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有向切换通信拓扑下多无人机分布式编队控制

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摘要: 本文对多无人机分布式时变编队控制问题进行了研究. 无人机之间的通信拓扑假定是有向和切换的. 基于自身状态与邻居状态的相对局部信息构建了分布式编队控制器. 通过引入一个恰当的编队误差向量, 将有向切换通信拓扑下的多无人机编队问题转化为一个切换系统的镇定问题. 基于Lyapunov稳定性分析方法得到了达成编队的充分性条件. 仿真实验结果验证了结论的有效性.

关键词: 无人机; 编队; 有向切换拓扑

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Distributed formation control for multiple unmanned aerial vehicles with directed switching communication topologies

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Abstract: We investigate the time-varying formation problem for unmanned aerial vehicles. The communication topology is assumed to be directed and switching. Distributed formation controllers are proposed using only relative state information of neighboring agents. By constructing a proper formation error vector, we convert the formation problem with switching communication topologies into a stabilization problem of a switched system. Sufficient conditions are obtained using common Lyapunov approach. Finally, a numerical simulation is provided to illustrate the effectiveness of the theoretical results.

Key words: unmanned aerial vehicles; formation; directed switching topologies

1 Introduction

Cooperative control of a group of unmanned aerial vehicles (UAVs) has drawn great attention for its broad potential applications in the military area such as cooperative targets search[1], cooperative attack[2] and battlefield persistent surveillance[3]. Formation control is an important issue in cooperative control for the UAVs. A formation is defined as a group of UAVs forming a particular configuration[4]. In this group, UAVs are coupled through communication topologies and achieve the formation throughout distributed controllers. It has been studied using different approaches such as leader-follower structure[5–6], virtual structure[7] and behavior approach[8].

Recently, the consensus or synchronization based approaches for formation problem have been developed. By performing a variable transformation, the formation problem can be converted into a consensus problem or a synchronization problem. In [9], the formation stability was investigated using consensus based approach. The feasibility problem of formation for higher-order systems was studied with fixed communication topologies in [10]. In [11], consensus based approach was employed to design the formation controller of a four-order flight dynamics. Note that, all the mentioned above articles focus on the formation problem with fixed formation shape. However, in real applications, time-varying formation may be required. This may happen when a group of UAVs keep surveilling a moving target or move through a narrow path. Motivated by this, some efforts have been made to study the time-varying formation problem. In [12], the time-varying formation problem of nonholonomic wheeled mobile robots was investigated using synchronization approach. The feedback information differential game approach was used to solve the formation problem in [13]. In [14], consensus based approach was employed to deal with formation problem of a group of UAVs with second order dynamics.

In many applications, the interaction topology a-

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mong agents may change dynamically. This may happen when the communication links among agents may be unreliable due to disturbance and obstacle, or subject to communication range limitations. In order to deal with this situation, the time-varying formation problems of UAVs with undirected switching interaction topologies were investigated in [15–16]. However, in real applications, the communications topologies are usually assumed to be directed. Thus, the conclusions obtained in [15–16] are not applicable to the case when the communications topologies are directed.

Motivated by the above observation, this paper investigates the time-varying formation problem of a group of UAVs with directed switching topologies. The communication topology is modelled by a directed graph which is assumed to be strongly connected and balanced. Based on a special property of the Laplacian matrix of the graph and a properly designed formation error vector, the formation problem with switching communication topologies is converted into a stabilization problem of a switched system. Then a common Lyapunov function is constructed. It is proved that if the formation configuration fulfils some condition and the feedback matrix is properly designed, the time-varying formation can be achieved. Compared with the results in [15–16], where the communication topology are restricted to be undirected, the topology condition is significantly relaxed here. Meanwhile, a quite different approach for analysis is used here without the need of performing any similarity transformation on the Laplacian matrix for analysis as that in [15–16]. The analysis complexity is reduced.

The remainder of this paper is organized as follows. In Section 2, some preliminaries and the problem formulation are provided. In Section 3, the main results are presented. In Section 4, a simulation example is presented. Section 5 is the conclusion.

2 Preliminaries and problem formulation

2.1 Preliminaries

Throughout this paper, $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times n}$ denote the set of $n\times n$ real and complex matrices, respectively. $\otimes$ denotes the Kronecker product. I_n is the $n\times n$ identity matrix. For $\mu\in\mathbb{C}$, the real part is $\mathrm{Re}(\mu)$. For a square matrix A, $\lambda(A)$ denotes the eigenvalues of matrix A. $A>B$ ($A\geqslant B$) means that $A-B$ is positive definite (respectively, positive semidefinite). $\max\{\lambda(A)\}$ ($\min\{\lambda(A)\}$) denotes the largest (smallest) eigenvalue of the matrix A.

A directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ contains the vertex set $\mathcal{V}=\{1,2,\cdots,N\}$, the directed edges set $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$, the weighted adjacency matrix $\mathcal{A}=[a_{ij}]_{N\times N}$ with nonnegative elements a_{ij}. $a_{ij}=1$ if there is a directed edge between vertex i and j, $a_{ij}=0$ otherwise. The set of neighbors of i is defined as $\mathcal{N}_i:=\{j\in\mathcal{V}:a_{ij}=1\}$. A directed path is a sequence of ordered edges of the form $(i_1,i_2),(i_2,i_3),\cdots$, where $i_j\in\mathcal{V}$. The Laplacian matrix of the topology $\mathcal{G}$ is defined as $\mathcal{L}=[\mathcal{L}_{ij}]_{N\times N}$, where $\mathcal{L}_{ii}=\sum\limits_{j\neq i}a_{ij}$ and $\mathcal{L}_{ij}=-a_{ij}$. Then 0 is an eigenvalue of $\mathcal{L}$ with 1_N as the eigenvector. A directed graph is called balanced if $\sum\limits_{j=1}^{N}a_{ij}=\sum\limits_{j=1}^{N}a_{ji}$. A directed graph is said to be strongly connected if there is a directed path between every pair of distinct vertices.

In this paper, the communication topology is molded by a directed graph and we assume that the communication topology is time-varying. Denote $\hat{\mathcal{G}}=\{\mathcal{G}^1,\mathcal{G}^2,\cdots,\mathcal{G}^p\},p\geqslant 1$ be the set of all possible directed topologies. We define the switching signal $\sigma(t)$, where $\sigma(t)$: $[0,+\infty)\to\mathcal{P}=\{1,2,\cdots,p\}$. $0=t_0<t_1<t_2<\cdots$ denote the switching instants of $\sigma(t)$. Let $\mathcal{G}^{\sigma(t)}\in\hat{\mathcal{G}}$ be the communication topology at time t. Across each time interval $[t_j,t_{j+1})$, $j\in\mathbb{Z}$, the graph $\mathcal{G}^{\sigma(t)}$ is fixed.

Lemma 1 Zero is a simple eigenvalue of $\mathcal{L}$ and all the other nonzero eigenvalues have positive real parts if and only if the graph $\mathcal{G}$ has a directed spanning tree[17].

Lemma 2 Suppose that the graph $\mathcal{G}$ is strongly connected and balanced. Then, $\mathcal{L}+\mathcal{L}^{\mathrm{T}}$ is positive semi-definite with zero being its simple eigenvalue[18–19].

2.2 Problem formulation

Consider a multi-agent system composed of N agents with following identical dynamics

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\ i=1,2,\cdots,N,\quad(1)$$

where $x_i(t)\in\mathbb{R}^n$, $u_i(t)\in\mathbb{R}^p$ are the state and the control input of the ith agent, respectively. A and B are constant system matrices with compatible dimensions.

Definition 1 The time-varying formation of system (1) is said to be achieved, if there exists a controller $u_i(t)$ such that

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)-h_{ij}(t)\|=0,\ i,j=1,2,\cdots,N,$$

where $h_{ij}(t)=h_i(t)-h_j(t)$, and $h(t)=[h_1^{\mathrm{T}}(t)\ \ h_2^{\mathrm{T}}(t)\ \ \cdots\ \ h_N^{\mathrm{T}}(t)]^{\mathrm{T}}\in\mathbb{R}^{nN}$ is the time-varying formation vector.

In order to achieve the formation, the following distributed controller based on local relative states information of neighbor agents is proposed

$$u_i(t) = cK \sum_{j=1}^{N} a_{ij}^{\sigma(t)}(x_j(t) - x_i(t) - h_{ji}(t)) + w_i(t),\ i = 1, 2, \cdots, N, \tag{2}$$

where $K \in \mathbb{R}^{p\times n}$ is the feedback matrix to be designed, c is the coupling strength to be selected, $a_{ij}^{\sigma(t)}$ is the element of the adjacency matrix $\mathcal{A}^{\sigma(t)} = [a_{ij}^{\sigma(t)}]_{N\times N}$ of the graph $\mathcal{G}^{\sigma(t)}$, $w_i(t)$ is the external command inputs depending on $h_i(t)$[16].

The closed-loop system dynamics of (1) with the controller (2) is

$$\dot{x}(t) = (I_N \otimes A - c\mathcal{L}^{\sigma(t)} \otimes BK)x(t) + (c\mathcal{L}^{\sigma(t)} \otimes BK)h(t) + (I_N \otimes B)w(t), \tag{3}$$

where

$$x(t) = [x_1(t)^{\mathrm{T}}\ x_2(t)^{\mathrm{T}}\ \cdots\ x_N(t)^{\mathrm{T}}]^{\mathrm{T}},$$
$$w(t) = [w_1^{\mathrm{T}}(t)\ w_2^{\mathrm{T}}(t)\ \cdots\ w_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$\mathcal{L}^{\sigma(t)} \in \mathbb{R}^{N\times N}$ is the Laplacian matrix of the graph $\mathcal{G}^{\sigma(t)}$, $h(t) = [h_1^{\mathrm{T}}(t)\ h_2^{\mathrm{T}}(t)\ \cdots\ h_N^{\mathrm{T}}(t)]^{\mathrm{T}}$ is the time-varying formation vector.

Assumption 1 In this paper, we assume that each possible directed graph $\mathcal{G}^{\sigma(t)} \in \hat{\mathcal{G}}$, $\sigma(t) = i$, $i \in \mathcal{P}$, is strongly connected and balanced.

3 Main results

In this section, we will show that the formation can be achieved with arbitrary switching topologies fulfilling Assumption 1 if the formation vector fulfils some conditions and the feedback matrix in the formation controller is properly designed. Before moving forwards, we first introduce following conclusions obtained in our earlier work[0].

Lemma 3 For a Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N\times N}$ of graph $\mathcal{G}$ and a full row rank matrix $E \in \mathbb{R}^{(N-1)\times N}$ defined as

$$E = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \tag{4}$$

there exists a matrix $M \in \mathbb{R}^{N\times(N-1)}$ such that $\mathcal{L} = ME$. Furthermore, if the graph has a directed spanning tree, M is full column rank and $\mathrm{Re}(\lambda(EM)) > 0$.

Lemma 4 Suppose that the graph $\mathcal{G}$ is strongly connected and balanced, then matrix $E(\mathcal{L} + \mathcal{L}^{\mathrm{T}})E^{\mathrm{T}}$ is positive definite, where E is defined in (4), $\mathcal{L}$ is the Laplacian matrix of graph $\mathcal{G}$.

Without loose of generality, the following formation error vector is introduced

$$\xi(t) = (E \otimes I_n)(x(t) - h(t)),$$

where E is defined in (4), $\xi(t) = [\xi_1^{\mathrm{T}}(t)\ \cdots\ \xi_{N-1}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\xi_i(t) = x_i(t) - h_i(t) - x_{i+1}(t) + h_{i+1}(t)$, $i = 1, 2, \cdots, N-1$, We can see that the formation is achieved if and only if $\xi(t) \to 0$ as $t \to \infty$.

Based on the property of the graph Lapalcian matrix obtained in Lemma 3, the closed-loop formation error system dynamics can be written as

$$\begin{aligned}
&\dot{\xi}(t) = \\
&(I_{N-1} \otimes A - cEM^{\sigma(t)} \otimes BK)(E \otimes I_n)x(t) + \\
&(cEM^{\sigma(t)} \otimes BK)(E \otimes I_n)h(t) - \\
&(E \otimes I_n)\dot{h}(t) + (I_{N-1} \otimes B)(E \otimes I_n)w(t) = \\
&(I_{N-1} \otimes A - cEM^{\sigma(t)} \otimes BK) \times \\
&(\xi(t) + (E \otimes I_n)h(t)) + \\
&(cEM^{\sigma(t)} \otimes BK)(E \otimes I_n)h(t) - \\
&(E \otimes I_n)\dot{h}(t) + (I_{N-1} \otimes B)(E \otimes I_n)w(t) = \\
&(I_{N-1} \otimes A - cEM^{\sigma(t)} \otimes BK)\xi(t) - \\
&\dot{H}(t) + (I_{N-1} \otimes A)H(t) + (I_{N-1} \otimes B)W(t),
\end{aligned} \tag{5}$$

where

$$H(t) = [H_1^{\mathrm{T}}(t)\ H_2^{\mathrm{T}}(t)\ \cdots\ H_{N-1}^{\mathrm{T}}(t)] = (E \otimes I_n) \cdot h(t),$$
$$H_i(t) = h_i(t) - h_{i+1}(t),\ i = 1, 2, \cdots, N-1,$$
$$W(t) = (E \otimes I_n)w(t),$$

$M^{\sigma(t)}$ is the corresponding matrix such that $\mathcal{L}^{\sigma(t)} = M^{\sigma(t)}E$.

From the definition of the formation error vector defined above, we can see that the formation problem of system (3) with switching communication topologies has been converted into a stabilization problem of the switched system (5).

Theorem 1 Suppose that Assumption 1 holds. The formation of multi-agent system (1) can be achieved with arbitrary switching topologies fulfilling Assumption 1 if following two conditions are fulfilled.

i) $\lim\limits_{t\to\infty}((I_{N-1} \otimes B)W(t) - \dot{H}(t) + (I_{N-1} \otimes A)H(t)) = 0$, where $H(t) = (E \otimes I_n)h(t)$, $W(t) = (E \otimes I_n)w(t)$, E is defined in (4).

ii) There exist a real scalar $c > 0$ and a positive definite matrix P such that

[0] LIU W, ZHOU S, QI Y, et al. Leaderless consensus of multi-agent systems with Lipschitz nonlinear dynamics and switching topologies. *Neurocomputing*, 2015, http://dx.doi.org/10.1016/j.neucom.2015.09.005.

$$A^{\mathrm{T}}P + PA - \frac{c\lambda_0}{\tilde{\phi}_1}PBB^{\mathrm{T}}P < 0, \tag{6}$$

where $\lambda_0 = \min\limits_{i\in\mathcal{P}}\{\lambda(E((\mathcal{L}^{(i)})^{\mathrm{T}} + \mathcal{L}^{(i)})E^{\mathrm{T}})\}$, $\tilde{\phi}_1 = \max\{\lambda(EE^{\mathrm{T}})\}$, and the feedback matrix is designed as $K = B^{\mathrm{T}}P$.

Proof Since condition (i) holds then the following work is to design the feedback gain matrices in (2) such that the following switched autonomous systems is stable

$$\dot{\xi}(t) = (I_{N-1}\otimes A - cEM^{\sigma(t)}\otimes BK)\xi(t), \tag{7}$$

where $\sigma(t) = i$, $i\in\mathcal{P}$.

Consider the following common Lyapunov candidate of the switched system (7)

$$V(t) = \xi(t)^{\mathrm{T}}((EE^{\mathrm{T}})^{-1}\otimes P)\xi(t),$$

where P is a positive solution of the inequality (6). Since E is a full row rank matrix defined in (4), $EE^{\mathrm{T}} > 0$.

The derivation of this Lyapunov candidate along the trajectory of system (7) is

$$\begin{aligned}\dot{V}(t) = {} & 2\xi(t)^{\mathrm{T}}(I_{N-1}\otimes A - cEM^{\sigma(t)}\otimes BK)^{\mathrm{T}}\times\\ & ((EE^{\mathrm{T}})^{-1}\otimes P)\xi(t).\end{aligned} \tag{8}$$

Substituting $K = B^{\mathrm{T}}P$ into (8) yields

$$\begin{aligned}\dot{V}(t) = {} & \xi(t)^{\mathrm{T}}((EE^{\mathrm{T}})^{-1}\otimes(A^{\mathrm{T}}P + PA))\xi(t) -\\ & c\xi(t)^{\mathrm{T}}(((EM^{\sigma(t)})^{\mathrm{T}}(EE^{\mathrm{T}})^{-1} +\\ & (EE^{\mathrm{T}})^{-1}EM^{\sigma(t)})\otimes PBB^{\mathrm{T}}P)\times\xi(t).\end{aligned} \tag{9}$$

In light of the fact that $M^{\sigma(t)} = \mathcal{L}^{\sigma(t)}E^{\mathrm{T}}(EE^{\mathrm{T}})^{-1}$, one has

$$\begin{aligned}& (EM^{\sigma(t)})^{\mathrm{T}}(EE^{\mathrm{T}})^{-1} + (EE^{\mathrm{T}})^{-1}EM^{\sigma(t)} =\\ & (EE^{\mathrm{T}})^{-1}E(\mathcal{L}^{\sigma(t)})^{\mathrm{T}}E^{\mathrm{T}}(EE^{\mathrm{T}})^{-1} +\\ & (EE^{\mathrm{T}})^{-1}E\mathcal{L}^{\sigma(t)}E^{\mathrm{T}}(EE^{\mathrm{T}})^{-1} =\\ & (EE^{\mathrm{T}})^{-1}E((\mathcal{L}^{\sigma(t)})^{\mathrm{T}} + \mathcal{L}^{\sigma(t)})E^{\mathrm{T}}(EE^{\mathrm{T}})^{-1}.\end{aligned} \tag{10}$$

According to Lemma 4, one can obtain that $E((\mathcal{L}^{\sigma(t)})^{\mathrm{T}} + \mathcal{L}^{\sigma(t)})E^{\mathrm{T}} > 0$, $\sigma(t) = i$, $i\in\mathcal{P}$. Let $\lambda_0 = \min\limits_{i\in\mathcal{P}}\{\lambda(E((\mathcal{L}^{(i)})^{\mathrm{T}} + \mathcal{L}^{(i)})E^{\mathrm{T}})\}$, then it follows from (10), one has

$$\begin{aligned}& (EM^{\sigma(t)})^{\mathrm{T}}(EE^{\mathrm{T}})^{-1} + (EE^{\mathrm{T}})^{-1}EM^{\sigma(t)} \geqslant\\ & \lambda_0((EE^{\mathrm{T}})^{-1})^2.\end{aligned} \tag{11}$$

It follows from (9) using (11) that

$$\begin{aligned}\dot{V}(t) \leqslant {} & \xi(t)^{\mathrm{T}}((EE^{\mathrm{T}})^{-1}\otimes(A^{\mathrm{T}}P + PA))\xi(t) -\\ & c\lambda_0\xi(t)^{\mathrm{T}}(((EE^{\mathrm{T}})^{-1})^2\otimes PBB^{\mathrm{T}}P)\xi(t).\end{aligned} \tag{12}$$

Since $EE^{\mathrm{T}} \leqslant \tilde{\phi}_1 I_{N-1}$, where $\tilde{\phi}_1 = \max\{\lambda(EE^{\mathrm{T}})\}$, one can obtain that

$$\dot{V}(t) \leqslant \xi(t)^{\mathrm{T}}((EE^{\mathrm{T}})^{-1}\otimes\varXi)\xi(t), \tag{13}$$

where $\varXi = A^{\mathrm{T}}P + PA - \dfrac{c\lambda_0}{\tilde{\varphi}_1}PBB^{\mathrm{T}}P$.

According to (6), one can obtain that $\dot{V}(t) < 0$. This means that $\lim\limits_{t\to\infty}\xi(t) = 0$. Thus, the formation is achieved. This completes the proof.

Remark 1 By constructing a proper formation error vector, the formation problem of system (3) with switching topologies is converted into a stabilization problem of a switched system (5). Then, based on the property of the topologies in Lemma 4, a common Lyapunov function is constructed. Compared with the results in [15–16] where the time-varying formation problem are also investigated, there are two different aspects. Firstly, in [15–16], the communication topologies are restricted to be undirected. In contrast, the communication topologies used here are assumed to be directed which can include the undirected communication topologies as a special case. Thus, the conclusion obtained here is more applicable in real applications. Secondly, the analysis method used here is quite different. In [15–16], a similarity transformation was performed on the Laplacian matrix for analysis. Here, based on the property of Laplacian matrix in Lemma 3 and the properly designed formation error vector, a more direct analysis method is employed without performing any similarity transformation. The analysis complexity is significantly reduced.

Remark 2 In order to deal with the external terms $-\dot{H}(t) + (I_{N-1}\otimes A)H(t)$ introduced by the system dynamics and the time-varying property of the formation, external command inputs $w_i(t)$ was introduced[16] in the formation controller (2). Actually, if the formation vector fulfils that $\dot{h}_i(t) = Ah_i(t) + Bw_i(t)$, $i = 1, 2, \cdots, N$, condition (i) in Theorem 1 holds. It is interesting that, in this case, it can be reconsidered in such a way that for each UAV, there is a virtual leader having following systems dynamics $\dot{h}_i(t) = Ah_i(t) + Bw_i(t)$ where $h_i(t)$ is the state and $w_i(t)$ is the control input of the virtual leader i. The virtual leader has the same system dynamics as the follower's but with different control inputs.

4 Simulations

In this section, we provide a simulation example to illustrate the effectiveness of the above theoretical results. A group of four UAVs is considered. The system dynamics matrices are defined as

$$x_1(t) = \begin{bmatrix} x_{\mathrm{x}}(t)\\ v_{\mathrm{x}}(t)\\ x_{\mathrm{y}}(t)\\ v_{\mathrm{y}}(t)\end{bmatrix},\ A = \begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix},\ B = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}.$$

$[x_{\mathrm{x}}(t)\ \ x_{\mathrm{y}}(t)]$ and $[v_{\mathrm{x}}(t)\ \ v_{\mathrm{y}}(t)]$ are the position and velocity vector, respectively. The time-varying formation is chosen as follows[15]:

$$h_i(t)=\begin{bmatrix} r\cos(\omega t+\frac{\pi}{2}(i-1)) \\ -2r\sin(\omega t+\frac{\pi}{2}(i-1)) \\ r\sin(\omega t+\frac{\pi}{2}(i-1)) \\ 2r\cos(\omega t+\frac{\pi}{2}(i-1)) \end{bmatrix},\ i=1,2,3,4,$$

where $r=10\,\mathrm{m}$ and $\omega=2\,\mathrm{rad/s}$. The formation configuration is a rotating regular square. The initial states of the UAVs are chosen as

$$x_1(0)=[9.84\ \ -0.11\ \ 0.19\ \ 0.07]^{\mathrm{T}},$$
$$x_2(0)=[-0.41\ \ 0.04\ \ 10.51\ \ 0.22]^{\mathrm{T}},$$
$$x_3(0)=[-10.47\ \ 0.08\ \ 0.48\ \ 0.02]^{\mathrm{T}},$$
$$x_4(0)=[-0.93\ \ -0.08\ \ -9.11\ \ -0.25]^{\mathrm{T}}.$$

The directed communication topologies

$$\hat{\mathcal{G}}=\{\mathcal{G}^1,\mathcal{G}^2,\mathcal{G}^3,\mathcal{G}^4\}$$

are given in Fig.1. Clearly, each directed topology is strongly connected and balanced.

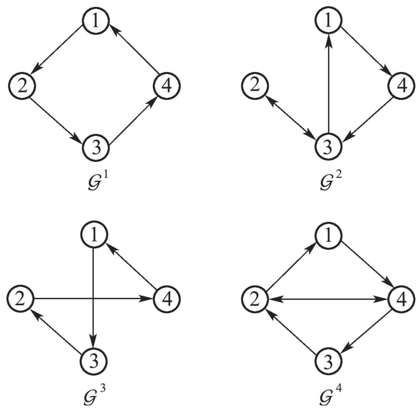

Fig. 1 Communication topologies $\{\mathcal{G}^1,\mathcal{G}^2,\mathcal{G}^3,\mathcal{G}^4\}$

Thus we can get $\lambda_0=1.6754$, $\tilde{\phi}_1=3.4142$. Solving the inequality (6) with $c=0.5$, we get a feasible solution

$$P=\begin{bmatrix} 2.3063 & 1.0644 & 0 & 0 \\ 1.0644 & 7.1721 & 0 & 0 \\ 0 & 0 & 15.9691 & 0 \\ 0 & 0 & 0 & 2.3236 \end{bmatrix}.$$

According to Theorem 1, the feedback matrix can be chosen as $K=B^{\mathrm{T}}P$. The external command inputs are chosen satisfying that $Bw_i(t)=\dot{h}_i(t)-Ah_i(t)$.

The topologies are switching as $\mathcal{G}^1\to\mathcal{G}^2\to\mathcal{G}^3\to\mathcal{G}^4\to\mathcal{G}^3\to\mathcal{G}^1$. Fig.3 show the formation errors trajectories of all the agents with switching signal shown in Fig.2. The time-varying formation trajectories of the UAV are shown in Fig.4. We can see that the time-varying formation has been achieved.

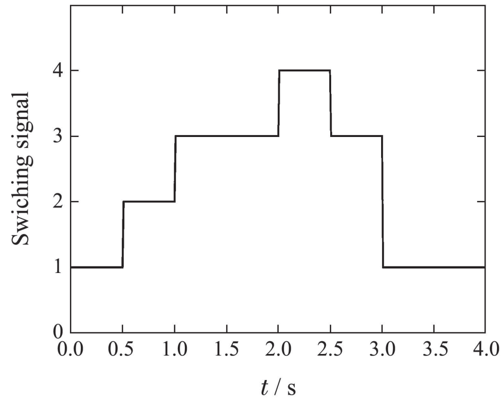

Fig. 2 Switching signal $\sigma(t)$

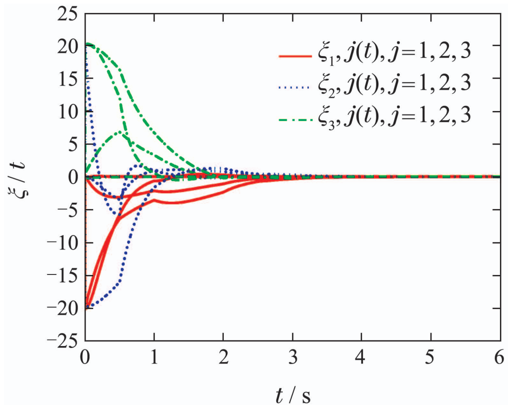

Fig. 3 Formation errors $\xi(t)=(E\otimes I_n)(x(t)-h(t))$

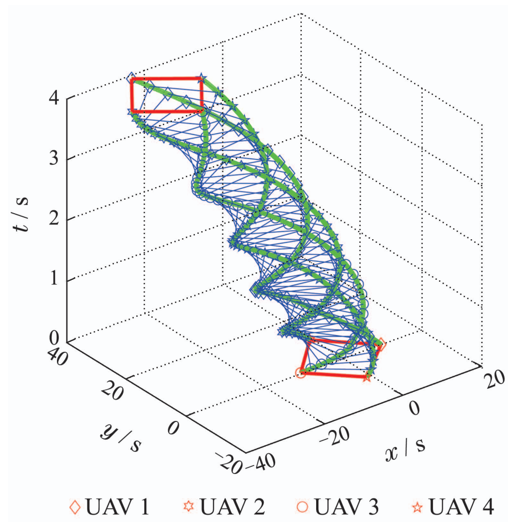

Fig. 4 The formation trajectories of the UAVs

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