第 32 卷第 10 期
2015 年 10 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 32 No. 10
Oct. 2015

DOI: 10.7641/CTA.2015.50220

有向图下非线性无人机群自适应合围控制

余 瑶, 任 昊, 张 兰, 孙长银†

(北京科技大学 自动化学院, 北京 100083)

摘要: 本文研究了有向图下具有非线性和干扰的无人机群的分布式合围控制问题. 其中仅部分跟随者是领导者的邻居, 对于每一个跟随者, 至少存在一条从领导者到这个跟随者有向路径. 文中假设无人机的空气动力学特性是非线性不确定的, 并且领导者的输出是时变的. 结合反推设计方法提出了仅利用邻居信息的分布式合围控制方法, 使得跟随者的状态收敛于领导者状态所张成的凸包里. 利用神经网络函数逼近技术补偿无人机系统中的非线性不确定项, 通过李雅普诺夫稳定性理论证明了合围误差可以以任意收敛速度收敛到原点任意小的邻域. 最后通过仿真结果验证了控制协议的有效性.

关键词: 合围; 无人机群系统; 非线性不确定性; 自适应神经网络控制; 图论; 反推法

中图分类号: TP273 **文献标识码**: A

Distributed adaptive neural containment control for multi-UAV systems with nonlinear uncertainties under a directed graph

YU Yao, REN Hao, ZHANG Lan, SUN Chang-yin†

(School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China)

Abstract: We investigate the distributed containment control problem for multiple unmanned aerial vehicles (UAVs) systems with nonlinear uncertainties and bounded disturbances under a directed graph, where the leaders are neighbors of only a subset of the followers. For each follower, there exists at least one leader that has a directed path to the follower. It is assumed that aerodynamic characteristics of UAVs are nonlinear uncertainties, and the outputs of leaders are time-varying. A distributed containment control protocol combined with backstepping design method is proposed by using neighbors' information, so that the states of the followers will converge to the convex hull spanned by the dynamic leaders. The function approximation technique using neural networks is employed to compensate unknown nonlinear terms induced from the controller design procedure. By Lyapunov stability theorem, it is shown that the containment control errors will converge to an expected neighborhood of the origin with an arbitrary convergence rate. Simulation examples are presented to illustrate the effectiveness of the proposed control algorithm.

Key words: containment; multi-UAV systems; nonlinear uncertainties; adaptive neural control; graph theory; backstepping

1 Introduction

An unmanned aerial vehicle (UAV) is an aircraft with no on-board human pilot. UAVs are usually deployed for military and special operation applications, and used in numerous civil tasks falling within the dull, dirty and dangerous category. Potential and existing applications of UAVs include acrobatic aerial footage in filmmaking, forest fire detection, search and rescue missions. The research on cooperative control of multi-UAV systems[1] has drawn considerable interests in recent years due to several superiorities of multi-UAV systems in contrast with individual UAV systems, such as higher efficiency, better robustness, and larger survival probability. The distributed cooperative control[2–3] for multi-UAV systems requires that the protocol for each UAV use only limited local information to reach an overall goal in complicated environment, which makes the control a great challenge. To achieve the goals above, much progress has been made in study of consensus control[4–8], containment control[9–10], and formation control[11–13]. Among these control problems, consensus is an important and fundamental problem, which means that all agent states reach an agreement with their neigh-

Received 23 March 2015; accepted 18 July 2015.

†Corresponding author. E-mail: cys@ustb.edu.cn; Tel.: +86 13913036082.

Supported by National Outstanding Youth Science Foundation (61125306), National Natural Science Foundation (61473324), Beijing Higher Education Young Elite Teacher Project (YETP0378), Fundamental Research Funds for the Central Universities (FRF–TP–14–118A2) and Beijing Natural Science Foundation (4154079).

bors in a certain sense. Consensus is a typical collective behavior[14–15], which can be classified into leaderless consensus[16–17] and leader-following consensus[18–21]. In practice, it may be safer and more efficient for multi-UAV systems if there are multiple leaders and followers in battlefield environment, e.g., only some of UAVs are equipped with high precision sensors, so that other UAVs may be safer and easier to avoid obstacles if they converge to the convex hull.

For multiple leaders case, containment control problems arise, which require that the states of followers converge to the convex hull formed by the leaders. The containment control problem has recently gained much attention. Containment problems for first-order and second-order multi-agent systems have been studied in [9, 22–25]. In [9], the distributed containment control for double-integrator dynamics in the presence of both stationary and dynamic leaders was studied. In [22], the problem of distributed containment control of a group of mobile autonomous agents was discussed with multiple stationary or dynamic leaders under both fixed and switching directed network topologies. In [23], a stop-and-go strategy was proposed for a group of single-integrator agents to the convex polytope spanned by the leaders under a fixed undirected network topology. The finite-time containment control problem for second-order multi-agent systems under a fixed communication topology is addressed in [24]. For double-integrator dynamics, distributed containment control in the presence of both stationary and dynamic leaders was discussed in [25]. There have been also some results on the problem of distributed containment control for linear high-order multi-agent systems[26–27]. In [26], the behavior of multiple agents with linear dynamics was investigated by the study of interaction topologies. Formation-containment problem was transformed into asymptotic stability problem by using state transformation and state space decomposition approaches in [27]. All these results focused on containment control of linear multi-agent systems. For multi-UAV systems, there have been a few results of containment control[28]. In [28], formation-containment control problems for multi-UAV systems with directed interaction topologies were addressed, where each UAV system was modeled as a second order linear system.

Nonlinear uncertainties are inevitable in real systems due to imprecise measurements, unmodeled dynamics, external disturbances, etc. Therefore, different kinds of control strategies, such as backstepping[29–31], sliding mode control[32], neural networks[33–35], adaptive control[36], have been developed to deal with nonlinear systems. We notice that there are many results on tracking or consensus problems for multi-agent systems with nonlinear uncertainties[37–41], but there are only a few results on containment control problem with nonlinear uncertain agents. Despite these efforts, little progress has been made in distributed adaptive containment control for multi-UAV systems with nonlinear uncertainties under a directed graph, which is the main content of our research.

In this paper, containment control is investigated for multi-UAV systems with nonlinear uncertainty under a directed graph topology. Nonlinear uncertain aerodynamic characteristics of UAVs are considered. Matching condition is not satisfied. And the leaders are neighbors of only a subset of the followers. The dynamic characteristics of a single UAV are divided into nominal model and unknown nonlinear uncertainties. A distributed adaptive containment controller combined with backstepping method is proposed so that all of the followers converge to the dynamic convex hull spanned by the dynamic leaders. The function approximation technique using neural networks is employed to compensate unknown nonlinear terms. The stability of the closed-loop systems is analyzed via a Lyapunov-based method, which shows that the containment error can be reduced as small as desired.

This paper is organized as follows. Section 2 introduces useful results of the graph theory and the dynamics model of multi-UAV systems. In Section 3, an adaptive neural controller with backstepping design method is proposed and stability analysis is presented by Lyapunov function. In Section 4, simulation examples are presented to illustrate the analytical results. Finally, conclusions are drawn in Section 5.

2 Preliminaries

2.1 Graph theory

For $M + N$ UAVs, a directed graph $G \triangleq (\nu, \varepsilon)$ is a pair (ν, ε), where the set of nodes or vertices is $\nu \triangleq \{1, \cdots, M + N\}$ and the set of edges or arcs is $\varepsilon \subseteq \nu \times \nu$. An edge $(j, i) \in \varepsilon$ means that UAV i can obtain information from UAV j, but not vice versa where j and i are the parent node and child node, respectively. $N_i = \{j|(j, i) \in \varepsilon\}$ means the set of neighbors of the node i, which is the set of nodes with edges incoming to node i. A directed path from node i_1 to node i_k is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \cdots, (i_{k-1}, i_k)$ in a directed graph. A directed tree is a special directed graph where every node has exactly one parent except for the root and the root has directed paths to every other node. If there exists at least one UAV that has directed paths to all other UAVs, we can believe that the directed graph has a directed spanning tree[42–43].

For a directed graph G, the adjacent matrix $A = (a_{ij}) \in \mathbb{R}^{(M+N)\times(M+N)}$ is defined as $a_{ij} > 0$ if $(j,i) \in \varepsilon$, otherwise, $a_{ij} = 0$. Self-edges are not allowed, i.e., $a_{ii} = 0$. The (nonsymmetric) Laplacian matrix $L = D - A \in \mathbb{R}^{(M+N)\times(M+N)}$, where $D = \operatorname{diag}\{d_1, d_2, \cdots, d_M, d_{M+1}, \cdots, d_{M+N}\}$, $d_i = \sum\limits_{j=1,j\neq i}^{M+N} a_{ij}$ is the diagonal element of the degree matrix D.

Lemma 1 For a directed graph G, it is easy to see that zero is always an eigenvalue of L with 1 as a corresponding right eigenvector, and all of the nonzero eigenvalues of L have positive real parts. Besides, zero is a simple eigenvalue of L if and only if the directed graph G has a directed spanning tree[44].

2.2 Problem statement

It is assumed that there exist M followers, labeled as UAVs 1 to M , and N leaders, labeled as an UAV $M+1$ to $M+N$ under a directed communication graph topology. Each UAV should be equipped with a sensor suite to determine its own position and orientation in the inertial reference frame and receive the (relative) position information from its neighbors. Assume that all the UAVs have fixed attitudes. The translational dynamics of the ith UAV can be written as[45]

$$\begin{cases} \dot{p}_i = g_i(\Theta_i)v_i, \\ M_i\dot{v}_i = -D_i(v_i)v_i - R_i(\Theta_i) + \tau_i, \end{cases} \tag{1}$$

where $p_i = [X_i \ \ Y_i \ \ Z_i]^{\mathrm{T}} \in \mathbb{R}^3$, $\Theta_i = [\phi_i \ \ \theta_i \ \ \psi_i]^{\mathrm{T}}$ means position and attitude (described by Euler angles, i.e., roll ϕ_i, pitch θ_i, and yaw ψ_i) vectors in the inertial reference frame, respectively. $g_i(\Theta_i)$ is the kinematic transformation matrix from the body-fixed reference frame to the inertial reference frame, $v_i = [\mu_i, \nu_i, \omega_i]^{\mathrm{T}}$ is translational velocity vector in the body-fixed reference frame, M_i is the inertia matrix, $D_i(v_i)$ is the damping matrix, $R_i(\Theta_i)$ is the restoring force vector, and τ_i is the control force vector. For an angle $\alpha \in \mathbb{R}$, denote $s_\alpha = \sin\alpha$, $c_\alpha = \cos\alpha$ for brevity. $g_i(\Theta_i)$ is defined as

$$g_i(\Theta_i) = \begin{bmatrix} c_{\psi_i}c_{\theta_i} & -s_{\psi_i}c_{\phi_i}+s_{\phi_i}s_{\theta_i}c_{\psi_i} & s_{\psi_i}s_{\phi_i}+s_{\theta_i}c_{\psi_i}c_{\phi_i} \\ s_{\psi_i}c_{\theta_i} & c_{\psi_i}c_{\phi_i}+s_{\phi_i}s_{\theta_i}s_{\psi_i} & -c_{\psi_i}s_{\phi_i}+s_{\theta_i}s_{\psi_i}c_{\phi_i} \\ -s_{\theta_i} & s_{\phi_i}c_{\theta_i} & c_{\phi_i}c_{\theta_i} \end{bmatrix},$$

$$\begin{aligned} &M_i = \operatorname{diag}\{m_{i1}, m_{i2}, m_{i3}\}, \\ &D_i(v_i) = \operatorname{diag}\{d_{\mathrm{L}i1} + d_{Qi1}|\mu_i|, d_{\mathrm{L}i2} + d_{Qi2}|\nu_i|, \\ &\qquad d_{\mathrm{L}i3} + d_{Qi2}|\omega_i|\},\ m_{ij}, d_{\mathrm{L}ij}, d_{Qij} > 0, \\ &R_i(\Theta_i) = [(W_i - B_i)s_{\theta_i}, -(W_i - B_i)c_{\theta_i}s_{\phi_i}, \\ &\qquad -(W_i - B_i)c_{\theta_i}c_{\phi_i}]^{\mathrm{T}}, \end{aligned}$$

where W_i and B_i represent the gravitational and buoyancy forces, respectively.

For the position containment control of $M+N$ UAVs system, the dynamic models of M followers can be simplified as

$$\begin{cases} \dot{x}_{i,1} = g_i(\Theta_i)x_{i,2}, \\ \dot{x}_{i,2} = u_i + \varPhi_i(x_{i,1}, x_{i,2}, \kappa_i), \\ y_i = x_{i,1}, \end{cases} \tag{2}$$

where $i = 1, 2, \cdots, M$, $x_{i,1} = [X_i \ \ Y_i \ \ Z_i]^{\mathrm{T}}$, $\Theta_i = [\phi_i \ \ \theta_i \ \ \psi_i]^{\mathrm{T}}$ means position and attitude (described by Euler angles, i.e., roll ϕ_i, pitch θ_i, and yaw ψ_i) vectors in the inertial reference frame, respectively, $x_{i,2} \in \mathbb{R}^3$ is translational velocity vector in the body-fixed reference frame, $u_i \in \mathbb{R}^3$ is the input vector of the ith follower, $y_i \in \mathbb{R}^3$ is the output vector, which is the position vector of the ith follower in the inertial reference frame, $\varPhi_i(x_{i,1}, x_{i,2}, \kappa_i)$ are unknown nonlinear smooth functions, which include the aerodynamic forces, the pitch and elevation channels couplings, the parameters perturbation, and time-varying disturbances κ_i. It's supposed that the motions of N leaders are independent of the motions of M followers, the followers 1 to M have at least one neighbor, and the leaders $M+1$ to $M+N$ have no neighbors.

The communication topology for the M+N UAVs is considered as a directed graph $G \triangleq (\nu, \varepsilon)$ with $\nu \triangleq \{0, 1, 2, \cdots, M, M+1, \cdots, M+N\}$. We denote $\nu_{\mathrm{f}} = 1, 2, \cdots, M$ and $\nu_{\mathrm{l}} = \{M+1, \cdots, M+N\}$ as the node set of the followers and the leaders, respectively. It is considered that $\nu_{\mathrm{f}} \cup \nu_{\mathrm{l}} = \nu$ and $\nu_{\mathrm{f}} \cap \nu_{\mathrm{l}} = \varnothing$. To represent the communications among followers and the communications between the followers and the leaders, the Laplacian matrix L is defined as

$$L = \begin{bmatrix} \bar{L}_{\mathrm{f}} & \bar{L}_{\mathrm{l}} \\ 0_{N\times M} & 0_{N\times N} \end{bmatrix}, \tag{3}$$

where $\bar{L}_{\mathrm{f}} \in \mathbb{R}^{M\times M}$ is the matrix related to the communication among the M followers and $\bar{L}_{\mathrm{l}} \in \mathbb{R}^{N\times N}$ is the matrix related to the communication from the N leaders to the M followers.

The following technical lemmas and assumptions of bounded input and bounded output property for stable dynamic inequalities are considered for the convenience of stability analysis.

Remark 1 The set $\mathbb{C} \in \mathbb{R}^n$ is said to be convex if for any $x_1, x_2 \in \mathbb{C}$ and any $\partial \in [0,1]$, the point $\partial x_1 + (1-\partial)x_2$ is in $\mathbb{C}$. The convex hull $\mathrm{Co}(X)$ for a set of points $X = [x_1 \ \cdots \ x_n]$ id the minimal convex set containing all points in X and is defined as $\mathrm{Co}(X) = \sum\limits_{i=1}^{n} \partial_i = 1$[46].

Assumption 1 Both multiple dynamic leaders $r_j(t) \in \mathbb{C}^2$, $j = M+1, \cdots, M+N$ and its derivative $\dot{r}_j(t) \in \mathbb{R}^3$ are bounded, and the leaders $r_j(t)$ are only available for the ith follower satisfying $j \in \mathbb{N}_i$, $i = 1, \cdots, M$.

Assumption 2 The states $x_{i,1}$ and $x_{i,2}$ of the ith follower are only known and available for the j-th followers satisfying $i \in \mathbb{N}_j$, $i = 1, \cdots, M$, $j = 1, \cdots, M$ and $i \neq j$.

Assumption 3 For any one of the M followers, there exists at least one leader that has a directed path to that follower.

Lemma 2 According to Assumption 3, $\bar{L}_\mathrm{f}$ is a nonsingular M matrix, and thus it is invertible. Additionally, each entry of $-\bar{L}_\mathrm{f}^{-1}\bar{L}_\mathrm{l}$ is nonnegative and all row sums of $-\bar{L}_\mathrm{f}^{-1}\bar{L}_\mathrm{l}$ equal to one[47].

Let $r(t) = [r_{M+1}^\mathrm{T}(t) \ \cdots \ r_{M+N}^\mathrm{T}(t)]^\mathrm{T}$, $r_\mathrm{d}(t) = [r_{\mathrm{d},1}^\mathrm{T}(t) \ \cdots \ r_{\mathrm{d} \ M}^\mathrm{T}(t)]^\mathrm{T} = -(\bar{L}_\mathrm{f}^{-1}\bar{L}_\mathrm{l} \otimes I_\mathrm{p})r(t)$, where $r_{\mathrm{d},i}(t) \in \mathbb{R}^p$, $i = 1, \cdots, M$, $\otimes$ stands for Kronecker product, and I_p is an identity matrix of order p. For ith follower, $r_{\mathrm{d},i}(t)$ can be regard as a reference point in the convex hull spanned by the leaders. From Lemma 2, we get that $\inf\limits_{h(t) \in R(t)} \parallel r_{\mathrm{d},i}(t) - h(t) \parallel < \epsilon$, with $i = 1, \cdots, M$ for all $t \geqslant 0$, where $R(t) = \mathrm{Co}\{r_{M+1}(t), \cdots, r_{M+N}(t)\}$, and ϵ is a positive constant which can be made sufficiently small, while all signals in the total class-loop systems are bounded. Therefore, the containment control problem can be regarded as the tracking problem of each follower such that $\parallel y_i(t) - r_{\mathrm{d},i}(t) \parallel = \|y(t) + (\bar{L}_\mathrm{f}^{-1}\bar{L}_\mathrm{l} \otimes I_\mathrm{p})r(t)\| < \bar{\epsilon}$, where $i = 1, \cdots, M$ and $\bar{\epsilon}$ is a positive constant which can be made sufficiently small[48]. When each follower tracks to its own reference point, we can consider that all followers converge to the convex hull spanned by the leaders.

As we know, radial basis function (RBF) neural networks are usually employed as the function approximator to model nonlinear functions, which we believe to have reached a fairly good point in terms of stability and flexibility. The following RBF NN is used to approximate the continuous function $h(Z)$: $\mathbb{R}^q \to \mathbb{R}$,

$$h_{nn}(Z) = W^\mathrm{T} S(Z), \tag{4}$$

where the input vector $Z \in \varOmega \subset \mathbb{R}^q$, weight vector $W = [w_1 \ w_2 \ \cdots \ w_l]^\mathrm{T} \in \mathbb{R}^k$, the number of neuron $k > 1$; and $S(Z) = [s_1(Z) \ s_2(Z) \ \cdots \ s_k(Z)]^\mathrm{T}$ with

$$s_i(Z) = \exp[\frac{-(Z-\mu_i)^\mathrm{T}(Z-\mu_i)}{\varphi_i^2}], \tag{5}$$

where $i = 1, 2, \cdots, k$, $\mu_i = [\mu_{i1} \ \mu_{i2} \ \cdots \ \mu_{iq}]^\mathrm{T}$ is the center of the receptive field and φ_i is the width of the Gaussian function[49].

It has been proven that the aforementioned RBFNN can approximate any smooth function over a compact set to arbitrarily any accuracy as

$$h(Z) = W^{*\mathrm{T}} S(Z) + \xi, \ \forall Z \in \varOmega_Z, \tag{6}$$

where the compact set $\varOmega_Z \subset \mathbb{R}^q$, Z is the input variables of the NNs, W^* is the ideal constant weights vector, and ξ is the bounded function approximation error. If Z remain within some prefixed compact set $\varOmega_Z$ which have a sufficiently large size, there exists controllers with sufficiently large number of NN nodes such that all the signals in the closed-loop remain bounded.

For convenience of analysis, the ideal constant weights W^* is defined as the value of that minimizes for all, i.e.,

$$W^* \triangleq \arg\min_{W \in \mathbb{R}^k} \{\sup_{Z \in \varOmega_Z} |h(Z) - W^\mathrm{T} S(Z)|\}, \tag{7}$$

and there exists W^* such that $|\xi| \leqslant \xi^*$ with constant $\xi^* > 0$ for all $Z \in \varOmega_Z$.

3 Adaptive containment controller design and stability analysis

For the second-order system of the ith follower, the idea of backstepping is employed to design controllers. The design procedure of the ith follower contains two steps. Based on the distributed dynamic surface design, we define the graph based error surfaces $s_{i,k}$, $k = 1, 2$, for the ith follower as

$$\begin{cases} s_{i,1} = \sum\limits_{j=1}^{M} a_{ij}(y_i - y_j) + \\ \qquad \sum\limits_{j=M+1}^{M+N} a_{ij}(y_i - r_j), \\ s_{i,2} = x_{i,2} - v_{i,2}, \end{cases} \tag{8}$$

where $i = 1, \cdots, M$, $v_{i,2}$ are the virtual control.

Step 1 The derivative of $s_{i,1}$ along (2) and (8) is

$$\begin{aligned} \dot{s}_{i,1} = &\sum_{j=1}^{M} a_{ij}(\dot{y}_i - \dot{y}_j) + \sum_{j=M+1}^{M+N} a_{ij}(\dot{y}_i - \dot{r}_j) = \\ &-\sum_{j=1}^{M} a_{ij} g_j(\varTheta_j) x_{j,2} - \sum_{j=M+1}^{M+N} a_{ij}\dot{r}_j + \\ &(\sum_{j=1}^{M} a_{ij} + \sum_{j=M+1}^{M+N} a_{ij}) g_i(\varTheta_i) x_{i,2} = \end{aligned}$$

$$-\sum_{j=1}^{M} a_{ij} g_j(\Theta_j) x_{j,2} - \sum_{j=M+1}^{M+N} a_{ij} \dot{r}_j + d_i g_i(\Theta_i)(s_{i,2} + v_{i,2}). \tag{9}$$

Choose the Lyapunov candidate function $V_{i,1}$ as

$$V_{i,1} = \frac{1}{2} s_{i,1}^2. \tag{10}$$

To stabilize (9), the distributed first virtual control law $v_{i,2}$ for the ith follower is designed as

$$v_{i,2} = \frac{1}{d_i}(-\rho s_{i,1} + \sum_{j=1}^{M} a_{ij} g_j(\Theta_j) x_{j,2} + \sum_{j=M+1}^{M+N} \dot{r}_j), \tag{11}$$

where ρ is a positive constant.

Substituting (11) into (9), one obtains

$$\dot{s}_{i,1} = -\rho s_{i,1} + d_i g_i(\Theta_i) s_{i,2}. \tag{12}$$

Differentiating (10) along (12) yields

$$\dot{V}_{i,1} = -\rho s_{i,1}^2 + d_i g_i(\Theta_i) s_{i,1} s_{i,2}. \tag{13}$$

Step 2 Using (2) and (8), the derivative of $s_{i,2}$ can be represented by

$$\dot{s}_{i,2} = u_i + \Phi_i(x_{i,1}, x_{i,2}, \kappa_i) - \dot{v}_{i,2}. \tag{14}$$

To stabilize (14), there exists a desired feedback control

$$u_i^* = -\rho s_{i,2} - (\Phi_i(x_{i,1}, x_{i,2}, \kappa_i) - \dot{v}_{i,2}),$$

where $\dot{v}_{i,2}$ can be expressed as

$$\dot{v}_{i,2} = \sum_{j=1}^{M} \frac{\partial v_{i,2}}{\partial x_{j,1}}(x_{j,2} + \Phi_i(x_{i,1}, x_{i,2}, \kappa_i)) + \sum_{j=M+1}^{M+N} \frac{\partial v_{i,2}}{\partial r_j} \dot{r}_j.$$

For the desired feedback control u_i^*, from Assumption 1 we can get that

$$h_i(Z_i) = \Phi_i(x_{i,1}, x_{i,2}, \kappa_i) - \dot{v}_{i,2}$$

are smooth functions, which denote the unknown part of u_i^*, where

$$Z_i = [x_{i,1}^{\mathrm{T}}\ x_{i,2}^{\mathrm{T}}\ \kappa_i\ (\frac{\partial v_{i,2}}{\partial x_{i,1}})^{\mathrm{T}}\ \cdots\ (\frac{\partial v_{i,2}}{\partial x_{M,1}})^{\mathrm{T}}, \frac{\partial v_{i,2}}{\partial r_{M+1}} \dot{r}_{M+1}\ \cdots\ \frac{\partial v_{i,2}}{\partial r_{M+N}} \dot{r}_{M+N}]^{\mathrm{T}}.$$

By employing an RBF neural network $W_i^{\mathrm{T}} S_i(Z_i)$ to approximate $h_i(Z_i)$, u_i^* can be expressed

$$u_i^* = -\rho s_{i,2} - W_i^{*\mathrm{T}} S_i(Z_i) - \xi_i,$$

where $W_i^{*\mathrm{T}}$ denote the ideal constants weights, and $|\xi_i| \leqslant \xi_i^*$ is the approximation error with constant $\xi_i^* > 0$.

Since W_i^* is unknown, u_i^* cannot be realized in practice. Consider

$$u_i = -\rho s_{i,2} - \hat{W}_i^{\mathrm{T}} S_i(Z_i). \tag{15}$$

Then, we have

$$\dot{s}_{i,2} = u_i + \Phi_i(x_{i,1}, x_{i,2}, \kappa_i) - \dot{v}_{i,2} = -\rho s_{i,2} - \tilde{W}_i^{\mathrm{T}} S_i(Z_i) + \xi_i. \tag{16}$$

Consider the Lyapunov function candidate $V_{i,2}$ as

$$V_{i,2} = \frac{1}{2} s_{i,2}^2 + \frac{1}{2} \tilde{W}_i^{\mathrm{T}} \Gamma_i^{\mathrm{T}} \tilde{W}_i. \tag{17}$$

The derivative of $V_{i,2}$ is

$$\dot{V}_{i,2} = s_{i,2}\dot{s}_{i,2} + \tilde{W}_i^{\mathrm{T}} \Gamma_i^{\mathrm{T}} \dot{\hat{W}}_i = -\rho s_{i,2}^2 + s_{i,2}\xi_i - \tilde{W}_i^{\mathrm{T}} S_i(Z_i) s_{i,2} + \tilde{W}_i^{\mathrm{T}} \Gamma_i^{\mathrm{T}} \dot{\hat{W}}_i. \tag{18}$$

Consider the adaptation law for $\hat{W}_i$ as

$$\dot{\tilde{W}}_i = \dot{\hat{W}}_i = \Gamma_{W_i}[S_i(Z_i) s_{i,2} - \sigma_i \hat{W}_i], \tag{19}$$

where $\sigma_i > 0$ and $\Gamma_{W_i} = \Gamma_{W_i}^{\mathrm{T}} > 0$ are design constants, $\tilde{W}_i = \hat{W}_i - W_i^*$.

Let $\rho = \rho_1 + \rho_2$, where ρ_1 and $\rho_2 > 0$. Then, (18) becomes

$$\dot{V}_{i,2} = -\rho_1 s_{i,2}^2 - \rho_2 s_{i,2}^2 + s_{i,2}\xi_i - \sigma_i \tilde{W}_i^{\mathrm{T}} \hat{W}_i. \tag{20}$$

By completion of squares, one has

$$-\sigma_i \tilde{W}_i^{\mathrm{T}} \hat{W}_i = -\sigma_i \tilde{W}_i^{\mathrm{T}}(\tilde{W}_i + W_i^*) \leqslant -\sigma_i \|\tilde{W}_i\|^2 + \sigma_i \|\tilde{W}_i\| \|W_i^*\| \leqslant -\frac{\sigma_i \|\tilde{W}_i\|^2}{2} + \frac{\sigma_i \|W_i^*\|^2}{2} - \rho_2 s_{i,2}^2 + s_{i,2}\xi_i \leqslant \frac{\xi_i^2}{4\rho_2} \leqslant \frac{\xi_i^{*2}}{4\rho_2}. \tag{21}$$

Then one has the following inequality

$$\dot{V}_{i,2} \leqslant -\rho_1 s_{i,2}^2 - \frac{\sigma_i \|\tilde{W}_i\|^2}{2} + \delta_i, \tag{22}$$

where $\delta_i \leqslant \sigma_i \|W_i^*\|^2/2 + \xi_i^{*2}/4\rho_2$.

Let $V_i = V_{i,1} + V_{i,2}$. If we choose

$$\rho = \rho^* + \frac{d_i g_i(\Theta_i)}{2},\ \rho_1 = \rho_1^* + \frac{d_i g_i(\Theta_i)}{2},$$

where $\rho^* \geqslant \frac{\gamma}{2}$, $\rho_1^* \geqslant \frac{\gamma}{2}$, γ is a positive constant, and choose σ_i and Γ_i such that $\sigma_i \geqslant \gamma\lambda\{\Gamma_i^{-1}\}$, then from (13) and (22) we have the following inequality:

$$\dot{V}_i = -\rho s_{i,1}^2 + d_i g_i(\Theta_i) s_{i,1} s_{i,2} - \rho s_{i,2}^2 - \frac{\sigma_i \|\tilde{W}_i\|^2}{2} + \delta_i \leqslant -(\rho - \frac{d_i g_i(\Theta_i)}{2}) s_{i,1}^2 - (\rho_1 - \frac{d_i g_i(\Theta_i)}{2}) s_{i,2}^2 - \frac{\sigma_i \tilde{W}_i^{\mathrm{T}} \Gamma_i^{\mathrm{T}} \tilde{W}_i}{2} + \delta_i = -\rho^* s_{i,1}^2 - \rho_1^* s_{i,2}^2 - \frac{\sigma_i \tilde{W}_i^{\mathrm{T}} \Gamma_i^{\mathrm{T}} \tilde{W}_i}{2} + \delta_i \leqslant -\gamma(\frac{1}{2} s_{i,1}^2 + \frac{1}{2} s_{i,2}^2 + \frac{1}{2} \tilde{W}_i^{\mathrm{T}} \Gamma_i^{\mathrm{T}} \tilde{W}_i) + \delta_i =$$

$$-\gamma V_i + \delta_i. \tag{23}$$

Let $V = \sum_{i=1}^{M} V_i$. Its derivative is

$$\dot{V} = \sum_{i=1}^{M} \dot{V}_i \leqslant \sum_{i=1}^{M} (-\gamma V_i + \delta_i) \leqslant -\gamma V + \delta, \tag{24}$$

where $\delta = \sum_{i=1}^{M} \delta_i$ is positive constant.

The inequality (24) implies $\dot{V} < 0$ on $V > \delta/\gamma$. Therefore, if $V(0) \leqslant \delta/\gamma$, then $V(t) \leqslant \delta/\gamma$ for all $t \geqslant 0$. In addition, one has that $V(t) \leqslant \mathrm{e}^{-\gamma t} V(0) + \delta/\gamma(1-\mathrm{e}^{-\gamma t})$. Using $\|s_1\|^2/2 \leqslant V(t)$ with $s_1 = [s_{1,1}^{\mathrm{T}} \ \cdots \ s_{M,1}^{\mathrm{T}}]^{\mathrm{T}}$, one gets $\|s_1\|^2 \leqslant 2\mathrm{e}^{-\gamma t} V(0) + 2\delta/\gamma(1-\mathrm{e}^{-\gamma t})$. Therefore, as time increases, all error surfaces $\|s_1\|$ exponentially converge to the compact set $\Omega_{\mathrm{s}} = \{s_1 | \|s_1\| \leqslant \sqrt{2\delta/\gamma}\}$. The compact set Ω_{s} can be kept arbitrarily small by increasing γ. Then, from $s_1 = (\bar{L}_{\mathrm{f}} \otimes I_{\mathrm{p}})y + (\bar{L}_{\mathrm{l}} \otimes I_{\mathrm{p}})r$ where $y = [y_1^{\mathrm{T}} \ \cdots \ y_M^{\mathrm{T}}]^{\mathrm{T}}$, since s_1 can be reduced sufficiently small, we know that the follower output vector $y(t)$ converge to the convex hull spanned by the dynamic leaders $r(t)$, i.e., $\|y(t) + (\bar{L}_{\mathrm{f}}^{-1} \bar{L}_{\mathrm{l}} \otimes I_{\mathrm{p}}) r(t)\| < \bar{\epsilon}$. So the containment control errors in the overall closed-loop system can be converged to an adjustable neighborhood of the origin with an arbitrary convergence rate.

4 Simulation results

In this section, three cases are preformed to illustrate the effectiveness of the proposed approach. A multi-UAV system with 3 leaders and 4 followers in a three-dimensional space is considered. Fig.1 shows the topological directed graph of the interaction among the leaders (L5 to L7) and the followers (F1 to F4).

It is assumed that $a_{ij} = 1$ if $(j, i) \in \varepsilon$, otherwise, $a_{ij} = 0$. Then one can get the adjacent matrix A as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and the degree matrix $D = \mathrm{diag}\{2, 2, 2, 2, 0, 0, 0\}$. The dynamics models of M followers are given by (2). The RBF neural network approximation technique is used to estimate unknown nonlinear functions $\varPhi_i(x_{i,1}, x_{i,2}, \kappa_i)$, which include the aerodynamic forces, couplings, the parameters perturbation, and time-varying disturbances. It is assumed that the bound of nonlinear smooth functions $|\varPhi_i(x_{i,1}, x_{i,2}, \kappa_i)| = |x_{i,1}| + |x_{i,2}| + 3$. The containment control input (15) of the proposed approach is applied.

Case 1 We consider the trajectories of UAVs are in a 3-D space. The reference trajectories of three leaders are set to be static, i.e., the position states of leaders L5, L6 and L7 are $r_{\mathrm{L}5} = [-1 \ 1 \ 1]^{\mathrm{T}}$, $r_{\mathrm{L}6} = [5 \ 5 \ -5]^{\mathrm{T}}$ and $r_{\mathrm{L}7} = [3 \ -3 \ 3]^{\mathrm{T}}$, respectively. The initial positions of the four followers are random. In the feedback controller (15), we set $\rho = 50$. The states trajectories of the followers in simulations are shown in Fig.2.

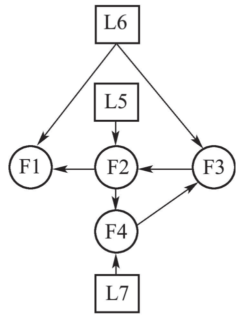

Fig. 1 Topological directed graph

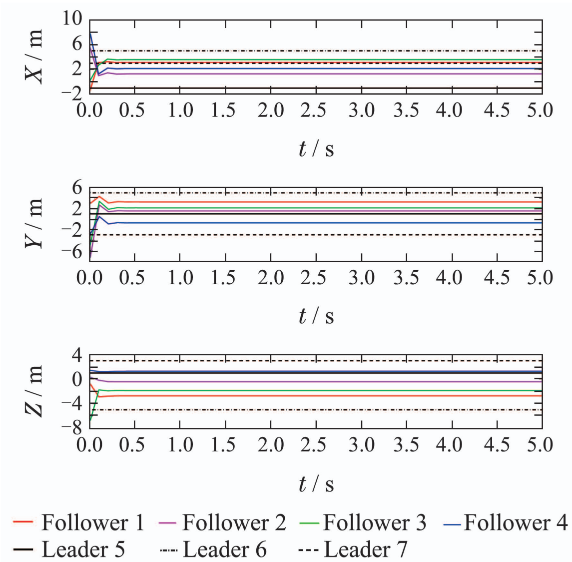

Fig. 2 The states trajectories of UAVs for Case 1

Case 2 Compared with Case 1, the reference trajectories one of three leaders to is assumed to be sine or cosine functions, and other two leaders keep static, i.e., the reference trajectories of leaders L5, L6 and L7 are $r_{\mathrm{L}5} = [-1 \ 1 \ 1]^{\mathrm{T}}$, $r_{\mathrm{L}6} = [5 \ 5 \ -5]^{\mathrm{T}}$ and $r_{\mathrm{L}7}(t) = [2+\sin(5t) \ \ 3+\cos(5t) \ \ -2+\sin t^2]^{\mathrm{T}}$, respectively. The same distributed containment control protocol is applied. The states trajectories of UAVs is shown in Fig. 3.

Case 3 In order to further illustrate the versatility of the proposed approach, the reference trajectories of three leaders are set to periodic functions. We choose the reference trajectories of leaders L5, L6 and L7 as $r_{\mathrm{L5}}(t) = [5+2\sin(5t+1)\ \ 10+\cos(8t-2)\ \ 3+\sin 10t+\cos(7t+1)]^{\mathrm{T}}$, $r_{\mathrm{L6}}(t) = [10+\cos(8t-2)\ \ 3+\sin 10t+\cos(7t+1)\ \ 5+2\sin(5t+1)]^{\mathrm{T}}$ and $r_{\mathrm{L7}}(t) = [3+\sin 10t+\cos(7t+1)\ \ 5+2\sin(5t+1)\ \ 10+\cos(8t-2)]^{\mathrm{T}}$, respectively. One practical problem corresponding to this model is the secure landing of multi-UAV systems on the deck of a galley. The multi-UAV systems have to protect itself from wave-induced oscillations which are usually described by a sinusoid signal with variable frequency. The same distributed containment control protocol is applied. Figure 4 shows the states trajectories of UAVs in this simulation.

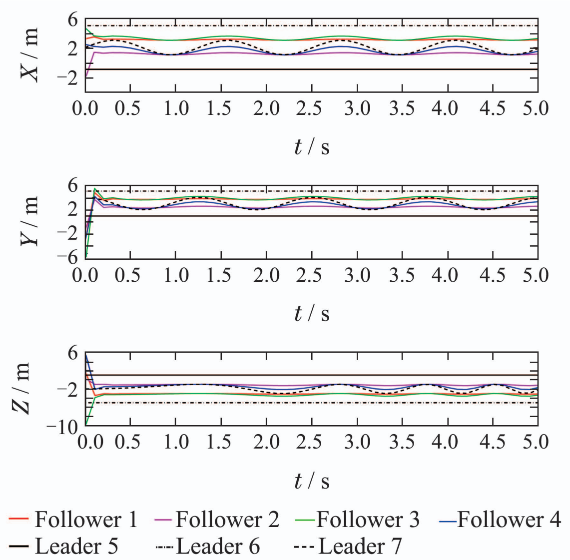

Fig. 3 The states trajectories of UAVs for Case 2

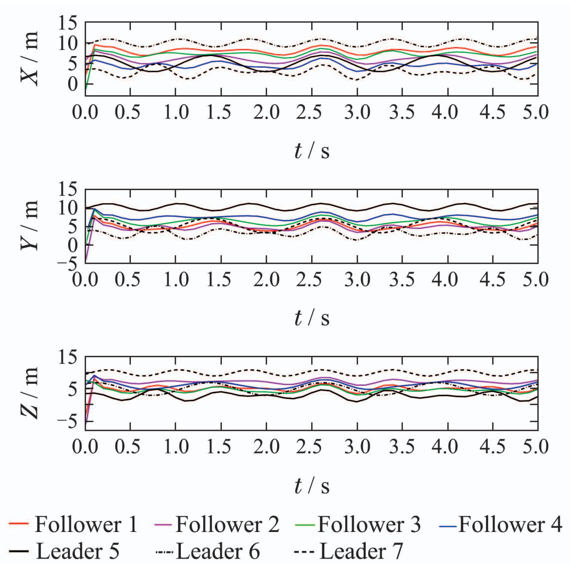

Fig. 4 The states trajectories of UAVs for Case 3

In Cases 1, 2, 3, the same containment control protocol is applied to multi-UAV systems with different types of reference trajectories of three leaders. It is obvious that all followers can converge to the static convex hull spanned by leaders quickly and the containment control protocol designed by the proposed method is robust and effective.

5 Conclusions

In this paper, distributed containment control problem for multi-UAV systems with nonlinear uncertainty under a directed graph topology is considered. The dynamic characteristics of a single UAV are divided into nominal model and unknown nonlinear function. A distributed adaptive containment controller based on the dynamic surface design is proposed so that all of the followers converge to the dynamic convex hull spanned by the dynamic leaders. Neural networks method is used as the function approximation technique to compensate unknown nonlinear terms derived from the controller design procedure for the followers. It has been shown that the system has achieved semi-global uniform ultimate boundedness and the containment control error can converge to any small neighbourhood of the origin with an arbitrary convergence rate. Simulation examples has been given to verify the algorithms.

References:

[1] DUAN H, LI P. *Bio-inspired Computation in Unmanned Aerial Vehicles* [M]. Berlin: Springer, 2014: 14 – 25.

[2] GUAN Y Q, JI Z J, ZHANG L, et al. Recent developments on controllability of multi-agent systems [J]. *Control Theory & Applications*, 2015, 32(4): 421 – 431.

[3] HONG Y G, ZHAI C. Dynamic coordination and distributed control design of multi-agent systems [J]. *Control Theory & Applications*, 2011, 28(10): 1506 –1512.

[4] SUN Y G, WANG L. Consensus of multi-agent systems in directed networks with nonuniform time-varying delays [J]. *IEEE Transactions on Automatic Control*, 2009, 54(7): 1607 – 1613.

[5] KURIKI Y, NAMERIKAWA T. Consensus-based cooperative formation control with collision avoidance for a multi-UAV system [C] //*American Control Conference*. Portland, USA: IEEE, 2014: 2077 – 2082.

[6] ZHOU Y, YU X, SUN C, et al. Robust synchronisation of second-order multi-agent system via pinning controlp [J]. *IET Control Theory & Applications*, doi: 10. 1049/iet-cta, 2014: 0295.

[7] ZHOU Y, YU X, SUN C, et al. Higher order finite-time consensus protocol for heterogeneous multi-agent systems [J]. *International Journal of Control*, 2015, 88(2): 285 – 294.

[8] XI J X, YU Y, LIU G B, et al. Guaranteed-cost consensus for singular multi-agent systems with switching topologies [J]. *IEEE Transactions on Circuits and Systems*, 2014, 61(5): 1531 – 1542.

[9] CAO Y, STUART D, REN W, et al. Distributed containment control for multiple autonomous vehicles with double-integrator dynamics: algorithms and experiments [J]. *IEEE Transactions on Control Systems Technology*, 2011, 19(4): 929 – 938.

[10] LI Z, DUAN Z, REN W, et al. Containment control of linear multi-agent systems with multiple leaders of bounded inputs using distributed continuous controllers [J]. *International Journal of Robust and Nonlinear Control*, doi: 10. 1002/rnc, 2014: 3195.

[11] CUI R, GE S S, VOON E, et al. Leader–follower formation control of underactuated autonomous underwater vehicles [J]. *Ocean Engineering*, 2010, 37(17): 1491 – 1502.

[12] ABDESSAMEUD A, TAYEBI A. Formation control of VTOL unmanned aerial vehicles with communication delays [J]. *Automatica*, 2011, 47(11): 2383 – 2394.

[13] WANG Q, TIAN Y P, XU Y J. Globally asymptotically stable formation control of three agents [J]. *Journal of Systems Science and Complexity*, 2012, 25(6): 1068 – 1079.

[14] REN W. Information consensus in multivehicle cooperative control [J]. *IEEE Control Systems Magazine*, 2007, 27(2): 71 – 82.

[15] CHEN W S, WEN C Y, HUA S Y, et al. Distributed cooperative adaptive identification and control for a group of continuous-time systems with a cooperative PE condition via consensus [J]. *IEEE Transactions on Automatic Control*, 2014, 59(1): 91 – 106.

[16] RAO S, GHOSE D. Sliding mode control-based autopilots for leaderless consensus of unmanned aerial vehicles [J]. *IEEE Transactions on Control Systems Technology*, 2014, 22(5): 1964 – 1972.

[17] ZHENG Z, SPRY S C, GIRARD A R. Leaderless formation control using dynamic extension and sliding control [C] //*The 17th IFAC World Congress*. Korea, South: IFAC, 2008: 16027 – 16032.

[18] CAI N, CAO J, KHAN M J. A controllability synthesis problem for dynamic multi-agent systems with linear high-order protocol [J]. *International Journal of Control, Automation and Systems*, 2014, 12(6): 1366 – 1371.

[19] MENG Z, LIN Z, REN W. Robust cooperative tracking for multiple non-identical second-order nonlinear systems [J]. *Automatica*, 2013, 49(8): 2363 – 2372.

[20] GALZI D, SHTESSEL Y. UAV formations control using high order sliding modes [C] //*American Control Conference*. Minneapolis, USA: IEEE, 2006: 4249 – 4254.

[21] YOO S J. Distributed consensus tracking for multiple uncertain nonlinear strict-feedback systems under a directed graph [J]. *IEEE Transactions on Neural Networks and Learning Systems*, 2013, 24(4): 666 – 672.

[22] CAO Y, REN W, EGERSTEDT M. Distributed containment control with multiple stationary or dynamic leaders in fixed and switching directed networks [J]. *Automatica*, 2012, 48(8): 1586 – 1597.

[23] JI M, FERRARI-TRECATE G, EGERSTEDT M, et al. Containment control in mobile networks [J]. *IEEE Transactions on Automatic Control*, 2008, 53(8): 1972 – 1975.

[24] ZHAO Y, DUAN Z, WEN G, et al. Distributed finite-time containment control for second-order multi-agent systems [C] //*The 31st Chinese Control Conference*. Hefei: IEEE, 2012: 6202 – 6207.

[25] CAO Y, STUART D, REN W, et al. Distributed containment control for double-integrator dynamics: algorithms and experiments [C] //*American Control Conference*. Marriott Waterfront, USA: IEEE, 2010: 3830 – 3835.

[26] LIU H, XIE G, WANG L. Containment of linear multi-agent systems under general interaction topologies [J]. *Systems & Control Letters*, 2012, 61(4): 528 – 534.

[27] DONG X, SHI Z, LU G, et al. Formation-containment analysis and design for high-order linear time-invariant swarm systems [J]. *International Journal of Robust and Nonlinear Control*, 2014: DOI: 10.1002/rnc.3274.

[28] YU B, DONG X, SHI Z, et al. Formation-containment control for unmanned aerial vehicle swarm system [C] //*The 33rd Chinese Control Conference*. Nanjing: IEEE, 2014: 1517 – 1523.

[29] YU Y, SUN C Y, JIAO Z. Robust-decentralized tracking control for a class of uncertain MIMO nonlinear systems with time-varying delays [J]. *International Journal of Robust and Nonlinear Control*, 2014, 24(18): 3474 – 3490.

[30] YU Y, LU G, SUN C, et al. Robust backstepping decentralized tracking control for a 3-DOF helicopter [J]. *Nonlinear Dynamics*, 2015, doi: 10.1007/s11071-015-2209-8.

[31] SUN C, YU Y. *Robust Control for Strict-Feedback for Nonlinear Systems and Its Application* [M]. Beijing: Science Press, 2014: 19 – 43.

[32] SUN C, MU C, ZHANG R. *Hypersonic Vehicles Terminal Sliding Mode Control Technology* [M]. Beijing: Science Press, 2013: 1 – 23.

[33] WANG D, HUANG J. Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form [J]. *IEEE Transactions on Neural Networks*, 2005, 16(1): 195 – 202.

[34] LI T, GUO L, SUN C, et al. Further results on delay-dependent stability criteria of neural networks with time-varying delays [J]. *IEEE Transactions on Neural Networks*, 2008, 19(4): 726 – 730.

[35] HU X, SUN C, ZHANG B. Design of recurrent neural networks for solving constrained least absolute deviation problems [J]. *IEEE Transactions on Neural Networks*, 2010, 21(7): 1073 – 1086.

[36] JIANG Z P, REPPERGER D W. New results in decentralized adaptive non-linear stabilization using output feedback [J]. *International Journal of Control*, 2001, 74(7): 659 – 673.

[37] CAI N, CAO J W, MA H Y, et al. Swarm stability analysis of nonlinear dynamical multi-agent systems via relative Lyapunov function [J]. *Arabian Journal for Science and Engineering*, 2014, 39(3): 2427 – 2434.

[38] WANG X L, HONG Y G, HUANG J, et al. A distributed control approach to a robust output regulation problem for multi-agent linear systems [J]. *IEEE Transactions on Automatic Control*, 2010, 55(12): 2891 – 2895.

[39] LI W, ZHANG J F. Distributed practical output tracking of high-order stochastic multi-agent systems with inherent nonlinear drift and diffusion terms [J]. *Automatica*, 2014, 50(12): 3231 – 3238.

[40] LI Z, DUAN Z, LEWIS F L. Distributed robust consensus control of multi-agent systems with heterogeneous matching uncertainties [J]. *Automatica*, 2014, 50(3): 883 – 889.

[41] HOU Z G, CHENG L, TAN M. Decentralized robust adaptive control for the multiagent system consensus problem using neural networks [J]. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 2009, 39(3): 636 – 647.

[42] TAN F X, GUAN X P, LIU D R. Consensus protocol in networked multi-agent systems with non-balanced topology [J]. *Control Theory & Applications*, 2009, 26(10): 1087 – 1092.

[43] PAN H, NIAN X H, GUO L. Consensus control of higher-order multi-agent systems with delays [J]. *Control Theory & Applications*, 2013, 30(7): 909 – 915.

[44] LIN Z, FRANCIS B, MAGGIORE M. Necessary and sufficient graphical conditions for formation control of unicycles [J]. *IEEE Transactions on Automatic Control*, 2005, 50(1): 121 – 127.

[45] LI S, WANG X. Finite-time consensus and collision avoidance control algorithms for multiple AUVs [J]. *Automatica*, 2013, 49(11): 3359 – 3367.

[46] BOYD S, VANDENBERGHE L. *Convex Optimization* [M]. Cambridge: Cambridge University Press, 2004: 35 – 47.

[47] MENG Z, REN W, YOU Z. Distributed finite-time attitude containment control for multiple rigid bodies [J]. *Automatica*, 2010, 46(12): 2092 – 2099.

[48] YOO S J. Distributed adaptive containment control of uncertain nonlinear multi-agent systems in strict-feedback form [J]. *Automatica*, 2013, 49(7): 2145 – 2153.

[49] GE S S, WANG C. Adaptive neural control of uncertain MIMO nonlinear systems [J]. *IEEE Transactions on Neural Networks*, 2004, 15(3): 674 – 692.

作者简介:

余　瑶　(1982–), 女, 博士, 副教授, 目前研究方向为非线性控制、多智能体系统、鲁棒控制、时延系统, E-mail: yuyao@ustb.edu.cn;

任　昊　(1991–), 男, 硕士研究生, 目前研究方向为无人机群系统分布式协同控制, E-mail: renh618@126.com;

张　兰　(1968–), 女, 硕士生导师, 副教授, 目前研究方向为自适应信号处理、多智能体系统, E-mail: zhanglan2013@ustb.edu.cn;

孙长银　(1975–), 男, 博士生导师, 教授, 目前研究方向为智能控制、飞行控制、模式识别、最优理论、多智能体系统等, E-mail: cys@ustb.edu.cn.