSE计算结果



(c) CKF算法的RMSE计算结果



(d) CKF–RDSCN算法的RMSE计算结果

图 3 $S_k = 0.1, 0.2, \cdots, 0.7$和$p_k = 0.1, 0.2, \cdots, 0.9$情况下的EKF, UKF, CKF以及CKF–RDSCN性能比较

Fig. 3 Comparison with EKF, UKF, CKF and CKF–RDSCN in $S_k = 0.1, 0.2, \cdots, 0.7$ and $p_k = 0.1, 0.2, \cdots, 0.9$

6 结论(Conclusions)

针对随机量测时滞和同步相关噪声等非理想条件下的状态估计问题, 本文提出了一种高斯滤波框架形式的最优估计算法. 该算法以贝叶斯估计方法为基础, 利用高斯条件分布性质, 修正噪声同步相关条件下的状态一步预测值, 并采用以量测噪声作为状态增量的方式, 来达到对时滞量测信息的一步预测的目的. 与已有方法不同的是, 所提算法将对上述干扰因素的解决, 统一到高斯滤波框架下, 且保留了经典高斯滤波算法中一步预测的方式, 使其数值形式适用于高动态条件下的应用要求. 需要指出的是, 一般形式的高斯滤波框架是所提出的改进框架在系统处于理想条件时的特例, 后者较前者具有更广阔的适用范围. 此外, 本文以三阶球径容积法则为高斯加权积分的近似方法, 给出了所提出算法的次优形式——CKF–RDSCN. 然而, 其数值实现并不局限于CKF–RDSCN, 采用不同的近似方法, 可以给出不同的实现. 仿真表明, 同EKF, UKF和CKF 算法相比, CKF–RDSCN算法具有较高的精度和较强的数值稳定性, 有效地解决了随机量测时滞和同步相关噪声存在条件下的状态估计问题.

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