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多智能体系统的自适应群集分布式优化

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摘要: 本文对具有非线性函数群集行为的连续时间多智能体系统的分布式优化问题进行了研究. 这篇文章旨在说明所有智能体的速度和位置可以渐近一致, 并且速度达到最优, 从而使局部代价函数之和最小. 在这个研究中, 每个智能体只知道与其对应的代价函数. 首先, 文章对局部代价函数作了一些假设; 第二, 设计了一个分布式控制法则和更新律, 该控制法则仅仅依赖于自己和邻居的速度. 然后证明了多智能体系统的稳定性以及在最小化局部代价函数之和的同时所有智能体可以避免碰撞. 最后, 使用一个仿真案例来说明所获得的分析结果.

关键词: 代价函数优化; 分布式优化; 自适应多智能体系统; 群集

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Distributed optimization for adaptive flocking of multi-agent systems

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Abstract: A distributed optimization problem is investigated for continuous-time multi-agent systems with flocking behavior of a nonlinear continuous function. This paper aims at showing that the velocities and positions of all agents can be the same asymptotically and the velocity is optimal, thus minimizing the sum of local cost functions. In this study, each cost function can only be known to its corresponding agent. Firstly, the paper makes some assumptions about the local cost function; Secondly, a distributed control rule and updating laws are designed, in which each agent depends only on its own velocity and neighbor's velocity. Then, the stability of the multi-agent systems is proved and the agents can avoid inter-agent collision while minimizing the sum of local cost functions. Finally, using a simulation case to illustrate the obtained analytical results.

Key words: cost function optimization; distributed optimization; adaptive multi-agent system; flocking

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1 Introduction

In recent years, distributed optimization has drawn great attention of scholars in the field of control, including consensus-based flocking, distributed optimization, etc[1–7]. Minimizing the function which is composed of the sum of local cost functions is the purpose of distributed optimization in distributed manner.

Previous researches focused on the distributed optimization problems and most of them were about discrete-time algorithms. For example, Nedic et al. [8] gave a discrete-time sub-gradient optimization rule where each agent is restricted to lie in different convex sets. References [9–10] analyzed distributed optimization problems with inequality-equality constrains or other discrete-time rules in order to ensure that all agents converge to the optimal point. At present, the distributed optimization problem has aroused many scholars' interests in continuous-time algorithm. Wang and Elia[11] proposed a novel distributed continuous-time rule for distributed convex optimization by introducing a dynamic integrator. Based on the work[11], Refs. [12–13] studied continuous-time distributed optimization problem by strengthening conditions. Lin et al.[14] studied a convex optimization problem of continuous-

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time multi-agent system for non-uniform gradient gain, finite-time convergence, etc. Distributed optimization rule based on the edge and the adaptive design method based on nodes are proposed in [15]. Wang et al.[16] obtained finite-time consensus theoretically via the proposed non-smooth but continuous forms of distributed coordination controllers. The goal of reference [17] is to extend adaptive control with multiple models to the class of systems where the unknown parameters change rapidly and frequently. The distributed optimization problem was studied for high-order integrator dynamics in [18]. Although many articles have studied distributed optimization, many problems need to be solved, such as adaptive distributed optimization. A good adaptive controller can readjust its characteristics under the changing environment. On one hand, it is beneficial to the stable operation of the system. On the other hand, it can enhance the system's efficiency. The relevant scholars have shown great interest in the research of the flocking problems for multi-agent systems.

Recently, flocking in nonlinear systems has been further studied. Referecnces [19–20] showed that global Lipchitz condition works and all followers knew the information of the virtual leader in nonlinear systems. Yang et al.[21] studied distributed velocity optimization of time-varying functions with flocking behavior. It is worth noting that internal structure and other factors are changing in the working process of actual system. These factors might have a negative influence on the system. If we don't take the influence of these factors into account, the adaptability of the system in practical engineering is uncertain. Considering the actual situation, the study of distributed optimization with adaptive flocking of multi-agent systems is necessary in theory and practice.

The innovation of this paper comes as follows: we design an adaptive distributed optimization algorithm for adaptive flocking of multi-agent system to study how a group of agents can achieve optimization cooperatively. This idea can be seen as the extension of flocking. In the distributed optimization algorithm, there is no pre-given trajectory or leader, which is different from the consensus tracking problems.

Here is the structure of the paper. We give some notations, concepts and the preliminaries in Section 2. An adaptive distributed controller is designed for multi-agent system with flocking behavior. Then the system's stability and the optimization are achieved in Section 3. The results we have obtained are illustrated by a numerical case in Section 4. Finally, the full text content is summarized.

2 Notations and preliminaries

Here are some notations that will be used in this article. Denote $\mathbf{1}_m = (1\ \ 1\ \ \cdots\ \ 1)^{\mathrm{T}}$, $\mathbf{0}_m = (0\ \ 0\ \ \cdots\ \ 0)^{\mathrm{T}}$. A^{T} is the transpose matrix of A and x^{T} represents the transpose vector of x. And the unit matrix is $I_n \in \mathbb{R}^{n\times n}$. $\mathcal{N} = \{1, 2, \cdots, N\}$. $A \bigotimes B$ is defined as the Kronecker product of matrix A and B. Let $\|x\|_p$ denote the p-norm of $x \in \mathbb{R}^n$. The gradient and Hessian of function g are ∇g and H, respectively.

In generally, $\mathcal{G} = (\mathcal{V}, \varepsilon)$ is an undirected graph which is composed of a series of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$ and a set of links ε. If i and j can be joined through a link (i, j), then denote by $(i, j) \in \varepsilon$. $N_i = \{j \in \mathcal{V} : (j, i) \in \varepsilon\}$ stands for neighbor set of node i. If each pair of nodes in $\mathcal{G}$ has a link connected to each other, we say that the graph $\mathcal{G}$ is connected. $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is a weighted adjacent matrix of the graph $\mathcal{G}$, which meets the conditions: 1) $a_{ii} = 0$; 2) $a_{ij} = a_{ji} > 0$, if $(i, j) \in \varepsilon$; 3) $a_{ij} = 0$, if $(i, j) \notin \varepsilon$. The degree matrix of the graph $\mathcal{G}$ is $\bigwedge = \mathrm{diag}\{d_1, d_2, \cdots, d_N\} \in \mathbb{R}^{n\times n}$, where $d_i = \sum\limits_{j=1, j\neq i}^{N} a_{ij}$ for $i \in \mathbb{N}$. D is the correlation matrix related to the graph $\mathcal{G}$. Let $L = \bigwedge - A$ be a symmetric Laplacian matrix and $L = DD^{\mathrm{T}}$. The eigenvalues of L are defined as $\lambda_1(L), \lambda_2(L), \cdots, \lambda_N(L)$, then the well-known result is given for $0 = \lambda_1(L) \leqslant \lambda_2(L) \leqslant \cdots \leqslant \lambda_N(L)$ when the graph G is connected. The graph $\mathcal{G}$ is connected if and only if $\lambda_1(L) = 0$ with the associated eigenvector $\mathbf{1}_N = (1\ \ 1\ \ \cdots\ \ 1)^{\mathrm{T}}$, and all the rest are positive eigenvalues[22].

Lemma 1[22] Considering a continuous differentiable convex function $g(s) : \mathbb{R}^n \to \mathbb{R}$. Then $x \in \mathbb{R}^n$ is a global minimum of $g(s)$ if and only if $\lim\limits_{s\to x} \dfrac{\partial g(s)}{\partial s} = 0$.

Definition 1[22] $g(v)$ is σ-strongly ($\sigma > 0$) convex if and only if

$$(v_1 - v_2)(\nabla g(v_1) - \nabla g(v_2)) \geqslant \sigma \|v_1 - v_2\|^2$$

for $\forall v_1, v_2 \in \mathbb{R}^n,\ v_1 \neq v_2$. If $g(v)$ is σ-strongly convex and twice differentiable on x, then $H(x) \geqslant \sigma I_n$.

Lemma 2[23] The second smallest eigenvalue $\lambda_2(L)$ of the Laplacian matrix L associated with the undirected connected graph G satisfies $\lambda_2(L) = \min\limits_{x^{\mathrm{T}}1_N = 0, x\neq \mathbf{0}} \dfrac{x^{\mathrm{T}}Lx}{x^{\mathrm{T}}x}$.

We consider a group of $N (N \geqslant 2)$ agents described by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = f(v_i(t)) + u_i(t),\ i = 1, 2, \cdots, N, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$ represent the position vector and velocity vector of the agent i, respectively. And $u_i \in \mathbb{R}^n$ is the controller of the agent i, continuous function $f(v_i)$ is non-linear part of the system.

The goal of this paper is to devise a controller u_i for (1) that enables all agents to achieve the optimal state using local interactive information, in order to solve the following convex optimization problem:

$$\min \sum_{i=1}^{N} g_i(s),\ s \in \mathbb{R}^n, \tag{2}$$

where $g_i(s) : \mathbb{R}^n \to \mathbb{R}$ is a local cost function and it can only be known by agent i and satisfies the following assumption.

Assumption 1 The function $g_i(s) : \mathbb{R}^n \to \mathbb{R}$ is convex, differentiable and satisfies

$$g_i(s) = s^{\mathrm{T}} A_i s, \tag{3}$$

where $\sum_{i=1}^{N} A_i = B$ is positive definite and reversible matrix.

The above problem is transformed into minimizing function $\sum_{i=1}^{N} g_i(v_i)$, that is,

$$\min_{v \in \mathbb{R}^n} \sum_{i=1}^{N} g_i(v_i),\ \text{s.t. } v_i = v_j. \tag{4}$$

The following assumption is given to achieve the goal.

Assumption 2 Each function $g_i(v_i)$ has a nonempty optimal solution set, that is, there exists $v_i^* \in \mathbb{R}^n$ satisfies that $g_i(v_i^*)$ is minimum.

Definition 2 The potential function V_{ij} is a differentiable non-negative function of $\|x_i - x_j\|$ which satisfies the following:

1) $V_{ij} = V_{ji}$ has a unique minimum in $\|x_i - x_j\| = d_{ij}$, where d_{ij} is a desired distance between agents i and j and $R > \max\limits_{i,j} d_{ij}$;

2) $V_{ij} \to \infty$, if $\|x_i - x_j\| \to 0$;

3) $$\begin{cases} \dfrac{\partial V_{ij}}{\partial(\|x_i - x_j\|)} = 0, & \|x_i(0) - x_j(0)\| \geqslant R, \\ & \|x_i - x_j\| \geqslant R, \\ \dfrac{\partial V_{ij}}{\partial(\|x_i - x_j\|)} \to \infty, & \|x_i(0) - x_j(0)\| < R, \\ & \|x_i - x_j\| \to R. \end{cases}$$

Definition 3 (Flocking[24]) A group of mobile agents is said to (asymptotically) flock, when all agents attain the same velocity vector, distance between the agents is stabilized and no collision between them occurs.

Definition 4 (Dynamic graphs[24]) We call $G(t) = (V, E(t))$ a dynamic graph consisting of a set of vertices $V = \{1, 2, \cdots, N\}$ indexed by the set of agents and a time varying set of links $E(t) = \{(i,j) | i, j \in V\}$, such that for any $0 < r < R$.

1) if $0 < \|x_i(t) - x_j(t)\| \leqslant r$, then $(i,j) \in E(t)$;

2) if $r < \|x_i(t) - x_j(t)\| < R$, from Fig. 1, when time t_0 to t_1, then $(i,j) \notin E(t)$; when time t_1 to t_2, then (i,j) is a new edge being added to $E(t)$;

3) if $R \leqslant \|x_i(t) - x_j(t)\|$, then $(i,j) \notin E(t)$.

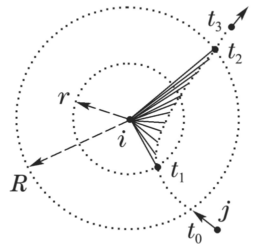

Fig. 1 The switching process of dynamic graphs according to Definition 4

Undirected dynamic graph $G(t)$ meets that $(i,j) \in E(t)$ and $(j,i) \in E(t)$, which is critical in the article. Moreover, the graph is connected in this paper.

Definition 5 (Graph Connectivity) We say that a dynamic graph $G(t)$ is connected at time t if there exists a path, that is, any two vertices can form continuous edges in $G(t)$.

3 Main results

In this section, we presented a distributed adaptive algorithm with flocking behavior. Then we prove that the velocities of the agents reach consensus and the velocities of the agents are optimal while minimizing the function $\sum_{i=1}^{N} g_i(v_i)$.

$$\begin{cases} u_i = -(P^2)^{-1} \sum\limits_{j=1}^{N} a_{ij} \nabla_{x_i} V_{ij} - \sum\limits_{j=1}^{N} a_{ij}(v_i - v_j) - \\ \qquad d_i \sum\limits_{j=1}^{N} a_{ij} \mathrm{sgn}(v_i - v_j) + \phi_i, \\ \dot{d}_i = e_{v_i}^{\mathrm{T}} (P \otimes I_N) k_i \sum\limits_{j=1}^{N} a_{ij} \mathrm{sgn}(v_i - v_j), \end{cases} \tag{5}$$

where

$$\phi_i = -H_i^{-1}(v_i) \nabla g_i(v_i).$$

The coefficients $d_i > 0$ and $k_i > 0$, $\mathrm{sgn}(\cdot)$ is the signum function. It need to be pointed out that ϕ_i relies just on the velocity of agent i. V_{ij} is defined in Definition 2. $\nabla_{x_i} V_{ij}$ corresponds to a vector in the direction of the negative gradient of V_{ij}. Let $V = [{v_1}^{\mathrm{T}}\ {v_2}^{\mathrm{T}}\ \cdots\ {v_N}^{\mathrm{T}}]^{\mathrm{T}}$, $d = [d_1\ d_2\ \cdots\ d_N]^{\mathrm{T}}$. We also define $P = I_N - \dfrac{1}{N} \mathbf{1_N} {1_N}^{\mathrm{T}}$. $e_V(t) = (P \otimes I_N)V$ denotes a consistent error vector. Obviously, zero as a simple eigenvalue of P, the right eigenvector corresponding to 0 is $\mathbf{1_N}$, and the other eigenvalues are 1 with the multiplicity $N-1$. Then $e_V(t) = 0$ is equivalent to $v_i = v_j$.

Theorem 1 By taking the controller (5), if $\hat{d}_i \geqslant$

$\dfrac{\|(P\otimes I_N)F\|_2}{\sqrt{\lambda_2(L)}}$, the velocities of all agents in the group described in (1) asymptotically approach the same and collision is avoided.

Proof The system (1) with controller (5) can be written as

$$\begin{cases}\dot{X}=V,\\ \dot{V}=-(L\otimes I_N)V-\\ \quad d^{\mathrm{T}}(D\otimes I_N)\mathrm{sgn}((D^{\mathrm{T}}\otimes I_N)V)-\\ \quad (P^{-2})\begin{pmatrix}\sum\limits_{j\in N_i}\dfrac{\partial V_{1j}}{\partial x_1}\\ \vdots\\ \sum\limits_{j\in N_i}\dfrac{\partial V_{Nj}}{\partial x_N}\end{pmatrix}+F,\end{cases} \tag{6}$$

where

$$F_i=f(v_i)+\phi_i,$$
$$F=[F_1^{\mathrm{T}}\ F_2^{\mathrm{T}}\ \cdots\ F_N^{\mathrm{T}}]^{\mathrm{T}}.$$

From $e_V(t)=(P\otimes I_N)V$, rewriting (6) as

$$\begin{cases}\dot{e}_X=e_V,\\ \dot{e}_V=-(L\otimes I_N)e_V-\\ \quad d^{\mathrm{T}}(D\otimes I_N)\mathrm{sgn}((D^{\mathrm{T}}\otimes I_N)e_V)-\\ \quad (P^{-1}\otimes I_N)\begin{pmatrix}\sum\limits_{j\in N_i}\dfrac{\partial V_{1j}}{\partial x_1}\\ \vdots\\ \sum\limits_{j\in N_i}\dfrac{\partial V_{Nj}}{\partial x_N}\end{pmatrix}+\\ \quad (P\otimes I_N)F.\end{cases} \tag{7}$$

Definite the following function:

$$V_G=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}V_{ij}+\frac{1}{2}e_V^{\mathrm{T}}e_V+\frac{1}{2}\sum_{i=1}^{N}\frac{(d_i-\hat{d}_i)^2}{k_i}, \tag{8}$$

where $\hat{d}_i$ is an arbitrary large constant.

The generalized time derivative of V_G is as follows:

$$\dot{V}_G=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\dot{V}_{ij}+e_V^{\mathrm{T}}\dot{e}_V+\sum_{i=1}^{N}\frac{(d_i-\hat{d}_i)}{k_i}\dot{d}_i. \tag{9}$$

Using Lemma 1 in [25], we can get

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\dot{V}_{ij}=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(\frac{\partial V_{ij}}{\partial x_i}\dot{x}_i+\frac{\partial V_{ij}}{\partial x_j}\dot{x}_j)=\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\frac{\partial V_{ij}}{\partial x_i}v_i.$$

From (7), we obtain

$$e_V^{\mathrm{T}}\dot{e}_V=-e_V^{\mathrm{T}}(L\otimes I_N)e_V-e_V^{\mathrm{T}}d^{\mathrm{T}}(D\otimes I_N)\mathrm{sgn}((D^{\mathrm{T}}\otimes I_N)e_V)-V^{\mathrm{T}}\begin{pmatrix}\sum\limits_{j\in N_i}\dfrac{\partial V_{1j}}{\partial x_1}\\ \vdots\\ \sum\limits_{j\in N_i}\dfrac{\partial V_{Nj}}{\partial x_N}\end{pmatrix}+e_V^{\mathrm{T}}(P\otimes I_N)F,$$

Moreover,

$$\begin{aligned}&\sum_{i=1}^{N}\frac{(d_i-\hat{d}_i)}{k_i}\dot{d}_i=\\ &\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(d_i-\hat{d}_i)e_{v_i}^{\mathrm{T}}\times(P\otimes I_N)\mathrm{sgn}(v_i-v_j)=\\ &\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}d_ie_{v_i}^{\mathrm{T}}(P\otimes I_N)\mathrm{sgn}(v_i-v_j)-\\ &\hat{d}_i\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}e_{v_i}^{\mathrm{T}}(P\otimes I_N)\mathrm{sgn}(v_i-v_j)=\\ &e_V^{\mathrm{T}}d^{\mathrm{T}}(D\otimes I_N)\mathrm{sgn}((D^{\mathrm{T}}\otimes I_N)e_V)-\\ &\hat{d}_i\|(D^{\mathrm{T}}\otimes I_N)e_V\|_1.\end{aligned}$$

Thus, we have

$$\begin{aligned}\dot{V}_G=&-e_V^{\mathrm{T}}(L\otimes I_N)e_V+e_V^{\mathrm{T}}(P\otimes I_N)F-\\ &\hat{d}_i\|(D^{\mathrm{T}}\otimes I_N)e_V\|_1\leqslant\\ &-e_V^{\mathrm{T}}(L\otimes I_N)e_V+\|(P\otimes I_N)F\|_2\|e_V\|_2-\\ &\hat{d}_i\sqrt{e_V^{\mathrm{T}}(DD^{\mathrm{T}}\otimes I_N)e_V}\leqslant\\ &-e_V^{\mathrm{T}}(L\otimes I_N)e_V+\|(P\otimes I_N)F\|_2\|e_V\|_2-\\ &\hat{d}_i\sqrt{\lambda_2(L)}\|e_V\|_2=\\ &-e_V^{\mathrm{T}}(L\otimes I_N)e_V+(\|(P\otimes I_N)F\|_2-\\ &\hat{d}_i\sqrt{\lambda_2(L)})\|e_V\|_2\leqslant\\ &-e_V^{\mathrm{T}}(L\otimes I_N)e_V.\end{aligned}$$

Clearly, if $\hat{d}_i\geqslant\dfrac{\|(P\otimes I_N)F\|_2}{\sqrt{\lambda_2(L)}}$, then $\dot{V}_G\leqslant-e_V^{\mathrm{T}}(L\otimes I_N)e_V$. It is known from the characteristics of L to get $\dot{V}_G\leqslant 0$ which shows that $V_G\geqslant 0$ is monotonically decreasing. So the boundness of e_V, d_i can be seen. Further $V_{ij}, e_V\in\mathcal{L}_\infty$ can be derived. By integrating both side of $\dot{V}_G\leqslant-e_V^{\mathrm{T}}(L\otimes I_N)e_V$, we can see that $e_V\in\mathcal{L}_2$. By using the Barbalat's lemma, we get $e_V\to 0$ which is equivalent to the all agents' velocities become consensus as $t\to\infty$, i.e. $\lim\limits_{t\to\infty}(v_i(t)-\dfrac{1}{N}\sum\limits_{k=1}^{N}v_k(t))=\lim\limits_{t\to\infty}e_{V_i}(t)=0$.

Because $v_i-v_j=0$ for $i,j=1,2,\cdots,N$, we can get

$$\frac{\mathrm{d}}{\mathrm{d}t}\|x_i - x_j\|^2 = 2(x_i - x_j)^{\mathrm{T}}(v_i - v_j) = 0,$$

which shows that the distance is invariant. Hence, all agents become consensus and the collision is avoided.

QED.

Remark 1 The assumption $\hat{d}_i \geqslant \frac{\|(P \otimes I_N)F\|_2}{\sqrt{\lambda_2(L)}}, \forall i$ in Theorem 1 is regarded as a bound on the difference between the agents' internal signals. If $\|f(v_i)\|$, $\|\nabla g_i\|$ and $\|\frac{\partial \nabla g_i}{\partial t}\|, \forall i$ are bounded, then $\|F\|_2$ is bounded.

Theorem 2 Suppose that $H_i(v_i) = H_j(v_j)$, $\forall i, j \in \mathbb{N}$. For system (1) with controller (5) the problem (4) is solved if the function is designed as $f(v_i) = \hat{A}v_i$ where matrix $\hat{A}$ satisfies

$$B^{\mathrm{T}}\hat{A} - \sigma^{-1}B^{\mathrm{T}}B < 0. \tag{10}$$

Proof It is proved that the system is stable in Theorem 1. Next let us prove the optimization.

By denoting $v^*(t) = \frac{1}{N}\sum_{i=1}^{N} v_i(t)$, we have

$$\frac{\mathrm{d}v^*(t)}{\mathrm{d}t} = \frac{1}{N}\sum_{i=1}^{N}\dot{v}_i(t) = \frac{1}{N}\sum_{i=1}^{N}(f(v_i) + N\phi_i) = \frac{1}{N}\sum_{i=1}^{N}(f(v_i) - NH_i^{-1}(v_i)\nabla g_i(v_i)) = \frac{1}{N}\sum_{i=1}^{N}(\hat{A}v_i - NH_i^{-1}(v_i)A_iv_i) = (\hat{A} - \sum_{i=1}^{N}H_i^{-1}(v_i)A_i)v_i.$$

Therefore, we can obtain

$$\frac{\partial\sum_{i=1}^{N}g_i(v^*)}{\partial t} = (\frac{\partial\sum_{i=1}^{N}g_i(v^*)}{\partial v^*})^{\mathrm{T}}\frac{\mathrm{d}v^*}{\mathrm{d}t} = (\sum_{i=1}^{N}A_iv^*)^{\mathrm{T}}(\hat{A} - \sum_{i=1}^{N}H_i^{-1}(v^*)A_i)v^* = (v^*)^{\mathrm{T}}[(\sum_{i=1}^{N}A_i)^{\mathrm{T}}\hat{A} - (\sum_{i=1}^{N}A_i)^{\mathrm{T}}\sum_{i=1}^{N}H_i^{-1}(v^*)A_i]v^*.$$

By $\sum_{i=1}^{N}A_i = B$, we can get

$$\frac{\partial\sum_{i=1}^{N}g_i(v^*)}{\partial t} = (v^*)^{\mathrm{T}}(B^{\mathrm{T}}\hat{A} - \sigma^{-1}B^{\mathrm{T}}B)v^* < 0.$$

Note that $\sum_{i=1}^{N}g_i(v^*(t))$ is lower bounded according to Assumption 2. $\lim_{t\to\infty}\frac{\partial\sum_{i=1}^{N}g_i(v^*(t))}{\partial v^*(t)} = 0$ is got by $B^{\mathrm{T}}\hat{A} - \sigma^{-1}B^{\mathrm{T}}B < 0$ and LaSalle's invariance principle. Then it follows from Lemma 1 that $v^*(t)$ minimizes the differentiable convex function $\sum_{i=1}^{N}g_i(v_i(t))$ as $t \to \infty$. QED.

Corollary 1 Under controller (5), if the nonlinear continuous function is designed as $f(v_i) = 0$, all agents in the system (1) converge to the same velocity and collision between agents is avoided. Moreover, the problem (2) is solved.

Proof From Theorem 1, we know that all agents' velocities and the distance are invariant in the stable state.

By denoting $v^*(t) = \frac{1}{N}\sum_{i=1}^{N}v_i(t)$, one has

$$\frac{\mathrm{d}v^*(t)}{\mathrm{d}t} = \frac{1}{N}\sum_{i=1}^{N}\dot{v}_i(t) = \frac{1}{N}\sum_{i=1}^{N}N\phi_i = -\sum_{i=1}^{N}H_i^{-1}(v_i)\nabla g_i(v_i) = -\sum_{i=1}^{N}H_i^{-1}(v_i)A_iv_i.$$

Then, we can get

$$\frac{\partial\sum_{i=1}^{N}g_i(v^*)}{\partial t} = (\frac{\partial\sum_{i=1}^{N}g_i(v^*)}{\partial v^*})^{\mathrm{T}}\frac{\mathrm{d}v^*}{\mathrm{d}t} = (\sum_{i=1}^{N}A_iv^*)^{\mathrm{T}}(-\sum_{i=1}^{N}H_i^{-1}(v^*)A_iv^*) = (\sum_{i=1}^{N}A_iv^*)^{\mathrm{T}}(-\sigma^{-1}Bv^*) = -(v^*)^{\mathrm{T}}(\sigma^{-1}B^{\mathrm{T}}B)v^* < 0.$$

It follows by Theorem 1 that $v^*(t)$ minimizes the differentiable convex function $\sum_{i=1}^{N}g_i(v_i(t))$ as $t \to \infty$.

QED.

4 Numerical simulations

In this section, we give an example with five agents. Assume that link range $R = 5$, which means that two agents are adjacent when their distance is less than R. The agent's task is to minimize the function $\sum_{i=1}^{N}g_i(v_i(t), t)$, where $v_i(t) = (v_{x_i}(t)\ \ v_{y_i}(t))^{\mathrm{T}}$ is the coordinate of agent i in 2D plane.

In the example, we use controller (5) for the system (1), the nonlinear function is designed by

$$f(v_i) = \hat{A}v_i,$$

where

$$\hat{A} = \begin{bmatrix} -0.05\mathrm{i} & 0.01\mathrm{i} \\ 0.01 + 0.02\mathrm{i} & -0.04\mathrm{i} \end{bmatrix}$$

We randomly choose the initial velocity of each agent and mark it with dots. The initial status of the agent is shown in Fig. 2. The full line between the two dots in the figure represents the path of the adjacent agent, while the arrow represents the velocity of the agent. The final stable state configuration is shown in Fig. 3. The error between the different velocity components and the optimal velocity is described in Fig. 4, and the error curves of different velocity components are represented by different lines. From the graph, we can see that the velocities of all agents converge to the optimal velocity.

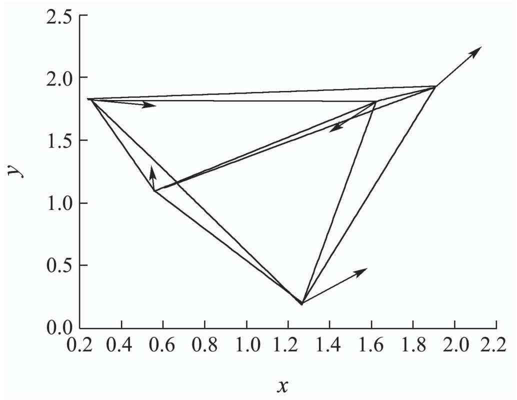

Fig. 2 Initial configuration of agents

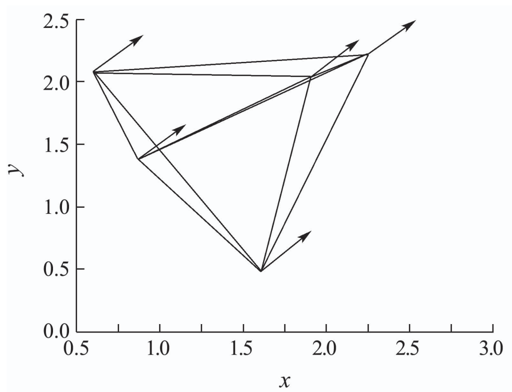

Fig. 3 Final configuration of agents

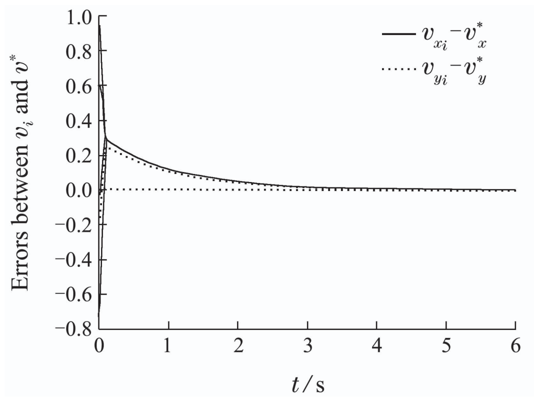

Fig. 4 Errors between agents' and the optimal velocity

5 Conclusions

In this paper, we analyze a distributed optimization problem for adaptive flocking of multi-agent systems. A novel multi-agent adaptive controller is presented to achieve the goal proposed in this paper. First, it proves that the agents can asymptotically achieve flocking behavior, that is, the agents can avoid collision when they have same velocity and invariant distance. Furthermore, it shows that all agents' velocities are asymptotically optimal while minimizing the total cost function. Finally, in order to be recognized for the theoretical results we obtained, we use a simulation case to verify the above results. Our further study will focus on the adaptive flocking of non-linear multi-agent systems with time delay.

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