具有混合时滞的四元数神经网络全局同步性控制

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摘要: 本文研究了具有混合时滞的四元数神经网络的全局同步性问题. 在不要求网络的激活函数可分离为2个复数函数或4个实数函数的情况下, 通过选取合适的Lyapunov-Krasovskii泛函, 并运用驱动–响应同步、自由权矩阵方法和矩阵不等式技巧, 给出了网络全局同步性的充要条件, 建立了同步控制器的设计方法. 给出的同步性判据是四元数和复数两种形式的线性矩阵不等式(LMI). 同时在注记中与已有的结果进行了对比. 最后, 通过一个数值仿真算例验证了理论分析的有效性.

关键词: 四元数神经网络; 混合时滞; Lyapunov-Krasovskii泛函; 线性矩阵不等式; 全局同步性

引用格式: 刘丽缤, 游星星, 高小平. 具有混合时滞的四元数神经网络全局同步性控制. 控制理论与应用, 2019, 36(8): 1360 – 1368

DOI: 10.7641/CTA.2019.80837

Global synchronization control of quaternion-valued neural networks with mixed delays

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Abstract: This paper focuses on the issue of global synchronization for the quaternion-valued neural networks (QVNNs) with mixed time-varying delays. In the case where the activation function of the network is not required to be separated into two complex parts or four real parts. Several new sufficient conditions for the proposed neural networks are established in terms of Lyapunov-Krasovskii functional approach, drive-response synchronization method, linear matrix inequality technique. Meanwhile the design method of the synchronous controller is established. The synchronization criteria of linear matrix inequalities are provided in the form of quaternion-valued and complex-valued linear matrix inequality (LMI). In the remark, the results are compared with the existing ones. Finally, a numerical simulation example is used to verify the validity of theoretical analysis.

Key words: quaternion-valued neural networks; mixed time-varying delays; Lyapunov-Krasovskii functional; linear matrix inequality; global synchronization

Citation: LIU Libin, YOU Xingxing, GAO Xiaoping. Global synchronization control of quaternion-valued neural networks with mixed delays. *Control Theory & Applications*, 2019, 36(8): 1360 – 1368

1 引言

近年来, 由于神经网络的学习功能、联想记忆功能、鲁棒特性等被广泛用于模式识别、联想记忆、优化计算等多方面[1–3], 因而有大量学者对实值神经网络(real-valued neural networks, RVNNs)、复值神经网络(complex-valued neural networks, CVNNs) 进行了深入地研究并得到许多研究成果[4–9]. 然而对于更多的应用, 比如有常用的彩色图像处理[10]、阵列信号处理[11]、风速预报[12]等, 实值神经网络、复值神经网络已经不能满足需求, 而四元数作为复数理论在某种意义上的扩展在这些方面发挥了极大的优势.

爱尔兰数学家威廉·卢云·哈密顿(William Rowan Hamilton)首次提出四元数概念并开始研究. 由于四元数乘法不符合交换律, 这使得对四元数的研究要比复

收稿日期: 2018−10−29; 录用日期: 2019−04−28.
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本文责任编委: 张化光.
重庆研究生教育创新基金项目(CYS18230)资助.
Supported by the Graduate Scientific Research and Innovation Foundation Project of Chongqing (CYS18230).

数困难得多, 这也是四元数发展缓慢的原因之一. 四元数神经网络(quaternion-valued neural networks, QVNNs)在高维数据应用领域具有复值与实值神经网络不可取代的优势[11–17], 因此对四元数域上神经网络的动态行为分析成为近年的一个研究热点. 文献[11–12]给出了四元数场上行列式的新定义, 从定义中得到了右(或左)线性方程的Cramer解和逆平方矩阵的存在条件以及在联想记忆中的应用. 文献[13]基于同胚映射定理和Lyapunov定理, 通过四元数模的不等式技术, 得到了平衡点存在性、唯一性和全局鲁棒稳定性的充分条件. 文献[14]利用Lyapunov稳定性理论和四元数矩阵理论, 研究了具有泄漏、离散和分布时滞的QVNNs的状态估计问题. 文献[15–16]采用类似于文献[14]的研究方法, 通过把QVNNs系统分解为两个CVNNs系统, 得到了一类具有时滞的QVNNs的平衡点全局μ稳定性的充分条件. 文献[17]研究了具有时间延迟的QVNNs的多稳态问题, 利用不等式技术, 提出了延迟QVNNs的有界性和全局吸引性的充分条件. 而同步作为一种物理现象, 普遍存在于自然界和实际系统中. 自1990年Pecora和Carollt提出驱动–响应同步概念以来, 学者们对神经网络在实数域、复数域上的同步性问题进行了广泛研究, 但对QVNNs同步性问题的研究尚不多见. 此外, 一部分学者采用将四元数的激活函数分解为2个复数函数或者4个实数函数的方法, 在网络的激活函数不能分解时, 其获得的结果是无效的.

基于以上分析, 本文在假设QVNNs的状态、连接权矩阵、激活函数以及输出都是定义在四元数域上, 并且不要求网络的激活函数可分解的情况下, 通过选择合适的Lyapunov-Krasovskii泛函, 研究了具有混合时滞QVNNs的全局同步性控制问题, 并给出了网络全局同步性的不等式判据. 相比已有文献, 本文的贡献主要体现在3方面:

1) 讨论了具有混合时滞的神经网络在四元数域上的全局同步性, 从而把神经网络的同步性从复数域上推广到四元数域上. 与已有研究[4,7]相比, 本文的不同之处在于QVNNs的状态、连接权矩阵、激活函数以及输出都是定义在四元数域上的, 并且本文考虑了分布时滞.

2) 与文献[15–16]相比, 本文不要求神经网络的激活函数分解为2个复数函数或4个实数函数, 而是考虑为一个整体. 从而解决了四元数乘法在应用中不可交换的难题, 并且利用不等式等技术给出了网络全局同步性问题的充分性判据, 保证了四元数所携带信息最小限度的丢失.

3) 将得到的四元数线性矩阵不等式(quaternion-valued linear matrix inequality, QVLMI)等效地转换为复数线性矩阵不等式(complex-valued linear matrix inequality, CVLMI), 从而能用数学软件MATLAB 直接、高效地求解.

本文的结构组织如下: 第2节给出了相关的预备知识, 包括四元数代数介绍、QVNNs模型及基本引理; 第3节给出了QVNNs同步性的充分性判据及推论; 在第4节中通过仿真例子验证了所得结论的有效性; 最后总结了论文所做的工作.

2　预备知识

2.1　四元数代数

四元数的定义是跟实数域相结合的, 一个四元数可以写成下面的形式:

$$q = q_0 + q_1\mathrm{i} + q_2\mathrm{j} + q_3\mathrm{k} \in \mathbb{Q},$$

其中: 系数$q_0, q_1, q_2, q_3 \in \mathbb{R}$, 并且虚数单位i, j, k满足条件 $\mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = \mathrm{ijk} = -1$, $\mathrm{ij} = -\mathrm{ji} = \mathrm{k}$, $\mathrm{jk} = -\mathrm{kj} = \mathrm{i}$, $\mathrm{ki} = -\mathrm{ik} = \mathrm{j}$, 从上面定义可知四元数乘法不可交换.

令2个四元数分别为$p = p_0 + p_1\mathrm{i} + p_2\mathrm{j} + p_3\mathrm{k}$, $q = q_0 + q_1\mathrm{i} + q_2\mathrm{j} + q_3\mathrm{k}$, 则它们的和定义为

$$\begin{aligned} p + q = &(p_0 + q_0) + (p_1 + q_1)\mathrm{i} + (p_2 + q_2)\mathrm{j} + \\ &(p_3 + q_3)\mathrm{k}. \end{aligned}$$

根据四元数的乘法规则, 定义p, q的乘积如下:

$$\begin{aligned} pq = &(p_0q_0 - p_1q_1 - p_2q_2 - p_3q_3) + \\ &(p_0q_1 + p_1q_0 + p_2q_3 - p_3q_2)\mathrm{i} + \\ &(p_0q_2 + p_2q_0 + p_3q_1 - p_1q_3)\mathrm{j} + \\ &(p_0q_3 + p_3q_0 + p_1q_2 - p_2q_1)\mathrm{k}. \end{aligned}$$

此外, 四元数$q = q_0 + q_1\mathrm{i} + q_2\mathrm{j} + q_3\mathrm{k}$的共轭转置用$q^*$或$\bar{q}$表示, 定义如下:

$$q^* = \bar{q} = q_0 - q_1\mathrm{i} - q_2\mathrm{j} - q_3\mathrm{k}.$$

q的模用$|q|$表示, 定义如下:

$$|q| = \sqrt{qq^*} = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}.$$

令$\mu = [\mu_1\ \ \mu_2\ \ \cdots\ \ \mu_n]^{\mathrm{T}} \in \mathbb{Q}^n$, 则用$|\mu| = [|\mu_1|\ \ |\mu_2|\ \ \cdots\ \ |\mu_n|]^{\mathrm{T}}$表示$\mu$的模, $\|\mu\| = \sqrt{\sum_{i=1}^{n} |\mu_i|^2}$表示$\mu$的范数. 如果一个矩阵$A \in \mathbb{Q}^{n\times n}$, 存在另一个矩阵$B \in \mathbb{Q}^{n\times n}$, 满足$AB = BA = I$, 则矩阵$A$是可逆的; 矩阵$A \in \mathbb{Q}^{n\times n}$满足$A = A^*$, 那么$A$矩阵是一个Hermitian矩阵; 定义$q \in \mathbb{Q}^n$为任意非零向量, A为Hermitian矩阵, 满足$q^*Aq > 0$, 则矩阵A正定.

2.2　模型表示及基本引理

本文考虑如下具有混合时滞的QVNNs模型:

$$\begin{aligned} \dot{x}(t) = &-Dx(t-\delta) + Ag(x(t)) + \\ &Bg(x(t-\tau_1(t))) + \\ &C\int_{t-\tau_2(t)}^{t} g(x(s))\mathrm{d}s + J,\ t \geqslant 0, \end{aligned} \tag{1}$$

式中: $x(t)=[x_1(t)\ \ x_2(t)\ \ \cdots\ \ x_n(t)]^{\mathrm{T}}\in\mathbb{Q}^n$表示神经元状态向量; $D=\mathrm{diag}\{d_1,d_2,\cdots,d_n\}\in\mathbb{R}^{n\times n}$且$d_j>0,j=1,2,\cdots,n$, 表示自反馈连接权矩阵; $A=(a_{ij})_{n\times n}\in\mathbb{Q}^{n\times n}$, $B=(b_{ij})_{n\times n}\in\mathbb{Q}^{n\times n}$和$C=(c_{ij})_{n\times n}\in\mathbb{Q}^{n\times n},i,j=1,2,\cdots,n$, 分别为连接权矩阵, 离散时变时滞连接权矩阵和分布时变时滞连接权矩阵; $J=[J_1\ \ J_2\ \ \cdots\ \ J_n]^{\mathrm{T}}\in\mathbb{Q}^n$表示外部输入常数向量; 泄漏时滞$\delta\geqslant 0$为常量; $\tau_1(t)$和$\tau_2(t)$分别表示离散时变时滞和分布时变时滞;

$$g(x(t))=[g_1(x_1(t))\ \ g_2(x_2(t))\ \ \cdots\ \ g_n(x_n(t))]^{\mathrm{T}}\in\mathbb{Q}^n$$

为向量值激活函数. 模型(1)的初始条件为

$$x(s)=\varphi(s),\ s\in[-\tau,0],$$

式中: $\tau=\max\{\tau_1(t),\tau_2(t),\delta\}$, $\varphi(s)$在$[-\tau,0]$有界且连续.

令模型(1)为驱动QVNNs, 则响应QVNNs如下:

$$\begin{aligned}\dot{y}(t)=&-Dy(t-\delta)+Ag(y(t))+\\&Bg(y(t-\tau_1(t)))+\\&C\int_{t-\tau_2(t)}^{t}g(y(s))\mathrm{d}s+J+\mu(t),\ t\geqslant 0.\end{aligned}\tag{2}$$

模型(2)的初始条件为

$$y(s)=\psi(s),\ s\in[-\tau,0],$$

式中: $\tau=\max\{\tau_1(t),\tau_2(t),\delta\}$, $\psi(s)$在$[-\tau,0]$有界且连续. $u(t)$是设计合理的控制器.

令$e(t)=x(t)-y(t)$, $f(e(t))=g(x(t))-g(y(t))$, 控制器为$\mu(t)=-K_1e(t)+K_2e(t-\delta)$, 其中$K_i\in\mathbb{Q}^{n\times n},i=1,2$, 待确定. 由模型(1)和模型(2)可以得到误差QVNNs模型, 模型如下:

$$\begin{aligned}\dot{e}(t)=&K_1e(t)-(D+K_2)e(t-\delta)+\\&Af(e(t))+Bf(e(t-\tau_1(t)))+\\&C\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s,\ t\geqslant 0,\end{aligned}\tag{3}$$

其中初始条件为

$$e(s)=\varPsi(s)=\varphi(s)-\psi(s),\ s\in[-\tau,0],$$

式中: $\tau=\max\{\tau_1(t),\tau_2(t),\delta\}$, $\varPsi(s)$在$[-\tau,0]$有界且连续.

本文给出假设如下:

假设 1 激活函数$f_i(\cdot)$是连续的, 并且对任意$\alpha_1,\alpha_2\in\mathbb{Q}$, 存在$\gamma_i\in\mathbb{R},i=1,2,\cdots,n$, 有

$$|f_i(\alpha_1)-f_i(\alpha_2)|\leqslant\gamma_i|\alpha_1-\alpha_2|,$$

令$\varGamma=\mathrm{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_n\}$.

假设 2 离散时变时滞函数$\tau_1(t)$是非负连续可微的, 并且满足$\dot{\tau}_1(t)\leqslant\varepsilon\leqslant 1$, ε是一个正常数.

定义 1 如果误差QVNNs(3)是全局稳定的, 那么驱动QVNNs(1)和响应QVNNs(2)是全局同步的.

定义 2 对于模型(3)中的任意解$e(t)=[e_1(t)\ \ e_2(t)\ \ \cdots\ \ e_n(t)]^{\mathrm{T}}$, 如果存在常数$M\geqslant 0$, 使得

$$\|e(t)\|\leqslant M\sup_{s\in[-\tau,0]}\|\varphi(s)-\psi(s)\|$$

成立, 那么就说QVNNs(3)是全局稳定的.

引理 1[13–14] 令$x,y\in\mathbb{Q}$; $A,B,P\in\mathbb{Q}^{n\times n}$; P是一个正定的Hermitian矩阵, 则

1) $|x+y|\leqslant|x|+|y|,\ |xy|=|x||y|$;

2) $(AB)^*=B^*A^*$;

3) $(AB)^{-1}=B^{-1}A^{-1}$, 其中A,B是可逆矩阵;

4) $(A^*)^{-1}=(A^{-1})^*$, 其中A是可逆矩阵;

5) 任意四元数q可以唯一表示成$q=c_1+c_2\times\mathrm{j}$; $c_1,c_2\in\mathbb{C}$;

6) 任意一个矩阵$C\in\mathbb{C}^{n\times n}$, 满足$\mathrm{j}C=\bar{C}\mathrm{j}$或$\mathrm{j}C\mathrm{j}^*=\bar{C}$;

7) 存在可逆矩阵$Q\in\mathbb{Q}^{n\times n}$, 使得$P=Q^*Q$.

引理 2[13–14] 令$A=A_1+A_2\mathrm{j}$, $B=B_1+B_2\mathrm{j}$, $A_1,A_2,B_1,B_2\in\mathbb{C}^{n\times n}$, 则有

1) $A^*=A_1^*-A_2^{\mathrm{T}}\mathrm{j}$;

2) $AB=(A_1B_1-A_2\bar{B}_2)+(A_1B_2+A_2\bar{B}_1)\mathrm{j}$.

引理 3[13–14] 对正定的Hermitian常数矩阵$W\in\mathbb{Q}^{n\times n}$, $W\geqslant 0$和标量函数$w:[a,b]\rightarrow\mathbb{Q}^n,\ a\leqslant b$, 有

$$\begin{aligned}&(\int_a^b w(s)\mathrm{d}s)^*W(\int_a^b w(s)\mathrm{d}s)\leqslant\\&(b-a)\int_a^b w^*(s)Ww(s)\mathrm{d}s.\end{aligned}$$

注 1 本文中“T”, “$*$”和“$\lozenge$”分别表示矩阵的转置、矩阵的共轭转置和斜对称矩阵块的负转置; 对于矩阵$A\geqslant B(A>B)$表示$A-B$是半正定的(正定的); $\lambda_{\min}(A)$和$\lambda_{\max}(A)$分别表示矩阵A的最小特征值和最大特征值.

3 主要结果

定理 1 在假设1与假设2成立的条件下, 如果存在正定Hermitian矩阵$P_1,P_2,P_3,P_4,P_5\in\mathbb{Q}^{n\times n}$, 四元数矩阵$Q,S,T$, 正定对角阵$G_1\in\mathbb{R}^{n\times n}$, 使得不等式

$$\begin{pmatrix}\varOmega_{11}&\varOmega_{12}&\varOmega_{13}&D^*P_1D&QA&QB&QC\\ *&\varOmega_{22}&\varOmega_{23}&-P_1D&QA&QB&QC\\ *&*&\varOmega_{33}&-D^*P_1D&QA&QB&QC\\ *&*&*&-P_3&0&0&0\\ *&*&*&*&\varOmega_{55}&0&0\\ *&*&*&*&*&\varOmega_{66}&0\\ *&*&*&*&*&*&-\dfrac{P_5}{\tau}\end{pmatrix}<0\tag{4}$$

成立, 其中:

$$\Omega_{11}=-D^*P_1-P_1D+P_2+\delta^2P_3+S+S^*+\Gamma^*G_1\Gamma,$$
$$\Omega_{12}=S^*-Q+P_1,$$
$$\Omega_{13}=-QD-T+S^*+P_1D,$$
$$\Omega_{22}=-Q-Q^*,\ \Omega_{23}=-QD-T-Q^*,$$
$$\Omega_{33}=-QD-T-D^*Q^*-T^*-P_2,$$
$$\Omega_{55}=P_4+\tau P_5-G_1,\ \Omega_{66}=-(1-\varepsilon)P_4,$$

且控制器系数被设计为

$$K_1=Q^{-1}S,\ K_2=Q^{-1}T,$$

则QVNNs(1)和QVNNs(2)是全局同步的.

证　构造Lyapunov-Krasovskii泛函

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t), \tag{5}$$

其中:

$$V_1(t)=(e(t)-D\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*P_1\times(e(t)-D\int_{t-\delta}^{t}e(s)\mathrm{d}s),$$
$$V_2(t)=\int_{t-\delta}^{t}e^*(s)P_2e(s)\mathrm{d}s+\delta\int_0^{\delta}\int_{t-\mu}^{t}e^*(s)P_3e(s)\mathrm{d}s\mathrm{d}u,$$
$$V_3(t)=\int_{t-\tau_1(t)}^{t}f^*(e(s))P_4f(e(s))\mathrm{d}s,$$
$$V_4(t)=\int_{-\tau}^{0}\int_{t+s}^{t}f^*(e(y))P_5f(e(y))\mathrm{d}y\mathrm{d}s.$$

计算$V(t)$沿着模型(3)的导数, 由引理3可得

$$\begin{aligned}\dot V_1(t)=&(\dot e(t)-De(t)+De(t-\delta))^*\times P_1(e(t)-D\int_{t-\delta}^{t}e(s)\mathrm{d}s)+\\&(e(t)-D\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*P_1\times(\dot e(t)-De(t)+De(t-\delta))=\\&\dot e^*(t)P_1e(t)+e^*(t)P_1\dot e(t)+\\&e^*(t)(-D^*P_1-P_1D)e(t)+\\&e^*(t-\delta)D^*P_1e(t)+\\&e^*(t)P_1De(t-\delta)+\\&e^*(t)D^*P_1D(\int_{t-\delta}^{t}e(s)\mathrm{d}s)+\\&(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*D^*P_1De(t)+\\&\dot e^*(t)(-P_1D)(\int_{t-\delta}^{t}e(s)\mathrm{d}s)+\\&(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*(-D^*P_1)\dot e(t)+\\&e^*(t-\delta)(-D^*P_1D)(\int_{t-\delta}^{t}e(s)\mathrm{d}s)+\\&(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*(-D^*P_1D)e(t-\delta),\end{aligned}\tag{6}$$

$$\begin{aligned}\dot V_2(t)=&e^*(t)P_2e(t)-e^*(t-\delta)P_2\times e(t-\delta)+\delta\int_0^{\delta}\big[e^*(t)P_3e(t)-e^*(t-u)P_3e(t-u)\big]\mathrm{d}u=\\&e^*(t)(P_2+\delta^2P_3)e(t)-\\&e^*(t-\delta)P_2e(t-\delta)-\\&\delta\int_{t-\delta}^{t}e^*(s)P_3e(s)\mathrm{d}s\leqslant\\&e^*(t)(P_2+\delta^2P_3)e(t)+e^*(t-\delta)\times(-P_2)e(t-\delta)+(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*\times(-P_3)(\int_{t-\delta}^{t}e(s)\mathrm{d}s),\end{aligned}\tag{7}$$

$$\begin{aligned}\dot V_3(t)=&f^*(e(t))P_4f(e(t))+f^*(e(t-\tau_1(t)))\times(-(1-\dot\tau(t))P_4)f(e(t-\tau_1(t)))\leqslant\\&f^*(e(t))P_4f(e(t))+f^*(e(t-\tau_1(t)))\times(-(1-\varepsilon)P_4)f(e(t-\tau_1(t))),\end{aligned}\tag{8}$$

$$\begin{aligned}\dot V_4(t)=&\int_{-\tau}^{0}f^*(e(t))P_5f(e(t))\mathrm{d}s-\int_{-\tau}^{0}f^*(e(t+s))P_5f(e(t+s))\mathrm{d}s\leqslant\\&\tau f^*(e(t))P_5f(e(t))+\int_{t-\tau_2(t)}^{t}f^*(e(s))(-P_5)f(e(s))\mathrm{d}s\leqslant\\&f^*(e(t))\tau P_5f(e(t))+(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)^*(-\frac{P_5}{\tau})\times(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s).\end{aligned}\tag{9}$$

于是, 将式(6)–(9)相加, 并由假设2可得

$$\begin{aligned}\dot V(t)\leqslant&e^*(t)(-D^*P_1-P_1D+P_2+\delta^2P_3)e(t)+\\&\dot e^*(t)P_1e(t)+e^*(t)P_1\dot e(t)+\\&e^*(t-\delta)D^*P_1e(t)+e^*(t)P_1De(t-\delta)+\\&e^*(t-\delta)(-P_2)e(t-\delta)+\\&f^*(e(t))(P_4+\tau P_5)f(e(t))+\\&f^*(e(t-\tau_1(t)))(-(1-\varepsilon)P_4)\times f(e(t-\tau_1(t)))+\\&\dot e^*(t)(-P_1D)(\int_{t-\delta}^{t}e(s)\mathrm{d}s)+\\&(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*(-D^*P_1)\dot e(t)+\\&e^*(t)D^*P_1D(\int_{t-\delta}^{t}e(s)\mathrm{d}s)+\\&(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*D^*P_1De(t)+\end{aligned}$$

$$e^*(t-\delta)(-D^*P_1D)(\int_{t-\delta}^{t}e(s)\mathrm{d}s)+$$
$$(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*(-D^*P_1D)e(t-\delta)+$$
$$(\int_{t-\delta}^{t}e(s)\mathrm{d}s)^*(-P_3)(\int_{t-\delta}^{t}e(s)\mathrm{d}s)+$$
$$(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)^*(-\frac{P_5}{\tau})\times$$
$$(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s). \tag{10}$$

通过采用自由权矩阵得到

$$0=(Q^*e(t)+Q^*e(t-\delta)+Q^*\dot e(t))^*H+$$
$$H^*(Q^*e(t)+Q^*e(t-\delta)+Q^*\dot e(t)),$$

其中由式(3)可知

$$H=-\dot e(t)+K_1e(t)-(D+K_2)e(t-\delta)+$$
$$Af(e(t))+Bf(e(t-\tau_1(t)))+$$
$$C\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s.$$

因此得到

$$0=e^*(t)(K_1^*Q^*-Q)\dot e(t)+\dot e^*(t)(QK_1-Q^*)\times$$
$$e(t)+e^*(t)(QK_1+K_1^*Q^*)e(t)+$$
$$e^*(t)(-Q(D+K_2)+K_1^*Q^*)e(t-\delta)+$$
$$e^*(t-\delta)(QK_1-(D+K_2)^*Q^*)e(t)+$$
$$e^*(t-\delta)(-Q-(D+K_2)^*Q^*)\dot e(t)+$$
$$\dot e^*(t)(-Q(D+K_2)-Q^*)e(t-\delta)+$$
$$e^*(t-\delta)(-Q(D+K_2)-(D+K_2)^*Q^*)\times$$
$$e(t-\delta)+\dot e^*(t)(-Q-Q^*)\dot e(t)+e^*(t)QA\times$$
$$f(e(t))+f^*(e(t))A^*Q^*e(t)+e^*(t-\delta)\times$$
$$QAf(e(t))+f^*(e(t))A^*Q^*e(t-\delta)+$$
$$\dot e^*(t)QAf(e(t))+f^*(e(t))A^*Q^*\dot e(t)+$$
$$e^*(t)QBf(e(t-\tau_1(t)))+$$
$$f^*(e(t-\tau_1(t)))B^*Q^*e(t)+$$
$$e^*(t-\delta)QBf(e(t-\tau_1(t)))+$$
$$f^*(e(t-\tau_1(t)))B^*Q^*e(t-\delta)+$$
$$\dot e^*(t)QBf(e(t-\tau_1(t)))+$$
$$f^*(e(t-\tau_1(t)))B^*Q^*\dot e(t)+$$
$$e^*(t)QC(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)+$$
$$(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)^*C^*Q^*e(t)+$$
$$e^*(t-\delta)QC(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)+$$
$$(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)^*C^*Q^*e(t-\delta)+$$
$$\dot e^*(t)QC(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)+$$
$$(\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)^*C^*Q^*\dot e(t). \tag{11}$$

由假设1得

$$0\leqslant e^*(t)\varGamma^*G_1\varGamma e(t)-f^*(e(t))G_1f(e(t)). \tag{12}$$

将式(10)–(12)相加可得

$$\dot V(t)\leqslant W^*(t)\varOmega W(t), \tag{13}$$

其中:

$$W(t)=(e(t),\dot e(t),e(t-\delta),\int_{t-\delta}^{t}e(s)\mathrm{d}s,f(e(t)),$$
$$f(e(t-\tau_1(t))),\int_{t-\tau_2(t)}^{t}f(e(s))\mathrm{d}s)^*,$$

$$\varOmega=\begin{pmatrix}\varOmega_{11}&\varOmega_{12}&\varOmega_{13}&D^*P_1D&QA&QB&QC\\ *&\varOmega_{22}&\varOmega_{23}&-P_1D&QA&QB&QC\\ *&*&\varOmega_{33}&-D^*P_1D&QA&QB&QC\\ *&*&*&-P_3&0&0&0\\ *&*&*&*&\varOmega_{55}&0&0\\ *&*&*&*&*&\varOmega_{66}&0\\ *&*&*&*&*&*&-\dfrac{P_5}{\tau}\end{pmatrix},$$

$$S=QK_1,\ T=QK_2,$$

其中:

$$\varOmega_{11}=-D^*P_1-P_1D+P_2+\delta^2P_3+$$
$$S+S^*+\varGamma^*G_1\varGamma,$$
$$\varOmega_{12}=S^*-Q+P_1,$$
$$\varOmega_{13}=-QD-T+S^*+P_1D,$$
$$\varOmega_{22}=-Q-Q^*,\ \varOmega_{23}=-QD-T-Q^*,$$
$$\varOmega_{33}=-QD-T-D^*Q^*-T^*-P_2,$$
$$\varOmega_{55}=P_4+\tau P_5-G_1,\ \varOmega_{66}=-(1-\varepsilon)P_4.$$

所以由式(4)和式(13)得

$$\dot V(t)\leqslant 0,\ t\geqslant 0. \tag{14}$$

从而可知$V(t)$在$t\geqslant 0$上是单调不增函数, 于是有

$$V(t)\leqslant V(0)=(e(0)-D\int_{-\delta}^{0}e(s)\mathrm{d}s)^*\times$$
$$P_1(e(0)-D\int_{-\delta}^{0}e(s)\mathrm{d}s)+$$
$$\int_{-\delta}^{0}e^*(s)P_2e(s)\mathrm{d}s+$$
$$\delta\int_{0}^{\delta}\int_{-\mu}^{0}e^*(s)P_3e(s)\mathrm{d}u\mathrm{d}s+$$
$$\int_{-\tau_1(0)}^{0}f^*(e(s))P_4f(e(s))\mathrm{d}s+$$
$$\int_{-\tau}^{0}\int_{s}^{0}f^*(e(y))P_5f(e(y))\mathrm{d}y\mathrm{d}s\leqslant$$
$$(\|P_1\|+2\delta\|P_1D\|+\delta^2\|D^*P_1D\|+$$

$$\delta\|P_2\|+\delta^3\|P_3\|+\tau\|P_4\|\max_{1\leqslant i\leqslant n}\{\gamma_i^2\}+\tau^2\|P_5\|\max_{1\leqslant i\leqslant n}\{\gamma_i^2\})(\sup_{s\in[-\tau,0]}\|\Psi(s)\|)^2=M_1(\sup_{s\in[-\tau,0]}\|\Psi(s)\|)^2, \tag{15}$$

其中

$$M_1=\|P_1\|+2\delta\|P_1D\|+\delta^2\|D^*P_1D\|+\delta\|P_2\|+\delta^3\|P_3\|+\tau\|P_4\|\max_{1\leqslant i\leqslant n}\{\gamma_i^2\}+\tau^2\|P_5\|\max_{1\leqslant i\leqslant n}\{\gamma_i^2\}.$$

又因为

$$V(t)\geqslant V_1(t)\geqslant\lambda_{\max}(P_1)\|e(t)-D\int_{t-\delta}^t e(s)\mathrm{d}s\|^2, \tag{16}$$

所以

$$\|e(t)\|\leqslant\|D\|\int_{t-\delta}^t\|e(s)\|\mathrm{d}s+\frac{\sqrt{M_1}}{\sqrt{\lambda_{\max}(P_1)}}\sup_{s\in[-\tau,0]}\|\Psi(s)\|. \tag{17}$$

由Gronwall不等式

$$\|e(t)\|\leqslant\frac{\sqrt{M_1}}{\sqrt{\lambda_{\max}(P_1)}}\sup_{s\in[-\tau,0]}\|\Psi(s)\|\mathrm{e}^{\delta\|D\|},$$

即

$$\|e(t)\|\leqslant M\sup_{s\in[-\tau,0]}\|\Psi(s)\|, \tag{18}$$

其中

$$M=\frac{\sqrt{M_1}}{\sqrt{\lambda_{\max}(P_1)}}\mathrm{e}^{\delta\|D\|}.$$

故由定义1和定义2知QVNNs(1)和QVNNs(2)是全局同步的.　　证毕.

注 2　神经网络的同步性问题得到了学者的广泛研究, 文献[4, 7]虽然也研究了同步性问题, 但其研究的模型是在复数域上的, 并且还未考虑分布时滞, 当本文的模型中分布时滞项参数为零时, 可以考虑为将文献[4]中模型从复数域到四元数域上的推广. 其次, 具有时滞的QVNNs模型也在文献[13–16]中得到研究, 文献[13]针对模型参数不确定情况研究了具有泄漏时滞和离散常数时滞QVNNs的鲁棒稳定性问题, 但没有考虑模型在离散时变时滞和分布时变时滞下的情况. 文献[14]虽然包含了泄漏、离散、分布时滞, 与本文的研究方向不同, 该文章研究的是QVNNs的状态估计问题. 文献[15–16]通过把QVNNs模型分解为两个CVNNs模型, 利用不等式等技术给出了若干个全局稳定充分判据. 但这种方法首先会使得模型的维数成倍增加, 可能会导致计算的复杂性; 其次, 四元数信号携带了信号的振幅信息和相位信息, 分解后可能会丢失一些与之关联的信息. 因此, 本文把QVNNs作为一个整体考虑研究了它的同步性问题, 在一定程度上扩展和改进了文献[4, 15–16]的结果.

注 3　研究QVNNs比RVNNs和CVNNs更复杂有以下原因: 一是四元数由4个部分组成(1个实部和3个虚部), 这表明四元数的结构比复数更复杂; 另一个原因是四元数乘法不满足交换律. 因此, 一种有效的处理方法是将QVLMI分解为CVLMI, 但它会增加LMI的维度.

由于LMI(4)是定义在四元数域上的, 该结果不能通过MATLAB的LMI工具箱直接处理. 本文考虑将QVLMI转换成CVLMI. 基于引理1和引理2, 参数A, B, C, K_1, K_2, Q, S, T表示复数对, 即

$$A=A_1+A_2\mathrm{j},\ B=B_1+B_2\mathrm{j},\ C=C_1+C_2\mathrm{j},$$
$$Q=Q_1+Q_2\mathrm{j},\ S=S_1+S_2\mathrm{j},\ T=T_1+T_2\mathrm{j},$$

并且$A_1, A_2, B_1, B_2, C_1, C_2, S_1, S_2, T_1, T_2\in\mathbb{C}^{n\times n}$, 使得$\Omega$被分解为$\Omega_1$和$\Omega_2$, 避免了模型维数的增加.

推论 1　在假设1–2成立的条件下, 如果存在正定的Hermitian矩阵$P_{11}, P_{21}, P_{31}, P_{41}, P_{51}\in\mathbb{C}^{n\times n}$, 斜对称矩阵$P_{12}, P_{22}, P_{32}, P_{42}, P_{52}\in\mathbb{C}^{n\times n}$, 正定对角阵$G_1\in\mathbb{R}^{n\times n}$, 复值矩阵$Q_1, Q_2, S_1, S_2, T_1, T_2$, 使得不等式

$$\begin{pmatrix}P_{11} & -P_{12}\\ \bar{P}_{12} & \bar{P}_{11}\end{pmatrix}>0,\ \begin{pmatrix}P_{21} & -P_{22}\\ \bar{P}_{22} & \bar{P}_{21}\end{pmatrix}>0, \tag{19}$$

$$\begin{pmatrix}P_{31} & -P_{32}\\ \bar{P}_{32} & \bar{P}_{31}\end{pmatrix}>0,\ \begin{pmatrix}P_{41} & -P_{42}\\ \bar{P}_{42} & \bar{P}_{41}\end{pmatrix}>0, \tag{20}$$

$$\begin{pmatrix}P_{51} & -P_{52}\\ \bar{P}_{52} & \bar{P}_{51}\end{pmatrix}>0,\ \begin{pmatrix}\Omega_1 & -\Omega_2\\ \bar{\Omega}_2 & \bar{\Omega}_1\end{pmatrix}<0 \tag{21}$$

成立, 其中:

$$\Omega_1=\begin{pmatrix}
\Omega_{11}^{(1)} & \Omega_{12}^{(1)} & \Omega_{13}^{(1)} & D^*P_{11}D & \Omega_{15}^{(1)} & \Omega_{16}^{(1)} & \Omega_{17}^{(1)}\\
* & \Omega_{22}^{(1)} & \Omega_{23}^{(1)} & -P_{11}D & \Omega_{25}^{(1)} & \Omega_{26}^{(1)} & \Omega_{27}^{(1)}\\
* & * & \Omega_{33}^{(1)} & -D^*P_{11}D & \Omega_{35}^{(1)} & \Omega_{36}^{(1)} & \Omega_{37}^{(1)}\\
* & * & * & -P_{31} & 0 & 0 & 0\\
* & * & * & * & \Omega_{55}^{(1)} & 0 & 0\\
* & * & * & * & * & \Omega_{66}^{(1)} & 0\\
* & * & * & * & * & * & -\dfrac{P_{51}}{\tau}
\end{pmatrix},$$

$$\Omega_2=\begin{pmatrix}
\Omega_{11}^{(2)} & \Omega_{12}^{(2)} & \Omega_{13}^{(2)} & D^*P_{12}D & \Omega_{15}^{(2)} & \Omega_{16}^{(2)} & \Omega_{17}^{(2)}\\
\Diamond & \Omega_{22}^{(2)} & \Omega_{23}^{(2)} & -P_{12}D & \Omega_{25}^{(2)} & \Omega_{26}^{(2)} & \Omega_{27}^{(2)}\\
\Diamond & \Diamond & \Omega_{33}^{(2)} & -D^*P_{12}D & \Omega_{35}^{(2)} & \Omega_{36}^{(2)} & \Omega_{37}^{(2)}\\
\Diamond & \Diamond & \Diamond & -P_{32} & 0 & 0 & 0\\
\Diamond & \Diamond & \Diamond & \Diamond & \Omega_{55}^{(2)} & 0 & 0\\
\Diamond & \Diamond & \Diamond & \Diamond & \Diamond & \Omega_{66}^{(2)} & 0\\
\Diamond & \Diamond & \Diamond & \Diamond & \Diamond & \Diamond & -\dfrac{P_{52}}{\tau}
\end{pmatrix},$$

其中:

$$\Omega_{11}^{(1)} = -D^*P_{11} - P_{11}D + P_{21} + \delta^2 P_{31} + S_1 + S_1^* + \Gamma^* G_1 \Gamma,$$
$$\Omega_{11}^{(2)} = -D^*P_{12} - P_{12}D + P_{22} + \delta^2 P_{32} + S_2 - S_2^{\mathrm{T}},$$
$$\Omega_{12}^{(1)} = S_1^* - Q_1 + P_{11},$$
$$\Omega_{12}^{(2)} = -S_2^{\mathrm{T}} - Q_2 + P_{12},$$
$$\Omega_{13}^{(1)} = -Q_1 D - T_1 + S_1^* + P_{11}D,$$
$$\Omega_{13}^{(2)} = -Q_2 D - T_2 - S_2^{\mathrm{T}} + P_{12}D,$$
$$\Omega_{15}^{(1)} = \Omega_{25}^{(1)} = \Omega_{35}^{(1)} = Q_1 A_1 - Q_2 \bar{A}_2,$$
$$\Omega_{15}^{(2)} = \Omega_{25}^{(2)} = \Omega_{35}^{(2)} = Q_1 A_2 + Q_2 \bar{A}_1,$$
$$\Omega_{16}^{(1)} = \Omega_{26}^{(1)} = \Omega_{36}^{(1)} = Q_1 B_1 - Q_2 \bar{B}_2,$$
$$\Omega_{16}^{(2)} = \Omega_{26}^{(2)} = \Omega_{36}^{(2)} = Q_1 B_2 + Q_2 \bar{B}_1,$$
$$\Omega_{17}^{(1)} = \Omega_{27}^{(1)} = \Omega_{37}^{(1)} = Q_1 C_1 - Q_2 \bar{C}_2,$$
$$\Omega_{17}^{(2)} = \Omega_{27}^{(2)} = \Omega_{37}^{(2)} = Q_1 C_2 + Q_2 \bar{C}_1,$$
$$\Omega_{22}^{(1)} = -Q_1 - Q_1^*,\ \Omega_{22}^{(2)} = -Q_2 + Q_2^{\mathrm{T}},$$
$$\Omega_{23}^{(1)} = -Q_1 D - T_1 - Q_1^*,$$
$$\Omega_{23}^{(2)} = -Q_2 D - T_2 + Q_2^{\mathrm{T}},$$
$$\Omega_{33}^{(1)} = -Q_1 D - T_1 - D^* Q_1^* - T_1^* - P_{21},$$
$$\Omega_{33}^{(2)} = -Q_2 D - T_2 + D^* Q_2^{\mathrm{T}} + T_2^{\mathrm{T}} - P_{22},$$
$$\Omega_{55}^{(1)} = P_{41} + \tau P_{51} - G_1,\ \Omega_{55}^{(2)} = P_{42} + \tau P_{52},$$
$$\Omega_{66}^{(1)} = -(1-\varepsilon)P_{41},\ \Omega_{66}^{(2)} = -(1-\varepsilon)P_{42}.$$

控制器系数被设计为

$$K_1 = Q^{-1}S,\ K_2 = Q^{-1}T,\ Q = Q_1 + Q_2\mathrm{j},$$
$$S = S_1 + S_2\mathrm{j},\ T = T_1 + T_2\mathrm{j},$$

则QVNNs(1)和QVNNs(2)是全局同步的.

4 数值仿真实例

考虑如下具有两个神经元的QVNNs模型:

$$\dot{x}(t) = -Dx(t-\delta) + Ag(x(t)) + Bg(x(t-\tau_1(t))) + C\int_{t-\tau_2(t)}^{t} g(x(s))\mathrm{d}s + J,\ t \geqslant 0, \tag{22}$$

其中:

$D = \mathrm{diag}\{3.6910, 1.2339\},\ A = (a_{ij})_{2\times 2},$
$B = (b_{ij})_{2\times 2},\ C = (c_{ij})_{2\times 2},\ J = [0\ \ 0]^{\mathrm{T}},\ \delta = 0.2,$
$\tau_1(t) = 0.1 + 0.1\cos(4t),\ \tau_2(t) = 0.1|\sin(7t)|,$
$g_1(x(t)) = g_2(x(t)) = 0.2\tanh(x(t)),\ \forall x(t) \in \mathbb{Q}^n,$
$a_{11} = -1.2171 + 1.8709\mathrm{i} - 0.4868\mathrm{j} + 0.7484\mathrm{k},$
$a_{12} = 0.4057 + 0.8205\mathrm{i} + 0.1623\mathrm{j} + 0.3282\mathrm{k},$
$a_{21} = -0.4057 + 1.8338\mathrm{i} - 0.1623\mathrm{j} + 0.7335\mathrm{k},$
$a_{22} = 1.6228 + 1.7873\mathrm{i} + 0.6491\mathrm{j} + 0.7149\mathrm{k},$
$b_{11} = -0.0347 + 0.7057\mathrm{i} - 0.0278\mathrm{j} + 0.5646\mathrm{k},$
$b_{12} = 0.0695 + 0.0197\mathrm{i} + 0.0556\mathrm{j} + 0.0158\mathrm{k},$
$b_{21} = 0.0232 + 1.6263\mathrm{i} + 0.0185\mathrm{j} + 1.3011\mathrm{k},$
$b_{22} = -0.0463 + 0.2778\mathrm{i} - 0.0371\mathrm{j} + 0.2222\mathrm{k},$
$c_{11} = -0.2468 + 0.3794\mathrm{i} - 0.0987\mathrm{j} + 0.1517\mathrm{k},$
$c_{12} = 0.0823 + 0.1664\mathrm{i} + 0.0329\mathrm{j} + 0.0666\mathrm{k},$
$c_{21} = -0.0823 + 0.3718\mathrm{i} - 0.0329\mathrm{j} + 0.1487\mathrm{k},$
$c_{22} = 0.3291 + 0.3624\mathrm{i} + 0.1316\mathrm{j} + 0.1450\mathrm{k}.$

显然A的复数部分如下:

$$A_1 = \begin{bmatrix} -1.2171 + 1.8709\mathrm{i} & 0.4057 + 0.8205\mathrm{i} \\ 0.4057 + 0.8205\mathrm{i} & 1.6228 + 1.7873\mathrm{i} \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.4868 + 0.7484\mathrm{i} & 0.1623 + 0.3282\mathrm{i} \\ -0.1623 + 0.7335\mathrm{i} & 0.6491 + 0.7149\mathrm{i} \end{bmatrix}.$$

由于篇幅限制B, C的复数部分就不一一列举了. 响应QVNNs模型如下:

$$\dot{y}(t) = -Dy(t-\delta) + Ag(y(t)) + Bg(y(t-\tau_1(t))) + C \times \int_{t-\tau_2(t)}^{t} g(y(s))\mathrm{d}s + J + \mu(t),\ t \geqslant 0, \tag{23}$$

令$e(t) = x(t) - y(t), f(e(t)) = g(x(t)) - g(y(t))$, 控制器为$\mu(t) = -K_1 e(t) + K_2 e(t-\delta)$. 由模型(22)和模型(23)可以得到误差QVNNs模型, 模型如下:

$$\dot{e}(t) = K_1 e(t) - (D + K_2)e(t-\delta) + Af(e(t)) + Bf(e(t-\tau_1(t))) + C\int_{t-\tau_2(t)}^{t} f(e(s))\mathrm{d}s,\ t \geqslant 0. \tag{24}$$

根据此定义, 当$\Gamma = \mathrm{diag}\{0.2, 0.2\},\ \tau = 0.2,\ \varepsilon = 0.4$时, 满足假设1–2. 此外, 运用MATLAB工具箱YALMIP, 求解LMIs(19)–(21)可得

$$P_{11} = \begin{bmatrix} 8.6070 + 0.0000\mathrm{i} & -0.2140 - 0.0552\mathrm{i} \\ -0.2141 + 0.0552\mathrm{i} & 18.3868 + 0.0000\mathrm{i} \end{bmatrix},$$
$$P_{12} = \begin{bmatrix} 0.0000 + 0.0000\mathrm{i} & -0.0002 + 0.0003\mathrm{i} \\ 0.0002 - 0.0003\mathrm{i} & 0.0000 + 0.0000\mathrm{i} \end{bmatrix},$$
$$P_{21} = \begin{bmatrix} 40.3496 + 0.0000\mathrm{i} & -1.1298 - 0.2921\mathrm{i} \\ -1.1298 + 0.2921\mathrm{i} & 41.0726 + 0.0000\mathrm{i} \end{bmatrix},$$
$$P_{22} = \begin{bmatrix} 0.0000 + 0.0000\mathrm{i} & -0.0011 + 0.0015\mathrm{i} \\ 0.0011 - 0.0015\mathrm{i} & 0.0000 + 0.0000\mathrm{i} \end{bmatrix},$$
$$P_{31} = \begin{bmatrix} 559.9600 + 0.0000\mathrm{i} & 4.2000 + 1.0900\mathrm{i} \\ 4.2000 - 1.0900\mathrm{i} & 227.3000 + 0.0000\mathrm{i} \end{bmatrix},$$
$$P_{32} = \begin{bmatrix} 0.0000 + 0.0000\mathrm{i} & 0.0029 - 0.0019\mathrm{i} \\ -0.0029 + 0.0019\mathrm{i} & 0.0000 + 0.0000\mathrm{i} \end{bmatrix},$$

$$P_{41}=\begin{bmatrix}52.2500+0.0000\mathrm{i} & -4.1300+5.6200\mathrm{i}\\ -4.1300-5.6200\mathrm{i} & 169.7000+0.0000\mathrm{i}\end{bmatrix},$$

$$P_{42}=\begin{bmatrix}0.0000+0.0000\mathrm{i} & -0.0056+5.4591\mathrm{i}\\ 0.0056-5.4591\mathrm{i} & 0.0000+0.0000\mathrm{i}\end{bmatrix},$$

$$P_{51}=\begin{bmatrix}35.0261+0.0000\mathrm{i} & -0.3045+0.2241\mathrm{i}\\ -0.3045-0.2241\mathrm{i} & 42.4793+0.0000\mathrm{i}\end{bmatrix},$$

$$P_{52}=\begin{bmatrix}0.0000+0.0000\mathrm{i} & -0.0005+0.2088\mathrm{i}\\ 0.0005-0.2088\mathrm{i} & 0.0000+0.0000\mathrm{i}\end{bmatrix},$$

$$Q_1=\begin{bmatrix}11.9059-0.0643\mathrm{i} & -6.6822-1.7352\mathrm{i}\\ -6.8106+1.8103\mathrm{i} & 7.6018-0.0009\mathrm{i}\end{bmatrix},$$

$$Q_2=\begin{bmatrix}0.0031-0.0009\mathrm{i} & -0.0079+0.0100\mathrm{i}\\ 0.0060-0.0091\mathrm{i} & 0.0002+0.0002\mathrm{i}\end{bmatrix},$$

$$S_1=\begin{bmatrix}-25.1330+0.0199\mathrm{i} & 6.8403+1.7606\mathrm{i}\\ 7.1751-1.8719\mathrm{i} & -26.2489+0.0134\mathrm{i}\end{bmatrix},$$

$$S_2=\begin{bmatrix}-0.0007+0.0002\mathrm{i} & 0.0108-0.0126\mathrm{i}\\ -0.0078+0.0104\mathrm{i} & -0.0020+0.0024\mathrm{i}\end{bmatrix},$$

$$T_1=\begin{bmatrix}-40.5734+0.1878\mathrm{i} & 2.8469+0.7729\mathrm{i}\\ 18.8357-5.0131\mathrm{i} & -7.7009-0.0231\mathrm{i}\end{bmatrix},$$

$$T_2=\begin{bmatrix}-0.0099+0.0030\mathrm{i} & 0.0003-0.0024\mathrm{i}\\ -0.0158+0.0246\mathrm{i} & 0.0017-0.0025\mathrm{i}\end{bmatrix},$$

$$G_1=\mathrm{diag}\{103.8033, 308.0326\}.$$

因此, 可以得到控制器系数K_1, K_2如下:

$$K_1=\begin{pmatrix}K_{111} & K_{112}\\ K_{121} & K_{122}\end{pmatrix},\ K_2=\begin{pmatrix}K_{211} & K_{212}\\ K_{221} & K_{222}\end{pmatrix},$$

其中:

$$K_{111}=-3.341-0.01486\mathrm{i}+0.0006\mathrm{j}-0.0001\mathrm{k},$$
$$K_{112}=-2.945-0.782\mathrm{i}-0.0028\mathrm{j}+0.0042\mathrm{k},$$
$$K_{121}=-2.053+0.5359\mathrm{i}+0.0022\mathrm{j}-0.0028\mathrm{k},$$
$$K_{122}=-6.277+0.0017\mathrm{i}-0.0002\mathrm{j}+0.0008\mathrm{k},$$
$$K_{211}=-4.154-0.0064\mathrm{i}+0.0002\mathrm{j}+0.0000\mathrm{k},$$
$$K_{212}=-0.7104-0.1864\mathrm{i}-0.0006\mathrm{j}+0.0011\mathrm{k},$$
$$K_{221}=-1.245+0.3239\mathrm{i}+0.0014\mathrm{j}-0.0017\mathrm{k},$$
$$K_{222}=-1.694-0.0010\mathrm{i}+0.0000\mathrm{j}-0.0000\mathrm{k}.$$

由定理1和推论1知, QVNNs(22)和QVNNs(23)是全局同步的. QVNNs(22)的初值为

$$x_1(t)=1.1623+2.2595\mathrm{i}+1.2294\mathrm{j}+0.6191\mathrm{k},$$
$$x_2(t)=1.0811-1.2983\mathrm{i}+4.9630\mathrm{j}+5.9425\mathrm{k};$$

QVNNs(23)的初值为

$$y_1(t)=-0.5032-5.4384\mathrm{i}-5.2108\mathrm{j}+2.8266\mathrm{k},$$
$$y_2(t)=4.0764-4.4743\mathrm{i}+3.1454\mathrm{j}-1.7500\mathrm{k};$$

对应QVNNs(24)的初值为

$$e_1(t)=1.6655+7.6979\mathrm{i}+6.4402\mathrm{j}-0.5671\mathrm{k},$$
$$e_2(t)=-2.9953+3.1760\mathrm{i}+1.8176\mathrm{j}+0.4517\mathrm{k}.$$

如图1–4所示, 状态变量$e(t)$的时间响应轨线验证了QVNNs(22)和QVNNs(23)的同步性.

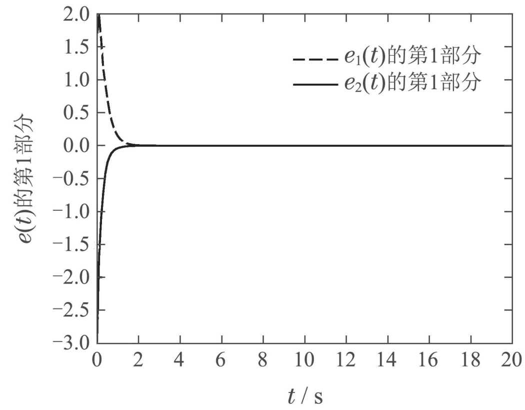

图 1　模型(24)第1部分状态响应轨线

Fig. 1　The first part of the state response trajectory for Model (24)

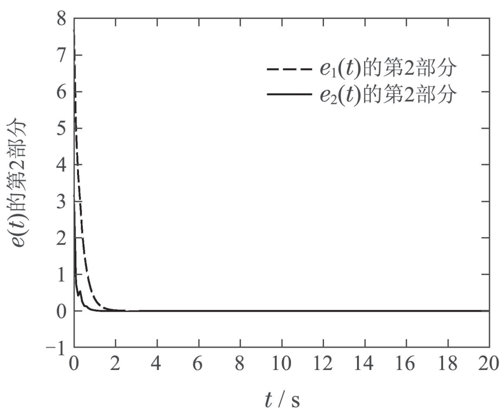

图 2　模型(24)第2部分状态响应轨线

Fig. 2　The second part of the state response trajectory for Model (24)

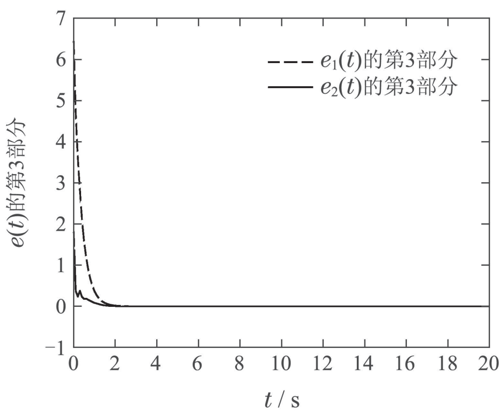

图 3　模型(24)第3部分状态响应轨线

Fig. 3　The third part of the state response trajectory for Model (24)

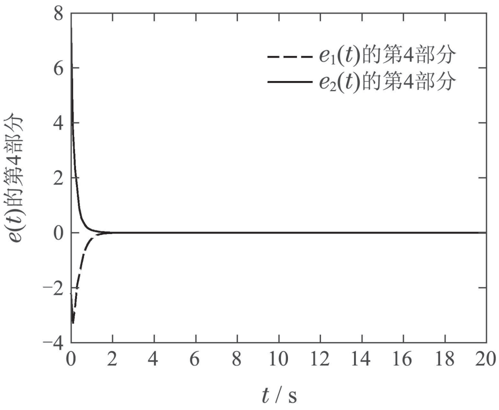

图 4 模型(24)第4部分状态响应轨线

Fig. 4 The fourth part of the state response trajectory for Model (24)

5 结论

本文研究了具有混合时滞的QVNNs全局同步性问题. 通过构造合适的Lyapunov-Krasovskii泛函, 得到了网络全局同步性的充分判据和同步控制器的设计方法. 此外, 为了便于验证, 把所得的QVLMI结果分解为CVLMI的形式. 最后通过一个数值仿真算例验证了本文理论分析的有效性.

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