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带有控制饱和约束的航天器交会输出反馈控制

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摘要: 针对航天器交会问题存在外部干扰和输入饱和的情况, 本文提出了一个输出反馈跟踪控制器. 仅利用测量得到的相对位置信息, 设计了一个滑模观测器用来估计相对角速度, 并根据该估计值设计了一个鲁棒反步控制律. 通过引入一个辅助系统, 对输入饱和情况进行了分析. 采用Lyapunov稳定性理论, 证明了本文提出的该控制器能够保证位置和速度跟踪误差的一致有界性. 最后通过数值分析验证了所设计的输出反馈控制器的有效性.

关键词: 航天器交会; 输出反馈; 输入饱和; 滑模观测器

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Output feedback control for spacecraft rendezvous with control saturation

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Abstract: An output feedback tracking control law is developed for spacecraft rendezvous problem in the presence of external disturbance and input saturation. By only using the measurable relative position, a sliding-mode-based observer is designed to estimate the relative velocity. With the estimated information, a robust backstepping control law is designed, where an auxiliary system is introduced to analyze the saturated input. With the Lyapunov framework, the proposed controller is proved to ensure the ultimate boundedness of the tracking errors of position and velocity. Numerical simulation demonstrates the effect of the designed control law.

Key words: spacecraft rendezvous; output feedback; input saturation; sliding-mode-based observer

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1 Introduction

Spacecraft rendezvous is one of the fundamental maneuvers in many space missions. Recently, great developments for spacecraft rendezvous control law design have been witnessed, and many control schemes have been proposed[1–6]. However, many spacecraft rendezvous models in literatures are simplified to match the forms of design methods for convenience. Furthermore, full-state measurements of a spacecraft are always assumed to be available, i.e., the position and the velocity. But this can not be achieved all the time, especially when some failures occur in velocity sensors which may result in wrong measurements. Another practical issue of significant importance in many applications is the physical constraint of actuators, that is, input saturation, which imposes limitations on the magnitude of the control input. Inspired by these three issues, this paper presents an output feedback controller by using fully nonlinear relative dynamics in the presence of external disturbances and input saturation.

Output-feedback controller design for spacecraft rendezvous has attracted enormous amount of attention all the time. In [7], an dynamic filter is established to generate a pseudo-velocity tracking error signal to facilitate the output feedback control law design, which was synthesized based on Lyapunov theory. In [8], a dynamic output feedback controller is designed to place the closed-loop poles within a specified disc, and computed by using LMIs to satisfy multi-objective requirements. In [9], the spacecraft rendezvous system was augmented with a passivity-based filter to generate pseudo-velocity estimates, with relative position error taken as the filter input. In [10], the author proposed a sliding-mode observer to reconstruct the full states with only the output measurement available for a class of nonlinear systems.

Control input saturation is a very practical issue that

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can not be neglected in control law design, and various approaches are proposed to tackle this problem. In [11], a signal with non-achievable portion of control signal was filtered to produce the magnitude, rate, and bandwidth limited control input, and then, a controller was designed based on backstepping scheme. In [12], a dead zone operator based model was considered as the thrust saturation phenomenon, and when the control law is designed as based on the dead zone model, the saturation constraint was directly satisfied. In [13], the author proposed a continuous dynamic scheduling approach to the stabilization of linear systems with input saturation. In [14], an auxiliary system was introduced for the convenience of input constraint effect analysis, and when the state of the auxiliary system was in a domain, the non-existence of input saturation is guaranteed.

It is worth pointing out that most research deals with spacecraft rendezvous either output feedback design or input saturation only. Further effort is needed to research the integrated design for the two problems. The main motivation and contribution of this paper is to attempt to tackle the two problems together. In this paper, a velocity-free tracking control law is designed for spacecraft rendezvous described by the fully nonlinear Clohessy-Wiltshire (C–W) equations in the presence of input saturation and external disturbance. Because sliding mode control has strong robustness and can achieve finite-time convergence, a sliding-mode-based observer is designed to estimate the relative velocity by only using the measurable relative position. In addition, an auxiliary system is introduced to analyze the saturated input. The rest of the paper is organized as follows: In Section 2, the fully nonlinear spacecraft rendezvous model is presented. In Section 3, a sliding-mode observer is designed, and a tracking controller is proposed. Numerical simulation is presented to demonstrate the effect of the derived controller in Section 4, and conclusions are given in Section 5.

2 Mathematical model and problem formulation

2.1 Mathematical model

Usually in spacecraft rendezvous problems, the local-vertical-local-horizontal (LVLH) frame is used to describe the relative orbit dynamics by attaching its origin to the center of mass of the target. The X-axis points radially outward from its orbit, the Y-axis is perpendicular to X along its direction of motion, and the Z-axis completes the right-handed coordinate system. This frame is taken as the reference target trajectory for the chaser spacecraft.

The relative motion between the chaser and the target in the LVLH coordinate can be described by the fully nonlinear Clohessy-Wiltshire (C–W) equations, shown as follows[9]:

$$\begin{cases} \ddot{x} = 2\dot{\theta}\dot{y} + \ddot{\theta}y + \dot{\theta}^2 x - \dfrac{\mu(r_\mathrm{c}+x)}{\rho^3} + \\ \quad \dfrac{\mu}{r_\mathrm{c}^2} + \dfrac{1}{m}F_\mathrm{x} + d_\mathrm{x}, \\ \ddot{y} = -2\dot{\theta}\dot{x} - \ddot{\theta}x + \dot{\theta}^2 y - \dfrac{\mu y}{\rho^3} + \dfrac{1}{m}F_\mathrm{y} + d_\mathrm{y}, \\ \ddot{z} = -\dfrac{\mu z}{\rho^3} + \dfrac{1}{m}F_\mathrm{z} + d_\mathrm{z}, \end{cases} \tag{1}$$

where $x,\ y,\ z$ stand for the relative position of chaser with respect to the target, r_c and $\rho = \sqrt{(r_\mathrm{c}+x)^2+y^2+z^2}$ represent the distance from the center of the Earth to the target and the chaser respectively, θ denotes the latitude angle of the target, and μ is the gravity constant. $F_i(i=x,y,z)$ is the control input force acting on the chaser, and m is the mass of the chaser. $d_i(i=x,y,z)$ is the external disturbance with known upper bound on the norm d^*, that is, $\|d\| \leqslant d^*$.

Let $\boldsymbol{x}_1 = [x\ \ y\ \ z]^\mathrm{T}$, $\boldsymbol{x}_2 = [\dot{x}\ \ \dot{y}\ \ \dot{z}]^\mathrm{T}$, $\boldsymbol{F} = [F_\mathrm{x}\ F_\mathrm{y}\ F_\mathrm{z}]^\mathrm{T}$, and $\boldsymbol{d} = [d_\mathrm{x}\ d_\mathrm{y}\ d_\mathrm{z}]^\mathrm{T}$, then the system can be rewritten in the following equivalent form:

$$\begin{cases} \dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2, \\ \dot{\boldsymbol{x}}_2 = A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 + \boldsymbol{g} + \dfrac{1}{m}\boldsymbol{F} + \boldsymbol{d}, \end{cases} \tag{2}$$

where

$$A_1 = \begin{bmatrix} \dot{\theta}^2 - \dfrac{\mu}{\rho^3} & \ddot{\theta} & 0 \\ -\ddot{\theta} & \dot{\theta}^2 - \dfrac{\mu}{\rho^3} & 0 \\ 0 & 0 & -\dfrac{\mu}{\rho^3} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 2\dot{\theta} & 0 \\ -2\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{g} = \begin{bmatrix} -\dfrac{\mu r_\mathrm{c}}{\rho^3} + \dfrac{\mu}{r_\mathrm{c}^2} \\ 0 \\ 0 \end{bmatrix}.$$

2.2 Problem formulation

Considering the spacecraft rendezvous relative dynamics model described by Eq.(1) with known initial position of the chaser relative to the target, for any given desired relative position trajectory $\boldsymbol{x}_\mathrm{d}$, the objective of this paper is to design a control law without velocity measurement, that is, $\boldsymbol{x}_2$ is not required, to guarantee that all states in the closed-loop system are uniformly ultimately bounded in the presence of external disturbance $\boldsymbol{d}$ and input constraint, that is, each actuator can generate the force of no more than F_max.

3 Design of tracking controller without velocity measurements

In this section, we will derive a control velocity-measurement-free law based on sliding-mode observer for autonomous spacecraft rendezvous. Before giving the main results, some notations and lemmas are necessary to be introduced first.

Notation: $\|\cdot\|$ represents the Euclidean norm of vectors or induced norm for matrices. For a given vector $\boldsymbol{x} = [x_1 \;\; x_2 \;\; x_3]^{\mathrm{T}} \in \mathbb{R}^3$ and a scalar $\alpha \in \mathbb{R}$, define $\boldsymbol{x}^{\alpha} = [x_1^{\alpha} \;\; x_2^{\alpha} \;\; x_3^{\alpha}]^{\mathrm{T}}$, $\operatorname{sgn}\boldsymbol{x} = [\operatorname{sgn}x_1 \;\; \operatorname{sgn}x_2 \;\; \operatorname{sgn}x_3]^{\mathrm{T}}$, and $\operatorname{sig}\boldsymbol{x}^{\alpha} = [|x_1|^{\alpha}\operatorname{sgn}x_1 \;\; |x_2|^{\alpha}\operatorname{sgn}x_2 \;\; |x_3|^{\alpha}\operatorname{sgn}x_3]^{\mathrm{T}}$; here, "$\operatorname{sgn}(\cdot)$" denotes the signum function.

Lemma 1[15] For any $a_i \in \mathbb{R},\ i = 1, 2, \cdots, n$,

$$(|a_1| + |a_2| + \cdots + |a_n|)^{\upsilon} \leqslant |a_1|^{\upsilon} + |a_2|^{\upsilon} + \cdots + |a_n|^{\upsilon},$$

where υ is a real number and $\upsilon \in (0, 1]$.

Lemma 2[16] Suppose $a_1, a_2, \cdots, a_n$ and $0 < \upsilon < 2$ are all positive numbers, then the following inequality holds:

$$(\alpha_1^2 + a_2^2 + \cdots + a_n^2)^{\upsilon} \leqslant (a_1^{\upsilon} + a_2^{\upsilon} + \cdots + a_2^{\upsilon})^2.$$

Lemma 3[17] For system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}),\ \boldsymbol{f} : D \to \mathbb{R}^n,\ \boldsymbol{f}(0) = 0$, suppose there exist a continuously differentiable function: $V : D \to \mathbb{R}$, real numbers $k > 0$ and $\alpha \in (0, 1)$, and a neighborhood $U \subset D$ of the origin such that V is positive definite on U and $\dot{V} + kV^{\alpha}$ is negative semidefinite on U, where $\dot{V} = \dfrac{\partial V(\boldsymbol{x})}{\partial \boldsymbol{x}}\boldsymbol{f}(\boldsymbol{x})$. The origin is a finite-time-stable equilibrium of the system. Moreover, if T is the settling time, then

$$T(\boldsymbol{x}) \leqslant \frac{1}{k(1-\alpha)}V^{1-\alpha}(\boldsymbol{x}) \tag{3}$$

for all x in some open neighborhood of the origin.

Lemma 4[16] An extended Lyapunov description of finite-time stability can be given with the form of fast terminal sliding mode as

$$\dot{V}(\boldsymbol{x}) + \lambda_1 V(\boldsymbol{x}) + \lambda_2 V^{\alpha}(\boldsymbol{x}) \leqslant 0,$$

where $\lambda_1 > 0,\ \lambda_2 > 0,\ 0 < \alpha < 1$, and the settling time can be given by

$$T \leqslant \frac{1}{\lambda_1(1-\alpha)} \ln \frac{\lambda_1 V^{1-\alpha}(\boldsymbol{x}_0) + \lambda_2}{\lambda_2}, \tag{4}$$

where $V(\boldsymbol{x}_0)$ is the initial value of $V(\boldsymbol{x})$.

Lemma 5 Consider the nonlinear system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$, where $\boldsymbol{x}$ is a state vector, $\boldsymbol{x}$ is the input vector. Suppose that there exist a Lyapunov function $V(\boldsymbol{x})$ and scalars $\alpha, \beta, \gamma, \eta \in \mathbb{R}^+,\ 0 < \gamma < 1$, and $0 < \vartheta < \infty$, such that $\dot{V}(\boldsymbol{x}) \leqslant -\alpha V(\boldsymbol{x}) - \beta V^{\gamma}(\boldsymbol{x}) + \vartheta$. Then, the trajectory of this system is practical fast finite-time stable (PFFS). Moreover, the trajectory of the system is bounded in finite time as

$$\lim_{t\to T} \boldsymbol{x} \in \{V(\boldsymbol{x}) \leqslant \min\{\frac{\vartheta}{\alpha(1-\theta)}, \frac{\vartheta^{1/\gamma}}{\beta^{1/\gamma}(1-\theta)^{1/\gamma}}\}\}. \tag{5}$$

where $0 < \theta \leqslant 1$. And the time needed to reach Eq.(5) is bounded by

$$T \leqslant \frac{1}{\alpha\theta(1-\gamma)} \ln \frac{\alpha V^{1-\gamma}(\boldsymbol{x}_0) + \beta}{\beta}, \tag{6}$$

where $V(\boldsymbol{x}_0)$ is the initial value of $V(\boldsymbol{x})$.

Proof There exists a scalar $0 < \theta \leqslant 1$, such that $\dot{V}(\boldsymbol{x}) \leqslant -\alpha V(\boldsymbol{x}) - \beta V^{\gamma}(\boldsymbol{x}) + \vartheta$ can be expressed as follows:

$$\dot{V}(\boldsymbol{x}) \leqslant -\theta[\alpha V(\boldsymbol{x}) + \beta V^{\gamma}(\boldsymbol{x})] - (1-\theta)[\alpha V(\boldsymbol{x}) + \beta V^{\gamma}(\boldsymbol{x})] + \vartheta. \tag{7}$$

If $-(1-\theta)[\alpha V(\boldsymbol{x}) + \beta V^{\gamma}(\boldsymbol{x})] + \vartheta \leqslant 0$, it is obtained that $\dot{V}(\boldsymbol{x}) \leqslant -\theta[\alpha V(\boldsymbol{x}) + \beta V^{\gamma}(\boldsymbol{x})]$. According to Lemma 4, $V(\boldsymbol{x})$ converges into a invariant set satisfying $\alpha V(\boldsymbol{x}) + \beta V^{\gamma}(\boldsymbol{x}) \leqslant \dfrac{\vartheta}{1-\theta}$ in finite time. Therefore, the trajectory of the closed-loop system is bounded as

$$\lim_{\theta\to\theta_0,\, t\to T} \boldsymbol{x} \in \{V(\boldsymbol{x}) \leqslant \min\{\frac{\vartheta}{\alpha(1-\theta)}, \frac{\vartheta^{1/\gamma}}{\beta^{1/\gamma}(1-\theta)^{1/\gamma}}\}\}, \tag{8}$$

where $0 < \theta_0 < 1$, and the time needed is

$$T \leqslant \frac{1}{\alpha\theta(1-\gamma)} \ln \frac{\alpha V^{1-\gamma}(\boldsymbol{x}_0) + \beta}{\beta}, \tag{9}$$

where $V(\boldsymbol{x}_0)$ is the initial value of $V(\boldsymbol{x})$. QED.

Remark 1 The range of $\boldsymbol{x}$ and T estimated above is conservative, which arises from the following two aspects. When $\theta_0 \in (0, 1)$ is specified a fixed value, $\dot{V}(\boldsymbol{x}) < 0$ after the time calculated from Eq.(9), then $V(\boldsymbol{x})$ continues to decrease. Therefore, the ultimate bound of $\boldsymbol{x}$ is smaller than that in Eq.(8), and the time needed is longer. On the other hand, the conservative estimation of $\boldsymbol{x}$ in Eq.(8) comes from solving the inequality $\alpha V(\boldsymbol{x}) + \beta V^{\gamma}(\boldsymbol{x}) \leqslant \dfrac{\vartheta}{1-\theta}$ due to $\gamma \in (0, 1)$.

3.1 Sliding-mode observer design

Position sensors are equipped to measure the relative position between the chaser and the target; therefore, $\boldsymbol{x}_1$ is available. The output of system (2) defined by $\boldsymbol{x}_{\mathrm{o}} = \boldsymbol{x}_1$ is thus available. To estimate states $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ in finite time, define $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ as the corresponding estimates, and a sliding-mode observer is proposed as follows[18]:

$$\dot{\hat{\boldsymbol{x}}}_1 = \hat{\boldsymbol{x}}_2 - \boldsymbol{x}_{\mathrm{v}}, \tag{10}$$

$$\dot{\hat{\boldsymbol{x}}}_2 = A_1\hat{\boldsymbol{x}}_1 + A_2\hat{\boldsymbol{x}}_2 + \boldsymbol{g} + \frac{1}{m}\boldsymbol{F} - \lambda_{\mathrm{o}2}\boldsymbol{x}_{\mathrm{v}} - \lambda_{\mathrm{o}3}\operatorname{sig}^{\alpha}(\boldsymbol{x}_{\mathrm{v}}), \tag{11}$$

where $\boldsymbol{x}_{\mathrm{v}} = \lambda_{\mathrm{o}1}\operatorname{sgn}\boldsymbol{\varepsilon}_{\mathrm{o}1},\ \boldsymbol{\varepsilon}_{\mathrm{o}1} = \hat{\boldsymbol{x}}_1 - \boldsymbol{x}_1$ denotes the observer error about position, $\lambda_{\mathrm{o}i}(i = 1, 2, 3)$ and $\alpha \in (0, 1)$ are positive scalars. Define $\boldsymbol{\varepsilon}_{\mathrm{o}2} = \hat{\boldsymbol{x}}_2 - \boldsymbol{x}_2$, the following theorem can be given.

Theorem 1 Consider the system (2) in combination with observer (10) and (11). Choose α and observer gains $\lambda_{\mathrm{o}i}(i = 1, 2, 3)$ such that $\lambda_{\mathrm{o}1} > \varepsilon_0$, $\lambda_{\mathrm{o}2} > ||A_2|| + \dfrac{d^{*2}}{4\epsilon}$ and $\lambda_{\mathrm{o}3} > 0$, where ϵ is a small positive scalar. Then, the observer error $\boldsymbol{\varepsilon}_{\mathrm{o}1}$ converges to zero in finite time, that is, $\boldsymbol{\varepsilon}_{\rm o1} \equiv 0$ for all $t > T_{\rm o1}$, and $\boldsymbol{\varepsilon}_{\rm o2}$ is practical fast finite-time stable (PFFS), that is, $||\boldsymbol{\varepsilon}_{\rm o2}|| \leqslant \varepsilon_0$ for all time $t > T_{\rm o2}$. And the expressions of ε_0, $T_{\rm o1}$ and $T_{\rm o2}$ are presented in the following proof.

Proof The observer error dynamics can be obtained from Eqs.(2)(10) and Eq.(11) as follows:

$$\dot{\boldsymbol{\varepsilon}}_{\rm o1} = \boldsymbol{\varepsilon}_{\rm o2} - \boldsymbol{\varepsilon}_{\rm v}, \tag{12}$$

$$\dot{\boldsymbol{\varepsilon}}_{\rm o2} = A_1\boldsymbol{\varepsilon}_{\rm o1} + A_2\boldsymbol{\varepsilon}_{\rm o2} - \boldsymbol{d} - \lambda_{\rm o2}\boldsymbol{x}_{\rm v} - \lambda_{\rm o3}{\rm sig}(\boldsymbol{x}_{\rm v})^{\alpha}. \tag{13}$$

Consider a candidate Lyapunov function as

$$V_{\rm o1}(t) = \frac{1}{2}\boldsymbol{\varepsilon}_{\rm o1}^{\rm T}\boldsymbol{\varepsilon}_{\rm o1}.$$

From Eq.(12) and Lemma 1, the derivative of it can be then written as

$$\dot{V}_{\rm o1} = \boldsymbol{\varepsilon}_{\rm o1}^{\rm T}\dot{\boldsymbol{\varepsilon}}_{\rm o1} \leqslant \boldsymbol{\varepsilon}_{\rm o1}^{\rm T}\boldsymbol{\varepsilon}_{\rm o2} - \lambda_{\rm o1}\|\boldsymbol{\varepsilon}_{\rm o1}\| \leqslant -\|\boldsymbol{\varepsilon}_{\rm o1}\|(\lambda_{\rm o1} - \|\boldsymbol{\varepsilon}_{\rm o2}\|).$$

One could choose $\lambda_{\rm o1} > \|\boldsymbol{\varepsilon}_{\rm o2}\| + l_{\rm o1}$, where $l_{\rm o1}$ is a positive scalar. Then, we have

$$\dot{V}_{\rm o1} \leqslant -l_{\rm o1}\|\boldsymbol{\varepsilon}_{\rm o1}\| = -\sqrt{2}l_{\rm o1}V_{\rm o1}^{1/2}.$$

By Lemma 3, $V_{\rm o1}$ converges to zero in finite time, that is, sliding surface $\boldsymbol{\varepsilon}_{\rm o1} = 0$ can be reached in finite time $T_{\rm o1}$, where $T_{\rm o1} = t_0 + \dfrac{\|\boldsymbol{\varepsilon}_{\rm o1}(t_0)\|}{l_{\rm o1}}$ according to Eq.(3). Choosing $\lambda_{\rm o1} > \max\limits_{t\in[t_0,T_{\rm o1}]}\|\boldsymbol{\varepsilon}_{\rm o2}(t)\|$, it then allows a fixed value for $\lambda_{\rm o1}$ to ensure $\|\boldsymbol{\varepsilon}_{\rm o1}\|$ reaching sliding surface. On the surface, $\boldsymbol{\varepsilon}_{\rm o1} = \dot{\boldsymbol{\varepsilon}}_{\rm o1} = 0$, from Eq.(12), we have the equivalent control $\boldsymbol{\varepsilon}_{\rm o2} = (\boldsymbol{x}_{\rm v})_{\rm eq}$.

After time $T_{\rm o1}$, substituting $(\boldsymbol{x}_{\rm v})_{\rm eq}$ into Eq.(13) yields the following observer error dynamics:

$$\dot{\boldsymbol{\varepsilon}}_{\rm o1} = 0, \tag{14}$$

$$\dot{\boldsymbol{\varepsilon}}_{\rm o2} = A_2\boldsymbol{\varepsilon}_{\rm o2} - \boldsymbol{d} - \lambda_{\rm o2}\boldsymbol{\varepsilon}_{\rm o2} - \lambda_{\rm o3}{\rm sig}^{\alpha}(\boldsymbol{\varepsilon}_{\rm o2}). \tag{15}$$

Consider the candidate Lyapunov function this time as $V_{\rm o2}(t) = \dfrac{1}{2}\boldsymbol{\varepsilon}_{\rm o2}^{\rm T}\boldsymbol{\varepsilon}_{\rm o2}$. Then, we can obtain the derivative of $V_{\rm o2}$ by Eq.(14) and Eq.(15):

$$\begin{aligned}&\dot{V}_{\rm o2} = \boldsymbol{\varepsilon}_{\rm o2}^{\rm T}\dot{\boldsymbol{\varepsilon}}_{\rm o2} = \\ &\boldsymbol{\varepsilon}_{\rm o2}^{\rm T}(A_2\boldsymbol{\varepsilon}_{\rm o2} - \boldsymbol{d} - \lambda_{\rm o2}\boldsymbol{\varepsilon}_{\rm o2} - \lambda_{\rm o3}{\rm sig}^{\alpha}(\boldsymbol{\varepsilon}_{\rm o2})) \leqslant \\ &-(\lambda_{\rm o2} - \|A_2\|)\|\boldsymbol{\varepsilon}_{\rm o2}\|^2 - \lambda_{\rm o3}\|\boldsymbol{\varepsilon}_{\rm o2}\|^{\alpha+1} + \|\boldsymbol{\varepsilon}_{\rm o2}\|d^* \leqslant \\ &-(2\lambda_{\rm o2} - 2\|A_2\| - \frac{d^{*2}}{2\epsilon})V_{\rm o2} - \\ &2^{\frac{\alpha+1}{2}}\lambda_{\rm o3}V_{\rm o2}^{\frac{\alpha+1}{2}} + \epsilon,\end{aligned}$$

where ϵ is a small positive constant. Choosing $\lambda_{\rm o2} > \|A_2\| + \dfrac{d^{*2}}{4\epsilon} + l_{\rm o2}$ with $l_{\rm o2}$ being a positive scalar, gives

$$\dot{V}_{\rm o2} \leqslant -\gamma_{\rm o1}V_{\rm o2} - \gamma_{\rm o2}V_{\rm o2}^{\beta} + \epsilon,$$

where $\beta = \dfrac{\alpha+1}{2}$, $\gamma_{\rm o1} = 2\lambda_{\rm o2} - 2\|A_2\| - \dfrac{d^{*2}}{2\epsilon}$ and $\gamma_{\rm o2} = 2^{\beta}\lambda_{\rm o3}$. Employing Lemma 5 and choosing $\theta = 0.5$, $\boldsymbol{\varepsilon}_{\rm o2}$ can be then bounded as

$$\boldsymbol{\varepsilon}_{\rm o2} \in \varGamma = \{\boldsymbol{\varepsilon}_{\rm o2}|V_{\rm o2} \leqslant \min\{\frac{2\epsilon}{\gamma_{\rm o1}}, \frac{\epsilon^{\frac{1}{\beta}}}{\gamma_{\rm o2}^{\frac{1}{\beta}}(1/2)^{\frac{1}{\beta}}}\}\}$$

in finite time $T_{\rm o2}$, where

$$T_{\rm o2} = T_{\rm o1} + \frac{2}{\gamma_{\rm o1}(1-\beta)}\ln\frac{\gamma_{\rm o1}V_{\rm o2}^{1-\beta}(T_{\rm o1}) + \gamma_{\rm o2}}{\gamma_{\rm o2}}$$

by Eq.(6). Therefore, $\varGamma$ is a region of attraction and $\|\boldsymbol{\varepsilon}_{\rm o2}\| \leqslant \boldsymbol{\varepsilon}_0$ for all $t \geqslant T_{\rm o2}$, where $\boldsymbol{\varepsilon}_0$ is given by

$$\varepsilon_0 \triangleq \max\{\min\{\frac{2\epsilon^{\frac{1}{2}}}{\gamma_{\rm o1}^{\frac{1}{2}}}, \frac{2^{\frac{1+\beta}{2\beta}}\epsilon^{\frac{1}{2\beta}}}{\gamma_{\rm o2}^{\frac{1}{2\beta}}}\}, \|\boldsymbol{\varepsilon}_{\rm o2}(0)\|\}.$$

The inequality $\lambda_{\rm o1} > \|\boldsymbol{\varepsilon}_{\rm o2}\| + l_{\rm o1}$ is thus always satisfied if $\lambda_{\rm o1} > \varepsilon_0$, and the finite-time convergence of $\boldsymbol{\varepsilon}_{\rm o1}$ can be guaranteed. This theorem is proved thereby.

QED.

Remark 2 The existence of signum function in the observer, i.e., in Eqs.(10) and (11), may lead to undesirable chattering. The problem can be attenuated by introducing a 'sigmoid function' [19]

$${\rm sgn}\,\sigma \approx \frac{\sigma}{|\sigma| + \delta_0},$$

where δ_0 is a small positive scalar.

3.2 Tracking controller design

Employing the estimated states in the observer Eqs.(10) and (11), with the help of Lyapunov stability, a tracking control law for the spacecraft rendezvous system can be designed based on backstepping approach in the absence of velocity measurements, and the magnitudes of the input forces are guaranteed to be bounded by $F_{\max}$. We first define the change of coordinates as follows:

$$\boldsymbol{z}_1 = \hat{\boldsymbol{x}}_1 - \boldsymbol{x}_{\rm d}, \tag{16}$$

$$\boldsymbol{z}_2 = \hat{\boldsymbol{x}}_2 + (c + 0.5\eta)\boldsymbol{z}_1 - \dot{\boldsymbol{x}}_{\rm d}, \tag{17}$$

where c and η are positive scalars.

Design a control law as

$$\boldsymbol{F} = {\rm sat}(\boldsymbol{v}, F_{\max}), \tag{18}$$

where $\boldsymbol{v}$ is the input signal of the controller. From Eq.(18), the forces acting on the chaser have the upper and lower limit. Here, an auxiliary system is introduced to analyze the saturated input conveniently[14]:

$$\dot{\boldsymbol{x}}_{\rm a} = \begin{cases} -k_1\boldsymbol{x}_{\rm a} - \dfrac{g(\Delta\boldsymbol{F})}{\|\boldsymbol{x}_{\rm a}\|^2}\boldsymbol{x}_{\rm a} - \dfrac{\Delta\boldsymbol{F}}{m}, & \|\boldsymbol{x}_{\rm a}\| \geqslant \delta, \\ 0, & \|\boldsymbol{x}_{\rm a}\| < \delta, \end{cases} \tag{19}$$

where $\Delta\boldsymbol{F} = \boldsymbol{F} - \boldsymbol{v}$, $g(\Delta\boldsymbol{F}) = \dfrac{\|\Delta\boldsymbol{F}\|^2}{m^2}$, k_1 is a positive scalar, $\boldsymbol{x}_{\rm a} \in \mathbb{R}^3$ is the state of the auxiliary system, and δ is a positive parameter to be designed.

The introduced mathematical treatment requirement deals with the input saturation in the following two situations: 1) If there exists input saturation, the state of the auxiliary system satisfies the condition $\|\boldsymbol{x}_{\mathrm{a}}\| \geqslant \delta$. 2) While $\|\boldsymbol{x}_{\mathrm{a}}\| < \delta$ represents the other case, that is, there does not exist actuator saturation. The auxiliary system is introduced to analyze the effect of input constraint, and $\boldsymbol{x}_{\mathrm{a}}$ is used to design the controller input. The stability of the closed-loop system can be guaranteed by Lyapunov function, which is then given by rigorous proof of Theorem 2.

Theorem 2 Considering the spacecraft rendezvous system denoted by Eq.(1) with the sliding-mode observer Eqs.(10) and (11), design the input signal $\boldsymbol{v}$ of controller Eq.(18) as follows:

$$\begin{aligned}\boldsymbol{v} = m\{&-A_1\hat{\boldsymbol{x}}_1 - A_2\hat{\boldsymbol{x}}_2 - \boldsymbol{g} + \ddot{\boldsymbol{x}}_{\mathrm{d}} - \\ &(c+0.5\eta)[\boldsymbol{z}_2 - (c+0.5\eta)\boldsymbol{z}_1] - \\ &\boldsymbol{z}_1 + \boldsymbol{\chi} - k_2\boldsymbol{z}_2 - k_3\boldsymbol{x}_{\mathrm{a}}\},\end{aligned} \tag{20}$$

where $\boldsymbol{\chi} = -0.5\eta[\lambda_{\mathrm{o2}}^2 + \lambda_{\mathrm{o3}}^2 + (c+0.5\eta)^2]\boldsymbol{z}_2$, k_2 and k_3 are positive control gains satisfying

$$k_2 - 1 > 0,\ k_1 - 0.5k_3^2 - 0.5 > 0. \tag{21}$$

Then, the relative position of the closed-loop rendezvous system can follow the desired trajectories, and the tracking error satisfies $\|\boldsymbol{x}_1 - \boldsymbol{x}_{\mathrm{d}}\| < \varepsilon^*$ for all $t > T^*$, where ε^* and T^* is given in the proof.

Proof If the initial condition $\boldsymbol{x}_1(0) = \hat{\boldsymbol{x}}_1(0)$, $\boldsymbol{\varepsilon}_{\mathrm{o1}} \equiv 0$ for all $t > 0$, and $\boldsymbol{\varepsilon}_{\mathrm{o2}} = (\boldsymbol{x}_{\mathrm{v}})_{\mathrm{eq}}$ holds. From Eqs.(16) and (17), the following derivatives can be obtained

$$\begin{aligned}\dot{\boldsymbol{z}}_1 &= \boldsymbol{z}_2 - (c+0.5\eta)\boldsymbol{z}_1 - \boldsymbol{\varepsilon}_{\mathrm{o2}},\\ \dot{\boldsymbol{z}}_2 &= \dot{\hat{\boldsymbol{x}}}_2 + (c+0.5\eta)[\boldsymbol{z}_2 - (c+0.5\eta)\boldsymbol{z}_1 - \\ &\quad \boldsymbol{\varepsilon}_{\mathrm{o2}}] - \ddot{\boldsymbol{x}}_{\mathrm{d}}.\end{aligned}$$

Case 1 There exists input saturation, that is, $\|\boldsymbol{x}_{\mathrm{a}}\| \geqslant \delta$. Choose a candidate Lyapunov function as $V_1(t) = \dfrac{1}{2}\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1 + \dfrac{1}{2}\boldsymbol{z}_2^{\mathrm{T}}\boldsymbol{z}_2 + \dfrac{1}{2}\boldsymbol{x}_{\mathrm{a}}^{\mathrm{T}}\boldsymbol{x}_{\mathrm{a}}$, then combining Eqs.(11) and (20), yields

$$\begin{aligned}&\dot{V}_1(t) = \\ &\boldsymbol{z}_1^{\mathrm{T}}\dot{\boldsymbol{z}}_1 + \boldsymbol{z}_2^{\mathrm{T}}\dot{\boldsymbol{z}}_2 + \boldsymbol{x}_{\mathrm{a}}^{\mathrm{T}}\dot{\boldsymbol{x}}_{\mathrm{a}} = \\ &-(c+0.5\eta)\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1 - \boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{\varepsilon}_{\mathrm{o2}} - k_1\|\boldsymbol{x}_{\mathrm{a}}\|^2 - \\ &\frac{\|\Delta\boldsymbol{F}\|^2}{m^2} - \boldsymbol{x}_{\mathrm{a}}^{\mathrm{T}}\frac{\Delta\boldsymbol{F}}{m} + \boldsymbol{z}_2^{\mathrm{T}}[-\lambda_{\mathrm{o2}}\boldsymbol{\varepsilon}_{\mathrm{o2}} - \\ &\lambda_{\mathrm{o3}}|\boldsymbol{\varepsilon}_{\mathrm{o2}}|^{\alpha}\mathrm{sgn}\,\boldsymbol{\varepsilon}_{\mathrm{o2}} - (c+0.5\eta)\boldsymbol{\varepsilon}_{\mathrm{o2}} + \\ &\frac{\Delta\boldsymbol{F}}{m} + \boldsymbol{\chi} - k_2\boldsymbol{z}_2 - k_3\boldsymbol{x}_{\mathrm{a}}].\end{aligned} \tag{22}$$

To estimate the bound of the above equality, the following inequalities can be obtained by employing Young's inequality.

$$-\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{\varepsilon}_{\mathrm{o2}} \leqslant \frac{\eta}{2}\|\boldsymbol{z}_1\|^2 + \frac{1}{2\eta}\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2, \tag{23}$$

$$-\boldsymbol{x}_{\mathrm{a}}^{\mathrm{T}}\frac{\Delta\boldsymbol{F}}{m} \leqslant \frac{1}{2}\|\boldsymbol{x}_{\mathrm{a}}\|^2 + \frac{1}{2}\frac{\|\Delta\boldsymbol{F}\|^2}{m^2}, \tag{24}$$

$$-\lambda_{\mathrm{o2}}\boldsymbol{z}_2^{\mathrm{T}}\boldsymbol{\varepsilon}_{\mathrm{o2}} \leqslant \frac{\eta\lambda_{\mathrm{o2}}^2}{2}\|\boldsymbol{z}_2\|^2 + \frac{1}{2\eta}\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2, \tag{25}$$

$$\begin{aligned}&-\lambda_{\mathrm{o3}}\boldsymbol{z}_2^{\mathrm{T}}|\boldsymbol{\varepsilon}_{\mathrm{o2}}|^{\alpha}\mathrm{sgn}\,\boldsymbol{\varepsilon}_{\mathrm{o2}} \leqslant \\ &\frac{\eta\lambda_{\mathrm{o3}}^2}{2}\|\boldsymbol{z}_2\|^2 + \frac{3}{2\eta}\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^{2\alpha},\end{aligned} \tag{26}$$

$$\begin{aligned}&-(c+0.5\eta)\boldsymbol{z}_2^{\mathrm{T}}\boldsymbol{\varepsilon}_{\mathrm{o2}} \leqslant \\ &\frac{\eta(c+0.5\eta)^2}{2}\|\boldsymbol{z}_2\|^2 + \frac{1}{2\eta}\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2,\end{aligned} \tag{27}$$

$$\boldsymbol{z}_2^{\mathrm{T}}\frac{\Delta\boldsymbol{F}}{m} \leqslant \frac{1}{2}\|\boldsymbol{z}_2\|^2 + \frac{\|\Delta\boldsymbol{F}\|^2}{2m^2}, \tag{28}$$

$$-k_3\boldsymbol{z}_2^{\mathrm{T}}\boldsymbol{x}_{\mathrm{a}} \leqslant \frac{1}{2}\|\boldsymbol{z}_2\|^2 + \frac{k_3^2}{2}\|\boldsymbol{x}_{\mathrm{a}}\|^2. \tag{29}$$

Then, substituting the above inequalities Eqs.(23)–(29) into Eq.(22), and considering the expression of $\boldsymbol{\chi}$, yields that $\dot{V}_1(t)$ is bounded as follows:

$$\begin{aligned}\dot{V}_1(t) \leqslant &-c\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1 - (k_2-1)\|\boldsymbol{z}_2\|^2 - (k_1 - 0.5k_2^2 - \\ &0.5)\|\boldsymbol{x}_{\mathrm{a}}\|^2 + \frac{3}{2\eta}\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2 + \frac{3}{2\eta}\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^{2\alpha} \leqslant \\ &-m_1(\|\boldsymbol{z}_1\|^2 + \|\boldsymbol{z}_2\|^2 + \|\boldsymbol{x}_{\mathrm{a}}\|^2) + \\ &\frac{3}{2\eta}(\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2 + \|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^{2\alpha}) \leqslant \\ &-2m_1V(t) + \varepsilon_1,\end{aligned}$$

where $m_1 = \min\{c, k_2-1, k_1-0.5k_2^2-0.5\},\ \varepsilon_1 = \dfrac{3}{2\eta}(\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2 + \|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^{2\alpha})$. Using the comparison lemma in [20], $V_1(t)$ is thus bounded as

$$V_1(t) \leqslant V_1(T_{\mathrm{o2}})\mathrm{e}^{-2m_1t} + \frac{\varepsilon_1}{2m_1}(1 - \mathrm{e}^{-2m_1t}).$$

Therefore, there exists a finite time $T_1^* > T_{\mathrm{o2}}$ such that the tracking errors satisfy $\|\boldsymbol{z}_1\| = \|\boldsymbol{x}_1 - \boldsymbol{x}_{\mathrm{d}}\| \leqslant \varepsilon_1^*$ and $\|\boldsymbol{z}_2\| \leqslant \varepsilon_1^*$ for $\forall\varepsilon_1^* > \sqrt{\dfrac{\varepsilon_1}{m_1}}$.

Case 2 There does not exist input saturation, that is, $\|\boldsymbol{x}_{\mathrm{a}}\| < \delta$ and $\Delta\boldsymbol{F} = 0$. Due to $\dot{\boldsymbol{x}}_a = 0$, this time chooses the candidate Lyapunov function as $V_2(t) = \dfrac{1}{2}\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1 + \dfrac{1}{2}\boldsymbol{z}_2^{\mathrm{T}}\boldsymbol{z}_2$, and the derivative can be obtained as follows:

$$\begin{aligned}\dot{V}_2(t) = &-(c+0.5\eta)\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1 - \boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{\varepsilon}_{\mathrm{o2}} + \\ &\boldsymbol{z}_2^{\mathrm{T}}[-\lambda_{\mathrm{o2}}\boldsymbol{\varepsilon}_{\mathrm{o2}} - \lambda_{\mathrm{o3}}|\boldsymbol{\varepsilon}_{\mathrm{o2}}|^{\alpha}\mathrm{sgn}\,\boldsymbol{\varepsilon}_{\mathrm{o2}} - \\ &(c+0.5\eta)\boldsymbol{\varepsilon}_{\mathrm{o2}} + \boldsymbol{\chi} - k_2\boldsymbol{z}_2].\end{aligned} \tag{30}$$

Employing the Young's Inequality again, and substituting $\boldsymbol{\chi}$ into Eq.(30), it is obtained, by similar means, that $\dot{V}_2(t)$ is bounded as

$$\begin{aligned}\dot{V}_2(t) \leqslant &-c\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1 - (k_2-0.5)\|\boldsymbol{z}_2\|^2 + \\ &\frac{3}{2\eta}(\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2 + \|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^{2\alpha}) + \frac{k_3^2}{2}\delta^2 \leqslant \\ &-m_2V_2(t) + \varepsilon_2,\end{aligned}$$

where $m_2 = \min\{c, k_2-0.5\}$, and $\varepsilon_2 = \dfrac{3}{2\eta}(\|\boldsymbol{\varepsilon}_{\mathrm{o2}}\|^2 +$

$\|\boldsymbol{\varepsilon}_{o2}\|^{2\alpha}) + \dfrac{k_3^2}{2}\delta^2$. And

$$V_2(t) \leqslant V_2(T_{o2})\mathrm{e}^{-2m_2 t} + \frac{\varepsilon_2}{2m_2}(1 - \mathrm{e}^{-2m_2 t}).$$

Thus, there exists a finite time $T_2^* > T_{o2}$ such that $\|\boldsymbol{z}_1\| = \|\boldsymbol{x}_1 - \boldsymbol{x}_{\mathrm{d}}\| \leqslant \varepsilon_2^*$ and $\|\boldsymbol{z}_2\| \leqslant \varepsilon_2^*$ for $\forall \varepsilon_2^* > \sqrt{\dfrac{\varepsilon_2}{m_2}}$.

Combining the above two cases, the tracking errors satisfy $\|\boldsymbol{x}_1 - \boldsymbol{x}_{\mathrm{d}}\| \leqslant \varepsilon^*$ and $\|\boldsymbol{z}_2\| \leqslant \varepsilon^*$ for $\forall \varepsilon^* > \max\{\varepsilon_1^*, \varepsilon_2^*\}$ and $t > T^* = \min\{T_1^*, T_2^*\}$. In the proof, the saturation constraint always holds. This theorem is thus proved. QED.

4 Simulation results

In this section, the simulation about autonomous spacecraft rendezvous is performed to demonstrate the effectiveness of the proposed controller, and the parameters needed are formulated as follows.

Suppose that the target spacecraft is on the geosynchronous orbit of radius $r_{\mathrm{c}} = 42164$ km and the gravity constant is $\mu = 3.986 \times 10^{14}\,\mathrm{m^3/s^2}$. Thus, the target angular velocity is $\dot{\theta} = \sqrt{\mu/r_{\mathrm{c}}^3}$, and $\ddot{\theta} = 0$. The mass of chaser is $m = 300$ kg. It is pointed out that the disturbances acting on spacecrafts from other objects may result in periodic influence[21]. Therefore, the external disturbance in the rendezvous model is formulated as periodic functions and chosen as in the form[22]: $\boldsymbol{d} = [3\cos(0.2t) + 1\ \ 1.5\sin(0.2t) + 3\cos(0.2t)\ \ 3\sin(0.2t) + 1]^{\mathrm{T}} \times 10^{-5}\,\mathrm{m/s^2}$. The desired relative trajectory is the chaser flying around the target, and the reference is specified as $\boldsymbol{x}_{\mathrm{d}} = [0\ \ 1000\sin(0.002\pi)\ \ 1000\cos(0.002\pi)]^{\mathrm{T}}$ m. The observer parameters are chosen as $\lambda_{o1} = 1.5$, $\lambda_{o2} = 7.5$, $\lambda_{o3} = 1.5$ and $\alpha = 0.6$. Select the scalars in the auxiliary system as $k_1 = 3$, $\delta = 0.001$, and the controller gains are designed as $c = 0.01$, $\eta = 0.2$, $k_2 = 5$ and $k_3 = 0.75$. Besides, assume that the actuators fixed on the chaser can generate the maximum force of $F_{\max} = 200$ N. The initial states of the rendezvous dynamics are given as $\boldsymbol{x}_1 = [50\ \ -80\ \ 1100]^{\mathrm{T}}$ m, $\boldsymbol{x}_2 = [0.1\ \ 1\ \ -0.4]^{\mathrm{T}}$ m/s, and the observer initial values as $\hat{\boldsymbol{x}}_1 = \boldsymbol{x}_1$, $\hat{\boldsymbol{x}}_2 = [0\ \ 0\ \ 0]^{\mathrm{T}}$ m/s.

With the application of the proposed terminal sliding-mode observer Eqs.(10) and (11), Fig. 1–4 show the observer errors. It is seen that the estimated relative position error reaches the sliding surface $\boldsymbol{\varepsilon}_{o1} = 0$ in finite time at about 0.8 s, and obtains a high accuracy of $|\varepsilon_{o1i}| < 2 \times 10^{-9} (i = 1, 2, 3)$ after having a steady behavior from Fig. 2. About 0.2 s later, the velocity error is well converged, and bounded as $|\varepsilon_{o2i}| < 3 \times 10^{-8} (i = 1, 2, 3)$ ultimately, as shown in Fig. 4. The results demonstrate the finite-time convergence of $\boldsymbol{\varepsilon}_{o1}$ and $\boldsymbol{\varepsilon}_{o2}$ with high accuracy in the presence of external disturbance which is illustrated in Theorem 1.

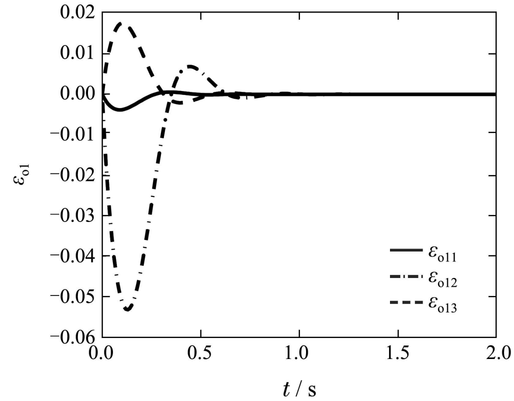

Fig. 1 The initial response of observer error ε_{o1}

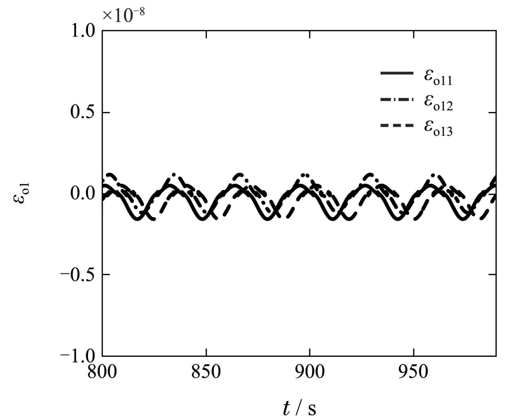

Fig. 2 The steady-state behavior of observer error ε_{o1}

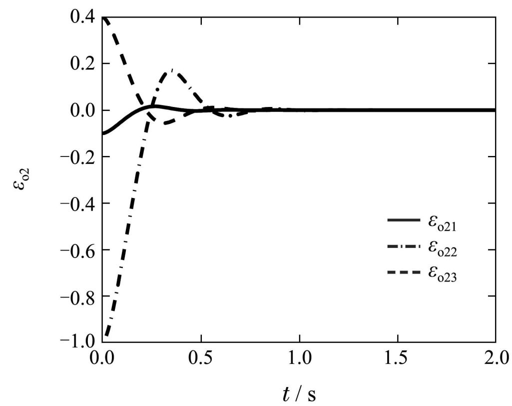

Fig. 3 The initial response of observer error ε_{o2}

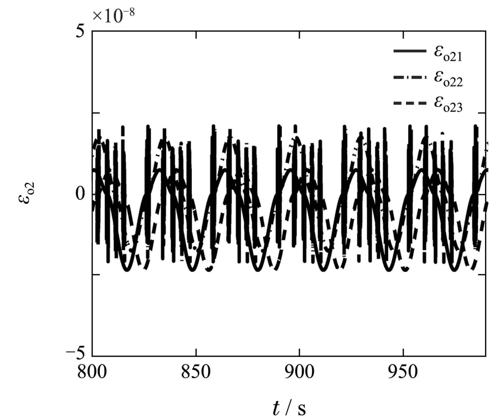

Fig. 4 The steady-state behavior of observer error ε_{o2}

Employing the designed controller described by Eqs.(18) and (20), the chaser flies around the target along the reference trajectory as shown in Fig. 5. According to Fig. 6–9, it is shown that relative motion tracks the desired $\boldsymbol{x}_{\mathrm{d}}$ precisely, and the tracking errors of position and velocity are $|x_{1i} - x_{\mathrm{d}i}| < 2 \times 10^{-5}$ and $|x_{2i} - \dot{x}_{\mathrm{d}i}| < 3 \times 10^{-5}$ respectively. This verifies the effectiveness of the controller even without velocity measurement.

Fig. 10 and Fig. 11 show the generated forces acting on the chaser. It is noted that the forces always satisfy the input constraint, and the controller prevents the actuator saturation effectively.

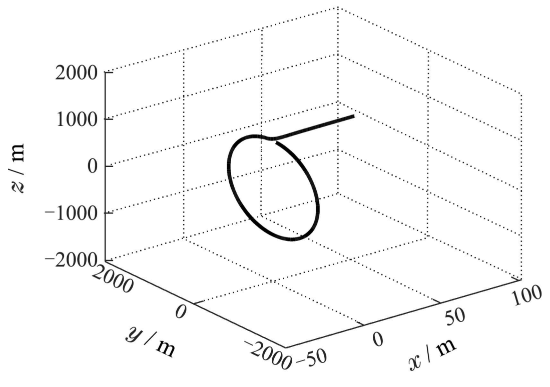

Fig. 5 The relative position between the chaser and the target

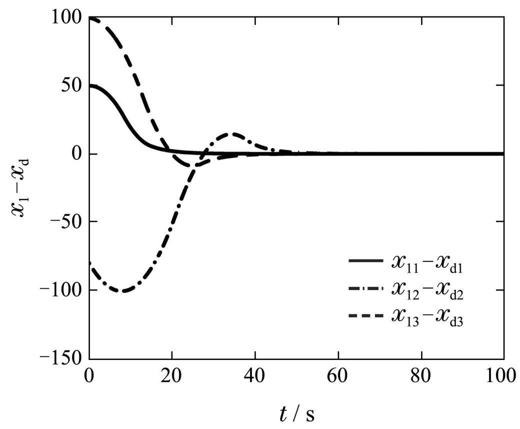

Fig. 6 The initial response of position tracking error $x_1 - x_{\mathrm{d}}$

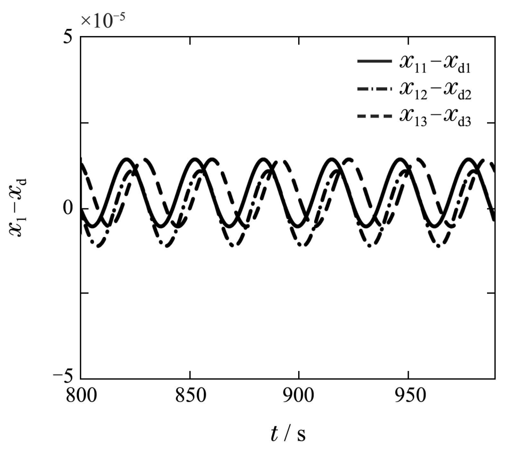

Fig. 7 The steady-state behavior of $x_1 - x_{\mathrm{d}}$

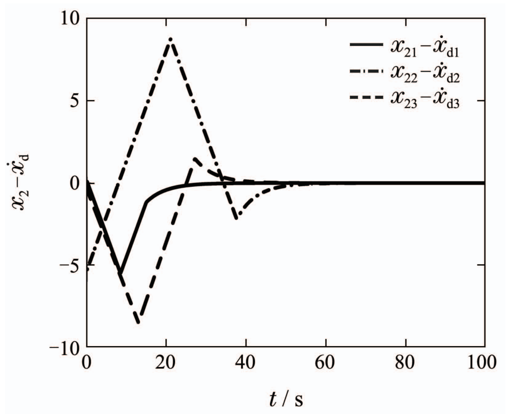

Fig. 8 The initial response of velocity tracking error $x_2 - \dot{x}_{\mathrm{d}}$

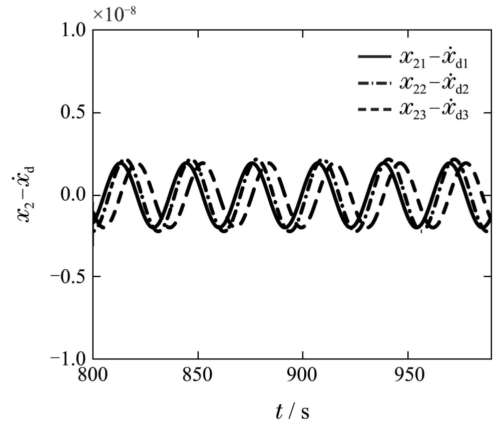

Fig. 9 The steady-state behavior of $x_2 - \dot{x}_{\mathrm{d}}$

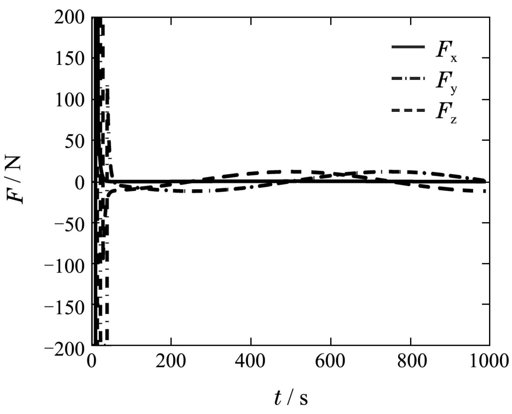

Fig. 10 The whole command control input F

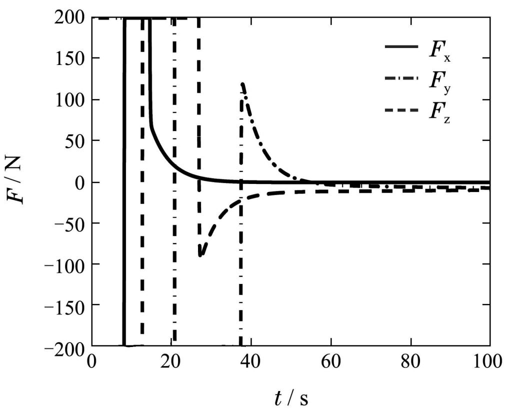

Fig. 11 The initial command control input F

5 Conclusions

In this paper, a tracking controller without velocity measurement is proposed for the autonomous rendezvous subjected to input constraint. The relative dynamics described by Clohessy-Wiltshire equations is fully nonlinear with external disturbance. Different from many approaches dealing with output feedback, a terminal sliding-mode observer is designed to estimate the relative velocity in finite time. In addition, to manage the input saturation, an auxiliary system is introduced in the design of the tracking controller. Numerical simulations are presented to validate the previous analysis; meanwhile, the estimation from observer and flying around can be achieved with high accuracy.

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