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带有网络拓扑优化的分布式预测控制方法

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摘要: 通常在大系统中, 全局信息优化的系统, 其性能要高于局部信息优化系统. 全局信息优化的算法由于大系统的复杂程度往往不可行. 所以通常会用分布式算法来解决此类问题. 在分布式算法中, 为了获得更好的系统性能, 要尽可能多的采用更多的信息信息交换, 然而这样会带来信息网络的负担增大. 本文在预测控制性能指标中引入通信代价, 并提出了一种随着系统状态变化的通信网络拓扑切换方法. 文中给出了该算法在供水管网动态模型中的仿真结果, 表明本方法的可行性.

关键词: 分布式系统; 预测控制; 拓扑优化

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Distributed model predictive control with optimal network topology for large scale systems

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Abstract: Generally, the performance of a large-scale system in global optimization is better than that in local optimization. To obtain a better performance, we need more information interchanges. However, in a distributed model prediction algorithm, this will increase the workload for an information network, making global optimization inapplicable in a large-scale system with complexity. To deal with this problem, we introduce the communication cost in the performance index and develop a novel method for switching the communication-network topology according to variations in system states. This method has been applied to the simulation experiment of a dynamical model of a water supply network system, demonstrating the feasibility of the proposed algorithm.

Key words: distributed system; model predictive control; topology optimization

1 Introduction

Centralized and decentralized control define the two extremes in distributing the decision making in a large scale system. Generally, the centralized control of systems may lead to a very complex computational problems or be less robust to disturbance[1]. The decentralized control can have a degrade performance using only local information in subsystems with coupling relationship. Distributed model predictive control (DMPC) in which the controllers can communicate information is an important control strategy at the current moment with the increasing of the degree of the complexity of control plants in the technological societal or environmental processes[2–4]. The application of the communication network allows the potential higher performance by exchanging information between controllers of all subsystems . That means the communication network is the key element in the distributed systems so that we have to improve the efficiency of the communication network.

Distributed MPC can be classified by some aspects such as information exchange protocol (i.e., non-iterative[5] and iterative algorithm[6]), the form of the performance index function (i.e., cooperative[7] and non-cooperative[8]), and so on[9]. In [5], the optimization performance index of each local MPC considers not only the performance of the related subsystems but also that of its neighbours. In [7], the each local MPC controller optimizes the same objective function in parallel without a coordinator, allowing the distributed optimization to be terminated at any iterate. In [8], an impacted-region optimization is used in the DMPC design by redefining the impact region of a subsystem according to the coordination strategy. In [10], the local

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PC controller needs the reference trajectories of the state variables of its neighbors by the neighbor-to-neighbor communication.

As all we know, existing DMPC algorithms mostly use a static network topology or a fixed composition of neighborhoods during the control process (eg. [11–12]). Motivated by these issues, this paper proposes a non-iterative and non-cooperative distributed MPC algorithm considering topology optimization.

In the existing literatures, there are some optimal control methods considering the design of communication network. [13] proposes an approach to simultaneous optimization of network topology and control law with respect to a cost function which combines a quadratic performance criterion with costs associated to the presence of topology structure. [14] extends the results to the case that some communication links are prone to failure by reconfiguring subsystem's local controllers. [15] and [16] use the game theory to analyze impact of each link and proposes the optimal communication topology, by the optimal control method. [17] proposes a novel two-layer control architecture that allows to jointly improve the system performance with the design of communication topology. [18] proposes a hierarchical control scheme for large-scale systems by two layers, in which the top layer is designed to find the optimal network topology.

In this paper, we propose a novel topology switching condition in the communication topology optimization for the DMPC algorithm and discuss the stability of the proposed algorithm. During the dynamic process, the each MPC controller has a time-varying communication topology for the current performance with the current state.

The rest of the paper is organized as follows. In Section 2, a description of distributed subsystems and communication network topology is provided. In Section 3, a novel Distributed MPC algorithm with communication cost is introduced. In Section 4, the stability analysis of the DMPC is derived. In Section 5, a simulation is given to show the feasibility of the algorithm.

2 Problem description

2.1 The description of distributed subsystems

The whole system is partitioned into several interconnected subsystems controlled by several controllers which are connected by network shown in the Fig.1. There are two levels in the whole system. The one is the control plant, the other is information network. The distributed controllers coordinates with each other by the information network. The topology of the communication network is described by the undirected graph $G = (\mathcal{N}, L)$ be an undirected graph with n nodes and m undirected edges. $\mathcal{N} = \{1, 2, \cdots, n\}$ denotes the sets of all vertices and $L = \{l^1, l^2, \cdots, l^m\}$ denotes the sets of all edges. For a system without constraints the discrete state space equation is written like this:

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

where $x(k+1) := [x_1^{\mathrm{T}} \ \cdots \ x_n^{\mathrm{T}}]^{\mathrm{T}}$ and $u(k+1) := [u_1^{\mathrm{T}} \ \cdots \ u_n^{\mathrm{T}}]^{\mathrm{T}}$. x_i and u_i $(i = 1, \cdots, n)$ are the local states and the inputs respectively. The whole system composes n subsystems. Generally each subsystem can be written as follow:

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + \sum_{j\neq i}(A_{ij}x_j + B_{ij}u_j(k)), \tag{2}$$

where $x_i \in \mathbb{R}^{n_{x_i}}$ is the local state of the subsystem $i, u_i \in \mathbb{R}^{n_{u_i}}$ is the input of the subsystem $i(i = 1, \cdots, n)$. A_{ij} represents the coupled states and B_{ij} represents the coupled inputs between subsystems. If A_{ij} or B_{ij} is zero, the subsystem i and the subsystem j does not have coupled states or coupled input sequences.

The topology mode $\Lambda \subseteq L$ means that the set of links are enabled. There are some communication components in a topology mode Λ which can be defined as $C \in \Lambda$ and also called coalitions. The dynamic equation for the subsystems in the same communication components is given by

$$x_C(k+1) = A_Cx_C(k) + B_Cu_C(k) + \sum_{j\in\mathcal{N}-C}(A_{Cj}x_j + B_{Cj}u_j(k)), \tag{3}$$

where $x_C = (x_i)_{i\in C}$ is the set of the states of the subsystems in the communication component C. $U_C = (u_i)_{i\in C}$ is the inputs in the communication component C made up by local input u_i. A_C and B_C are the corresponding matrices in the communication component C made up of local matrices.

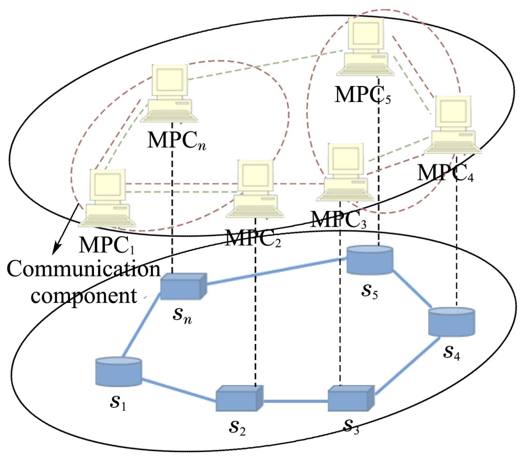

Fig. 1 Distributed predictive control with dynamic topology

2.2 The description of communication cost

For an edge l connecting node i and node j, we define a column vector $a_l \in \mathbb{R}^n$, where $a_{l_i} = 1$, $a_{l_j} = 1$, and other elements are zeros. We can get an adjacency matrix A which is used to illustrate the connection of a graph.

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{pmatrix}, \tag{4}$$

where $a_{i,j} \in \{1, 0\}$ are given by

$$a_{i,j} = \begin{cases} a_{l_i}, & l \in L, \\ 0, & \text{otherwise.} \end{cases}$$

For each communication link between the subsystem i and the subsystem j, which can be enabled or disabled, we define a performance index for the communication cost.

$$J_{c,ij} = w_{ij} * a_{i,j}, \tag{5}$$

where w_{ij} is a communication cost weight, which is chosen by the hardware costs, energy consumption for communicating information, distance between subsystems, or the number of hops in a multi-hop wireless network. For the global system, the total communication cost is written by

$$J_c = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} * a_{i,j}. \tag{6}$$

Obviously, we can see the global optimization performance is best but this way causes the highest communication cost. In the distributed controllers, not all interactions between subsystems are necessary. We use a performance index considering the communication cost to choose the optimal communication topology for the best performance for the current states.

3 The coordinated distributed model predictive control

3.1 Finite horizons of performance index

For a linear system without constraints, we define a performance index with terminal cost function as

$$J_i(k) = \sum_{t=0}^{M-1} [\| x_i(k+t|k) \|^2_{Q_i} + \| u_i(k+t|k) \|^2_{R_i}] + \| x_i(k+M|k) \|^2_{P_i}, \tag{7}$$

where M is the predictive horizon, P_i is the terminal penalty function for each subsystem. The matrix P_i must meet the conditions as

$$P_i > 0, \tag{8}$$

$$(A_i + B_i H_i)^{\mathrm{T}} P_i (A_i + B_i H_i) + Q_i + H_i^{\mathrm{T}} R_i H_i - P_i < 0, \tag{9}$$

where H_i is the LQ feedback control law after M steps from the initial state $x(k+M|k)$. Applying the Schur's complement to the inequation, we can get the LMI like this:

$$\begin{bmatrix} P_i^{-1} & G_i & P_i^{-1} Q_i^{1/2} & (P_i^{-1})^{\mathrm{T}} H_i^{\mathrm{T}} R_i^{1/2} \\ * & P_i^{-1} & 0 & 0 \\ * & * & I & 0 \\ * & * & * & I \end{bmatrix} > 0,$$

where $G_i = P_i^{-1} A_i^{\mathrm{T}} + (P_i^{-1})^{\mathrm{T}} H_i^{\mathrm{T}} B_i^{\mathrm{T}}$.

As mentioned before, the performance for subsystems in the same communication component is written as

$$J_C(k) = \sum_{t=0}^{M-1} [\| x_C(k+t|k) \|^2_{Q_C} + \| u_C(k+t|k) \|^2_{R_C}] + \| x_C(k+M|k) \|^2_{P_C}, \tag{10}$$

where Q_C and R_C are the weighting matrix for the states and inputs of the subsystem i respectively, P_C is the terminal penalty for each subsystem, which can be derived by the similar Eq.(9).

For the global system, the receding-horizon optimization is

$$\min_{u,\Lambda} \sum_{C \in \Lambda} J_C(k) + J_c(k). \tag{11}$$

The problem is a dynamic programming optimization with mixed integer variables, which is hard to solve. From the two parts of the equation, the first particularly stage cost does not contain explicitly the integer variables. Any topology mode Λ corresponds to a partition of the global system , which can produce different coalitions.

3.2 Topology switching condition

The discrete part of the Eq.(11) is up to the choice of the network topology. We define a new performance index:

$$J_T(k) = x_{\mathcal{N}}^{\mathrm{T}} P_\Lambda x_{\mathcal{N}} + |\Lambda(k)|, \tag{12}$$

where $|\Lambda(k)|$ is $J_c(k)$ of the topology mode Λ.

Assumption 1 For every subsystem, there exits a decoupled static feedback $u_i = H_i x_i$ such that $A_{di} = A_{ii} + B_{ii} H_i$ is Shur stable, and the close-loop system $x(k+1) = A_c x(k)$ is asymptotically stable, where $A_c = A + BH$ and $H = \mathrm{diag}\{H_1, H_2, \cdots, H_n\}$.

This assumption is usually found in the design method of stabilizing DMPC, e.g, in the paper [8, 10, 19]. this assumption inherently presumes that the coupling between the subsystems is sufficiently weak, and that each subsystem can be stabilized by a decentralized control $H_i x_i$. The feedback control law H_i is derived by the LMI in the book [20] in the continuous mode. Obviously, we can get the discrete time mode based on the continuous mode.

Based on these, we can get a theorem to calculate the P_Λ of the different communication topologies defined by Λ.

Theorem 1 Let $\varLambda \subseteq \mathcal{L}$ be a set of active links in a distributed control system. The dynamics of the whole system are given by $A_\varLambda = (A_{ij})_{i,j\in\mathcal{N}}$ and $B_\mathcal{N} = (B_{ij})_{i,j\in\mathcal{N}}$. And its stage cost weight is defined by $Q_\mathcal{N} = \mathrm{diag}\{Q_i\}_{i\in\mathcal{N}}$ and $R_\mathcal{N} = \mathrm{diag}\{R_i\}_{i\in\mathcal{N}}$. The local model predictive control law is $K_\varLambda$. If there exist matrices $P_\varLambda = P_\varLambda^\mathrm{T} = (P_{ij})_{i,j\in\mathcal{N}}$ such that the following constraints are satisfied

$$\begin{bmatrix} P_\varLambda^{-1} & G_\varLambda & P_\varLambda^{-1}Q_\mathcal{N}^{1/2} & (P_\varLambda^{-1})^\mathrm{T}K_\varLambda^\mathrm{T}R_\mathcal{N}^{1/2} \\ * & P_\varLambda^{-1} & 0 & 0 \\ * & * & I & 0 \\ * & * & * & I \end{bmatrix} > 0,$$

where $G_\varLambda = P_\varLambda^{-1}A_\mathcal{N}^\mathrm{T} + (P_\varLambda^{-1})^\mathrm{T}K_\varLambda^\mathrm{T}B_\mathcal{N}^\mathrm{T}$,

$$P_{ij} = 0,\ \forall i,j \in \mathcal{N}/\varLambda \text{ such that } i \in C,\ j \notin C, \tag{13}$$

the matrix $P_\varLambda$ satisfies

$$x_\mathcal{N}^\mathrm{T}P_\varLambda x_\mathcal{N} = \sum_{C\in\mathcal{N}/\varLambda} x_C^\mathrm{T}P_C^\varLambda x_C \geqslant \sum_{j\in N}\sum_{n=0}^{\infty} l_j(n), \tag{14}$$

and all the communication constraints imposed by the network mode $\varLambda$ and stabilize the whole system asymptotically. Besides, the diagonal elements of $P_\varLambda$ can be taken as the $\mathrm{diag}\{P_1,\cdots,P_C\}, C\in\mathcal{N}/\varLambda$.

Proof Given the local MPC law $u = K_\varLambda x_\mathcal{N}$ and the Eq.(1), we can get that

$$\begin{aligned} & x_\mathcal{N}(k+1) = \\ & A_\mathcal{N}x_\mathcal{N}(k) + B_\mathcal{N}K_\varLambda x_\mathcal{N}(k) = \\ & (A_\mathcal{N} + B_\mathcal{N}K_\varLambda)x_\mathcal{N}(k). \end{aligned} \tag{15}$$

So, the global stage cost can be written like this:

$$\begin{aligned} & J_\mathcal{N}(k) := \\ & x_\mathcal{N}(k)^\mathrm{T}Q_\mathcal{N}x_\mathcal{N}(k) + x_\mathcal{N}(k)^\mathrm{T}K_\varLambda^\mathrm{T}R_\mathcal{N}K_\varLambda x_\mathcal{N}(k) = \\ & x_\mathcal{N}(k)^\mathrm{T}(Q_\mathcal{N} + K_\varLambda^\mathrm{T}R_\mathcal{N}K_\varLambda)x_\mathcal{N}(k). \end{aligned} \tag{16}$$

Applying the Shur's theory to the LMI matrix and according to the Eq.(9), we can get a similar inequation

$$\begin{aligned} & (A_\mathcal{N} + B_\mathcal{N}K_\mathcal{N})^\mathrm{T}P_\mathcal{N}(A_\mathcal{N} + B_\mathcal{N}K_\mathcal{N}) + \\ & Q_\mathcal{N} + K_\mathcal{N}^\mathrm{T}R_\mathcal{N}K_\mathcal{N} - P_\mathcal{N} < 0. \end{aligned} \tag{17}$$

Substitute the Eq.(15) and the Eq.(16) into the Eq.(17), we can derive that

$$\begin{aligned} & x_\mathcal{N}(k+1)^\mathrm{T}P_\varLambda x_\mathcal{N}(k+1) - \\ & x_\mathcal{N}(k)^\mathrm{T}P_\varLambda x_\mathcal{N}(k) + J_\mathcal{N}(k) \leqslant 0. \end{aligned} \tag{18}$$

By the same way, we can get

$$\begin{aligned} & x_\mathcal{N}(k+M-1)^\mathrm{T}P_\varLambda x_\mathcal{N}(k+M-1) - \\ & x_\mathcal{N}(k+M-2)^\mathrm{T}P_\varLambda x_\mathcal{N}(k+M-2) + J_\mathcal{N}(k) \leqslant 0, \end{aligned} \tag{19}$$

sum these inequations form k to $k+M-1$, we can get

$$\begin{aligned} & x_\mathcal{N}(k+M-1)^\mathrm{T}P_\varLambda x_\mathcal{N}(k+M-1) + \\ & \sum_{t=k}^{t=k+M-1} J_\mathcal{N}(k) \leqslant x_\mathcal{N}(k)^\mathrm{T}P_\varLambda x_\mathcal{N}(k). \end{aligned} \tag{20}$$

That is

$$x_\mathcal{N}(k)^\mathrm{T}P_\varLambda x_\mathcal{N}(k) \geqslant \sum_{t=k}^{t=k+M-1} J_\mathcal{N}(t). \tag{21}$$

Under the same network topology, we can get the same MPC law, i.e., the same $P_\varLambda$. Then, according to the Eq.(21) we can get

$$x_\mathcal{N}(k+1)^\mathrm{T}P_\varLambda x_\mathcal{N}(k+1) \geqslant \sum_{t=k+1}^{t=k+M} J_\mathcal{N}(t). \tag{22}$$

From the Eq.(18), we can also get

$$x_\mathcal{N}(k+1)^\mathrm{T}P_\varLambda x_\mathcal{N}(k+1) \leqslant x_\mathcal{N}(k)^\mathrm{T}P_\varLambda x_\mathcal{N}(k). \tag{23}$$

The upper bound of the MPC performance index decreases under the same topology. Thus, we can get the global stability for the system with MPC controllers under the same communication topology.

Remark 1 The Theorem 1 supplies a method to get the upper bound parameter $P_\varLambda$ under the communication topology $\varLambda$ with the control law $K_\varLambda$. Different communication topology has different $P_\varLambda$ and different MPC control law $K_\varLambda$. The LMI matrix can not be calculated every sample time. The $P_\varLambda$ can be saved and reused when the corresponding communication topology is used again with the same $K_\varLambda$. If MPC control law $K_\varLambda$ changes, the $P_\varLambda$ needs to be recalculated even under the same topology.The MPC control law given in the Theorem 1 must be stable, otherwise, the LMIs may have no solution.

In the paper [13–15] and [16], they all use the Eq.(12) or similar equation as the topology switching condition, i.e., sum the communication links cost to the stage cost directly and find the minimum cost summation and the corresponding communication topology. Actually, the communication cost and the stage cost often represent different physical means and are not in the same order of magnitude. In some possible condition, the initial state is very small which leads to the stage cost smaller than the communication links cost. By the way, the on-off the communication frequently switching consumes much more energy than keeping it on. Thus, in order to get a more reasonable switching condition we propose a condition instead of the Eq.(12) like this:

$$\begin{aligned} & \frac{x_\mathcal{N}^\mathrm{T}P_{\varLambda(k_0)}x_\mathcal{N} - x_\mathcal{N}^\mathrm{T}P_{\varLambda(k_1)}x_\mathcal{N}}{|x_\mathcal{N}^\mathrm{T}P_{\varLambda(k_1)}x_\mathcal{N}|} > \\ & \frac{||\varLambda(k_0)| - |\varLambda(k_1)||}{|\varLambda(k_1)|}. \end{aligned} \tag{24}$$

That means we consider the relative change ratio of stage cost and the communication cost. If the relative change ratio of the stage cost is bigger than the relative change ratio of the communication links cost, the system change the communication network topology for the better performance.

Above all, we give out a novel DMPC algorithm (the related chart shown in the Fig. 2):

Algorithm 1

Step 1 Get the adjacency matrix A for the current network topology Λ, and find out the communication components C. Then, compute the MPC control law for each communication components. According to the Theorem 1, solve the LMI matrix for the P_Λ.

Step 2 Store the P_Λ, and calculate the stage cost and communication links cost for the topology Λ. Repeat this process for all possible topologies (i.e., 2^m).

Step 3 If the time t is a multiple of time interval T_Λ, each subsystem shares its states so that we can choose the optimal topology for the current state, according to the condition (24). Otherwise, the system still use the current communication topology.

Step 4 Each subsystem updates state in its communication component. Go to Step 3.

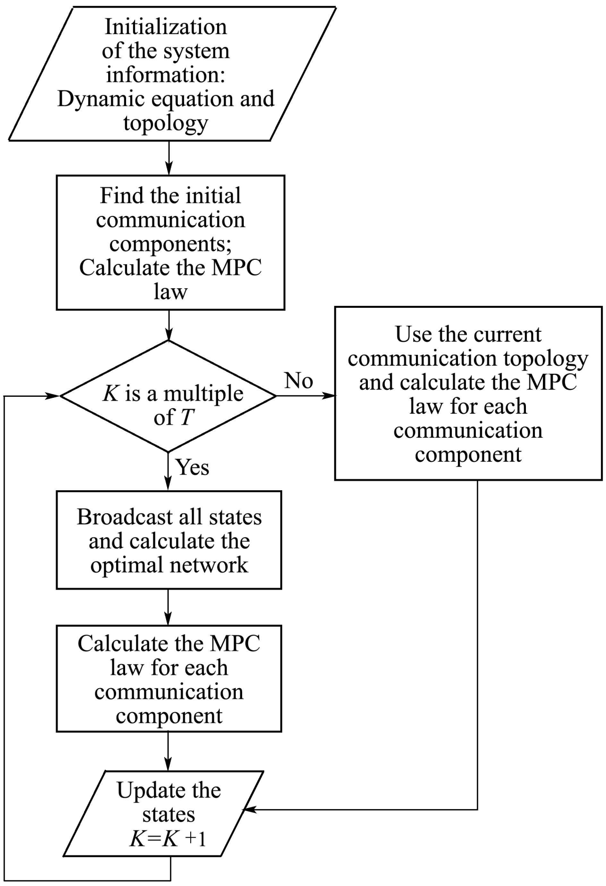

Fig. 2 Algorithm flow chart

Remark 2 The proposed algorithm shows a solution for a system without constraints. So, we can get the explicitly solution for the optimization in the MPC method. The number of the possible communication topology structures is 2^m. The numbers will increase as an exponential rate which can improve the complexity of finding the optimal topology structure. So the links in the topology must be the necessary in the system.

According to the Theorem 1, if the change of the communication topology can make relative bigger change of the performance, the condition works. In the Theorem 1, the $x_{\mathcal{N}}(k)^{\mathrm{T}} P_\Lambda x_{\mathcal{N}}(k)$ decreases with the time k which means there must be some k that $x_{\mathcal{N}}(k)^{\mathrm{T}} P_\Lambda x_{\mathcal{N}}(k) \approx 0$. The condition of the switching topology is not only the condition (24). The condition should be revised with that if $x_{\mathcal{N}}(k)^{\mathrm{T}} P_\Lambda x_{\mathcal{N}}(k) \approx 0$, the communication network topology chooses the least communication costs $\Lambda_{\min}$.

In order to complete the algorithm, we introduce the threshold δ which is a trivial constant. If $|x_{\mathcal{N}}(k)^{\mathrm{T}} P_\Lambda x_{\mathcal{N}}(k)| \leqslant \delta$ is satisfied, the system is in the stable situation and the network communication topology changes to the $\Lambda_{\min}$. The judgement of the threshold should be ahead of the the switching condition (24) so that it can reduce the unnecessary switch judgement.

3.3 Stability analysis

During proposed algorithm, we can see there could be some switchings in the dynamic process. It is known that the switching between different stable control plants can result in an unstable dynamic system. It is necessary to study the stability of the proposed algorithm.

We assume that the initial state is at the time k_0. We can get the performance upper bound $x_{\mathcal{N}}^{\mathrm{T}} P_{\Lambda(k_0)} x_{\mathcal{N}}$. According to the Theorem 1, the system can be asymptotically stable with the given local MPC control as long as the communication topology keeps the same structure because of the $x_{\mathcal{N}}^{\mathrm{T}} P_{\Lambda(k_0)} x_{\mathcal{N}}$ decreasing with the time.

If the switching condition (24) is satisfied, the topology needs to change for the better performance, assuming the switching time point k_1. From the condition (24), we know $x_{\mathcal{N}}^{\mathrm{T}} P_{\Lambda(k_0)} x_{\mathcal{N}} > x_{\mathcal{N}}^{\mathrm{T}} P_{\Lambda(k_1)} x_{\mathcal{N}}$. After the topology switching, $x_{\mathcal{N}}^{\mathrm{T}} P_{\Lambda(k_1)} x_{\mathcal{N}}$ decreases with time. It can be concluded that the performance index decreases with time under either the same communication topology structure or different communication topology structures. So, the system is asymptotically stable under the Theorem 1 and the switching condition (24).

Remark 3 From the proposed DMPC algorithm, we can see that there is an invisible supervision coordinator in the system which can know global system states every time interval T_Λ. The terminal penalty in performance index of local systems can reduce the complexity of calculation of the LMIs for the matrix P_Λ.

4 Validation for water supply systems

In this section we use a water supply network as an example[21] to show feasibility of the algorithm proposed in the paper. According to the book [21], the water supply network can be regarded as consisting of hydraulic elements interconnected at nodal points. The nodes with reservoir contain the dynamic elements. We focus on the dynamic nodes so that we assume that it is possible to manipulate the flow of the pipes that connect the tanks without loss of generality.

We choose a water supply network with 16 dynamic nodes. The example contains sixteen subsystems shown in Fig. 3, represented by circles. Among the sixteen subsystems, we choose four subsystems $\{1, 2, 5, 6\}$ which can use the proposed methods considering the topology optimization, and we choose four subsystems$\{3, 4, 7, 8\}$ use the decentralized MPC method. The remainder of subsystems$\{9, \cdots, 16\}$ are under the centralized MPC control method.

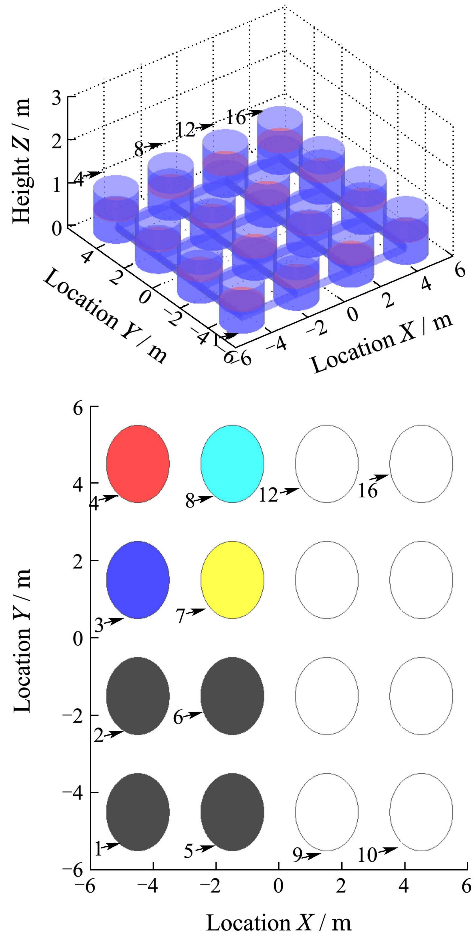

Fig. 3 Water supply network with 16 tanks

In Fig. 4, there are four subsystems $\{1, 2, 5, 6\}$ using the proposed method, the connection relationship between subsystems is also defined: there are four links in this example, represented by arrows with roman numbers. There are $2^4 = 16$ communication modes in the dynamic process because of four links in the system, represented by the arabic numbers $\{0 - 15\}$.

A link is enabled in the topology whose numbers appear next to the link in Fig. 4. For example, $\{1, 5, 6, 7\}$ means the link I is enabled in the mode $\{1, 5, 6, 7\}$. Besides, the mode $11 - 14$ means three of four links are enabled such that there is only one communication component in the system, and the mode 15 means all links are enabled. The mode $11 - 15$ has a same communication component. From Fig. 4, we also get $n = 4$ and $m = 4$. The state space equations of the subsystems in the system are defined as follow:

$$x_i(k+1) = x_i(k) + \frac{T_{\mathrm{s}}}{A_{ij}} \sum_{j \in N_i} u_{ij}, \qquad (25)$$

where x_i is the level of the water stored in tank i and A_i is its surface, T_{s} is the time step length, u_{ij} is the flow through the pipe connecting tank i and j, and N_i is the set of tanks connected to tank i.

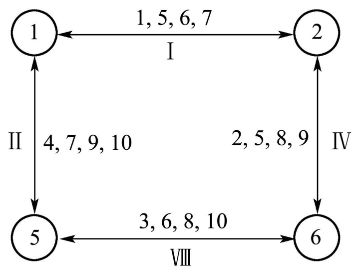

Fig. 4 Four subsystems from the water supply network with enable links

We choose $T_A = 0.6$, $T_{\mathrm{s}} = 0.15$ and that means every four simulation step the communication network topology is revised. The Weighting coefficient of the communication topology links is 1, i.e., $w_{ij} = 1$.

Assumption 2 For every subsystem, the predictive horizon M is chosen such that stability can be guaranteed without terminal constraint for a set of initial conditions.

This assumption is used to guarantee the stability for MPC controllers without the terminal functions in performance index. It is the special issue of Theorem 1. According to Assumption 2, we can choose the parameters of the local MPC. So, the predictive horizon and the control horizon are both 5, i.e., $M = 5$. Q_{c} and R_{c} are the unit matrices of the proper size.

The first communication topology is optimal at the $k = 0$ initial level of all tanks is 0.5. The total simulation time step is 300. At the time 150, the water reference of each tank has got a disturbance from 0.5 to $0.25 + 0.5 * \mathrm{random}$, which is the standard Gauss white noise.

From Fig. 5 and Fig. 6, the tank $\{1,\ 2,\ 5,\ 6\}$ using proposed method is better than tank 3, 4, 7, 8 using decentralized on issue the tracking reference obviously. In order to show the degree of tracking performance, we define the relative errors for each tank $\sqrt{\sum_k (\frac{y(k) - r(k)}{r(k)})^2}$. The results are shown in Table 1, compared with two extreme conditions.

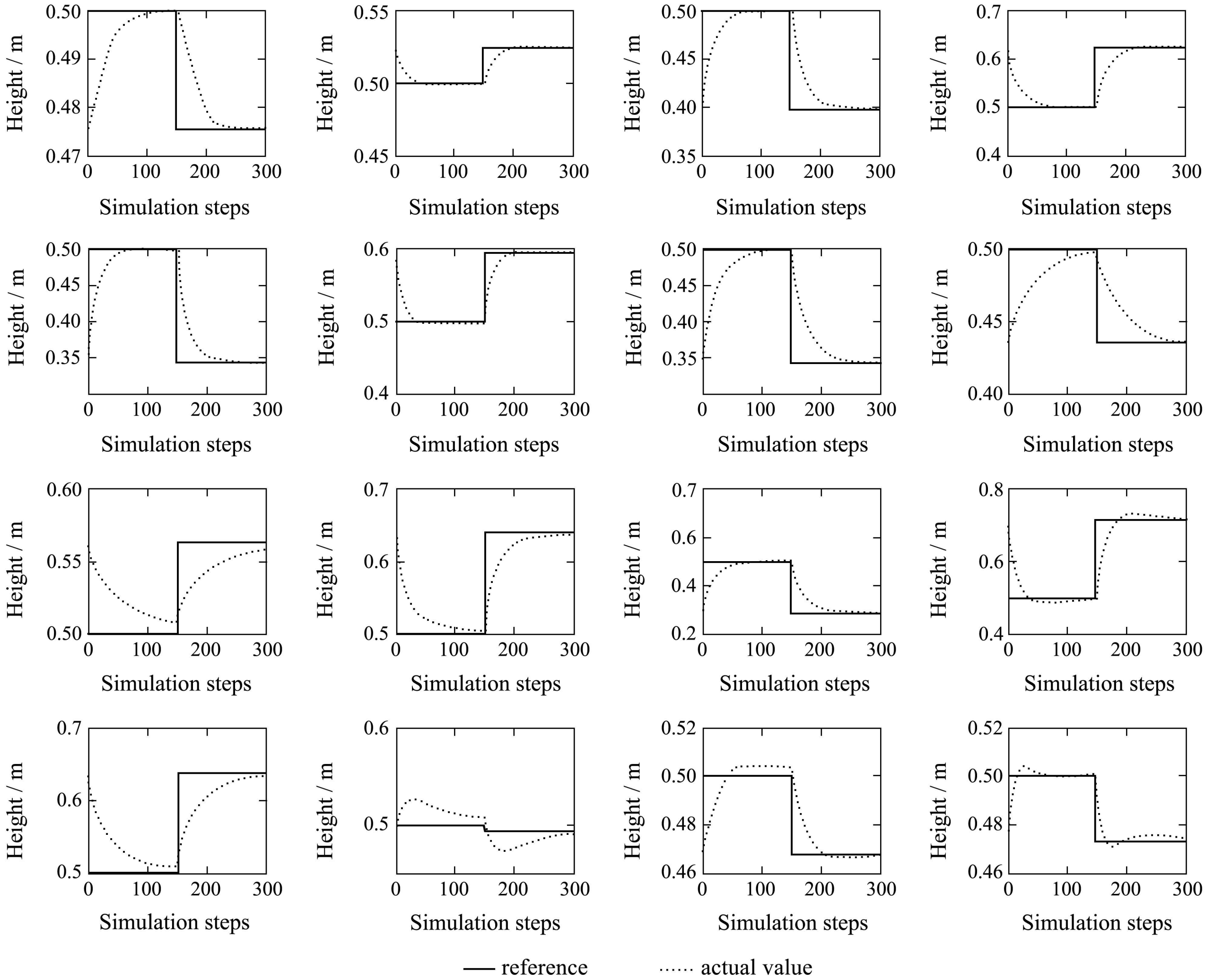

Fig. 5 States of the system

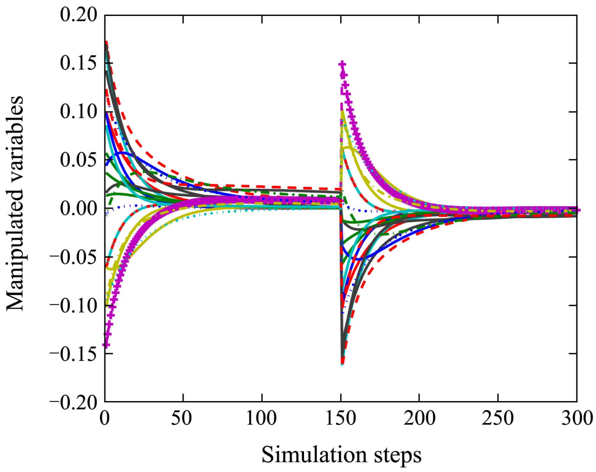

Fig. 6 Input vectors

Table 1 The relative errors

Proposed	Decentralized	Centralized
1.3399	1.5532	1.0618

Figure 7 shows the evolution of the communication topology, where the mode 12 shown in the figure means mode 11 including links $\{2, 3, 4\}$.

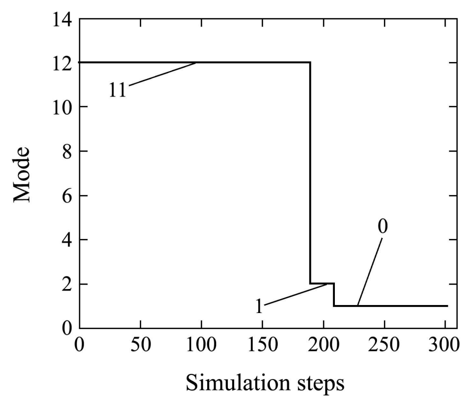

Fig. 7 Switching mode

At the beginning, the optimal topology mode is 11, after a time interval T_Λ, the optimal topology mode is 1, which means tank 1 and tank 2 cooperate together, and tank 5 and tank 6 work independently. At last, the optimal topology mode is 0, which means all tanks work independently, while the level tracking is stable. Considering the communication costs, the global MPC control has taken much redundant communication cost. The decentralized MPC controllers method taken least communication cost, but get the worst performance. The proposed DMPC algorithm has got a well trade-off between performance and the communication cost.

The simulation result shows the feasibility of the proposed method.

5 Conclusions

In this paper, we propose a novel distributed model predictive control considering the time-varying communication network topology of the system. We give out the design method of the DMPC and a stability analysis. The proposed algorithm has got a well balance trade-off between the performance and communication cost. There is a lot of space to improve the proposed algorithm. For instance, such as the coupling between subsystem is weak, and there is no constraint.

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