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利用FastSLAM框架的多自治水下航行器同时定位与跟踪算法

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摘要: 协同定位是共融机器人研究领域的重要问题. 协同定位方案的制定受限于机器人间信息交互的能力. 针对长时间通讯中断时多自治水下航行器(AUV)协同定位精度明显下降的问题, 借鉴同时定位与制图(SLAM) 方法, 提出了基于FastSLAM框架的同时定位与跟踪(SLAT)算法. 将主AUV视为非合作目标, 在从AUV上建立起一个关于主AUV的运动估计器, 利用从AUV上声呐传感器实时获取的相对量测信息, 在对主AUV运动状态估计的同时, 完成对从AUV自定位精度的提升. 仿真实验结果表明, 在长时间通讯中断发生的条件约束下, 相比于传统的航位推算方法, 所提出的SLATF1.0和2.0算法能够有效减小定位误差, 2.0算法对于探测精度变化等因素的影响具有更好适应性.

关键词: 协同定位; 自治水下航行器; 同时定位与制图; 同时定位与跟踪; 声呐; 传感器; 仿真; 误差

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Simultaneous localization and tracking algorithm utilizing FastSLAM framework for autonomous underwater vehicles

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Abstract: The cooperative localization is an important research question in the field of Tri-Co Robots study. The scheme of the cooperative localization algorithm depends on the ability of information interaction between the robots. To solve the problem that the cooperative localization accuracy is obviously reduced when the communication is interrupted for a long time between the autonomous underwater vehicles (AUV), the simultaneous localization and tracking (SLAT) algorithms based on the FastSLAM framework are developed in this research, borrowing the principle of the simultaneous localization and mapping (SLAM) algorithms. The master AUV is regarded as a non-cooperative target and a motion estimator used to track the master AUV is built in the slaver AUV. When the motion state of the master AUV is estimated, the improvement of the self localization accuracy of the slaver AUV is achieved, using the relative measurement information obtained from the sonar sensor on the slaver AUV in real time. The simulation experimental results show that the proposed SLATF1.0 and 2.0 algorithms can effectively reduce the localization errors compared to the conventional dead reckoning method under the condition of long-term communication interruption, and the 2.0 algorithm has better adaptability to the influence of the detection accuracy variety.

Key words: cooperative localization; autonomous underwater vehicles; simultaneous localization and mapping; simultaneous localization and tracking; sonar; sensors; simulation; errors

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1　引言

协同定位的研究开始于无人陆上航行器[1–2], 继而发展到水下机器人[3–4]. 研究方向大致可归为: 1) 协同定位模型及相关算法[5]; 2) 环境约束下误差建模与补偿[6]; 3) 多协同体编队结构设计[7–8] 等. 其目标都是通过优化协同算法和改善协同结构提高定位精度. 本质上, 协同定位是通过主从多自治水下航行器(autonomous underwater vehicles, AUV)间信息的共享[9]来

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提高从AUV的定位精度. 当AUV间通讯发生中断时, 从AUV一般只能通过自身携带的航位推算装置进行自定位. 如果共享信息长时间不能获取, 从AUV将会在内部传感器较大的过程噪声和非线性推算模型的共同作用下, 产生较大的定位累积误差. 为了保持从AUV仍能有较高的自定位精度水平, 可以如图1所示, 通过安装在从AUV上的声呐装置探测获取相关信息. 由此, 本文将面对一个与上面提到的3个经典问题不同的新问题, 或将之认为是1, 2两个问题的交叉问题, 一个以补偿误差为目标构建环境约束下协同定位模型及相关算法问题.

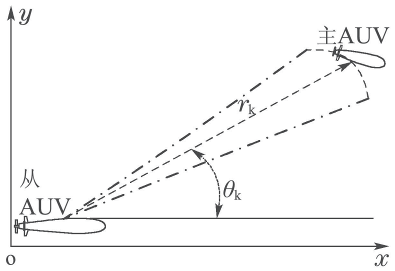

图 1 探测式信息获取协同定位示意

Fig. 1 The schematic diagram of the cooperative localization of detection information acquisition

高精度地捕捉主AUV的位置信息是高水平协同定位的关键[10]. 可以将主AUV视为非合作目标体, 在从AUV上建立起一个关于主AUV的运动估计模型[11]. 由于缺乏主AUV的本体测量数据, 估计模型不能采用航位推算模型, 而要转向选取不依赖交互信息的目标跟踪模型[12].

作为信息探测端的从AUV定位精度较低, 对主AUV的探测所形成的量测误差较大. 相对于主AUV较小的本体定位误差, 不准确的量测和不精准的跟踪模型将导致对主AUV较大的状态估计误差. 而在协同定位过程中, 较粗糙的主AUV的位置估计又会反过来降低从AUV的定位精度. 由此提出了一个关于通讯中断时协同定位的新问题: 当主、从AUV的状态都有很大的不确定性, 如何同时提高二者的位姿(位置)估计精度.

同时定位与制图(simultaneous localization and mapping, SLAM)方法[13]提供了一种解决此类估计问题的思路. 仿照SLAM方法, 本文在给出从AUV运动学模型、主AUV跟踪模型、传感器量测模型的基础上, 提出了2种基于FastSLAM[14]框架的同时定位与跟踪(simultaneous localization and tracking, SLAT)算法(简称SLATF1.0算法和SLATF2.0算法). 在1.0算法中, 参与后验估计的每个粒子都含有主从AUV的状态估计参量和粒子权值, 并设置了与主AUV数量相同的EKF, 利用EKF估计主AUV位置的同时, 使用粒子滤波器估算从AUV的位姿状态. 在2.0算法中, 为了解决传感器精度过高所造成的粒子枯竭问题, 建议分布不仅依靠于控制模型, 还融入了传感器当前的观测值. 本文还证明了在给定从AUV整个运动路径的前提下, 当环境中存在多个主AUV时, SLAT可以分解为一个从AUV和若干个彼此独立主AUV的状态估计. 在本文最后给出了仿真结果和分析.

2 运动学模型和量测模型

2.1 从AUV运动学模型

AUV概率运动学模型可概括为

$$p(X_{k+1}|X_k, u_k), \tag{1}$$

式中: X_k为位姿状态量, u_k为控制量. 由于深度信息可由深度仪准确测得, 定位问题可以简化在二维平面中进行研究. 控制向量可表述为$u_k=[V_k\ \ \phi_k]^{\mathrm T}$, $V_k = V_{mk}+\omega_{vk}$, $\phi_k=\phi_{mk}+\omega_{\phi k}$为运动速度和偏航角, $w_k=[\omega_{vk}\ \ \omega_{\phi k}]^{\mathrm T}$为过程噪声, $\mathrm E[w_k]=0$, $\mathrm E[w_k w_k^{\mathrm T}] = Q_k\delta_{kl}$.

从AUV运动学模型可表述为

$$\begin{cases} x^{\mathrm S}_{k+1}=x^{\mathrm S}_k+T^{\mathrm S}V_k\cos\ \phi_k,\\ y^{\mathrm S}_{k+1}=y^{\mathrm S}_k+T^{\mathrm S}V_k\sin\ \phi_k,\\ \phi^{\mathrm S}_{k+1}=\phi^{\mathrm S}_k+\Delta\phi^{\mathrm S}_{k+1|k}, \end{cases} \tag{2}$$

式中$T^{\mathrm S}$为采样周期. 运动学模型可简单描述为

$$X^{\mathrm S}_{k+1}=f_1(X^{\mathrm S}_k, u^{\mathrm S}_k). \tag{3}$$

2.2 主AUV跟踪模型

主AUV在二维平面时刻的状态向量用

$$X^{(\mathrm M)}_k=[x^{(\mathrm M)}_k\ \ y^{(\mathrm M)}_k\ \ \dot x^{(\mathrm M)}_k\ \ \dot y^{(\mathrm M)}_k\ \ \ddot x^{(\mathrm M)}_k\ \ \ddot y^{(\mathrm M)}_k]^{\mathrm T}$$

来描述. 状态跟踪运动模型可表示为

$$\begin{aligned} &X^{(\mathrm M)}_{k+1}=f_2(X^{(\mathrm M)}_k, u^{(\mathrm M)}_k, w^{(\mathrm M)}_k)=\\ &\varPhi^{(\mathrm M)}_k\cdot X^{(\mathrm M)}_k+E^{(\mathrm M)}_k\cdot u^{(\mathrm M)}_k+G^{(\mathrm M)}_k\cdot w^{(\mathrm M)}_k, \end{aligned} \tag{4}$$

式中: $\varPhi^{(\mathrm M)}_k$为状态转移概率, $u^{(\mathrm M)}_k$为控制量, $w^{(\mathrm M)}_k$为过程噪声. 式(4)具体模型描述见文献[12]. 在跟踪过程中, 如果作为目标体的主AUV的实际运动比较复杂, 可以采用交互多模型(interacting multiple model, IMM)方法[15]实现对主AUV稳定的航迹估计.

2.3 量测模型

利用从AUV声呐探测主AUV, 回波提供两维量测信息:

$$\begin{cases} r^{(\mathrm{S,M})}_k=\sqrt{(x^{(\mathrm S)}_k-x^{(\mathrm M)}_k)^2+(y^{(\mathrm S)}_k-y^{(\mathrm M)}_k)^2}+\Delta r^{(\mathrm{S,M})}_k,\\ \theta^{(\mathrm{S,M})}_k=\arctan\dfrac{y^{(\mathrm S)}_k-y^{(\mathrm M)}_k}{x^{(\mathrm S)}_k-x^{(\mathrm M)}_k}+\Delta\theta^{(\mathrm{S,M})}_k, \end{cases} \tag{5}$$

式中$\Delta r^{(\mathrm{S,M})}_k$和$\Delta\theta^{(\mathrm{S,M})}_k$为量测噪声. 设

$$\nu_k=[\Delta r^{(\mathrm{S,M})}_k\ \ \Delta\theta^{(\mathrm{S,M})}_k]^{\mathrm T},$$

有$\mathrm E[\nu_k]=0$, $\mathrm E[\nu_k\nu_k^{\mathrm T}]=R_k\delta_{kl}$.

量测模型可简单描述为

$$z_k = g(X_k^{(\mathrm{M})}, X_k^{(\mathrm{S})}) + \nu_k. \tag{6}$$

3 SLATF1.0算法

设定到$k+1$时刻从AUV的位姿状态更新为$X^{(\mathrm{S}),K+1}$,主AUV的位置状态更新为$\Theta_{k+1}^{\mathrm{M}}$, 仿效FastSLAM框架,联合估计问题可描述成

$$p(X^{(\mathrm{S}),K+1}, \Theta_{k+1}^{\mathrm{M}} | z^{k+1}, u^{(\mathrm{S}),K+1}, n^{k+1}), \tag{7}$$

其中:

$$\Theta_{k+1}^{\mathrm{M}} : \{X_1^{(\mathrm{M}),k+1}, \cdots, X_n^{(\mathrm{M}),k+1}\},$$

$$z^{k+1} : \{z_1, \cdots, z_{k+1}\},$$

$$u^{(\mathrm{S}),K+1} : \{u_1^{(\mathrm{S})}, \cdots, u_{k+1}^{(\mathrm{S})}\}, n^{k+1} : \{n_1, \cdots, n_{k+1}\}$$

表示量测与观测目标主AUV间的数据关联集.

当环境中只有一个主AUV时, 不考虑数据关联,由条件概率定义可得联合分布为

$$\begin{aligned} &p(X^{(\mathrm{S}),k+1}, X^{(\mathrm{M}),k+1} | z^{k+1}, u^{(\mathrm{S}),k+1}) = \\ &p(X^{(\mathrm{S}),k+1} | z^{k+1}, u^{(\mathrm{S}),k+1}) \cdot \\ &p(X^{(\mathrm{M}),k+1} | X^{(\mathrm{S}),k+1}, z^{k+1}, u^{(\mathrm{S}),k+1}). \end{aligned} \tag{8}$$

先考虑式(8)右端第1项, 使用粒子滤波器实现对从AUV位姿的估计. 设定滤波器有N个粒子, 采样时间$T^{(\mathrm{S})} = T^{(\mathrm{M})}$, 粒子$i$的构成可由下式表示:

$$\Gamma_k^i = \langle (X_k^{(\mathrm{S})})^i \, w_k^i \, (X_k^{(\mathrm{M})})^i \, (P_k^{(\mathrm{M})})^i \rangle, \tag{9}$$

式中: w_k^i为粒子权值, $(P_k^{(\mathrm{M})})^i$为粒子对主AUV状态估计误差协方差.

$k+1$时刻粒子i对从AUV的状态估计值为

$$(X_{k+1}^{(\mathrm{S})})^i \sim p(X_{k+1}^{(\mathrm{S})} | u_k^{(\mathrm{S})}, (X_k^{(\mathrm{S})})^i). \tag{10}$$

根据式(3), 可得

$$(X_{k+1}^{(\mathrm{S})})^i = f_1((X_k^{(\mathrm{S})})^i, u_k^{(\mathrm{S})}), \tag{11}$$

则粒子建议分布为

$$p(X^{(\mathrm{S}),k+1} | z^k, u^{(\mathrm{S}),k+1}). \tag{12}$$

对于式(8)右端的第2项, 由条件概率公式和马尔科夫准则:

$$\begin{aligned} &p(X^{(\mathrm{M}),k+1} | X^{(\mathrm{S}),k+1}, z^{k+1}, u^{(\mathrm{S}),k+1}) = \\ &p(X^{(\mathrm{M}),k} | X^{(\mathrm{S}),k}, z^k, u^{(\mathrm{S}),k}) \cdot \\ &p(X_{k+1}^{(\mathrm{M})} | X_k^{(\mathrm{M})}, X^{(\mathrm{S}),k+1}, z^{k+1}, u^{(\mathrm{S}),k+1}). \end{aligned} \tag{13}$$

根据上式可知, 求取$X^{(\mathrm{M}),k+1}$的过程可转化为在求得$X^{(\mathrm{M}),k}$的基础上求取$X_{k+1}^{(\mathrm{M})}$的过程. 根据贝叶斯准则和马尔科夫准则:

$$\begin{aligned} &p(X_{k+1}^{(\mathrm{M})} | X_k^{(\mathrm{M})}, X^{(\mathrm{S}),k+1}, z^{k+1}, u^{(\mathrm{S}),k+1}) = \\ &\eta p(z_{k+1} | X_{k+1}^{(\mathrm{M})}, X_{k+1}^{(\mathrm{S})}) \cdot \\ &p(X_{k+1}^{(\mathrm{M})} | X_k^{(\mathrm{M})}, X^{(\mathrm{S}),k}, z^k, u^{(\mathrm{S}),k}). \end{aligned} \tag{14}$$

在上式右边第2项由于缺少观测量z_{k+1}, 所以去除无影响量$X_{k+1}^{(\mathrm{S})}$和$u_{k+1}^{(\mathrm{S})}$. 根据式(4), 可以推算得到主AUV在$k+1$时刻的状态估计值为

$$(X_{k+1|k}^{(\mathrm{M})})^i = f_2((X_{k|k}^{(\mathrm{M})})^i, u_k^{(\mathrm{M})}, 0). \tag{15}$$

在$k+1$时刻主AUV的状态协方差的预测为

$$\begin{aligned} (P_{k+1|k}^{(\mathrm{M})})^i = \varPhi_k^{(\mathrm{M})} (P_{k|k}^{(\mathrm{M})})^i (\varPhi_k^{(\mathrm{M})})^{\mathrm{T}} + \\ G_k^{(\mathrm{M})} Q_k^{(\mathrm{M})} (G_k^{(\mathrm{M})})^{\mathrm{T}}, \end{aligned} \tag{16}$$

则有

$$\begin{aligned} &p(X_{k+1}^{(\mathrm{M})} | X_k^{(\mathrm{M})}, X^{(\mathrm{S}),k}, z^k, u^{(\mathrm{S}),k}) \sim \\ &N(X_{k+1}^{(\mathrm{M})}; (X_{k+1|k}^{(\mathrm{M})})^i, (P_{k+1|k}^{(\mathrm{M})})^i). \end{aligned} \tag{17}$$

使用$k+1$时刻传感器量测更新系统状态. 设粒子i利用状态估计值对量测值的估计为

$$\hat{z}_{k+1}^i = [\hat{r}_{k+1}^i \quad \hat{\theta}_{k+1}^i]^{\mathrm{T}} = g((X_{k+1}^{(\mathrm{S})})^i, (X_{k+1|k}^{(\mathrm{M})})^i), \tag{18}$$

则有

$$g(X_{k+1}^{(\mathrm{S})}, X_{k+1}^{(\mathrm{M})}) \approx \hat{z}_{k+1}^i + H_{k+1}^i (X_{k+1}^{(\mathrm{M})} - (X_{k+1|k}^{(\mathrm{M})})^i), \tag{19}$$

$$H_{k+1}^i = \frac{\partial g}{\partial X_{k+1}^{(\mathrm{M})}} \Big|_{X_{k+1}^{(\mathrm{M})} = (X_{k+1|k}^{(\mathrm{M})})^i, X_{k+1}^{(\mathrm{S})} = (X_{k+1}^{(\mathrm{S})})^i}, \tag{20}$$

$$\begin{aligned} &p(z_{k+1} | X_{k+1}^{(\mathrm{M})}, X_{k+1}^{(\mathrm{S})}) \sim \\ &N(z_{k+1}; \hat{z}_{k+1}^i + H_{k+1}^i (X_{k+1}^{(\mathrm{M})} - (X_{k+1|k}^{(\mathrm{M})})^i), R_{k+1}). \end{aligned} \tag{21}$$

按式(21), $k+1$时刻量测误差符合高斯分布, 可以使用EKF估计主AUV的即时状态$X_{k+1}^{(\mathrm{M})}$.

$k+1$时刻的新息和新息协方差为

$$V_{k+1}^i = z_{k+1} - \hat{z}_{k+1}^i, \tag{22}$$

$$S_{k+1}^i = H_{k+1}^i (P_{k+1|k}^{(\mathrm{M})})^i (H_{k+1}^i)^{\mathrm{T}} + R_{k+1}. \tag{23}$$

滤波器增益为

$$K_{k+1}^i = (P_{k+1|k}^{(\mathrm{M})})^i (H_{k+1}^i)^{\mathrm{T}} (S_{k+1}^i)^{-1}. \tag{24}$$

则对主AUV的状态向量及其协方差矩阵进行更新, 有

$$(X_{k+1|k+1}^{(\mathrm{M})})^i = (X_{k+1|k}^{(\mathrm{M})})^i + K_{k+1}^i V_{k+1}^i, \tag{25}$$

$$(P_{k+1|k+1}^{(\mathrm{M})})^i = [I - K_{k+1}^i H_{k+1}^i] (P_{k+1|k}^{(\mathrm{M})})^i. \tag{26}$$

重要性权值为后验分布函数与建议分布函数之比,利用条件概率公式和马尔科夫准则:

$$\begin{aligned} &w_{k+1}^i = \\ &\frac{p(X^{(\mathrm{S}),k+1} | z^{k+1}, u^{(\mathrm{S}),k+1})}{\pi(X^{(\mathrm{S}),k+1} | z^{k+1}, u^{(\mathrm{S}),k+1})} = \\ &\frac{\eta p(z_{k+1} | X^{(\mathrm{S}),k+1}, z^k, u^{(\mathrm{S}),k+1}))}{\pi(X_{k+1}^{(\mathrm{S})} | X^{(\mathrm{S}),k}, z^{k+1}, u^{(\mathrm{S}),k+1})} \times \\ &\frac{p(X^{(\mathrm{S}),k+1} | z^k, u^{(\mathrm{S}),k+1})}{\pi(X^{(\mathrm{S}),k} | z^{k+1}, u^{(\mathrm{S}),k+1})} = \end{aligned}$$

$$\frac{p(X^{(\mathrm{S}),k}\,|z^k,u^{(\mathrm{S}),k})}{\pi(X^{(\mathrm{S}),k}\,|z^k,u^{(\mathrm{S}),k})}\times \frac{\eta p(z_{k+1}|X^{(\mathrm{S}),k+1},z^k,u^{(\mathrm{S}),k+1})}{\pi(X^{(\mathrm{S})}_{k+1}|X^{(\mathrm{S}),k},z^{k+1},u^{(\mathrm{S}),k+1})}\times p(X^{(\mathrm{S})}_{k+1}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k+1}). \tag{27}$$

因为有建议分布

$$\pi(X^{(\mathrm{S}),k+1}|z^{k+1},u^{(\mathrm{S}),k+1})= p(X^{(\mathrm{S}),k+1}|z^k,u^{(\mathrm{S}),k+1}), \tag{28}$$

所以对于采样有

$$\pi(X^{(\mathrm{S})}_{k+1}|X^{(\mathrm{S}),k},z^{k+1},u^{(\mathrm{S}),k+1})= p(X^{(\mathrm{S})}_{k+1}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k+1}). \tag{29}$$

将式(29)代入式(27)中, 则有

$$w^i_{k+1}=\eta w^i_k p(z_{k+1}|X^{(\mathrm{S}),k+1},z^k,u^{(\mathrm{S}),k+1})\propto w^i_k W^i_{k+1}, \tag{30}$$

可有

$$W^i_{k+1}=\frac{1}{\sqrt{|2\pi S^i_{k+1}|}}\times \exp\{-\frac{1}{2}(V^i_{k+1})^{\mathrm{T}}(S^i_{k+1})^{-1}V^i_{k+1}\}. \tag{31}$$

归一化所有粒子的权值

$$\tilde{w}^i_{k+1}=\frac{w^i_{k+1}}{\sum\limits_{i=1}^{N} w^i_{k+1}}. \tag{32}$$

为了保证粒子的有效性, 估计过程中经常需要进行重抽样. 判断是否需进行重抽样的指标为

$$N_{\mathrm{eff}}=\frac{1}{\sum\limits_{i=1}^{N}(\tilde{w}^i_{k+1})^2}, \tag{33}$$

式中N_{eff}代表有效粒子数. 当N_{eff}小于设定最小粒子数时, 需要进行重采样.

4 主AUV位置估计的独立性

本文给出如下命题: 在基于FastSLAM算法框架的SLAT解决方案中, 在给定从AUV整个运动路径的前提下, 对某个主AUV的观测结果不包含其他主AUV的位置信息:

$$p(X^{(\mathrm{S}),k},\Theta^{\mathrm{M}}_k|z^k,u^{(\mathrm{S}),k},n^k)= p(X^{(\mathrm{S}),k}|z^k,u^{(\mathrm{S}),k},n^k)\times \prod_{n=1}^{L} p(X^{(\mathrm{M}),k}_n\,|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k), \tag{34}$$

即当存在多个主AUV时, SLAT可以被分解为一个从AUV和若干个彼此独立的主AUV的状态估计.

证 依据条件概率的定义, 从AUV位姿与主AUV位置的联合后验估计为

$$p(X^{(\mathrm{S}),k},\Theta^{\mathrm{M}}_k|z^k,u^{(\mathrm{S}),k},n^k)= p(X^{(\mathrm{S}),k}|z^k,u^{(\mathrm{S}),k},n^k)\times p(\Theta^{\mathrm{M}}_k\,\Big|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k), \tag{35}$$

则证明式(34)成立等价于证明

$$p(\Theta^{\mathrm{M}}_k|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k)= \prod_{n=1}^{L} p(X^{(\mathrm{M}),k}_n|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k). \tag{36}$$

根据贝叶斯公式可有

$$p(X^{(\mathrm{M}),k}_{n_k}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k)\overset{\mathrm{Bayes}}{=} \frac{p(z_k|X^{(\mathrm{M}),k}_{n_k},X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k)}{p(z_k|X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k)}\times p(X^{(\mathrm{M}),k}_{n_k}|X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k). \tag{37}$$

在没有获取到量测z_k的情况下, 当前时刻的从AUV位姿状态$X^{(\mathrm{S})}_k$、控制量$u^{(\mathrm{S})}_k$和数据关联n_k对$X^{(\mathrm{M}),k}_{n_k}$没有影响. 则有

$$\overset{\mathrm{Markov}}{=}\frac{p(z_k|X^{(\mathrm{M})}_{k,n_k},X^{(\mathrm{S})}_k,n_k)}{p(z_k|X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k)}\times p(X^{(\mathrm{M}),k}_{n_k}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1}). \tag{38}$$

当前量测不属于对被估计主AUV的观测结果时有

$$p(X^{(\mathrm{M}),k}_{n\neq n_k}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k)\overset{\mathrm{Markov}}{=} p(X^{(\mathrm{M}),k}_{n\neq n_k}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1}). \tag{39}$$

基于上述推导结果, 使用数学归纳法对命题进行证明. 首先, 因为在$k=0$时刻, 没有任何对主AUV的观测信息融入后验估计中, 所以$k=0$时命题成立. 设当$k>0$时, $k-1$时刻命题成立

$$p(\Theta^{\mathrm{M}}_{k-1}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1})= \prod_{n=1}^{L} p(X^{(\mathrm{M}),k-1}_n|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1}). \tag{40}$$

对式(36)左边应用贝叶斯公式并通过马尔科夫准则简化有

$$p(\Theta^{\mathrm{M}}_k|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k)= \frac{p(z_k|X^{(\mathrm{M})}_{k,n_k},X^{(\mathrm{S})}_k,n_k)}{p(z_k|X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k)}\times p(\Theta^{\mathrm{M}}_k|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1})\overset{\mathrm{Induction}}{=} \frac{p(z_k|X^{(\mathrm{M})}_{k,n_k},X^{(\mathrm{S})}_k,n_k)}{p(z_k|X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k)}\times \prod_{n=1}^{L} p(X^{(\mathrm{M}),k-1}_n|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1}). \tag{41}$$

将式(41)右边第2项进行分解可得

$$
\begin{aligned}
&= \frac{p(z_k|X_{k,n_k}^{(\mathrm{M})},X_k^{(\mathrm{S})},n_k)}{p(z_k|X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k)}\times\\
&p(X_{n_k}^{(\mathrm{M}),k-1}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1})\times\\
&\prod_{n\neq n_k}^{L}p(X_n^{(\mathrm{M}),k-1}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1}).
\end{aligned}\tag{42}
$$

当没有新的信息量注入时, 主AUV的后验概率分布不会发生改变, 则有

$$
\begin{aligned}
&p(X_n^{(\mathrm{M}),k-1}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1})=\\
&p(X_n^{(\mathrm{M}),k}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1}),
\end{aligned}\tag{43}
$$

于是有

$$
\begin{aligned}
&p(\Theta_k^{\mathrm{M}}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k)=\\
&\frac{p(z_k|X_{k,n_k}^{(\mathrm{M})},X_k^{(\mathrm{S})},n_k)}{p(z_k|X^{(\mathrm{S}),k},z^{k-1},u^{(\mathrm{S}),k},n^k)}\times\\
&p(X_{n_k}^{(\mathrm{M}),k}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1})\times\\
&\prod_{n\neq n_k}^{L}p(X_n^{(\mathrm{M}),k}|X^{(\mathrm{S}),k-1},z^{k-1},u^{(\mathrm{S}),k-1},n^{k-1})=\\
&p(X_{n_k}^{(\mathrm{M}),k}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k)\times\\
&\prod_{n\neq n_k}^{L}p(X_n^{(\mathrm{M}),k}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k)=\\
&\prod_{n=1}^{L}p(X_n^{(\mathrm{M}),k}|X^{(\mathrm{S}),k},z^k,u^{(\mathrm{S}),k},n^k),
\end{aligned}\tag{44}
$$

则式(34)得证.　　证毕.

5 SLATF2.0算法

如果AUV所携带的传感器探测精度过高, 即过程噪声明显高于传感器的观测噪声时, 建议分布函数将相对于后验分布失真, 1.0算法的性能将会逐渐退化: 运动模型所传播的粒子在一个广泛的区域, 但是只有一小部分粒子拥有值得重视的权值, 仅有一小部分采样值能存活到下一代. 观测噪声越小, 保留下来的粒子就越少, 足够准确的观测将导致滤波器瘫痪.

为了适应观测小噪声的情景, 本文接下来将构建一种新的同时定位与跟踪算法, 并称之为SLATF2.0算法. 为了保证粒子的分散度, 对AUV新位姿进行采样时, 改变建议分布的选取规则, 不仅依靠航位推算模型, 还要兼顾考虑传感器当前的观测值. 新算法将运动模型线性化, 使改进的建议分布能以近似的形式进行计算. 建构SLATF2.0算法过程中所考虑的主要因素是: 粒子抽取的程序和粒子权重的计算.

与1.0算法只从运动模型中抽取位姿信息不同, 2.0算法是从一个包含当前量测的运动模型中进行抽取:

$$
(X_{k+1}^{(\mathrm{S})})^i\sim p(X_{k+1}^{(\mathrm{S})}|u^{(\mathrm{S}),k},(X^{(\mathrm{S}),k})^i,z^{k+1}).\tag{45}
$$

这里, $(X^{(\mathrm{S}),k})^i$是路径上第i个粒子更新到k时刻的全部状态, 采样分布中明确地包含$k+1$时刻可利用的信息, 即当前量测z_{k+1}和最新控制量u_k.

对于采样的机制做更深入的分析. 首先, 新的运动模型能以已知分布的形式分解, 包括标准运动模型和测量模型, 而两个模型都可以进一步应用高斯特征估计. 采样分布用贝叶斯法则进行扩展:

$$
\begin{aligned}
&p(X_{k+1}^{(\mathrm{S})}|u^{(\mathrm{S}),k},(X^{(\mathrm{S}),k})^i,z^{k+1})\overset{\mathrm{Bayes}}{\propto}\\
&\eta p(z_{k+1}|X_{k+1}^{(\mathrm{S})},(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k},z^k)\times\\
&p(X_{k+1}^{(\mathrm{S})}|u^{(\mathrm{S}),k},(X^{(\mathrm{S}),k})^i,z^k).
\end{aligned}\tag{46}
$$

式(46)右边第2项中, 由于缺少z_{k+1}的参与, 从AUV的姿态$X_{k+1}^{(\mathrm{S})}$仅仅依赖于先前的姿态$X_k^{(\mathrm{S})}$和当前的控制量$u_k^{(\mathrm{S})}$, 则有

$$
\begin{aligned}
\overset{\mathrm{Markov}}{=}\ &\eta p(z_{k+1}|X_{k+1}^{(\mathrm{S})},(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k},z^k)\times\\
&p(X_{k+1}^{(\mathrm{S})}|u_k^{(\mathrm{S})},(X_k^{(\mathrm{S})})^i).
\end{aligned}\tag{47}
$$

根据离散型全概率公式, 将式(47)中的第1项写为含有$X_{k+1}^{(\mathrm{M})}$的形式, 并根据马尔科夫准则进行简化:

$$
\begin{aligned}
&p(z_{k+1}|X_{k+1}^{(\mathrm{S})},(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k},z^k)=\\
&\sum_{(X_{k+1}^{(\mathrm{M})})^i}p(z_{k+1}|X_{k+1}^{(\mathrm{M})},X_{k+1}^{(\mathrm{S})},(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k},z^k)\times\\
&p(X_{k+1}^{(\mathrm{M})}|X_{k+1}^{(\mathrm{S})},(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k},z^k)\overset{\mathrm{Markov}}{=}\\
&\sum_{(X_{k+1}^{(\mathrm{M})})^i}p(z_{k+1}|X_{k+1}^{(\mathrm{M})},X_{k+1}^{(\mathrm{S})})\times\\
&p(X_{k+1}^{(\mathrm{M})}|(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k-1},z^k).
\end{aligned}\tag{48}
$$

则式(46)可表述为

$$
\begin{aligned}
=\eta\sum_{(X_{k+1}^{(\mathrm{M})})^i}\Big[&\underbrace{p(z_{k+1}|X_{k+1}^{(\mathrm{M})},X_{k+1}^{(\mathrm{S})})}_{\sim N(z_{k+1};g(X_{k+1}^{(\mathrm{S})},X_{k+1}^{(\mathrm{M})}),R_{k+1})}\times\\
&\underbrace{p(X_{k+1}^{(\mathrm{M})}|(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k-1},z^k)}_{\sim N(X_{k+1}^{(\mathrm{M})};(X_{k+1|k}^{(\mathrm{M})})^i,(P_{k+1|k}^{(\mathrm{M})})^i)}\Big]\times\\
&\underbrace{p(X_{k+1}^{(\mathrm{S})}|u_k^{(\mathrm{S})},(X_k^{(\mathrm{S})})^i)}_{\sim N(X_{k+1}^{(\mathrm{S})};f_1((X_k^{(\mathrm{S})})^i,u_k^{(\mathrm{S})}),(P_{k+1|k}^{(\mathrm{S})})^i)},
\end{aligned}\tag{49}
$$

式中$(P_{k+1|k}^{(\mathrm{S})})^i$表述为

$$
(P_{k+1|k}^{(\mathrm{S})})^i=\varPhi_k^{(\mathrm{S})}(P_{k|k}^{(\mathrm{S})})^i(\varPhi_k^{(\mathrm{S})})^{\mathrm{T}}+G_k^{(\mathrm{S})}Q_k^{(\mathrm{S})}(G_k^{(\mathrm{S})})^{\mathrm{T}},\tag{50}
$$

其中: $\varPhi_k^{(\mathrm{S})}=\dfrac{\partial f_1}{\partial X_k^{(\mathrm{S})}}, G_k^{(\mathrm{S})}=\dfrac{\partial f_1}{\partial u_k^{(\mathrm{S})}}$.

主AUV的状态估计值参照式(15)–(16).

通过一阶泰勒扩展将观测函数g近似为一个线性项:

$$\begin{cases}\hat{X}_{k+1}^{(\mathrm{S})}=f_1((X_k^{(\mathrm{S})})^i,u_k^{(\mathrm{S})}),\\ \hat{z}_{k+1}=g(\hat{X}_{k+1}^{(\mathrm{S})},(X_{k+1|k}^{(\mathrm{M})})^i),\\ G_{X^{(\mathrm{M})}}=\nabla_{X_{k+1}^{(\mathrm{M})}}g(X_{k+1}^{(\mathrm{S})},X_{k+1}^{(\mathrm{M})}),\\ G_{X^{(\mathrm{S})}}=\nabla_{X_{k+1}^{(\mathrm{S})}}g(X_{k+1}^{(\mathrm{S})},X_{k+1}^{(\mathrm{M})}),\\ g(X_{k+1}^{(\mathrm{S})},X_{k+1}^{(\mathrm{M})})\approx G_{X^{(\mathrm{M})}}(X_{k+1}^{(\mathrm{M})}-(X_{k+1|k}^{(\mathrm{M})})^i)+\\ \qquad G_{X^{(\mathrm{S})}}(X_{k+1}^{(\mathrm{S})}-\hat{X}_{k+1}^{(\mathrm{S})})+\hat{z}_{k+1}.\end{cases}\tag{51}$$

根据线性项, 近似有

$$N(z_{k+1};\hat{z}_{k+1}+G_{X^{(\mathrm{S})}}X_{k+1}^{(\mathrm{S})}-G_{X^{(\mathrm{S})}}\hat{X}_{k+1}^{(\mathrm{S})},\underbrace{R_{k+1}+G_{X^{(\mathrm{M})}}(P_{k+1|k}^{(\mathrm{M})})^iG_{X^{(\mathrm{M})}}^{\mathrm{T}}}_{\Lambda_{k+1}}).\tag{52}$$

按1.0算法, 建议分布可以由式(49)的最右端项给出, 即高斯分布项$N(X_{k+1}^{(\mathrm{S})};(X_{k+1}^{(\mathrm{S})})^i,(P_{k+1|k}^{(\mathrm{S})})^i)$. 基于这个高斯项, 后验分布可以写作

$$p(X_{k+1}^{(\mathrm{S})}|(X^{(\mathrm{S}),k})^i,z^{k+1},u^{(\mathrm{S}),k})=\xi\exp\{-y_{k+1}\},\tag{53}$$

其中指数可以表示为

$$\begin{aligned}y_{k+1}=&\frac{1}{2}[(z_{k+1}-\hat{z}_{k+1}-G_{X^{(\mathrm{S})}}X_{k+1}^{(\mathrm{S})}+\\ &G_{X^{(\mathrm{S})}}\hat{X}_{k+1}^{(\mathrm{S})})^{\mathrm{T}}\Lambda_{k+1}^{-1}(z_{k+1}-\hat{z}_{k+1}-\\ &G_{X^{(\mathrm{S})}}X_{k+1}^{(\mathrm{S})}+G_{X^{(\mathrm{S})}}\hat{X}_{k+1}^{(\mathrm{S})})+(X_{k+1}^{(\mathrm{S})}-\hat{X}_{k+1}^{(\mathrm{S})})^{\mathrm{T}}\times\\ &((P_{k+1|k}^{(\mathrm{S})})^i)^{-1}(X_{k+1}^{(\mathrm{S})}-\hat{X}_{k+1}^{(\mathrm{S})})].\end{aligned}\tag{54}$$

上式表明指数是目标变量$X_{k+1}^{(\mathrm{S})}$的二次函数, 所以后验分布也是高斯项. 后验分布均值和方差为最小值和曲率, 可以通过计算y_{k+1}关于$X_{k+1}^{(\mathrm{S})}$的一阶和二阶微分项得到

$$\begin{aligned}\frac{\partial y_{k+1}}{\partial X_{k+1}^{(\mathrm{S})}}=&(G_{X^{(\mathrm{S})}}^{\mathrm{T}}\Lambda_{k+1}^{-1}G_{X^{(\mathrm{S})}}+(\Gamma_k^{(\mathrm{S})})^{-1})X_{k+1}^{(\mathrm{S})}-\\ &G_{X^{(\mathrm{S})}}^{\mathrm{T}}\Lambda_{k+1}^{-1}(z_{k+1}-\hat{z}_{k+1}+G_{X^{(\mathrm{S})}}\hat{X}_{k+1}^{(\mathrm{S})})-\\ &((P_{k+1|k}^{(\mathrm{S})})^i)^{-1}\hat{X}_{k+1}^{(\mathrm{S})},\end{aligned}\tag{55}$$

$$\frac{\partial^2 y_{k+1}}{\partial (X_{k+1}^{(\mathrm{S})})^2}=G_{X^{(\mathrm{S})}}^{\mathrm{T}}\Lambda_{k+1}^{-1}G_{X^{(\mathrm{S})}}+((P_{k+1|k}^{(\mathrm{S})})^i)^{-1}.\tag{56}$$

采样分布的方差$\Sigma_{X^{(\mathrm{S})},k+1}^i$可以通过对二阶微分求逆得到

$$\Sigma_{X^{(\mathrm{S})},k+1}^i=[G_{X^{(\mathrm{S})}}^{\mathrm{T}}\Lambda_{k+1}^{-1}G_{X^{(\mathrm{S})}}+((P_{k+1|k}^{(\mathrm{S})})^i)^{-1}]^{-1}.\tag{57}$$

采样均值可以通过使一阶微分为零获得

$$\mu_{X^{(\mathrm{S})}}^i=\Sigma_{X^{(\mathrm{S})},k+1}^iG_{X^{(\mathrm{S})}}^{\mathrm{T}}\Lambda_{k+1}^{-1}(z_{k+1}-\hat{z}_{k+1})+(\hat{X}_{k+1}^{(\mathrm{S})})^i.\tag{58}$$

由式(57)–(58)就确定了新的采样分布$p(X_{k+1}^{(\mathrm{S})}|u^{(\mathrm{S}),k},(X^{(\mathrm{S}),k})^i,z^{k+1})$的参数值.

因为使用了相对于1.0算法的一种不同的建议分布, 重要性权重也必须更新来反映这种变化. 在抽样过程中, 先前的路径以$(X^{(\mathrm{S}),k})^i$的形式产生, 根据的是后验分布$p((X^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k-1},z^k)$, 所以建议分布可以写作

$$\begin{aligned}&p((X^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k-1},z^k)\times\\ &p((X_{k+1}^{(\mathrm{S})})^i|u^{(\mathrm{S}),k},(X^{(\mathrm{S}),k})^i,z^{k+1}),\end{aligned}\tag{59}$$

重要性权重w_{k+1}^i写作

$$w_{k+1}^i=\frac{p((X^{(\mathrm{S}),k+1})^i|u^{(\mathrm{S}),k},z^{k+1})}{p((X^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k-1},z^k)}\times\frac{1}{p((X_{k+1}^{(\mathrm{S})})^i|u^{(\mathrm{S}),k},(X^{(\mathrm{S}),k})^i,z^{k+1})}.\tag{60}$$

分子能被条件概率拓展:

$$\begin{aligned}&=\frac{p((X_{k+1}^{(\mathrm{S})})^i|(X^{(\mathrm{S}),k})^i,u^{(\mathrm{S}),k},z^{k+1})}{p((X^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k-1},z^k)}\times\\ &\frac{p((X^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k},z^{k+1})}{p((X_{k+1}^{(\mathrm{S})})^i|u^{(\mathrm{S}),k},(X^{(\mathrm{S}),k})^i,z^{k+1})}=\\ &\frac{p((X^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k},z^{k+1})}{p((X^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k-1},z^k)}.\end{aligned}\tag{61}$$

再由贝叶斯法则拓展分子并利用马尔科夫准则:

$$\begin{aligned}&=\eta\frac{p(z_{k+1}|(X^{(\mathrm{S}),k})^i,z^k,u^{(\mathrm{S}),k})}{p((\hat{X}^{(\mathrm{S}),k})^i|u^{(\mathrm{S}),k-1},z^k)}\times\\ &p((X^{(\mathrm{S}),k})^i|z^k,u^{(\mathrm{S}),k-1})=\\ &\eta p(z_{k+1}|(X^{(\mathrm{S}),k})^i,z^k,u^{(\mathrm{S}),k}).\end{aligned}\tag{62}$$

利用两次全概公式给出权值关于$X_{k+1}^{(\mathrm{S})}$和$X_{k+1}^{(\mathrm{M})}$的表达式:

$$\begin{aligned}w_{k+1}^i=&\eta\sum_{X_{k+1}^{(\mathrm{S})}}p(z_{k+1}|X_{k+1}^{(\mathrm{S})},(X^{(\mathrm{S}),k})^i,z^k,u^{(\mathrm{S}),k})\times\\ &p(X_{k+1}^{(\mathrm{S})}|(X^{(\mathrm{S}),k})^i,z^k,u^{(\mathrm{S}),k})=\\ &\eta\sum_{X_{k+1}^{(\mathrm{S})}}\sum_{X_{k+1}^{(\mathrm{M})}}\underbrace{p(z_{k+1}|X_{k+1}^{(\mathrm{M})},X_{k+1}^{(\mathrm{S})})}_{\sim N(z_{k+1};g(X_{k+1}^{(\mathrm{S})},X_{k+1}^{(\mathrm{M})}),R_{k+1})}\times\\ &\underbrace{p(X_{k+1}^{(\mathrm{M})}|(X^{(\mathrm{S}),k})^i,z^k,u^{(\mathrm{S}),k-1})}_{\sim N(X_{k+1}^{(\mathrm{M})};(X_{k+1|k}^{(\mathrm{M})})^i,(P_{k+1|k}^{(\mathrm{M})})^i)}\times\\ &\underbrace{p(X_{k+1}^{(\mathrm{S})}|(X_k^{(\mathrm{S})})^i,u_k^{(\mathrm{S})})}_{\sim N(X_{k+1}^{(\mathrm{S})};f_1((X_k^{(\mathrm{S})})^i,u_k^{(\mathrm{S})}),(P_{k+1|k}^{(\mathrm{S})})^i)}.\end{aligned}\tag{63}$$

上式右边3项表达式都是高斯的, 分别是量测模型、主AUV预测模型、从AUV运动模型. 于是可以得到权值的近似表达

$$\begin{aligned}w_{k+1}^i\sim N(&z_{k+1};\hat{z}_{k+1},G_{X^{(\mathrm{S})}}(P_{k+1|k}^{(\mathrm{S})})^iG_{X^{(\mathrm{S})}}^{\mathrm{T}}+\\ &G_{X^{(\mathrm{M})}}(P_{k+1|k}^{(\mathrm{M})})^iG_{X^{(\mathrm{M})}}^{\mathrm{T}}+R_{k+1}),\end{aligned}\tag{64}$$

则第i个粒子的权重可表述为

$$\begin{cases} L_{k+1}^i = G_{X^{(\mathrm{S})}}(P_{k+1|k}^{(\mathrm{S})})^i G_{X^{(\mathrm{S})}}^{\mathrm{T}} + \\ \qquad G_{X^{(\mathrm{M})}}(P_{k+1|k}^{(\mathrm{M})})^i G_{X^{(\mathrm{M})}}^{\mathrm{T}} + R_{k+1}, \\ w_{k+1}^i = \left|2\pi L_{k+1}^i\right|^{-\frac{1}{2}} \exp\{\frac{1}{2}(z - \hat{z}_{k+1}^i)^{\mathrm{T}} \times \\ \qquad (L_{k+1}^i)^{-1}(z - \hat{z}_{k+1}^i)\}. \end{cases} \tag{65}$$

归一化所有粒子的权值和判断是否需要重采样如式(32)–(33). 同1.0算法类似, 利用$k+1$时刻量测, 并使用EKF方法更新主AUV状态估计值. 具体参见式(22)–(26).

6　算法实现流程

当主从AUV间通讯中断时, 根据先验知识, 抽取初始时刻的粒子Γ_0^i. 获取到量测z_{k+1}时, 按如下步骤估计主从AUV状态:

1) 由式(11)计算$(X_{k+1}^{(\mathrm{S})})^i$. 对于2.0算法, 将$(X_{k+1}^{(\mathrm{S})})^i$代入式(58)修正;

2) 由式(22)–(23)计算V_{k+1}^i和S_{k+1}^i, 并由式(25)–(26)估计$(X_{k+1|k+1}^{(\mathrm{M})})^i$和$(P_{k+1|k+1}^{(\mathrm{M})})^i$;

3) w_{k+1}^i计算. 1.0算法: 按式(30)–(31); 2.0算法: 按式(65);

4) 由式(32)正则化权值, 并由式(33)计算有效样本容量.

重复上述步骤直至AUV间通讯恢复.

7　仿真分析

图2显示了主、从AUV真实的运动轨迹. 设定仿真时长为400 s, 开始时主、从AUV沿与水平夹角$\frac{\pi}{12}$和$\frac{\pi}{6}$从左向右做直线运动. 从240 s开始, 主AUV按$\frac{\pi}{120}$ rad/s变化角度做曲线运动. 从AUV通过携带的声呐实时探测得到主AUV的距离与方位信息. 主从AUV均保持10 kn的运动速度, 声呐探测周期为2 s.

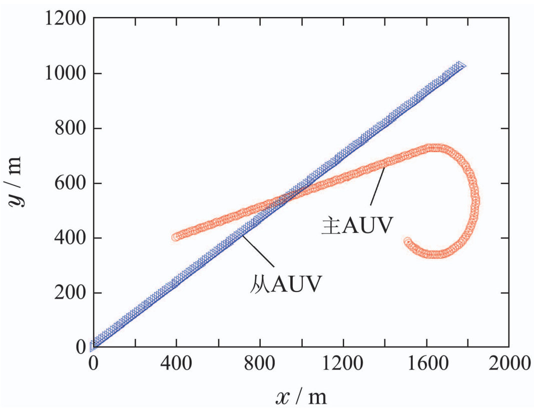

图 2　主从AUV的真实轨迹

Fig. 2　The true trajectory of the master and slaver AUV

采用3种方法估计从AUV的航行轨迹: ① 航位推算, 即只采用本体传感器和运动模型来估计位置; ② SLATF1.0算法; ③ SLATF2.0算法. SLATF算法中所使用粒子数$N=100$, 进行50次Monte Carlo仿真.

设定从AUV本体传感器噪声水平为$\sigma_{\mathrm{V}} = 2$ m/s, $\sigma_\phi = 0.3$ rad, 设置4种声呐传感器探测噪声水平如下:

① $\sigma_{\mathrm{r}}=15$ m, $\sigma_\theta=0.4$ rad;

② $\sigma_{\mathrm{r}}=10$ m, $\sigma_\theta=0.3$ rad;

③ $\sigma_{\mathrm{r}}=1$ m, $\sigma_\theta=0.06$ rad;

④ $\sigma_{\mathrm{r}}=0.5$ m, $\sigma_\theta=0.03$ rad.

其中: 定义①–②为大噪声情况, ③–④为小噪声情况. 各噪声水平下各算法的估计结果的均方根误差(root mean square error, RMSE)比较如表1. 相应的估计误差均值和方差的对比如图3, 其中柱状图为误差均值, 线棒为估计误差的方差.

表 1　不同探测噪声水平下各算法估计结果的RMSE

Table 1　RMSE of the estimation results of each algorithm under different detection noise levels

噪声水平	航位推算	1.0算法	2.0算法
(15 m, 0,4 rad)	59.9892	33.2032	27.7221
(10 m, 0.3 rad)	58.1659	29.1760	25.6497
(1 m, 0.06 rad)	59.0700	35.4430	20.2979
(0.5 m, 0.03 rad)	57.1940	39.7212	17.5921

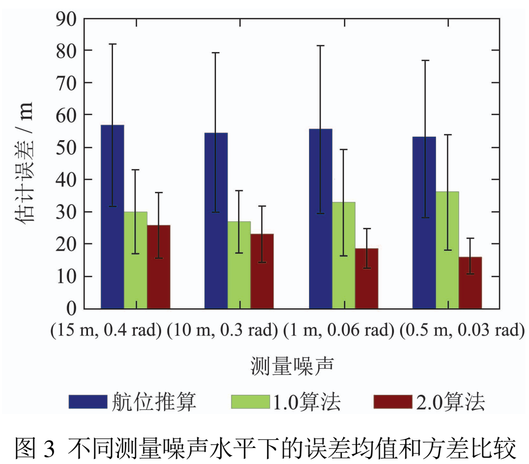

图 3　不同测量噪声水平下的误差均值和方差比较

Fig. 3　Comparison of mean and variance of error under at different measurement noise levels

表1中显示了4种不同探测噪声水平下3种估计方法对从AUV位置估计的RMSE, 本文提出的SLATF1.0和SLATF2.0算法在不同传感器探测噪声水平上均优于只利用内部量测信息进行估算的航位推算方法. 1.0算法在①–②两种较大噪声情况的RMSE与2.0算法差异不大, 但是在面对③–④两种较小噪声时, RMSE不减反增, 且噪声越小RMSE越大, 这与2.0算法随着噪声变小, RMSE逐渐变小不同. 图3显示2.0算法随着探测噪声变小, 误差均值和方差都变小; 而1.0算法在面

对小噪声情景时误差均值和方差都变大, 估计性能变差. 说明1.0算法在探测小噪声时估计效果明显变差, 而2.0算法的估计能力在任何噪声水平都很稳定.

为了进一步分析验证本文所提出算法的性能, 将探测精度为如上4种噪声水平的传感器分为两组, 分别对应①③和②④. 假设从AUV分别携带每组传感器对主AUV进行探测, 按照图2所示运动过程, 采用3种方法估计自身位置, 则各算法从AUV的位置估计误差比较如图4–5.

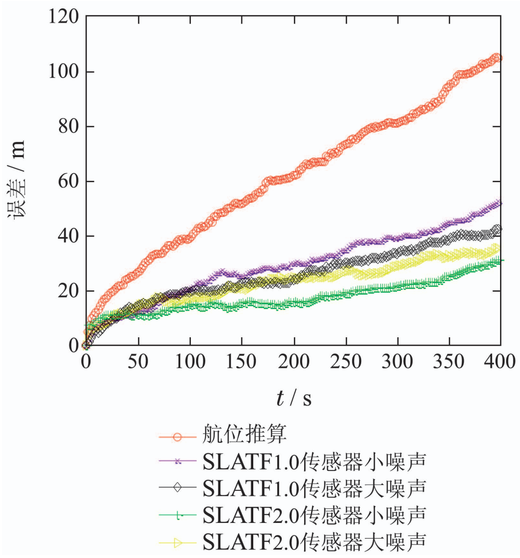

图 4 ①③不同测量噪声水平下的误差均值和方差比较

Fig. 4 Comparison of the slaver location estimation errors of ① and ③ two noise level algorithms

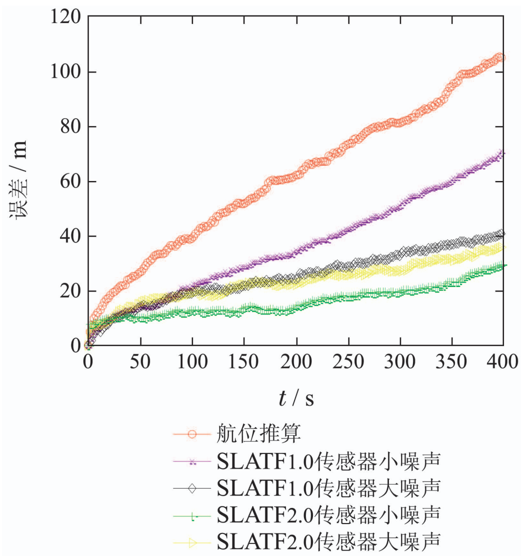

图 5 ②④两种噪声水平下各算法从AUV的位置估计误差比较

Fig. 5 Comparison of the slaver location estimation errors of ② and ④ two noise level algorithm

从数据上看, 图4–5具有较好的一致性. 在同一过程中, 面对大误差量测时2.0算法稍优于1.0算法; 面对小误差量测时, 2.0算法估计误差最小, 而1.0算法估计误差在4种算法与传感器的配组方案中最大且趋于发散. 由于两图为从AUV同一本体噪声水平下的50次Monte Carlo仿真生成结果, 具备可比较性, 则相比于图4, 在面对小误差情景时图5中1.0算法估计误差更大, 发散趋势明显, 说明1.0算法在更小的传感器探测噪声情况下估计效果更差.

为了更好地分析和比较1.0和2.0算法在传感器小噪声情况下的性能, 新设置3种小噪声传感器探测水平如下: ⑤ $\sigma_{\rm r}=1.5$ m, $\sigma_\theta=0.09$ rad; ⑥ $\sigma_{\rm r}=0.8$ m, $\sigma_\theta=0.05$ rad; ⑦ $\sigma_{\rm r}=0.3$ m, $\sigma_\theta=0.015$ rad. 假设从AUV装备噪声水平为③–⑦5种精度的传感器, 主从AUV沿图2路径运动, 分别使用1.0算法和2.0算法, 完成同时定位与跟踪过程. 则2种估计方法的RMSE和位置估计误差随噪声减小的变化趋势如图6所示.

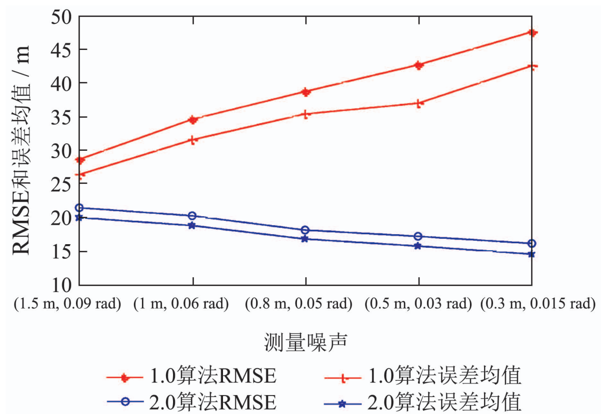

图 6 小噪声水平各算法RMSE和估计误差变化趋势

Fig. 6 Variation trend of RMSE and estimation errors of small noise level algorithms

从图6中可以看出, 在小噪声范畴内, 随着传感器探测噪声的减小, 1.0算法对从AUV位置估计的RMSE和估计误差均值逐渐增加, 而2.0算法逐渐减小. 说明相比于1.0算法, 改进后形成的2.0算法能充分适应传感器探测水平过高的情景.

从以上实验结果分析得到如下结论: 从AUV本体过程噪声水平保持不变, 当传感器的测量噪声明显减小后, SLATF1.0算法对于累积误差的抑制能力变低, 估计精度下降, 而SLATF2.0算法仍可以保持在一个较高的估计精度内. 说明将建议分布函数中融入传感器的当前观测值的SLATF2.0算法具备更高的估计精度和更稳定的估计性能.

8 结论

准确地描述广义环境并依托环境信息确定自身位姿是AUV真正实现水下自主航行的重要保证. 本文针对通讯长时间中断时协同定位中从AUV定位精度明显下降问题, 利用FastSLAM框架, 提出了基于探测式量测获取的同时定位与跟踪模型, 设计了2种SLATF

算法, 并进行了仿真实验分析. 本文证明或验证了如下结论:

1) 在给定某一从AUV运动路径的前提下, 多AUV同时定位与跟踪问题可以转化为这个从AUV和若干个彼此独立主AUV的状态估计问题.

2) 相比于传统的航位推算方法, 本文所提出的SLATF1.0和SLATF2.0算法能有效地提高从AUV的定位精度.

3) 相比于SLATF1.0算法在探测精度过高时估计性能出现严重下降的问题, 改进了粒子抽取规则的SLATF2.0算法对传感器精度变化具备更好的适应性.

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