DOI: 10.7641/CTA.2015.50264

通信延时和数据丢包下事件驱动的多智能体系统一致性研究

崔彦良[1], 费敏锐[1], 杜大军[1†], 李 慷[2]

(1. 上海市电站自动化技术重点实验室 上海大学 机电与自动化学院, 上海 200072;

2. 英国贝尔法斯特女王大学 电子电气与计算机学院, 英国 贝尔法斯特 BT9 5AH)

摘要: 本文研究受网络通信延时和数据随机丢包的多智能系统一致性问题, 探索事件驱动的分布式协同控制策略. 首先针对两类普遍应用的事件触发器, 提出了一个可用于选择触发策略的触发频率比较方法. 然后提出了分布式协同控制律以保证系统的渐近一致性, 并给出了相应的时滞依赖Markov切换控制器设计新方法. 本文所提的控制策略不仅保证系统一致性目标, 而且能显著减少通信数据传输量并降低控制器计算负担. 最后,通过仿真算例验证了所提方法的有效性.

关键词: 事件驱动; 渐近一致性; 多智能体系统; 通信延时; 数据丢包

中图分类号: TP273　　　**文献标识码**: A

Event-triggered consensus of multi-agent systems with data transmission delays and random packet dropouts

CUI Yan-liang[1], FEI Min-rui[1], DU Da-jun[1†], LI Kang[2]

(1.Shanghai Key Laboratory of Power Station Automation Technology, School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200072, China;

2. School of Electronics, Electrical Engineering and Computer Science, Queen's University Belfast, Belfast BT9 5AH, UK)

Abstract: This paper investigates the event-triggered consensus of multi-agent systems (MASs) with time delay and random packet dropout in data transmission. The event-triggered scheme is employed for broadcasting fewer necessary data to the neighbor agents through communication networks only when its threshold is violated. Based on prior topology information of the MASs, an approximate frequency comparison method is firstly proposed to choose the suitable one from two typical event-triggers. For guaranteeing the asymptotical consensus of MASs as well as enhancing system robustness against the communication drawbacks, a distributed Markov switching controller is designed. The sufficient delay dependent stability conditions are obtained and the corresponding controller design methods are subsequently presented. With the proposed strategy, it is shown that the amount of communication packages and the controller updates can be significantly reduced without introducing any significant negative effect on the consensus. Finally, the effectiveness of the proposed theoretical approach is validated through several numerical examples.

Key words: event-trigger; asymptotical consensus; multi-agent systems; time-delay; packet dropout

1 Introduction

Multi-agent systems (MASs) are coupled team of agents that exchange data through communication facility so as to solve problems which are beyond the individual capability. One of the fundamental characteristics for MASs is known as the consensus and consensus of MASs has found a variety of applications such as formation control[1], urban traffic[2], smart grid[3], robot system[4], missile guidance[5] and so on.

The objective of consensus for MASs is to probe feasible control protocol that enable the MASs to reach an agreement on certain interest[6]. To achieve the objective, the agents need to exchange their shared information via communication networks. Although the network brings convenience and effectiveness, however, a significant negative effect is that the communication induced drawbacks may degrade the control performance or even deteriorate the system stability. These include time-varying delays[7], data packet dropouts[8], quantization deviation[9], exogenous disturbances[10], switching topology[11], etc. To reduce the negative effect of the communication defections so as to improve the con-

Received 2 April 2015; accepted 19 June 2015.

†Corresponding author. E-mail: ddj@shu.edu.cn; Tel.: +86 21-56331934.

Supported by Key Project of Science and Technology Commission of Shanghai Municipality (14JC1402200, 15JC1401900), National Key Scientific Instrument and Equipment Development Project (2012YQ15008703), National Science Foundation of China (61473182) and Shanghai Rising-Star Program (13QA1401600).

sensus performance, such communication constraints should be properly considered in the design of consensus. For simplicity, only data transmission time-delay and packet dropout are considered in this paper.

In reality, not only the communication network is limited but also the energy resource and computation capability of the agent are inevitably constrained. Therefore, a better consensus strategy should consider saving the limited communication capacity and energy supply. To fulfil the above objectives, a possible solution is known as the event-triggered control. With the event-triggered scheme, the control tasks are only triggered by the occurrence of an event instead at a fixed sampling period. Due to reduced data transmissions and control task executions, the event-triggered control enables cost saving and communication reduction.

Recently, some event-triggered consensus schemes for MASs are reported. For example, an event-triggered consensus control for MASs with combinational measurements is investigated in [12]. A disturbance perturbed event-triggered mean square consensus is analyzed in [13]. In [14], an event-triggered control scheme for MASs with stochastic nonlinearities is discussed. In the aforementioned literatures[12–14], the event-triggers can be summarized as $\mathrm{e}^{\mathrm{T}}(kT)(I_N \otimes \varPhi)e(kT) = \delta x(kT)(I_N \otimes \varPhi)x(kT)$, where $x(kT)$ and $e(kT)$ denote the state and the error, respectively; δ and $\varPhi$ are parameters of event-trigger; I_N denotes an identity matrix. Apparently, the topology information is not involved in the above-mentioned literatures.

Different from the above-mentioned results, some articles focus on topology relevant event-triggered consensus, wherein the proposed event-trigger can be simplified as $\mathrm{e}^{\mathrm{T}}(kT)(I_N \otimes \varPhi)e(kT) = \delta x(kT)(L \otimes \varPhi)x(kT)$, where L is the Laplacian matrix. For example, an asymptotical event-triggered consensus issue for single integrator MASs is introduced in [15]. Two observer based event-triggered consensus are compared in [16]. Considering transmission time-delay, an asymptotical event-based consensus is proposed in [17], and a co-design event-triggered consensus is investigated in [18].

The above-mentioned event-triggers are all capable of reducing the communication burden and achieving the consensus of MASs. However, such event-triggers are employed without further justifications and explanations. A number of questions are yet to be answered, for example, which one will be suitable for given MASs? Is there any intrinsic relationship or difference between these schemes? Note that the event-trigger performance will be inevitably influenced by the controller and the MASs, therefore, it is difficult or even impossible to quantitatively assess the dynamic behavior of the event-triggers regardless of the controller and the MASs. Therefore, if one can establish a comparable relationship among them and a triggering behavior evaluation method, it may allow a reference point for designing the event-triggered consensus schemes.

Motivated by the above discussions, this paper further investigates the event-triggered asymptotical consensus control for MASs with data transmission time-delay and packet dropout. The contributions of this work are listed as follows: i) A feasible approximate selection criteria is given for choosing a more appropriate event-trigger. ii) A unified distributed consensus controller is proposed and the consensus is conveniently simplified to an equivalent asymptotical stability problem of a time-delayed Markov switching system.

The remainder of the paper is organized as follows: Section 2 introduces some fundamental notations and preliminaries. Section 3 gives the problem formulation. The main results are presented in Section 4. Simulation examples are illustrated in Section 5. Conclusion remarks are provided in Section 6.

2 Notations and preliminaries

The following notations are given which will be used throughout the literature. Let $\mathbb{R}$ and $\mathbb{N}$ denote the real numbers and the integer numbers, respectively. $\mathbb{R}^n$ is the n-dimensional Euclidean space and $\mathbb{R}^{n_1\times n_2}$ is the set of $n_1 \times n_2$ real matrices. The superscript "T" denotes the matrix transposition, the sign "$\otimes$" represents the matrix Kronecker product, the script "$*$" denotes the corresponding transposed matrix item. $I \in \mathbb{R}^{n\times n}$ denotes a n-dimensional unit matrix and $I_N = \mathrm{diag}\{\underbrace{I, I, \cdots, I}_{N}\}$ denotes a $(N\cdot n)\times(N\cdot n)$ diagonal matrix. $I_p = (\overbrace{\underbrace{0, 0, \cdots, I}_{p}, 0, \cdots, 0}^{N})^{\mathrm{T}}$ denotes a N–dimensional column vector with p-th element as I, and $I_{pq} = I_p - I_q$. $1_N = (\underbrace{1, 1, \cdots, 1}_{N})^{\mathrm{T}}$ and $0_N = (\underbrace{0, 0, \cdots, 0}_{N})^{\mathrm{T}}$ denote the N-dimensional column vector with all elements being either 1 or 0, respectively.

The information exchanged between N agents can be conveniently captured by a non-weighted direction graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ of order N, where $\mathscr{V} = \{1, 2, \cdots, N, N \in \mathbb{N}\}$ is the set of nodes, $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$ is the set of edges and $\mathscr{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$, $i, j \in \mathscr{V}$ is the adjacency matrix. An directed edge $(i, j) \in \mathscr{E}$ if agent i can obtain the information from agent j, and $a_{ij} = 1$ if $(i, j) \in \mathscr{E}$, otherwise, $a_{ij} = 0$. The set of neighbors of node i is denoted by $N_i = \{j \in \mathscr{V} | (i, j) \in \mathscr{E}\}$. The nonsymmetrical Laplacian matrix L associated with $\mathscr{A}$ and $\mathscr{G}$ is defined as $L = (L_{ij}) \in \mathbb{R}^{n\times n}$ where $l_{ij} = \sum\limits_{j=1}^{N} a_{ij}, \forall i = j$, otherwise, $l_{ij} = -a_{ij}$.

The following assumption will be used in the paper.

Assumption A1 The matrix pair of (A, B) is stabilizable.

3 Problem formulation

Consider a typical distributed MASs which is consisted by N homogeneous agents, each agent dynamics is described as[19–20]

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),$$

where $i = 1, 2, \cdots, N$, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices; $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are the state and the control input of agent i, respectively; $x_i(0) \in \mathbb{R}^n$ is the initial condition.

The objective is to design an event-trigger and a controller such that the MASs achieve consensus as[21–22]

$$\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0$$

for any initial condition $x_i(0)$, where $i, j = 1, 2, \cdots, N$.

For achieving the objective while reducing communication burden, an event-triggered decentralized cooperative control scheme is designed with the framework shown in Fig.1.

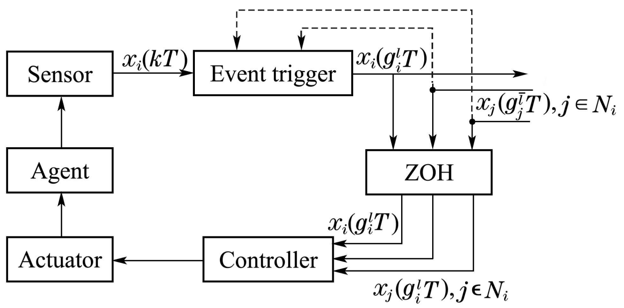


Fig. 1 Control strategy for agent i

In Fig.1, we assume that the sensor is time-driven while the controller and the actuator are event-driven. The sensor sends its sampled data at each sampling period of T to the event-trigger. Afterwards, the sampled data is authorized to be broadcasted to the zero order holder (ZOH) and neighbor agents by the event-trigger when its threshold is violated. The ZOH is employed to storage the latest received information. Accordingly, the controller calculates the control signal and sends it to the actuator. Finally, the actuator performs control action then completes the close-looped control.

Priori to the problem formulation, an event-triggered frequency evaluation method is proposed first, which will help to choose more appropriate one from two typical event-triggers.

3.1 Two comparable event-triggers

Let g_i^lT denotes the l^{th} data broadcasting time of agent i, the objective of event-trigger design is to determine the next broadcasting time instant of $g_i^{l+1}T$ by designing an appropriate event-triggered function. Generally, the event-triggered function is related to the local measurement error and/or neighbors information. A considerable issue is that each agent inevitably receives asynchronous neighbor information as $x_j(g_j^{\bar{l}}T)$, $j \in N_i$, where $g_j^{\bar{l}}T$ denotes the $\bar{l}^{\text{th}}$ broadcasting time instant of agent j. Such time unaligned information will introduce obstacles both in theoretical analysis and practical implementation. To overcome such defection, a ZOH is employed as shown in Fig.1. In this case, one can find that such asynchronous information can be synchronized, namely, $x_j(g_j^{\bar{l}}T) \to x_j(g_i^lT)$.

Accordingly, define

$$\begin{cases} X_i(g_i^lT + h_iT) = x_i(g_i^lT + h_iT) - x_i(g_i^lT), \\ Y_i(g_i^lT) = \sum\limits_{j\in N_i} a_{ij}(x_i(g_i^lT) - x_j(g_i^lT)), \end{cases}$$

where h_i denotes the broadcasting interval, it satisfies $h_i = \arg\min\limits_{h_i>0}(h_i > g_i^{l+1} - g_i^l)$, $h_i \in \mathbb{N}$. The above-introduced event-triggers are described as

$$X_i^{\mathrm{T}}(g_i^lT + h_iT)\ \varPhi_1 X_i(g_i^lT + h_iT) \geqslant \delta_1(x_i^{\mathrm{T}}(g_i^lT)\varPhi_1 x_i(g_i^lT)), \quad (1)$$

$$X_i^{\mathrm{T}}(g_i^lT + h_iT)\ \varPhi_2 X_i(g_i^lT + h_iT) \geqslant \delta_2(Y_i^{\mathrm{T}}(g_i^lT)\varPhi_2 Y_i(g_i^lT)), \quad (2)$$

where δ_k and $\varPhi_k$ are thresholds need to be determined, $k = 1, 2$. The structure of (1) is shown as the solid line in Fig.1, while the structure of (2) is illustrated as both of the solid and dash line in Fig.1.

Remark 1 From (1) and (2), one can easily conclude that there always exist a finite number of broadcasting time intervals within any time period. It ensures that Zeno phenomena is excluded[23–24].

Remark 2 In similar researches[15,18], the thresholds of event-triggers are designed as topology relevant parameters, i.e., $\delta_1 = \mathrm{diag}\{\delta_{11}, \cdots, \delta_{1N}\}$. By this method, the parameter conservativeness can be reduced. However, the later obtained result will be a nonlinear matrix inequality (NLMI) which is almost impossible to be decoupled. To design the controller, these parameters have to given as a prior. In addition, as far as the complex topology is concerned, it will induce the practical obstacles because recognizing all of the specific agents and setting their correct parameters will be a time-consuming task or even unrealistic. Therefore, we suggest the topology irrelevant event-trigger as in (1) and (2), although it sacrifices the conservativeness, but the benefit is that the parameters can be feasibly solved and the controller implementation becomes convenient.

Remark 3 In some literatures, the event-trigger is appended with the absolute threshold. For example, the decentralized event-trigger is designed as $\delta_i \leqslant \sigma z_i^{\mathrm{T}}\varTheta_i z_i + \eta$ in [25], where η is the given absolute threshold. In [26], the event-trigger function is designed as $f_i(\cdot) = d_i\|e_i(t)\| + b_i\|\tilde{e}_i(t)\| - \delta_i$, where the δ_i denote the given absolute threshold. In [27], the absolute event-trigger is designed as $e_i(t) \leqslant c_0 + c_1\mathrm{e}^{-\alpha t}$, where c_0, c_1 and α are the absolute thresholds. Because these absolute thresholds are employed, the event-triggered frequency will be lower than (1) and (2) when the state trajectories converge closer to the equilibrium point. However, the consequence is that the obtained consensuses will be inevitably bounded. For example, the final consequence deviations are $\lim\limits_{t\to\infty} \|x_i(t) - x_j(t)\| \leqslant \dfrac{N\eta}{\beta\lambda_{\min}P}$ in [25], $\lim\limits_{t\to\infty} \|\varepsilon(t)\| \leqslant \varDelta$ in [26] and $r = \dfrac{\|L\|\sqrt{N}c_0}{\lambda_2(L)}$ in [27]. It indicates that the control performance is sacrificed for reducing the event-triggered frequency. Moreover, concerning with the different absolute conditions, the event-triggered dynamic behavior will be changed or even diverged. Therefore, to guarantee the non-bounded consensus and without loss of generality, the absolute event-triggers are not considered.

Neglecting the controller effect, if

$$\delta_1(x_i^{\mathrm{T}}(g_i^lT)\varPhi_1 x_i(g_i^lT)) > \delta_2(Y_i^{\mathrm{T}}(g_i^lT)\varPhi_2 Y_i(g_i^lT))$$

holds, one can see that (1) has the lower triggered frequency than (2).

Define $\tilde{x}(t) = (x_1^{\mathrm{T}}(t) \ \cdots \ x_N^{\mathrm{T}}(t))^{\mathrm{T}}$, calculate the Euclide norm of the above inequality, one has

$$\|\tilde{x}^{\mathrm{T}}(t)(\delta_1 I_N \otimes \varPhi_1)\tilde{x}(t)\| > \|\tilde{x}^{\mathrm{T}}(t)(\delta_2 L^{\mathrm{T}}L \otimes \varPhi_2)\tilde{x}(t)\|.$$

Because

$$\begin{aligned}&\|\delta_1 I_N \otimes \varPhi_1\| = \\ &\mathrm{tr}^{\frac{1}{2}}(\delta_1^2 I_N \otimes \varPhi_1^{\mathrm{T}}\varPhi_1) = \sqrt{N}\delta_1 \mathrm{tr}^{\frac{1}{2}}(\varPhi_1^{\mathrm{T}}\varPhi_1), \\ &\|\delta_2 L^{\mathrm{T}}L \otimes \varPhi_2\| = \\ &\mathrm{tr}^{\frac{1}{2}}(\delta_2^2 L^{\mathrm{T}}L \otimes \varPhi_2^{\mathrm{T}}\varPhi_2) = \delta_2 \mathrm{tr}^{\frac{1}{2}}(L^{\mathrm{T}}L)\mathrm{tr}^{\frac{1}{2}}(\varPhi_2^{\mathrm{T}}\varPhi_2),\end{aligned}$$

and define

$$\begin{aligned}\chi_1 &= \sqrt{N}\delta_1 \mathrm{tr}^{\frac{1}{2}}(\varPhi_1^{\mathrm{T}}\varPhi_1), \\ \chi_2 &= \delta_2 \mathrm{tr}^{\frac{1}{2}}(L^{\mathrm{T}}L)\mathrm{tr}^{\frac{1}{2}}(\varPhi_2^{\mathrm{T}}\varPhi_2),\end{aligned}$$

if such that $\chi_1 > \chi_2$ holds, then (1) probably has the lower event-triggered frequency than (2) for given topology L.

As aforementioned, with different controllers, the event-triggered frequency will be different. Without loss of generality, we need to design a universal control strategy that can be used to cooperate with (1) or (2) for achieving the consensus.

3.2 Data transmission time-delay

Firstly, to guarantee the desired consensus, a typical distributed cooperative consensus law is chosen as

$$u_i(t) = -K\sum_{j=1}^{N} a_{ij}\left(x_i(g_i^lT) - x_j(g_i^lT)\right), \quad (3)$$

where K denotes the controller gain and $i, j \in \mathscr{V}$.

The agents exchange their information via communication networks, the inevitable data transmission time-delay will affect the control performance. Moreover, the controller performs the calculation if all of the necessary data are collected. It indicates that the controller has to wait in a delayed duration until the last necessary data is arrived. Let τ_{ij}^l denotes the transmission time-delay of l-th data that is originated from j and received by i, it is assumed that

$$0 \leqslant \tau = \max(\tau_{ij}^l), \quad (4)$$

where $i, j \in \{1, 2, \cdots, N\}$ and $l = 1, 2, \cdots$, then the control updating interval sector is described as $[g_i^lT + \tau, g_i^{l+1}T + \tau)$ and it can be divided as

$$[g_i^lT + \tau, g_i^{l+1}T + \tau) = \bigcup_{k=g_i^l}^{g_i^{l+1}-1} [kT + \tau, (k+1)T + \tau).$$

Define the disagreement vector as $e_i(kT) = x_i(kT) - x_i(g_i^lT)$, the augment vectors as $\bar{x}_i(t) = x_1(t) - x_i(t)$, $\bar{e}(kT) = e_1(kT) - e_i(kT)$, $E_1 = (1_{N-1}, -I_{N-1})$, $E_2 = (0_{N-1}^{\mathrm{T}}, -I_{N-1}^{\mathrm{T}})^{\mathrm{T}}$, $x(t) = (x_1^{\mathrm{T}}(t), \cdots, x_{N-1}^{\mathrm{T}}(t))^{\mathrm{T}}$, $e(t) = (e_1^{\mathrm{T}}(t), \cdots, e_{N-1}^{\mathrm{T}}(t))^{\mathrm{T}}$, $\bar{x}(t) = (\bar{x}_2^{\mathrm{T}}(t), \cdots, \bar{x}_{N-1}^{\mathrm{T}}(t))^{\mathrm{T}}$ and $\bar{e}(t) = (\bar{e}_2^{\mathrm{T}}(t), \cdots, \bar{e}_{N-1}^{\mathrm{T}}(t))^{\mathrm{T}}$, (3) is changed as

$$\begin{aligned}u_i(t) = -K\sum_{j=1}^{N} a_{ij}(&x_i(kT) - x_j(kT) - \\ &e_i(kT) + e_j(kT)).\end{aligned}$$

Therefore, we have the close-looped system as

$$\begin{aligned}\dot{\bar{x}}(t) = &(I_{N-1} \otimes A)\,\bar{x}(t) - \\ &\left(\bar{L} \otimes BK\right)(\bar{x}(kT) - \bar{e}(kT)),\end{aligned}$$

where $\bar{L} = E_1 L E_2 \in \mathbb{R}^{(N-1)\times(N-1)}$ and $t \in [kT + \tau, (k+1)T + \tau)$. The initial condition of $\bar{x}(t)$ is supplemented as $\bar{x}(\theta) = \bar{\varPsi}(\theta)$, where $\theta \in [-T-\tau, t_0]$, $\bar{\varPsi}(t_0) = \bar{x}(t_0) = (\bar{x}_1^{\mathrm{T}}(t_0), \cdots, \bar{x}_{N-1}^{\mathrm{T}}(t_0))^{\mathrm{T}}$, $\bar{\varPsi}(\cdot)$: $[t_0 - T - \tau, t_0] \to \mathbb{R}^{(N-1)\times n}$ denotes an appropriate absolutely continuous function defined in Banach space and it satisfies

$$\|\bar{\varPsi}\| = \max_{\theta\in[t_0-T-\tau, t_0]} \|\bar{\varPsi}(\theta)\| + \left(\int_{t_0-T-\tau}^{t_0} \|\dot{\bar{\varPsi}}(s)\|^2 \mathrm{d}s\right)^{\frac{1}{2}}.$$

3.3 Data packet dropouts

When data packet dropout event occurs, the agent links can be regarded as temporal disconnection[28]. In some similar researches[29–30], such dynamic phenomena is described as Markov processing. It is worth to point out that although single data packet incident is unpredictable, some helpful prior probability knowledge can be acquired. Therefore, we model the data packet dropouts as a state transform probability partly unknown homogeneous Markov chain. Let $r(k) \in S$ denotes the Markov chain, where $S = (1, 2, \cdots)$ denotes the finite set including s possible switching topologies. Let $\varPi = (\pi_{ij})$, $i, j \in S$ denotes the probability matrix and it satisfies

$$\begin{aligned}&\mathrm{P}\{r(t+\xi) = j | r(t) = i\} = \\ &\begin{cases}\pi_{ij}\xi + \mathrm{o}(\xi), & i \neq j, \\ 1 + \pi_{ii}\xi + \mathrm{o}(\xi), & i = j,\end{cases}\end{aligned} \quad (5)$$

where $\mathrm{P}\{\cdot\}$ is conditional probability, ξ is the i^{th} subsystem activation time and $\mathrm{o}(\xi)$ is the corresponding higher order infinitesimal to ξ, it satisfies $\lim\limits_{\xi\to 0}\dfrac{\mathrm{o}(\xi)}{\xi}=0$. The transition probabilities in $\varPi$ are partly unknown, for example $\varPi=\begin{pmatrix}\bar{\pi}_{11} & \pi_{12} & \cdots & \pi_{1s}\\ \pi_{21} & \bar{\pi}_{22} & \cdots & \pi_{2s}\\ \vdots & \vdots & \vdots & \vdots\\ \bar{\pi}_{s1} & \bar{\pi}_{s2} & \cdots & \bar{\pi}_{ss}\end{pmatrix}$, where $\bar{\pi}$ denotes the unknown information.

Accordingly, $\bar{L}$ is substituted with $\bar{L}_{\sigma}|_{\sigma=r(t)\in S}$, hence the general close-looped system is transformed to a Markov switching system as

$$\begin{aligned}\dot{\bar{x}}(t)=&(I_{N-1}\otimes A)\bar{x}(t)-\\&(\bar{L}_{\sigma}\otimes BK_{\sigma})(\bar{x}(kT)-\bar{e}(kT)),\end{aligned}\tag{6}$$

where $\bar{L}_{\sigma}=E_1L_{\sigma}E_2$ and $t\in[kT+\tau,(k+1)T+\tau)$.

Synthesizing all the above factors, the objective of the rest paper is to investigate a feasible design method for event-trigger (1) or (2) with the controller (3) to guarantee the close-looped system (6) achieving asymptotically stability under time-delays (4) and data packet dropouts (5).

Remark 4 Because the state vector $x_1(t)$ can be chosen from any agents, it can be concluded if $\lim\limits_{t\to\infty}\bar{x}(t)=0$ holds, the consensus of MASs is achieved. Therefore, the original consensus issue is conveniently simplified to the equivalent stability problem of (6).

4 Main result

Lemma 1 For any matrix $R=R^{\mathrm{T}}>0\in\mathbb{R}^{n\times n}$ and invertible matrix $P\in\mathbb{R}^{n\times n}$, the following inequality holds:

$$-(I_N\otimes P)(I_N\otimes R)^{-1}(I_N\otimes P)<I_N\otimes R-I_N\otimes 2P.$$

Proof When R is a positive symmetric matrix, the following inequality naturally holds: $((I_N\otimes R)-(I_N\otimes P))(I_N\otimes R)^{-1}((I_N\otimes R)-(I_N\otimes P))=(I_N\otimes P)(I_N\otimes R)^{-1}(I_N\otimes P)+(I_N\otimes R)-2(I_N\otimes P)>0$, then the proof for Lemma 1 is complete.

Define sets $U^r=U_k^r\cup U_{uk}^r$, $r\in S$, $U_k^r=\{\pi_{rj}|j\in S\}$, $U_{uk}^r=\{\bar{\pi}_{rj}|j\in S\}$ and $\pi_k^r=\sum\limits_{b\in U_k^a}\pi_{rb}$, then we have the following conclusion.

Lemma 2 For $r(t)$ in (5), the following inequality holds: $\sum\limits_{j=1}^{s}\pi_{rj}P_j\leqslant\sum\limits_{j=1}^{s}\tilde{\pi}_{rj}P_j$, where $\tilde{\pi}_{ri}=\pi_{ri},\forall\pi_{ri}\subset U_k^r$; otherwise, $\tilde{\pi}_{ri}=1-\pi_k^r,\forall\pi_{ri}\subset U_{uk}^r$.

Proof According to Markov property $\sum\limits_{b=1}^{s}\pi_{rb}=\sum\limits_{b\in U_k^a}\pi_{rb}+\sum\limits_{b\in U_{uk}^a}\bar{\pi}_{rb}=1$, define $P_k^r=\sum\limits_{b\in U_k^a}\pi_{rb}P_b$, then we have $\dfrac{\bar{\pi}_{rb}}{1-\pi_k^r}\leqslant 1$ and $\sum\limits_{b\in U_{uk}^r}\dfrac{\bar{\pi}_{rb}}{1-\pi_k^r}=1$. The elements in set U_k^r can be marked as $(\pi_{r1},\pi_{r2},\cdots,\pi_{ri})$, $0\leqslant i\leqslant s$, then the upper boundaries for unknown element in set U_{uk}^r can be substituted as $(\underbrace{1-\pi_k^r,1-\pi_k^r,\cdots,1-\pi_k^r}_{s-k})$. Therefore, we have $P_k^r+(1-\pi_k^r)\sum\limits_{b\in U_{uk}^a}P_b=\sum\limits_{j=1}^{s}\tilde{\pi}_{rj}P_j$, where $\tilde{\pi}_{ri}=\pi_{ri},\pi_{ri}\subset U_k^r$, $\tilde{\pi}_{ri}=1-\pi_k^r,\pi_{ri}\subset U_{uk}^r$. The proof for Lemma 2 is complete.

Theorem 1 Given a positive scalar $0<\delta_1<1$, if there exist the appropriate dimension symmetric matrices $\bar{P}_{\mathrm{r}}=\bar{P}_{\mathrm{r}}^{\mathrm{T}}>0$, $r\in S$, $\bar{Q}_j=\bar{Q}_j^{\mathrm{T}}\geqslant 0$, $\bar{R}_j=\bar{R}_j^{\mathrm{T}}\geqslant 0$, $j=1,2$ and any appropriate dimension matrices $\bar{M}_k$, $k=1,2,\cdots,6$, such that

$$\begin{pmatrix}\bar{\varGamma}_{11} & \bar{\varGamma}_{12} & 0 & 0 & \bar{\varGamma}_{15} & \bar{\varGamma}_{16} & \bar{\varGamma}_{17} & 0 & \bar{\varGamma}_{19} & \cdots & \bar{\varGamma}_{1,10}\\ * & \bar{\varGamma}_{22} & \bar{\varGamma}_{23} & \bar{\varGamma}_{24} & 0 & \bar{\varGamma}_{26} & \bar{\varGamma}_{27} & \bar{\varGamma}_{28} & 0 & 0 & 0\\ * & * & \bar{\varGamma}_{33} & 0 & 0 & 0 & 0 & \bar{\varGamma}_{38} & 0 & 0 & 0\\ * & * & * & \bar{\varGamma}_{44} & 0 & 0 & 0 & \bar{\varGamma}_{48} & 0 & 0 & 0\\ * & * & * & * & \bar{\varGamma}_{55} & \bar{\varGamma}_{56} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & \bar{\varGamma}_{66} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & \bar{\varGamma}_{77} & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & \bar{\varGamma}_{88} & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & \bar{\varGamma}_{99} & 0 & 0\\ * & * & * & * & * & * & * & * & * & \cdots & 0\\ * & * & * & * & * & * & * & * & * & * & \bar{\varGamma}_{10,10}\end{pmatrix}<0\tag{7}$$

holds, then (6) achieve asymptotically consensus under (1) and (3), where time-delays satisfy (4), data packet dropouts satisfy (5) and $K_{\mathrm{r}}=\bar{K}_{\mathrm{r}}\bar{P}_{\mathrm{r}}^{-1}$. Where:

$$\begin{aligned}
&\bar{\varGamma}_{11}=I_{N-1}\otimes(A\bar{P}_{\mathrm{r}}+\bar{P}_{\mathrm{r}}A^{\mathrm{T}}+\bar{Q}_1+\bar{U}_1+\bar{U}_1^{\mathrm{T}}),\\
&\bar{\varGamma}_{12}=-\bar{L}_{\mathrm{r}}\otimes B\bar{K}_{\mathrm{r}}+I_{N-1}\otimes(-\bar{U}_1+\bar{U}_2^{\mathrm{T}}),\\
&\bar{\varGamma}_{15}=\bar{L}_{\mathrm{r}}\otimes(B\bar{K}_{\mathrm{r}}),\ \bar{\varGamma}_{16}=I_{N-1}\otimes(\bar{P}_{\mathrm{r}}A^{\mathrm{T}}),\\
&\bar{\varGamma}_{17}=I_{N-1}\otimes(\tau+T)\bar{U}_1,\\
&\bar{\varGamma}_{19}=\sqrt{\tilde{\pi}_{r,1}}(I_{N-1}\otimes\bar{P}_1),\\
&\bar{\varGamma}_{1,10}=\sqrt{\tilde{\pi}_{r,s}}(I_{N-1}\otimes\bar{P}_s),\\
&\bar{\varGamma}_{22}=I_{N-1}\otimes(-\bar{U}_2-\bar{U}_2^{\mathrm{T}}+\bar{U}_3+\bar{U}_3^{\mathrm{T}}-\\
&\qquad\bar{U}_5-\bar{U}_5^{\mathrm{T}})+\lambda\delta_1E_2^{\mathrm{T}}E_2\otimes\varPhi_1,\\
&\bar{\varGamma}_{23}=I_{N-1}\otimes(\bar{U}_5-\bar{U}_6^{\mathrm{T}}),\\
&\bar{\varGamma}_{24}=I_{N-1}\otimes(-\bar{U}_3+\bar{U}_4^{\mathrm{T}}),\\
&\bar{\varGamma}_{26}=-\bar{L}_{\mathrm{r}}^{\mathrm{T}}\otimes(\bar{K}^{\mathrm{T}}B^{\mathrm{T}}),\\
&\bar{\varGamma}_{27}=I_{N-1}\otimes(\tau+T)\bar{U}_2,\\
&\bar{\varGamma}_{28}=I_{N-1}\otimes T(\bar{U}_3+\bar{U}_5),\\
&\bar{\varGamma}_{33}=I_{N-1}\otimes(\bar{Q}_2+\bar{U}_6+\bar{U}_6^{\mathrm{T}}),\\
&\bar{\varGamma}_{38}=I_{N-1}\otimes(T\bar{U}_6),\\
&\bar{\varGamma}_{44}=I_{N-1}\otimes(-\bar{Q}_2-\bar{U}_4-\bar{U}_4^{\mathrm{T}}),\\
&\bar{\varGamma}_{48}=I_{N-1}\otimes(T\bar{U}_4),\\
&\bar{\varGamma}_{55}=-I_{N-1}\otimes\varPhi+\lambda\delta_1E_2^{\mathrm{T}}E_2\otimes\varPhi_1,\\
&\bar{\varGamma}_{56}=\bar{L}_{\mathrm{r}}^{\mathrm{T}}\otimes(\bar{K}_{\mathrm{r}}^{\mathrm{T}}B^{\mathrm{T}}),\\
&\bar{\varGamma}_{66}=h^2(I_{N-1}\otimes((\tau+T)\bar{R}_1+T\bar{R}_2))-\\
&\qquad 2h(I_{N-1}\otimes\bar{P}_{\mathrm{r}}),
\end{aligned}$$

$\bar{\Gamma}_{77} = I_{N-1} \otimes (-(\tau+T)\bar{R}_1),$
$\bar{\Gamma}_{88} = I_{N-1} \otimes (-T\bar{R}_2),\ \bar{\Gamma}_{99} = -I_{N-1} \otimes \bar{P}_1,$
$\bar{\Gamma}_{10,10} = -I_{N-1} \otimes \bar{P}_s,\ \bar{L}_{\rm r} = E_1 \bar{L}_{\rm r} E_2.$

Proof When $t \in [kT+\tau, (k+1)T+\tau)$, define $\tau_{\rm d}(t) = t - kT$, we have $\tau_{\rm d}(t) \in [\tau, \tau+T)$, $\dot{\tau}_{\rm d}(t) = 1, t \neq kT$. Therefore, (6) is transformed to

$$\dot{\bar{x}}(t) = (I_{N-1} \otimes A)\bar{x}(t) - (\bar{L}_\sigma \otimes BK_\sigma)(\bar{x}(t - \tau_{\rm d}(t)) - \bar{e}(t-\tau_{\rm d}(t))).$$

Let $\sigma = r, r \in S$, when $t \in [kT+\tau, (k+1)T+\tau)$, we define a Lyapunov functional as

$$\begin{aligned} &V(t, \bar{x}(t), \dot{\bar{x}}(t)) = \\ &\bar{x}^{\rm T}(t)(I_{N-1} \otimes P_{\rm r})\bar{x}(t) + \\ &\int_{t-\tau_{\rm d}(t)}^{t} \bar{x}^{\rm T}(s)(I_{N-1} \otimes Q_1)\bar{x}(s){\rm d}s + \\ &\int_{t-\tau-T}^{t-T} \bar{x}^{\rm T}(s)(I_{N-1} \otimes Q_2)\bar{x}(s){\rm d}s + \\ &\int_{-\tau-T}^{0}\int_{t+\theta}^{t} \dot{\bar{x}}^{\rm T}(s)(I_{N-1} \otimes R_1)\dot{\bar{x}}(s){\rm d}\theta{\rm d}s + \\ &\int_{-\tau-T}^{-T}\int_{t+\theta}^{t} \dot{\bar{x}}^{\rm T}(s)(I_{N-1} \otimes R_2)\dot{\bar{x}}(s){\rm d}\theta{\rm d}s, \end{aligned}$$

where $P_{\rm r} = P_{\rm r}^{\rm T} > 0,\ P_{\rm r} \in \mathbb{R}^{(N-1)\times(N-1)},\ r \in S,\ Q_j = Q_j^{\rm T} \geqslant 0,\ R_j = R_j^{\rm T} \geqslant 0,\ Q_j, R_j \in \mathbb{R}^{(N-1)\times(N-1)},\ j = 1, 2.$

Calculating the differential of $V(t, \bar{x}(t), \dot{\bar{x}}(t))$ along the trajectory of (6), we have

$$\begin{aligned} &\dot{V}(t, \bar{x}(t), \dot{\bar{x}}(t)) = \\ &2\bar{x}^{\rm T}(t)(I_{N-1} \otimes P_{\rm r})\dot{\bar{x}}(t) + \\ &\bar{x}^{\rm T}(t)(I_{N-1} \otimes Q_1)\bar{x}(t) + \\ &\bar{x}^{\rm T}(t-T)(I_{N-1} \otimes Q_2)\bar{x}(t-T) - \\ &\bar{x}^{\rm T}(t-\tau-T)(I_{N-1} \otimes Q_2)\bar{x}(t-\tau-T) + \\ &(\tau+T)\dot{\bar{x}}^{\rm T}(t)(I_{N-1} \otimes R_1)\dot{\bar{x}}(t) + \\ &\tau\dot{\bar{x}}^{\rm T}(t)(I_{N-1} \otimes R_2)\dot{\bar{x}}(t) - \\ &\int_{t-\tau-T}^{t} \dot{\bar{x}}^{\rm T}(s)(I_{N-1} \otimes R_1)\dot{\bar{x}}(s){\rm d}s - \\ &\int_{t-\tau-T}^{t-T} \dot{\bar{x}}^{\rm T}(s)(I_{N-1} \otimes R_2)\dot{\bar{x}}(s){\rm d}s. \end{aligned}$$

For handling the latest two integral items, we add the following zero items into the above equality:

$$\begin{aligned} &2(\bar{x}^{\rm T}(t)(I_{N-1} \otimes U_1) + \bar{x}^{\rm T}(t-\tau_{\rm d}(t))(I_{N-1} \otimes U_2)) \cdot \\ &(\bar{x}(t) - \bar{x}(t-\tau_{\rm d}(t)) - \int_{t-\tau_{\rm d}(t)}^{t} \dot{\bar{x}}(s){\rm d}s), \\ &2(\bar{x}^{\rm T}(t-\tau_{\rm d}(t))(I_{N-1} \otimes U_3) + \\ &\bar{x}^{\rm T}(t-\tau-T)(I_{N-1} \otimes U_4)) \cdot \\ &(\bar{x}(t-\tau_{\rm d}(t)) - \bar{x}(t-\tau-T) - \int_{t-\tau-T}^{t-\tau_{\rm d}(t)} \dot{\bar{x}}(s){\rm d}s), \\ &2(\bar{x}^{\rm T}(t-\tau_{\rm d}(t))(I_{N-1} \otimes U_5) + \\ &\bar{x}^{\rm T}(t-\tau)(I_{N-1} \otimes U_6)) \cdot \\ &(\bar{x}(t-\tau) - \bar{x}(t-\tau_{\rm d}(t)) - \int_{t-\tau_{\rm d}(t)}^{t-\tau} \dot{\bar{x}}(s){\rm d}s), \end{aligned}$$

where $U_k \in \mathbb{R}^{(N-1)\times(N-1)},\ k = 1, 2, \cdots, 6.$

Define an augmented state as $\eta(t) = (\bar{x}^{\rm T}(t), \bar{x}^{\rm T}(t-\tau_{\rm d}(t)), \bar{x}^{\rm T}(t-\tau), \bar{x}^{\rm T}(t-\tau-T), \bar{\rm e}^{\rm T}(t-\tau_{\rm d}(t)))^{\rm T}$, then we have

$$\begin{aligned} &\dot{V}(t, \bar{x}(t), \dot{\bar{x}}(t)) = \\ &\eta^{\rm T}(t)\{I_1^{\rm T}(I_{N-1} \otimes (-U_1 + U_2^{\rm T}) + \\ &I_1^{\rm T}(I_{N-1} \otimes (P_{\rm r}A + A^{\rm T}P_{\rm r} + Q_1 + U_1 + U_1^{\rm T}))I_1 - \\ &I_1^{\rm T}(\bar{L}_{\rm r} \otimes P_{\rm r}BK_{\rm r})I_2 + I_1^{\rm T}(\bar{L}_{\rm r} \otimes P_{\rm r}BK_{\rm r})I_5 - \\ &I_2^{\rm T}(\bar{L}_{\rm r}^{\rm T} \otimes K_{\rm r}^{\rm T}B^{\rm T}P_{\rm r}^{\rm T})I_1 + I_5^{\rm T}(\bar{L}_{\rm r}^{\rm T} \otimes K_{\rm r}^{\rm T}B^{\rm T}P_{\rm r}^{\rm T})I_1 + \\ &I_2^{\rm T}(I_{N-1} \otimes (-U_2 - U_2^{\rm T} + U_3 + U_3^{\rm T} - U_5 - U_5^{\rm T}))I_2 + \\ &I_2^{\rm T}(I_{N-1} \otimes (U_5 - U_6^{\rm T}))I_3 + (\tau+T)X_1^{\rm T}R_1^{-1}X_1 + \\ &I_2^{\rm T}(I_{N-1} \otimes (-U_3 + U_4^{\rm T}))I_4 + TX_2^{\rm T}R_2^{-1}X_2 + \\ &I_3^{\rm T}(I_{N-1} \otimes (Q_2 + U_6 + U_6^{\rm T}))I_3 + TX_3^{\rm T}R_2^{-1}X_3 + \\ &I_4^{\rm T}(I_{N-1} \otimes (-Q_2 - U_4 - U_4^{\rm T}))I_4\}\eta(t) + \\ &\varXi^{\rm T}(I_{N-1} \otimes ((\tau+T)R_1 + TR_2))\varXi - \\ &\int_{t-\tau_{\rm d}(t)}^{t} \varPhi_1\bar{R}_1\varPhi_1^{\rm T}{\rm d}s - \int_{t-\tau-T}^{t-\tau_{\rm d}(t)} \varPhi_2\bar{R}_2\varPhi_2^{\rm T}{\rm d}s - \\ &\int_{t-\tau_{\rm d}(t)}^{t-\tau} \varPhi_3\bar{R}_2\varPhi_3^{\rm T}{\rm d}s, \end{aligned}$$

where

$$\begin{aligned} &\bar{R}_1 = I_{N-1} \otimes R_1^{-1},\ \bar{R}_2 = I_{N-1} \otimes R_2^{-1}, \\ &X_1 = (U_1^{\rm T}, U_2^{\rm T}, 0, 0)^{\rm T},\ X_2 = (0, U_3^{\rm T}, 0, U_4^{\rm T})^{\rm T}, \\ &X_3 = (0, U_5^{\rm T}, U_6^{\rm T}, 0)^{\rm T}, \\ &\varPhi_1 = \eta^{\rm T}(t)(I_{N-1} \otimes X_1) + \dot{\bar{x}}^{\rm T}(s)(I_{N-1} \otimes R_1), \\ &\varPhi_2 = \eta^{\rm T}(t)(I_{N-1} \otimes X_2) + \dot{\bar{x}}^{\rm T}(s)(I_{N-1} \otimes R_2), \\ &\varPhi_3 = \eta^{\rm T}(t)(I_{N-1} \otimes X_3) + \dot{\bar{x}}^{\rm T}(s)(I_{N-1} \otimes R_3) \end{aligned}$$

and

$$\varXi = (I_{N-1} \otimes A)I_1\eta(t) - (\bar{L}_{\rm r} \otimes BK_{\rm r})(I_2 - I_5)\eta(t).$$

It is easy to find that the last three integral items are less than 0 naturally. According to Lemma 2, we have $I_1^{\rm T}(I_{N-1} \otimes \sum\limits_{j=1}^{s} \pi_{rj}P_j)I_1 \leqslant I_1^{\rm T}(I_{N-1} \otimes (P_k^r + (1 - \pi_k^r)\sum\limits_{b \in U_{uk}^r} P_{\rm r}))I_1$. With the well-known Schur complement, one has $\dot{V}(t, \bar{x}(t), \dot{\bar{x}}(t)) \leqslant \eta^{\rm T}(t)\varGamma\eta(t)$, where

$$\varGamma = \begin{pmatrix} \varGamma_{11} & \varGamma_{12} & 0 & 0 & \varGamma_{15} & \varGamma_{16} & \varGamma_{17} & 0 & \varGamma_{19} & \cdots & \varGamma_{21} \\ * & \varGamma_{22} & \varGamma_{23} & \varGamma_{24} & 0 & \varGamma_{26} & \varGamma_{27} & \varGamma_{28} & 0 & 0 & 0 \\ * & * & \varGamma_{33} & 0 & 0 & 0 & 0 & \varGamma_{38} & 0 & 0 & 0 \\ * & * & * & \varGamma_{44} & 0 & 0 & 0 & \varGamma_{48} & 0 & 0 & 0 \\ * & * & * & * & 0 & \varGamma_{56} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \varGamma_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \varGamma_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \varGamma_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \varGamma_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \cdots & 0 \\ * & * & * & * & * & * & * & * & * & * & \varGamma_{10,10} \end{pmatrix}, \tag{8}$$

$$\varGamma_{11} = I_{N-1} \otimes (P_{\rm r}A + A^{\rm T}P_{\rm r} + Q_1 + U_1 + U_1^{\rm T}),$$

$\varGamma_{12} = -\bar{L}_{\rm r} \otimes BK_{\rm r} + I_{N-1} \otimes (-U_1 + U_2^{\rm T})$,
$\varGamma_{15} = \bar{L}_{\rm r} \otimes BK_{\rm r}, \varGamma_{16} = I_{N-1} \otimes A^{\rm T} P_{\rm r}$,
$\varGamma_{17} = I_{N-1} \otimes (\tau + T)U_1$,
$\varGamma_{19} = \sqrt{\bar{\pi}_{r,1}}(I_{N-1} \otimes P_1)$,
$\varGamma_{1,10} = \sqrt{\bar{\pi}_{r,s}}(I_{N-1} \otimes P_s)$,
$\varGamma_{22} = I_{N-1} \otimes (-U_2 - U_2^{\rm T} + U_3 + U_3^{\rm T} - U_5 - U_5^{\rm T})$,
$\varGamma_{23} = I_{N-1} \otimes (U_5 - U_6^{\rm T})$,
$\varGamma_{24} = I_{N-1} \otimes (-U_3 + U_4^{\rm T})$,
$\varGamma_{26} = -\bar{L}_{\rm r}^{\rm T} \otimes (K_{\rm r}^{\rm T} B^{\rm T}),\ \varGamma_{27} = I_{N-1} \otimes (\tau + T)U_2$,
$\varGamma_{28} = I_{N-1} \otimes T(U_3 + U_5)$,
$\varGamma_{33} = I_{N-1} \otimes (Q_2 + U_6 + U_6^{\rm T})$,
$\varGamma_{38} = I_{N-1} \otimes (TU_6)$,
$\varGamma_{44} = I_{N-1} \otimes (-Q_2 - U_4 - U_4^{\rm T})$,
$\varGamma_{48} = I_{N-1} \otimes (TU_4),\ \varGamma_{56} = \bar{L}_{\rm r}^{\rm T} \otimes (K_{\rm r}^{\rm T} B^{\rm T})$,
$\varGamma_{66} = I_{N-1} \otimes ((\tau + T)R_1 + TR_2) - I_{N-1} \otimes 2P_{\rm r}$,
$\varGamma_{77} = I_{N-1} \otimes (-(\tau + T)R_1)$,
$\varGamma_{88} = I_{N-1} \otimes (-TR_2)$,
$\varGamma_{99} = -I_{N-1} \otimes P_1, \varGamma_{10,10} = -I_{N-1} \otimes P_s$.

According to Lemma 1, we have $-I_{N-1} \otimes ((\tau + T)R_1 + TR_2))^{-1} < I_{N-1} \otimes P_{\rm r}^{-1}(I_{N-1} \otimes ((\tau + T)R_1 + TR_2))I_{N-1} \otimes P_{\rm r}^{-1} - 2I_{N-1} \otimes P_{\rm r}^{-1}$.

Define

$$J_1 = {\rm diag}\{\underbrace{I_{N-1} \otimes P_{\rm r}^{-1}, \cdots, I_{N-1} \otimes P_{\rm r}^{-1}}_{5}\},$$

$$J_2 = {\rm diag}\{\underbrace{I_{N-1} \otimes P_{\rm r}^{-1}, \cdots, I_{N-1} \otimes P_{\rm r}^{-1}}_{2+s}\},$$

$$J = {\rm diag}\{J_1, (I_{N-1} \otimes ((\tau + T)R_1 + TR_2))^{-1}, J_2\},$$

and performing congruence transformations to (8) by J. Define: $\bar{P}_{\rm r} = P_{\rm r}^{-1}$, $\bar{P}_i = \bar{P}_{\rm r} P_i \bar{P}_{\rm r}$, $i = 1, 2, \cdots, s$, $\bar{Q}_j = \bar{P}_{\rm r} Q_j \bar{P}_{\rm r}$, $\bar{R}_j = \bar{P}_{\rm r} R_j \bar{P}_{\rm r}$, $j = 1, 2$, $\bar{M}_k = \bar{P}_{\rm r} M_k \bar{P}_{\rm r}$, $k = 1, 2, \cdots, 6$, $\bar{K}_{\rm r} = K_{\rm r} \bar{P}_{\rm r}$, then the nonlinear matrix inequality $\varGamma < 0$ can be conveniently transformed to an equivalent linear matrix inequality (i.e., $\dot{V}(t, \bar{x}(t), \dot{\bar{x}}(t)) \leqslant \eta^{\rm T}(t) J^{\rm T} \varGamma J \eta(t)$).

When $t \in [kT + \tau, (k + 1)T + \tau)$, the event-triggered threshold is not violated, therefore, according to (1) we have $X_i^{\rm T}(g_i^l T + h_i T) \varPhi_1 X_i(g_i^l T + h_i T) < \delta_1 x_i^{\rm T}(g_i^l T) \varPhi_1 x_i(g_i^l T)$, where $X_i^{\rm T}(g_i^l T + h_i T) = e_i(kT)$ and $x_i(g_i^l T) = x_i(kT)$. Then the above inequality is equivalent to

$$\begin{aligned} &{\rm e}^{\rm T}(kT)(I_N \otimes \varPhi_1)e(kT) < \\ &x^{\rm T}(kT)(\delta_1 I_N \otimes \varPhi_1)x(kT). \end{aligned}$$

From the definition of $\bar{x}(t)$ and $\bar{e}(t)$, one has

$$\begin{aligned} &\bar{e}^{\rm T}(kT)(I_{N-1} \otimes \varPhi_1)\bar{e}(kT) = \\ &e^{\rm T}(kT)(E_1^{\rm T} E_1 \otimes I_n)e(kT) \leqslant \\ &\lambda {\rm e}^{\rm T}(kT)(I_N \otimes \varPhi_1)e(kT), \end{aligned}$$

$$\begin{aligned} &x^{\rm T}(kT)(\delta_1 \otimes \varPhi_1)x(kT) = \\ &\bar{x}^{\rm T}(kT)(E_2^{\rm T} \delta_1 E_2 \otimes \varPhi_1)\bar{x}(kT), \end{aligned}$$

where $\lambda = \lambda_{\max}(E_1^{\rm T} E_1)$, $\lambda_{\max}(\cdot)$ is the maximum eigenvalue function. Then the above inequality is equivalent to

$$\begin{aligned} &\bar{\rm e}^{\rm T}(kT)(I_{N-1} \otimes \varPhi_1)\bar{e}(kT) < \\ &\lambda \bar{x}^{\rm T}(kT)(E_2^{\rm T} \delta_1 E_2 \otimes \varPhi_1)\bar{x}(kT). \end{aligned}$$

Therefore

$$\begin{aligned} &\dot{V}(\cdot) = \\ &\dot{V}(\cdot) - \bar{\rm e}^{\rm T}(kT)(I_{N-1} \otimes \varPhi_1)\bar{e}(kT) + \\ &\bar{\rm e}^{\rm T}(kT)(I_{N-1} \otimes \varPhi_1)\bar{e}(kT) < \\ &\eta^{\rm T}(t) J^{\rm T} \varGamma J \eta(t) - \bar{\rm e}^{\rm T}(kT)(I_{N-1} \otimes \varPhi_1)\bar{e}(kT) + \\ &\lambda \bar{x}^{\rm T}(kT)(E_2^{\rm T} \delta_1 E_2 \otimes \varPhi_1)\bar{x}(kT). \end{aligned}$$

According to the Lyapunov stability theory, such that

$$\eta^{\rm T}(t) J^{\rm T} \varGamma J \eta(t) + \begin{pmatrix} I_2 \\ I_5 \end{pmatrix}^{\rm T} \begin{pmatrix} \varTheta_1 & 0 \\ * & \varTheta_2 \end{pmatrix} \begin{pmatrix} I_2 \\ I_5 \end{pmatrix} < 0$$

holds (i.e., (7)), where $\varTheta_1 = \lambda E_2^{\rm T} \delta_1 E_2 \otimes \varPhi_1$ and $\varTheta_2 = -I_{N-1} \otimes \varPhi_1 + E_2^{\rm T} \lambda \delta_1 E_2 \otimes \varPhi_1$, then the asymptotically stability of (6) is guaranteed. Therefore, the desired consensus can be achieved.

Theorem 2 Given a positive scalar $0 < \delta_2 < 1$, if there exist the appropriate dimension symmetric matrices $\bar{P}_{\rm r} = \bar{P}_{\rm r}^{\rm T} > 0$, $r \in S$; $\bar{Q}_j = \bar{Q}_j^{\rm T} \geqslant 0$, $\bar{R}_j = \bar{R}_j^{\rm T} \geqslant 0$, $j = 1, 2$ and any appropriate dimension matrices $\bar{M}_k$, $k = 1, 2, \cdots, 6$, such that

$$\begin{pmatrix} \bar{\varGamma}_{11} & \bar{\varGamma}_{12} & 0 & 0 & \bar{\varGamma}_{15} & \bar{\varGamma}_{16} & \bar{\varGamma}_{17} & 0 & \bar{\varGamma}_{19} & \cdots & \bar{\varGamma}_{1,10} \\ * & \tilde{\varGamma}_{22} & \bar{\varGamma}_{23} & \bar{\varGamma}_{24} & 0 & \bar{\varGamma}_{26} & \bar{\varGamma}_{27} & \bar{\varGamma}_{28} & 0 & 0 & 0 \\ * & * & \bar{\varGamma}_{33} & 0 & 0 & 0 & 0 & \bar{\varGamma}_{38} & 0 & 0 & 0 \\ * & * & * & \bar{\varGamma}_{44} & 0 & 0 & 0 & \bar{\varGamma}_{48} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\varGamma}_{55} & \bar{\varGamma}_{56} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \bar{\varGamma}_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \bar{\varGamma}_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \bar{\varGamma}_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \bar{\varGamma}_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \cdots & 0 \\ * & * & * & * & * & * & * & * & * & * & \bar{\varGamma}_{10,10} \end{pmatrix} < 0 \tag{9}$$

holds, then (6) achieve asymptotically consensus under (2) and (3), where time-delay satisfy (4), data packet dropouts satisfy (5) and $K_{\rm r} = \bar{K}_{\rm r} \bar{P}_{\rm r}^{-1}$. Where

$$\begin{aligned} \tilde{\varGamma}_{22} = {} & I_{N-1} \otimes (-\bar{U}_2 - \bar{U}_2^{\rm T} + \bar{U}_3 + \bar{U}_3^{\rm T} - \bar{U}_5 - \\ & \bar{U}_5^{\rm T}) + \lambda \delta_2 \tilde{L} \otimes \varPhi, \\ \tilde{\varGamma}_{55} = {} & -I_{N-1} \otimes \varPhi + \lambda \delta_2 \tilde{L} \otimes \varPhi, \end{aligned}$$

the other parameters have been defined in Theorem 1.

Proof The proof for Theorem 2 is almost same as the previous proof for Theorem 1, the only difference is that the event-trigger is changed to (2). According to (2), when $t \in [kT + \tau, (k + 1)T + \tau)$, we have $X_i^{\rm T}(g_i^l T + h_i T) \varPhi_2 X_i(g_i^l T + h_i T) < \delta_2 Y_i^{\rm T}(g_i^l T) \varPhi_2 Y_i(g_i^l T)$, then the above inequality can be reformulated as

$$\mathrm{e}^{\mathrm{T}}(kT)(I_N \otimes \Phi_2)e(kT) < x^{\mathrm{T}}(kT)(\delta_2 L^{\mathrm{T}} L \otimes \Phi_2)x(kT).$$

The right part of the above inequality can be written as

$$x^{\mathrm{T}}(kT)(\delta_2 L \otimes \Phi_2)x(kT) = \bar{x}^{\mathrm{T}}(kT)(\delta_2 \tilde{L} \otimes \Phi_2)\bar{x}(kT),$$

where $\tilde{L} = E_2^{\mathrm{T}} L^{\mathrm{T}} L E_2$.

Then the above inequality is equivalent to

$$\bar{\mathrm{e}}^{\mathrm{T}}(kT)(I_{N-1} \otimes \Phi_2)\bar{e}(kT) < \lambda \bar{x}^{\mathrm{T}}(kT)(\delta_2 \tilde{L} \otimes \Phi_2)\bar{x}(kT).$$

Therefore, we have

$$\begin{aligned}&\dot{V}(\cdot) = \\ &\dot{V}(\cdot) - \bar{\mathrm{e}}^{\mathrm{T}}(kT)(I_{N-1} \otimes \Phi_2)\bar{e}(kT) + \\ &\bar{\mathrm{e}}^{\mathrm{T}}(kT)(I_{N-1} \otimes \Phi_2)\bar{e}(kT) < \\ &\eta^{\mathrm{T}}(t)J^{\mathrm{T}}\Gamma J\eta(t) - \bar{\mathrm{e}}^{\mathrm{T}}(kT)(I_{N-1} \otimes \Phi_2)\bar{e}(kT) + \\ &\lambda \bar{x}^{\mathrm{T}}(kT)(\delta_2 \tilde{L} \otimes \Phi_2)\bar{x}(kT).\end{aligned}$$

According to the Lyapunov stability theory, such that

$$\eta^t(t)J^{\mathrm{T}}\Gamma J\eta(t) + \begin{pmatrix} I_2 \\ I_5 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \Theta_3 & 0 \\ * & \Theta_4 \end{pmatrix} \begin{pmatrix} I_2 \\ I_5 \end{pmatrix} < 0$$

holds (i.e., (9)), where $\Theta_3 = \lambda\delta_2\tilde{L} \otimes \Phi_2$ and $\Theta_4 = -I_{N-1} \otimes \Phi_2 + \lambda\delta_2\tilde{L} \otimes \Phi_2$, then the asymptotically stability of (6) is guaranteed, thus complete Theorem 2.

Remark 5 From (1) and (2), one can conclude that with larger values for δ_1 and δ_2, the event-triggered frequencies will become lower. Theoretically, without corrupting the consensus, there must exist tolerable upper boundaries of δ_1 and δ_2. Theorems 1 and 2 provide us a feasible method to acquire such maximal values. Namely, given $\bar{L}_\sigma$, Π, τ and T, set $\delta_1 = \delta_0 + \Delta\delta$ or $\delta_2 = \delta_0 + \Delta\delta$, where δ_0 and $\Delta\delta$ denote the initial value and the increasing step, respectively; the critical maximum of δ_1 and δ_2 can be connivently obtained by iteratively solving (7) and (9).

Remark 6 Note that the topology is supposed to be a directed graph, it is feasible for Theorems 1 and 2 to be extended to undirected circumstances. Generally, the connected graph is a fundamental assumption in most similar researches. However, in this work, such a condition is not strictly required. It implies that occasional disconnected subgraphs can be tolerated by the proposed method. Namely, given some disconnected sub-graphs, if there exist feasible solution of (7) or (9), the desired consensus still can be achieved.

5 Numerical examples

In this section, Example 1 is firstly given to validate the effectiveness of the proposed method. Afterwards, Example 2 is used to compare with [18].

5.1 Example 1

The supposed 4 switching topologies are shown in Fig. 2.

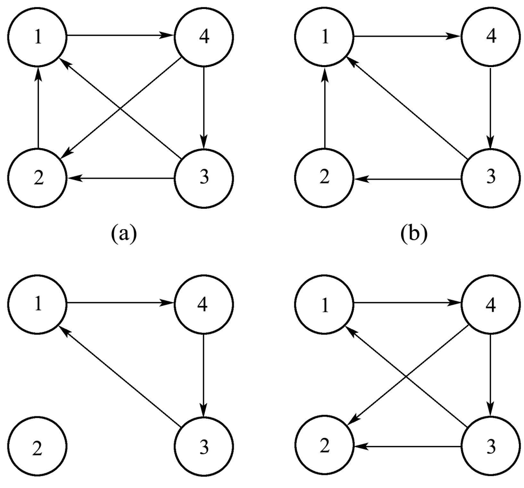


Fig. 2 Switching topologies

Figure 2(a) shows the initial connected graph, the other three graphs are employed to simulate possible data packet dropouts events. Fig. 2(c) illustrates that the agent 2 is disconnected from the topology, in this case, the disconnected graph is introduced in the network. The probability transition matrix Π is assumed as $\begin{pmatrix} 0.5 & ? & ? & ? \\ 0.65 & ? & ? & 0.1 \\ 0.65 & ? & ? & ? \\ 0.5 & ? & 0.1 & ? \end{pmatrix}$, where the notation “?” denotes the unknown information. Accordingly, we choose one instant (i.e., $r(t)$) as shown in Fig. 3.

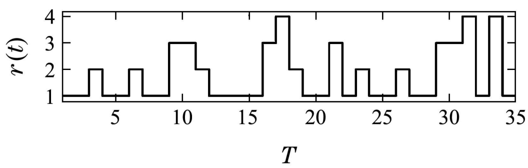


Fig. 3 Markov chain $r(t)$

Given $A = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.8 \end{pmatrix}$, $B = \begin{pmatrix} 0.2 \\ 0.3 \end{pmatrix}$, set $\tau = 0.02\,\mathrm{s}$ and $T = 0.01\,\mathrm{s}$, the state initial values are set as $x_1(0) = (-0.5, -0.5)^{\mathrm{T}}$, $x_2(0) = (-1.5, -1.5)^{\mathrm{T}}$, $x_3(0) = (0.5, -1.5)^{\mathrm{T}}$, $x_4(0) = (1.5, 1.0)^{\mathrm{T}}$. According to Theorems 1 and 2, we have the calculation results as listed in Tables 1 and 2, respectively.

Table 1 Calculation result

	Theorem 1	Theorem 2
	$\delta_1 = 0.41$	$\delta_2 = 0.12$
Φ	$\begin{pmatrix} 3.4140 & 0 \\ 0 & 3.4122 \end{pmatrix}$	$\begin{pmatrix} 4.7748 & 0.0001 \\ 0.0001 & 4.7731 \end{pmatrix}$
K_1	(0.0010, 0.0017)	(0.0015,0.0026)
K_2	(0.0009,0.0015)	(0.0014,0.0023)
K_3	(0.0011,0.0018)	(0.0014,0.0024)
K_4	(0.0011, 0.0018)	(0.0004,0.0008)

Table 2 Criterion value

	χ_1	χ_2
Topology 1	3.9580	3.2407
Topology 2	3.9580	2.8065
Topology 3	3.9580	1.9845
Topology 4	3.9580	2.8065

From Table 2, we know that the event-trigger (1) probably has lower triggering frequency than event-trigger (2). To verify the effectiveness, we first employ the event-trigger (1), the agent trajectories and the event-triggered time instants are shown in Figs. 4 and 5, respectively.

From Fig. 4, one can see that all of the trajectories are asymptotically attenuated, the curves converge to the equilibrium point in a relatively fast speed. In addition, the curves show a good agreement with the target consensus even some disconnect graphs are activated. The proposed method shows good robustness against the communication uncertainties.

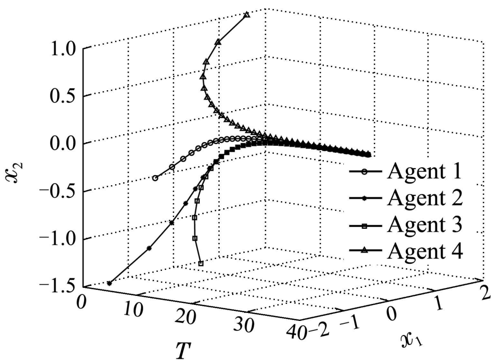
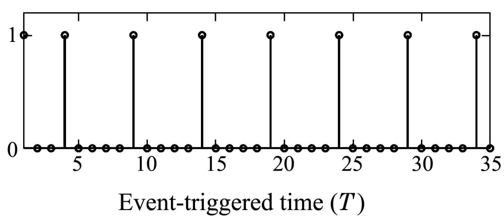
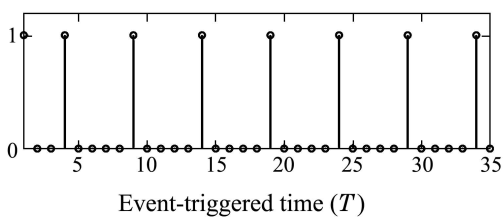


Fig. 4 Agent trajectories

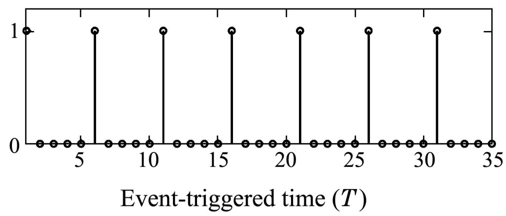
In Fig. 5, the vertical values are set as 1 only if the event-triggered threshold is violated, or 0 if not. One can clearly see that only a very small amount of data samples are broadcasted, accordingly, the communication burden as well as the controller updates are significantly reduced.



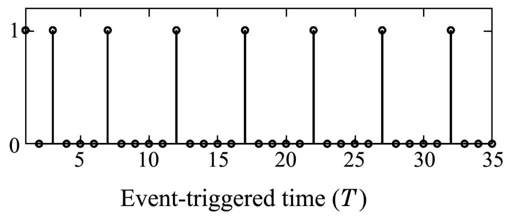
(a) Event-triggered time of Agent 1



(b) Event-triggered time of Agent 2



(c) Event-triggered time of Agent 3



(d) Event-triggered time of Agent 4

Fig. 5 Event-triggered time of the agents

The effectiveness of Theorem 1 is thus verified, now we proceed to verify Theorem 2. After employing the event-trigger (2) and by Theorem 2, the trajectory errors in comparison with the corresponding former one are illustrated in Fig. 6. The event-triggered time instants of each agent are shown as Fig. 5.

Figure 6 shows the trajectory error between two types of event-triggered consensus with the same validation example. Notice that the magnitude is quite small (10^{-3}), it indicates that the control performances among them have little differences. Table 2 shows that event-trigger (2) has lower triggering frequency than event-trigger (1). From Fig. 7, one can see that the event-triggered frequencies of (2) (i.e. 26%, 40%, 26%, 29%) are higher than the former one indeed (i.e. 23%, 23%, 20%, 23%), thus validating the effectiveness of the proposed methods.

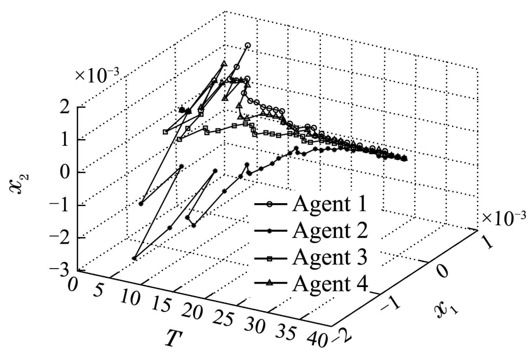
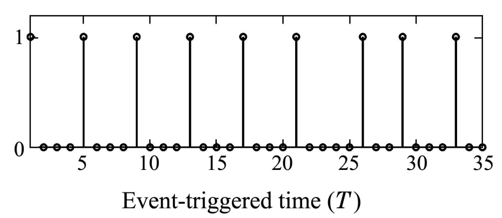
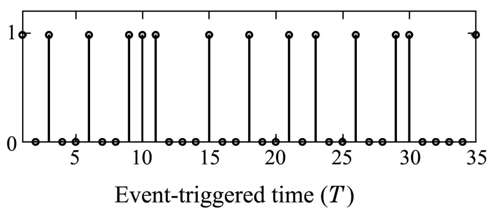


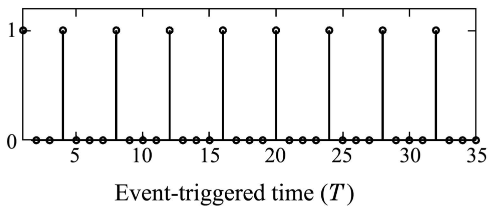
Fig. 6 Error trajectories



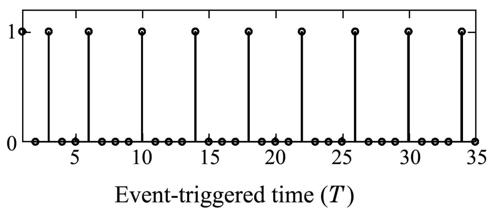
(a) Event-triggered time of Agent 1



(b) Event-triggered time of Agent 2



(c) Event-triggered time of Agent 3



(d) Event-triggered time of Agent 4

Fig. 7 Event-triggered time of the agents

5.2 Example 2

In [15,18], a similar consensus issue has been discussed with fixed topology (i.e., Fig. 2(a)). The event-triggers advocated in [15,18] are the same with (2). Because the results in [18] are superior than [15], therefore, we only compare with [18] wherein $A = \begin{pmatrix} 0 & 1 \\ 0 & -0.4 \end{pmatrix}$, $B = (0.8, 0.5)^{\mathrm{T}}$, $\tau = 0\,\mathrm{s}$, $T = 0.06\,\mathrm{s}$, $x_1(0) = (0, 1.0)^{\mathrm{T}}$, $x_2(0) = (0.5, 1.5)^{\mathrm{T}}$, $x_3(0) = (-1.0, 0)^{\mathrm{T}}$ and $x_4(0) = (0.5, -0.5)^{\mathrm{T}}$.

One can discover that by removing the rear part of (7) or (9) from 9th row and 9th column to the end, the rest part can handle the fixed topology situation. According to Theorem 2, we have $\delta_2 = 0.046$, $K = (0.2326, 0.1407)$, $\varPhi_2 = \begin{pmatrix} 452.3834 & -180.0247 \\ -180.0247 & 72.5753 \end{pmatrix}$. The agent trajectories and the triggered time instants are given in Figs. 8 and 9, respectively.

From Fig. 8, one can see that the trajectories fluctuate in a short time period and the curves converge to the equilibrium point quickly, the desired consensus is achieved. Fig. 9 illustrates the broadcasting instants of each agent, one can see that the event-triggered frequencies are reduced. The broadcasting ratios of agents are, respectively, 60%, 58%, 33% and 33%. It is higher than the ratios of [18], i.e., 9.8%, 7.8%, 6.6% and 9.0%. However, it can be estimated from Fig. 8 that the consensus is achieved in less than 3.6 s, which is lower than over 15.0 s in [18]. The proposed method thus shows a quicker converge rate.

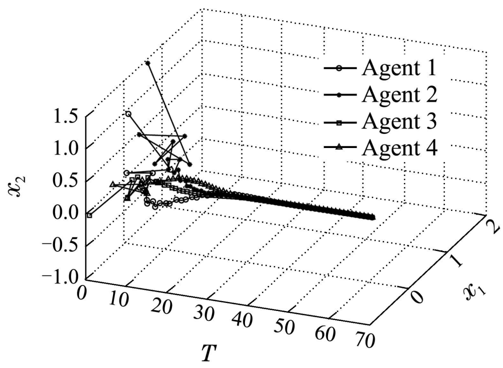
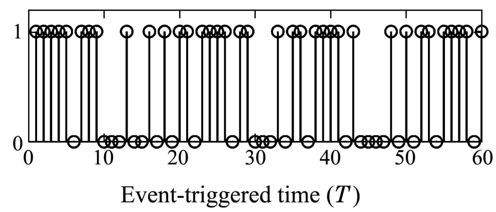
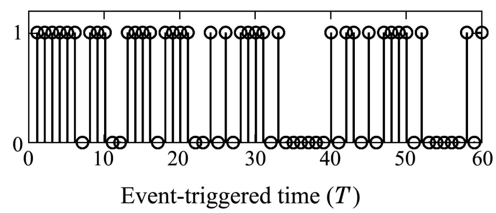


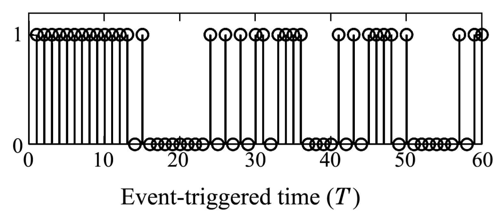
Fig. 8 Agent trajectories



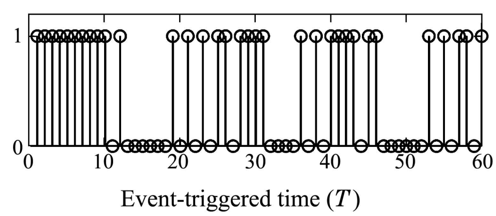
(a) Event-triggered time of Agent 1



(b) Event-triggered time of Agent 2



(c) Event-triggered time of Agent 3



(d) Event-triggered time of Agent 4

Fig. 9 Event-triggered time of the agents

6 Conclusions

This paper investigates the event-triggered consensus for MASs with communication constraints. An approximate event-triggered evaluation method is proposed which provides a reference for choosing the appropriate one from two typical event-triggers. Accordingly, a time-delay dependent Markov switching cooperative controller design method is developed. With the proposed scheme, the communication burden and the number of controller updates can be significantly reduced while the desired consensus of MASs can be preserved.

References:

[1] OH K K, PARK M C, AHN H S. A survey of multi-agent formation control [J]. *Automatica*, 2015, 53: 424 – 440.

[2] 王力, 李岱, 何忠贺, 等. 基于多智能体分群同步的城市路网交通控制 [J]. 控制理论与应用, 2014, 31(11): 1448 – 1456.
(WANG Li, LI Dai, HE Zhonghe, et al. Urban traffic network control based on cluster consensus of multi-agent systems [J]. *Control Theory & Applications*, 2014, 31(11): 1448 – 1456.)

[3] ZHANG W, LIU W X, WANG X, et al. Distributed multiple agent system based online optimal reactive power control for smart grids [J]. *IEEE Transactions on Smart Grid*, 2014, 5(5): 2421 – 2431.

[4] BENZERROUK A, ADOUANE L, MARTINET P. Stable navigation in formation for a multi-robot system based on a constrained virtual structure [J]. *Robotics Autonomous Systems*, 2014, 62(12): 1806 – 1815.

[5] SUN X J, ZHOU R, HOU D L, et al. Consensus of leader-followers system of multi-missile with time-delays and switching topologies [J]. *Optik*, 2014, 125(3): 1202 – 1208.

[6] 关永强, 纪志坚, 张霖, 等. 多智能体系统能控性研究进展 [J]. 控制理论与应用, 2015, 32(4): 421 – 431.
(GUAN Yongqiang, JI Zhijian, ZHANG Lin, et al. Recent developlents on controllability of multi-agent systems [J]. *Control Theory & Applications*, 2015, 32(4): 421 – 431.)

[7] ZHOU B, LIN Z L. Consensus of high-order multi-agent systems with large input and communication delays [J]. *Automatica*, 2014, 50(2): 452 – 464.

[8] ZHOU B, LIN Z L. Consensus of data-sampled multi-agent systems with random communication delay and packet loss [J]. *IEEE Transactions on Automatic Control*, 2010, 55(4): 939 – 943.

[9] LI T, XIE L H. Distributed coordination of multi-agent systems with quantized-observer based encoding-decoding [J]. *IEEE Transactions on Automatic Control*, 2012, 57(12): 3023 – 3037.

[10] NI Y H, LI X. Consensus seeking in multi-agent systems with multiplicative measurement noises [J]. *System & Control Letters*, 2013, 62(5): 430 – 437.

[11] LI W Q, XIE L H, ZHANG J F. Containment control of leader-following multi-agent systems with Markovian switching network topologies and measurement noises [J]. *Automatica*, 2015, 51: 263 – 267.

[12] FAN Y, FENG G, WANG Y, et al. Distributed event-triggered control of multi-agent systems with combinational measurements [J]. *Automatica*, 2013, 49(2): 671 – 675.

[13] HU A, CAO J D, HU M F, et al. Event-triggered consensus of multi-agent systems with noises [J]. *Journal of the Franklin Institute*, 2015, 352(9): 3489 – 3503.

[14] LI H J, MING C, SHEN S G, et al. Event-triggered control for multi-agent systems with randomly occurring nonlinear dynamics and time-varying delay [J]. *Journal of the Franklin Institute*, 2014, 351(5): 2582 – 2599.

[15] DIMAROGONAS D V, FRAZZOLI E, JOHANSSON K H. Distributed event-triggered control for multi-agent systems [J]. *IEEE Transactions on Automatic Control*, 2012, 57(5): 1291 – 1297.

[16] ZHANG H, FENG G, YAN H C, et al. Observer-based output feedback event-triggered control for consensus of multi-agent systems [J]. *IEEE Transactions on Industrial Electronics*, 2014, 61(9): 4885 – 4894.

[17] LI L L, HO D W C, XU S Y. A distributed event-triggered scheme for discrete-time multi-agent consensus with communication delays [J]. *IET Control Theory & Applications*, 2014, 8(10): 830 – 837.

[18] GUO G, DING L, HAN Q L. A distributed event-triggered transmission strategy for sampled-data for sampled-data consensus of multi-agent systems [J]. *Automatica*, 2014, 50(5): 1489 – 1496.

[19] 陈阳舟, 盖彦荣, 宋学君. 基于部分稳定性方法的离散时间多智能体系统的一致性 [J]. 控制理论与应用, 2014, 31(4): 438 – 443.
(CHEN Yangzhou, GE Yanrong, SONG Xuejun. Partial-stability-based approach to consensus problem in discrete-time multi-agent systems [J]. *Control Theory & Applications*, 2014, 31(4): 438 – 443.)

[20] 王振华, 徐娟娟, 张焕水. 受未知通信时滞影响的高阶多智能体系统的趋同 [J]. 控制理论与应用, 2015, 32(3): 295 – 303.
(WANG Zhenhua, XU Juanjuan, ZHANG Huanshui. Consensus of higher-order multi-agent systems with unknown communication delay [J]. *Control Theory & Applications*, 2015, 32(3): 295 – 303.)

[21] 孙一杰, 张国良, 张胜修, 等. 一类异构多智能体系统一致性协议的收敛性分析 [J]. 控制理论与应用, 2014, 31(11): 1524 – 1529.
(SUN Yijie, ZHANG Guoliang, ZHANG Shengxiu, et al. Convergence analysis for consensus protocol of heterogeneous multi-agent systems [J]. *Control Theory & Applications*, 2014, 31(11): 1524 – 1529.)

[22] 宋海裕, 俞立, 胡鸿翔. 牵制控制下的多智能体系统群一致性 [J]. 控制理论与应用, 2012, 29(6): 766 – 772.
(SONG Haiyu, YU Li, HU Hongxiang. Group consensus in multi-agent systems via pinning control [J]. *Control Theory & Applications*, 2012, 29(6): 766 – 772.)

[23] NOWZARI C, CORTES J. Zeno-free, distributed event-triggered communication and control for multi-agent average consensus [C] //*Proceedings of American Control Conference*. Portland: ACC, 2014: 2148 – 2153.

[24] PENG C, HAN Q L. A novel event-triggered transmission scheme and L_2 control co-design for sampled-data control system [J]. *IEEE Transactions on Automatic Control*, 2013, 58(10): 2620 – 2626.

[25] GARCIA E, CAO Y C, CASBEER D W. Decentralized event-triggered consensus with general linear dynamics [J]. *Automatica*, 2014, 50(10): 2633 – 2640.

[26] ZHU W, JIANG Z P. Event-based leader-following consensus of multi-agent systems with input time delay [J]. *IEEE Transactions on Automatic Control*, 2015, 60(5): 1362 – 1367.

[27] SEYBOTH G S, DIMAROGONAS D V, JPHANSSON K H. Event-based broadcasting for multi-agent average consensus [J]. *Automatica*, 2013, 49(1): 245 – 252.

[28] ZHANG Y, TIAN Y P. Maximum allowable loss probability for consensus of multi-agent systems over random weighted lossy networks [J]. *IEEE Transactions on Automatic Control*, 2012, 57(8): 2127 – 2132.

[29] WANG D, WANG J L, WANG L. H_∞ controller design of networked control systems with Markov packet dropouts [J]. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2013, 43(3): 689 – 697.

[30] QUEVEDO D E, NEŠIĆ D. Robust stability of packetized predictive control of nonlinear systems with disturbances and Markovian packet losses [J]. *Automatica*, 2012, 48(8): 1803 – 1811.

作者简介:

崔彦良 (1980–), 男, 博士研究生, 目前研究方向为多智能体系统控制和切换系统等, E-mail: cyl1600@126.com;

费敏锐 (1961–), 男, 教授, 博士生导师, 研究方向为网络化控制及其应用等, E-mail: mrfei@staff.shu.edu.cn;

杜大军 (1978–), 男, 副研究员, 研究方向为多智能体系统控制、网络控制系统的滤波与先进控制等, E-mail: ddj@shu.edu.cn;

李 慷 (1968–), 男, 教授, 博士生导师, 研究方向为非线性系统建模、神经网络、生物智能等, E-mail: k.li@qub.ac.uk.