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一类非仿射非线性时滞系统的动态状态反馈镇定

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摘要: 本文讨论了一类非仿射非线性时滞系统的全局镇定问题. 通过引入辅助积分系统和构造合适的Lyapunov-Krosovskii泛函, 提出了一种基于反推法的时滞无关动态状态反馈控制器, 所提控制方法无需时滞的任何先验知识. 利用Lyapunov稳定性理论证明了该控制策略能够保证非仿射时滞系统状态渐近收敛于原点, 且所有闭环信号全局有界. 一个仿真实例进一步验证了所得控制方案的可行性与有效性.

关键词: 非仿射系统; 非线性系统; 时滞; 动态状态反馈; 反推法; Lyapunov-Krosovskii泛函

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Dynamic state feedback stabilization for a class of nonaffine nonlinear systems with unknown time delays

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Abstract: We discuss the problem of global stabilization for a class of nonaffine nonlinear systems with unknown time delays. By introducing an auxiliary integrator system and constructing an appropriate Lyapunov-Krosovskii functional, a delay-independent dynamic state feedback controller is explicitly proposed via a backstepping approach, which does not require a priori knowledge of the time delays. Lyapunov stability theory is employed to prove that the states of the nonaffine time delay systems converge asymptotically to the origin and all the closed loop signals are globally bounded. A numerical example is further provided to illustrate the feasibility and effectiveness of the proposed control scheme.

Key words: nonaffine systems; nonlinear systems; time delay; dynamic state feedback; backstepping; Lyapunov-Krosovskii functional

1 引言(Introduction)

近年来, 非仿射非线性系统控制研究吸引了越来越多研究者的关注, 取得了许多富有意义的研究成果[1–16]. 文献[1]探讨了一类仿射输入纯反馈非线性系统控制问题, 提出了一种基于反推法的自适应控制算法, 然而, 该结果仅能保证系统局部渐近稳定. 文献[2]进一步讨论了该类系统的全局控制问题, 通过定义每个状态的参考轨迹, 给出了一种自适应反推控制方法. 针对一类泰勒级数型非仿射非线性系统, 文献[3]提出了一种有界光滑状态反馈控制算法, 该控制器可保证系统全局渐近镇定. 文献[4–5]研究了一类高阶非仿射非线性系统, 提出了一种新的可保证系统全局镇定的反推控制法–增加幂次积分法. 利用增加幂次积分法, 文献[6]研究了一类控制系数未知高阶非线性不确定系统的镇定问题, 设计了一种自适应状态反馈控制算法. 针对一类多输入多输出非仿射纯反馈非线性系统, 文献[7]运用隐函数定理和中值定理, 证明了该系统在一定条件下存在理想控制律, 并给出了该理想控制律的近似结果. 文献[8–11]分析了一类非仿射非线性系统, 提出了一种新的反推控制设计方法, 与标准反推法将状态x_{i+1}视为第i个子系统的虚拟控制变量不同, 该方法将第i个子系统的非仿射函数视为第i个子系统虚拟控制变量, 有效克服系统中的非仿射结构给控制器设计带来的困难. 针对模型信息完全未知的非仿射非线性系统控制设计问题, 常用的方法是利用隐函数定理确定理想控制律的存在性, 结合中值定理[12–14]、泰勒级数展开[15]和压缩映像原理[16]等方法, 将非仿射非线性系统转化为具有广义模型误差的

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仿射非线性系统, 然后利用神经网络或模糊系统逼近理想控制器, 最后按仿射非线性系统的设计思路给出相关控制算法. 然而, 基于通用逼近器的智能控制方法不能保证非仿射系统的全局性能.

在实际控制系统中, 时滞现象往往无法避免[17]. 时滞的出现常常会引起系统性能变差, 甚至不稳定[18], 因此, 时滞系统稳定性分析与控制器设计一直是控制领域的研究难点与热点. 分析时滞系统控制的主要工具包括Lyapunov–Krasovskii泛函和Lyapunov–Razumikhin函数. 文献[19]探讨了一类严格反馈非线性时滞系统, 通过构造合适的Lyapunov–Krasovskii泛函, 提出了一种基于反推法的鲁棒镇定控制器, 然而, 该方法中的虚拟控制律并不容易得到[20]. 针对一类参数化严格反馈非线性时滞系统, 文献[21]利用Lyapunov–Krasovskii泛函补偿未知时滞函数项, 给出了一种时滞相关的鲁棒自适应控制算法, 该控制器可保证闭环系统全局稳定. 文献[22]分析了一类非线性时滞系统自适应反馈控制问题, 提出了一种基于LaSalle–Razumikhin的方法, 通过构造Lyapunov–Razumikhin函数, 设计了一种时滞无关自适应镇定控制器, 其缺点是时滞函数项必须满足关于x_{1d}的线性增长条件. 基于更弱的时滞函数假设条件, 文献[23–27]分别讨论了非线性时滞系统的鲁棒自适应控制. 然而, 上述非线性时滞系统的结果大都是针对仿射非线性时滞系统. 与仿射非线性时滞系统相比, 由于非仿射特性与时滞的双重作用, 使得非仿射非线性时滞系统控制研究变得异常困难. 针对非仿射非线性时滞系统, 文献[28–33]利用隐函数定理和中值定理, 将非仿射时滞系统转化为仿射时滞系统, 提出了几种不同的时滞无关智能控制算法, 然而, 该类时滞无关控制算法仅能保证闭环系统半全局稳定. 据作者所知, 已有文献中暂无关于非仿射非线性时滞系统全局控制的结果报道.

本文分析了一类非仿射非线性时滞系统全局镇定问题. 通过引入辅助积分系统, 将非仿射输入非线性时滞系统转化为仿射输入时滞增广系统. 按照文献[8–11]提出的反推设计方法, 构造合适的Lyapunov–Krosovskii泛函补偿未知时滞函数项, 提出了一种鲁棒动态状态反馈控制算法. 所得控制策略保证了系统全局镇定, 且闭环系统所有信号有界. 最后, 一个数值实例对本文方法的有效性的进行了验证.

2 问题描述(Problem formulations)

考虑如下一类非仿射非线性时滞系统:

$$\begin{cases}\dot{x}_i(t)=f_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))+h_i(t,\bar{\boldsymbol{x}}_n(t),\\ \qquad \bar{\boldsymbol{x}}_n(t-\tau_n)),\ i=1,\cdots,n-1,\\ \dot{x}_n(t)=f_n(\bar{\boldsymbol{x}}_n(t),u(t))+h_n(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n)),\\ \boldsymbol{x}(t)=\boldsymbol{\varphi}(t),\ -\tau\leqslant t\leqslant 0,\end{cases} \tag{1}$$

其中: $\bar{\boldsymbol{x}}_i(t)=[x_1(t),\cdots,x_i(t)]^{\mathrm{T}}\in\mathbb{R}^i$, $i=1,\cdots,n$; $\boldsymbol{x}(t)=\bar{\boldsymbol{x}}_n(t)=[x_1(t),\cdots,x_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$为系统状态向量, $u(t)$为系统控制输入; $\bar{\boldsymbol{x}}_n(t-\tau_n)=[x_1(t-\tau_1),\cdots,x_n(t-\tau_n)]^{\mathrm{T}}\in\mathbb{R}^n$为时滞状态向量, τ_i为未知时滞, 且$\tau_i\leqslant\tau$, τ为未知上界常数, $i=1,\cdots,n$; $f_i(\cdot)$为已知光滑函数, 且$f_i(\mathbf{0})=0$, $h_i(\cdot)$为未知函数, $i=1,\cdots,n$; $\boldsymbol{\varphi}(t)$为初始状态向量连续函数.

注 1 系统(1)是一类更一般的非线性时滞系统, 不仅包含了应用广泛的严格反馈非线性时滞系统, 而且能够准确描述工业过程中很多具有非仿射特性的系统, 如风电机组系统[34]、化学系统[35]和飞行器系统[36]等. 值得指出的是, 由于系统同时具有非仿射结构与时滞作用, 使得大多数适合于严格反馈时滞系统的控制器设计方法很难直接应用于非仿射时滞系统控制.

控制目标: 设计控制器$u(t)$, 使系统状态$\boldsymbol{x}(t)$渐近收敛于原点, 且保证闭环系统的所有信号有界.

为了达到上述控制目标, 对系统(1)作如下假设:

假设 1 光滑函数$f_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))$满足

$$\left|\frac{\partial f_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))}{\partial x_{i+1}(t)}\right|\geqslant\epsilon,\ i=1,\cdots,n, \tag{2}$$

其中: ϵ为已知正常数, $x_{n+1}(t)=u(t)$.

注 2 假设1是系统(1)全局可控的一个基本条件[8–11], 其中ϵ仅为约束正常数, 不影响文中控制器设计.

假设 2[27] 未知函数$h_i(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n))$满足

$$|h_i(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n))|\leqslant g_i(\bar{\boldsymbol{x}}_i(t))\omega_i(x_1(t-\tau_1)), \tag{3}$$

其中: $g_i(\cdot)$和$\omega_i(\cdot)$为已知光滑函数, 且$\omega_i(0)=0$, $i=1,\cdots,n$.

引理 1[37] 对任意光滑函数$f(x):\mathbb{R}\to\mathbb{R}$, 下列恒等式成立:

$$f(x)-f(0)=\varpi(x)x, \tag{4}$$

其中$\varpi(x)=\int_0^1\frac{\partial f(s)}{\partial s}|_{s=\beta x}\mathrm{d}\beta$为光滑函数.

引理 2 对于任意的有界光滑函数$\psi_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))$, 若

$$\left|\frac{\partial\psi_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))}{\partial x_{i+1}(t)}\right|\geqslant\epsilon>0,$$

则$x_{i+1}(t)$亦有界, 其中$x_{n+1}(t)=u(t)(i=1,\cdots,n)$.

证 $\psi_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))$为光滑函数, 根据中值定理, 至少存在一点$\zeta(t)$ ($\zeta(t)\in(\min(0,x_{i+1}(t)),\max(0,x_{i+1}(t)))$), 使得

$$\psi_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))-\psi_i(\bar{\boldsymbol{x}}_i(t),0)=\frac{\partial\psi_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))}{\partial x_{i+1}(t)}|_{x_{i+1}(t)=\zeta(t)}(x_{i+1}(t)-0), \tag{5}$$

又

$$|\frac{\partial\psi_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))}{\partial x_{i+1}(t)}|_{x_{i+1}(t)=\zeta(t)}|\geqslant\epsilon>0. \quad (6)$$

由于$\psi_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))$为有界光滑函数, 则等式(5)左边亦有界. 故$x_{i+1}(t)$有界.

3 控制器设计(Controller design)

为克服时滞系统(1)控制器设计中的非仿射输入问题, 引入一个辅助积分系统$\dot{u}(t)=v(t)$, 将原n阶非仿射输入系统转换为$n+1$阶仿射输入增广系统, 则增广系统表示为

$$\begin{cases}\dot{x}_i(t)=f_i(\bar{\boldsymbol{x}}_i(t),x_{i+1}(t))+h_i(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n)),\\ \qquad i=1,\cdots,n-1,\\ \dot{x}_n(t)=f_n(\bar{\boldsymbol{x}}_n(t),u(t))+h_n(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n)),\end{cases} \quad (7)$$

$$\dot{u}(t)=v(t), \quad (8)$$

其中$v(t)$为辅助控制输入.

不同于标准反推法设计[38], 本文采用如下坐标变换[8–11]:

$$z_1(t)=x_1(t), \quad (9)$$

$$z_i(t)=f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))-\alpha_{i-1}(t),\quad i=2,\cdots,n+1, \quad (10)$$

其中$\alpha_{i-1}(t)$为第$i-1$个子系统的虚拟控制律.

步骤 1 由式(7)(9)–(10), 可得

$$\dot{z}_1(t)=z_2(t)+\alpha_1(t)+h_1(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n)). \quad (11)$$

由式(3)(11), 可得

$$\begin{aligned}&z_1(t)\dot{z}_1(t)\leqslant\\&z_1(t)(z_2(t)+\alpha_1(t))+\\&\frac{r_1}{4}z_1^2(t)g_1^2(x_1(t))+\frac{1}{r_1}\omega_1^2(x_1(t-\tau_1)),\end{aligned} \quad (12)$$

其中r_1为正的设计常数.

选择虚拟控制律$\alpha_1(t)$为

$$\begin{aligned}\alpha_1(t)=&-c_1z_1(t)-\frac{r_1}{4}z_1(t)g_1^2(x_1(t))-\\&\sum_{i=1}^{n}(\frac{1}{r_i}+\sum_{j=i+1}^{n+1}\frac{1}{k_{ji}})z_1(t)\varpi_i^2(x_1(t)),\end{aligned} \quad (13)$$

其中: c_1,r_i,k_{ji}为正的设计常数, $i=1,\cdots,n$, $j=i+1,\cdots,n+1$; 光滑函数

$$\varpi_i(x_1(t))=\int_0^1\frac{\partial\omega_i(s)}{\partial s}|_{s=\beta x_1(t)}\mathrm{d}\beta.$$

由假设2和引理1得到, 即

$$\begin{aligned}\omega_i(x_1(t))=&x_1(t)\int_0^1\frac{\partial\omega_i(s)}{\partial s}|_{s=\beta x_1(t)}\mathrm{d}\beta=\\&z_1(t)\varpi_i(x_1(t)).\end{aligned} \quad (14)$$

把式(13)代入式(12), 可得

$$\begin{aligned}z_1(t)\dot{z}_1(t)\leqslant&-c_1z_1^2(t)+z_1(t)z_2(t)+\\&\frac{1}{r_1}\omega_1^2(x_1(t-\tau_1))-\\&\sum_{i=1}^{n}(\frac{1}{r_i}+\sum_{i+1}^{n+1}\frac{1}{k_{ji}})z_1^2(t)\varpi_i^2(x_1(t)).\end{aligned} \quad (15)$$

步骤 $\boldsymbol{i}$ ($i=2,\cdots,n$) 对$z_i(t)=f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))-\alpha_{i-1}(t)$求导, 可得

$$\begin{aligned}\dot{z}_i(t)=&\sum_{j=1}^{i-1}\phi_{(i-1)j}(t)\dot{x}_j(t)+\\&\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))}{\partial x_i(t)}\dot{x}_i(t)=\\&\sum_{j=1}^{i-1}\phi_{(i-1)j}(t)(f_j(\bar{\boldsymbol{x}}_j(t),x_{j+1}(t))+\\&h_j(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n)))+\\&\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))}{\partial x_i(t)}(z_{i+1}(t)+\\&\alpha_i(t)+h_i(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n))),\end{aligned} \quad (16)$$

其中

$$\phi_{(i-1)j}(t)=\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))}{\partial x_j(t)}-\frac{\partial\alpha_{i-1}(t)}{\partial x_j(t)}. \quad (17)$$

根据式(3)和式(16), 可得

$$\begin{aligned}&z_i(t)\dot{z}_i(t)\leqslant\\&z_i(t)\sum_{j=1}^{i-1}\phi_{(i-1)j}(t)f_j(\bar{\boldsymbol{x}}_j(t),x_{j+1}(t))+\\&z_i^2(t)\sum_{j=1}^{i-1}\frac{k_{ij}}{4}\phi_{(i-1)j}^2(t)g_j^2(\bar{\boldsymbol{x}}_j(t))+\\&\sum_{j=1}^{i-1}\frac{1}{k_{ij}}\omega_j^2(x_1(t-\tau_1))+\\&z_i(t)\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))}{\partial x_i(t)}\times\\&(z_{i+1}(t)+\alpha_i(t))+\frac{1}{r_i}\omega_i^2(x_1(t-\tau_1))+\\&\frac{r_i}{4}z_i^2(t)g_i^2(\bar{\boldsymbol{x}}_i(t))(\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))}{\partial x_i(t)})^2,\end{aligned} \quad (18)$$

k_{ij},r_i为正的设计常数, $i=2,\cdots,n$, $j=1,\cdots,i-1$.

选择虚拟控制律$\alpha_i(t)$为

$$\begin{aligned}\alpha_i(t)=&\frac{1}{\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))}{\partial x_i(t)}}\{-c_iz_i(t)-\\&\frac{\partial f_{i-2}(\bar{\boldsymbol{x}}_{i-1}(t))}{\partial x_{i-1}(t)}z_{i-1}(t)-\\&\sum_{j=1}^{i-1}\phi_{(i-1)j}(t)f_j(\bar{\boldsymbol{x}}_j(t),x_{j+1})-\\&z_i(t)\sum_{j=1}^{i-1}\frac{k_{ij}}{4}\phi_{(i-1)j}^2(t)g_j^2(\bar{\boldsymbol{x}}_j(t))-\\&\frac{r_i}{4}z_i(t)g_i^2(\bar{\boldsymbol{x}}_i(t))\times\end{aligned}$$

$$(\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_{i-1}(t),x_i(t))}{\partial x_i(t)})^2\}, \tag{19}$$

其中: $\dfrac{\partial f_0(x_1(t))}{\partial x_1(t)}=1$, c_i为正设计常数, $i=2,\cdots,n$.

将式(19)代入式(18), 可得

$$\begin{aligned}&z_i(t)\dot{z}_i(t)\leqslant\\&-c_iz_i^2(t)-\frac{\partial f_{i-2}(\bar{\boldsymbol{x}}_{i-1}(t))}{\partial x_{i-1}(t)}z_i(t)z_{i-1}(t)+\\&\sum_{j=1}^{i-1}\frac{1}{k_{ij}}\omega_j^2(x_1(t-\tau_1))+\\&\frac{\partial f_{i-1}(\bar{\boldsymbol{x}}_i(t))}{\partial x_i(t)}z_i(t)z_{i+1}(t)+\frac{1}{r_i}\omega_i^2(x_1(t-\tau_1)).\end{aligned} \tag{20}$$

步骤 $n+1$ 这一步将得到实际控制输入$u(t)$. 对$z_{n+1}(t)=f_n(\bar{\boldsymbol{x}}_n(t),u(t))-\alpha_n(t)$求导, 可得

$$\begin{aligned}\dot{z}_{n+1}(t)=&\sum_{j=1}^{n}\phi_{nj}(t)\dot{x}_j(t)+\\&\frac{\partial f_n(\bar{\boldsymbol{x}}_n(t),u(t))}{\partial u(t)}v(t)=\\&\sum_{j=1}^{n}\phi_{nj}(t)(f_j(\bar{\boldsymbol{x}}_j(t),x_{j+1}(t))+\\&h_j(t,\bar{\boldsymbol{x}}_n(t),\bar{\boldsymbol{x}}_n(t-\tau_n)))+\\&\frac{\partial f_n(\bar{\boldsymbol{x}}_n(t),u(t))}{\partial u(t)}v(t),\end{aligned} \tag{21}$$

其中

$$\phi_{nj}(t)=\frac{\partial f_n(\bar{\boldsymbol{x}}_n(t),u(t))}{\partial x_j(t)}-\frac{\partial \alpha_n(t)}{\partial x_j(t)}. \tag{22}$$

由式(3)和式(16), 可得

$$\begin{aligned}&z_{n+1}(t)\dot{z}_{n+1}(t)\leqslant\\&z_{n+1}(t)\sum_{j=1}^{n}\phi_{nj}(t)f_j(\bar{\boldsymbol{x}}_j(t),x_{j+1}(t))+\\&z_{n+1}^2(t)\sum_{j=1}^{n}\frac{k_{(n+1)j}}{4}\phi_{nj}^2(t)g_j^2(\bar{\boldsymbol{x}}_j(t))+\\&\sum_{j=1}^{n}\frac{1}{k_{(n+1)j}}\omega_j^2(x_1(t-\tau_1))+\\&z_{n+1}(t)\frac{\partial f_n(\bar{\boldsymbol{x}}_n(t),u(t))}{u(t)}v(t),\end{aligned} \tag{23}$$

其中: $k_{(n+1)j}$为正的设计常数, $j=1,\cdots,n$.

辅助控制律$v(t)$设计为

$$\begin{aligned}v(t)=&\frac{1}{\dfrac{\partial f_n(\bar{\boldsymbol{x}}_n(t),u(t))}{u(t)}}\{-c_{n+1}z_{n+1}(t)-\\&\frac{\partial f_{n-1}(\bar{\boldsymbol{x}}_n(t))}{\partial x_n(t)}z_n(t)-\\&\sum_{j=1}^{n}\phi_{nj}(t)f_j(\bar{\boldsymbol{x}}_j(t),x_{j+1}(t))-\\&z_{n+1}(t)\sum_{j=1}^{n}\frac{k_{(n+1)j}}{4}\phi_{nj}^2(t)g_j^2(\bar{\boldsymbol{x}}_j(t))\},\end{aligned} \tag{24}$$

其中c_{n+1}为正的设计常数.

对任意初始控制条件$u(0)$, 将式(24)代入式(8), 即可得到实际控制律$u(t)$.

根据式(23)和式(24), 可得

$$\begin{aligned}z_{n+1}\dot{z}_{n+1}\leqslant&-c_{n+1}z_{n+1}^2(t)-\\&\frac{\partial f_{n-1}(\bar{\boldsymbol{x}}_n(t))}{\partial x_n(t)}z_{n+1}(t)z_n(t)+\\&\sum_{j=1}^{n}\frac{1}{k_{(n+1)j}}\omega_j^2(x_1(t-\tau_1)).\end{aligned} \tag{25}$$

4 稳定性分析(Stability analysis)

由式(8)和式(24)构成的控制律, 可以得到如下稳定性定理.

定理 1 对满足假设1和2的一类非仿射非线性时滞系统(1), 采用控制律(8)和式(24), 则闭环系统所有信号有界, 且系统状态$\boldsymbol{x}(t)$收敛于原点.

证 构造如下Lyapunov–Krosovskii泛函:

$$\begin{aligned}V(t)=&\frac{1}{2}\sum_{i=1}^{n+1}z_i^2(t)+\\&\sum_{i=1}^{n}\int_{t-\tau_i}^{t}(\frac{1}{r_i}+\sum_{j=i+1}^{n+1}\frac{1}{k_{ji}})\omega_i^2(x_1(\xi))\mathrm{d}\xi.\end{aligned} \tag{26}$$

对$V(t)$沿时间t求导, 可得

$$\begin{aligned}\dot{V}(t)\leqslant&\sum_{i=1}^{n+1}z_i(t)\dot{z}_i(t)+\sum_{i=1}^{n}(\frac{1}{r_i}+\sum_{j=i+1}^{n+1}\frac{1}{k_{ji}})\\&\omega_i^2(x_1(t))-\sum_{i=1}^{n}(\frac{1}{r_i}+\sum_{j=i+1}^{n+1}\frac{1}{k_{ji}})\\&\omega_i^2(x_1(t-\tau_i)).\end{aligned} \tag{27}$$

根据式(15)(20)(25), 可得

$$\dot{V}(t)\leqslant-\sum_{i=1}^{n+1}c_iz_i^2(t). \tag{28}$$

由式(28)可知, $V(t)$是一个不增函数, 因此, $z_1(t),\cdots,z_{n+1}(t)$有界. 同时, 根据坐标变换式(9)–(10), $z_1(t)$, $z_2(t)$有界可知$x_1(t)$, $\alpha_1(t)$, $f_1(x_1(t),x_2(t))$有界, 由引理2亦可得$x_2(t)$有界, 从而$f_1(x_1(t),x_2(t))$和$\alpha_1(t)$的各偏导数有界; 同理, 可推得$x_3(t),\cdots,x_n(t)$, $\alpha_2(t),\cdots,\alpha_n(t)$, $u(t)$, $v(t)$均有界. 另外, 由Barbalat引理, 可得$\lim\limits_{t\to\infty}z_i(t)=0$, $i=1,\cdots,n+1$, 结合式(9)–(10)(13)(19)和(24), 推得$\lim\limits_{t\to\infty}x_i(t)=0$, $i=1,\cdots,n$.

证毕.

注 3 系统(1)控制器设计的困难主要来自于系统的非仿射结构和未知时滞的双重作用, 本文利用文献[8–11]提出的反推法, 在步骤1的虚拟控制律α_1设计中引入时滞补偿项

$$-\sum_{i=1}^{n}(\frac{1}{r_i}+\sum_{j=i+1}^{n+1}\frac{1}{k_{ji}})z_1(t)\varpi_i^2(x_1(t)),$$

有效解决了系统(1)的全局镇定问题.

注 4　本文所提控制算法同样适用于严格反馈非线性时滞系统控制器设计, 相较于已有文献[21–27]中的控制方法, 其优点是可以任意选取控制输入初始状态$u(0)$.

注 5　针对非仿射非线性时滞系统, 已有文献[28–33]中通常的方法是利用隐函数定理和中值定理, 将非仿射时滞系统转化为仿射时滞系统, 再按照仿射时滞系统的设计思路给出控制策略, 然而, 该方法忽略了系统非仿射的本质属性, 因此, 即使模型信息完全已知, 其所得控制算法亦仅能保证闭环系统半全局稳定. 与此不同, 本文从系统非仿射结构出发, 结合文献[8–11]提出的反推法与构造合适的Lyapunov-Krosovskii泛函, 给出了一种新的能保证系统全局稳定的控制器设计方法.

5　仿真实例(Simulation example)

考虑如下二阶非仿射非线性时滞系统:

$$\begin{cases}\dot{x}_1(t)=f_1(x_1(t),x_2(t))+h_1(t,\bar{\boldsymbol{x}}_2(t),\bar{\boldsymbol{x}}_2(t-\tau_2)),\\ \dot{x}_2(t)=f_2(\bar{\boldsymbol{x}}_2(t),u(t))+h_2(t,\bar{\boldsymbol{x}}_2(t),\bar{\boldsymbol{x}}_2(t-\tau_2)),\end{cases} \tag{29}$$

其中:

$$f_1(x_1(t),x_2(t))=x_1(t)+x_2(t)+x_2^3(t),$$

$$f_2(\bar{\boldsymbol{x}}_2(t),u(t))=x_1(t)x_2(t)+u(t)+u^3(t),$$

$$h_1(t,\bar{\boldsymbol{x}}_2(t),\bar{\boldsymbol{x}}_2(t-\tau_2))=x_1(t)\cos(x_2(t))x_1(t-1),$$

$$h_2(t,\bar{\boldsymbol{x}}_2(t),\bar{\boldsymbol{x}}_2(t-\tau_2))=x_2(t)x_1^3(t-1)\sin(x_2(t-2)).$$

根据假设2, 选取

$$g_1(x_1(t))=x_1(t),\ h_1(x_1(t-1))=x_1(t-1),$$

$$g_2(\bar{\boldsymbol{x}}_2(t))=x_2(t),\ h_2(x_1(t-1))=x_1^3(t-1).$$

由引理1得$\varpi_1(x_1(t))=1,\ \varpi_2(x_1(t))=x_1^2(t)$.

本次仿真中, 系统初始条件$x_1(t_0)=1, x_2(t_0)=-2, t_0\in[-2,0]$. 选择设计参数$c_1=c_2=c_3=1,\ r_1=r_2=1,\ k_{21}=k_{31}=k_{32}=1$, 初始控制输入$u(0)=0.5$. 所得仿真结果如图1–2所示.

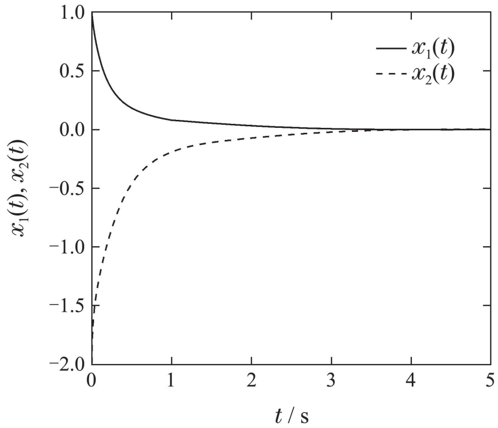

图 1　状态$x_1(t), x_2(t)$

Fig. 1　States $x_1(t), x_2(t)$

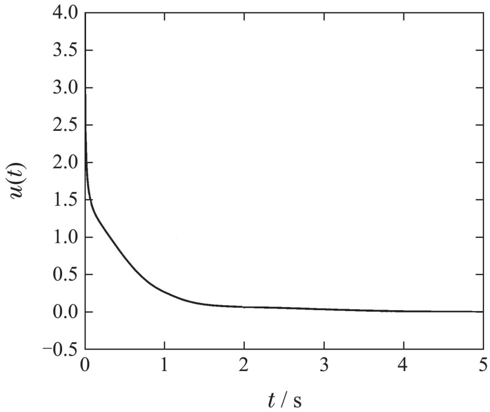

图 2　控制输入$u(t)$

Fig. 2　Control input $u(t)$

图1为系统状态$x_1(t)$和$x_2(t)$的响应曲线, 图2为控制输入$u(t)$. 仿真结果显示系统状态渐近收敛于原点, 且闭环信号有界.

6　结论(Conclusions)

本文解决了一类非仿射非线性时滞系统的全局镇定问题. 通过引入一个积分辅助系统, 将原n阶非仿射输入时滞系统转化为$n+1$阶仿射输入增广时滞系统. 按照反推设计方法, 得到了一种时滞无关的动态状态反馈控制算法. 理论分析与仿真结果表明, 该控制器保证了系统全局镇定, 且闭环系统所有信号有界. 考虑到本文中关于未知时滞函数项的假设过强, 下一步工作将探讨更弱化条件下非仿射非线性时滞系统的控制器设计.

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