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不确定非线性系统自适应动态事件触发输出反馈镇定

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摘要: 本文研究了一类不确定非线性系统的动态事件触发输出反馈镇定问题. 显著不同的是系统具有依赖于不可测状态的增长且增长率为输出的未知多项式. 尽管已有一些连续自适应控制器, 但需要巧妙融合非线性状态观测器、系统未知性的动态补偿以及非线性的抵御, 因此这些控制器具有一定的脆弱性, 不能平凡地拓展到不连续情形(采样误差导致). 为此, 首先通过引入动态高增益和基于高增益的观测器来分别抵御未知增长率和重构系统不可测状态. 进而, 意识到静态事件触发机制的无效性, 通过引入动态事件触发机制, 成功设计出了事件触发输出反馈控制器, 确保了系统状态的全局有界性和收敛性. 数值仿真验证了所设计控制器的有效性.

关键词: 非线性系统; 大范围不确定性; 输出未知多项式增长率; 事件触发控制; 自适应控制; 动态事件触发机制; 全局输出反馈镇定

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Adaptive stabilization via dynamic event-triggered output feedback for uncertain nonlinear systems

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Abstract: This paper is devoted to the global adaptive stabilization via dynamic event-triggered output feedback for a class of uncertain nonlinear systems. Remarkably, the systems admit unmeasured states dependent growth with the rate of unknown polynomial-of-output. Although some continuous adaptive controllers have been proposed, they cannot be trivially extended to the discontinuous (caused by sampled error) context since their fragility stemmed from the skillful integration of nonlinear observer to unmeasured states, dynamic compensation to system unknowns and domination to nonlinearities. To solve the problem, a dynamic high gain and a high-gain-based observer are first introduced to counteract the unknown growth rate and reconstruct the unmeasured system states, respectively. Then noting the ineffectiveness of static event-triggering mechanisms, an event-triggered output-feedback controller is successfully designed by introducing a dynamic event-triggering mechanism to achieve the global boundedness and convergence of the system states. A numerical example is provided to illustrate the validity of the designed controller.

Key words: nonlinear systems; large uncertainties; unknown polynomial-of-output growth rate; event-triggered control; adaptive control; dynamic event-triggering mechanism; global output-feedback stabilization

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1 Introduction

Adaptive and observer-based feedback for some uncertain nonlinear systems have been extensively investigated over the past decades[1–4]. With the popularity of networked control systems, event-triggered feedback control has received increasing interest during the past decade due to its advantage in saving communication/computation resources while guaranteeing desired system performance[5–18]. And various of event-triggering mechanisms have appeared which, in terms of the thresholds therein, can be classified into: absolute, relative, time-varying and dynamic types. However, it is rather challenging for systems admitting unmeasured states dependent growth with the rate of unknown polynomial-of-output to achieve global stabilization via event-triggered output-feedback scheme. Although the systems in [13] admit large uncertainties, the absolute/relative threshold schemes proposed merely guarantee the boundedness rather than the convergence. In addition, the time-varying event-triggered schemes in [15–16] achieved output regulation or stabilization, but the system in [16] is the strict output feedback type and doesn't allow large uncertainties. In [17–18], dynamic event-triggered schemes were pro-

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posed to achieve global stabilization, but the systems therein exclude any large uncertainties. When large uncertainties are allowed in nonlinear systems, the global stabilization via dynamic event-triggered output feedback becomes much more challenging: An effective compensation mechanism needs not only suitably introducing to counteract the large uncertainties but also delicately integrating with a dynamic event-triggering mechanism to handle the sampling error and to achieve the desired system performance.

This paper is concerned with the global stabilization via dynamic event-triggered output feedback for a class of uncertain nonlinear systems. Because of the unmeasured states dependent growth with the rate of unknown polynomial-of-output, the existing event-triggered schemes fail to guarantee the desired system performance. In fact, the existing (static) absolute threshold schemes merely guarantee the global boundedness rather than the convergence[13]. Although the (static) relative threshold schemes can enhance the control objective from boundedness to convergence, the large uncertainties are not allowed[7–8]. Inspired by works[16–18], for the possibility of further saving of computation/communication resources, a dynamic event-triggered output-feedback control scheme is proposed to achieve the global stabilization of the uncertain nonlinear systems. Detailedly, an observer with dynamic high gain is first introduced to reconstruct the unmeasured system states, and meanwhile the unknown growth rate is counteracted by the dynamic high gain. Then, noting the ineffectiveness of static event-triggering mechanisms, we propose a dynamic event-triggering mechanism. By flexibly integrating these compensation and dynamic strategies, an adaptive event-triggered output-feedback controller is successfully designed to achieve the global boundedness and convergence of the system states.

The remainder of this paper is organized as follows. Section 2 formulates the system model and control objective. A dynamic event-triggered controller is proposed in Section 3 to achieve global output-feedback stabilization. Section 4 presents some useful implications and summarizes the main results of this paper. Section 5 provides a simulation example and Section 6 gives some concluding remarks.

2 Problem formulation

Consider the global stabilization via event-triggered output feedback for the following uncertain nonlinear system:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x, u, t),\ i = 1, \cdots, n-1, \\ \dot{x}_n = u + f_n(x, u, t), \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1\ \ \cdots\ \ x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state vector with the initial value $x(0) = x_0$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control input and system output, respectively; unknown functions $f_i : \mathbb{R}^n \times \mathbb{R} \times [0, +\infty) \to \mathbb{R}$, $i = 1, \cdots, n$, called the system nonlinearities, are locally Lipschitz in the first argument and continuous in the rest ones.

Assumption 1 There exist a known positive constant p and an unknown positive constant θ such that

$$|f_i(t, x, u)| \leqslant \theta(1 + |x_1|^p) \sum_{j=1}^{i} |x_j|,\ i = 1, \cdots, n.$$

Remarkably, Assumption 1 makes system (1) admit unmeasured states dependent growth with the rate of unknown polynomial-of-output (the large uncertainties are reflected by the unknown θ), which is essentially different from the related works[13, 16–18]. Specifically, the works[13, 16] have the nonlinearities merely lying on the system output while the systems in [16–18] allowing not any uncertainties or merely weak uncertainties.

Detailedly, the global stabilization to be established is that for system (1) under Assumption 1, an event-triggered output-feedback controller with dynamic event-triggering mechanism will be designed such that all the signals of the closed-loop system are globally bounded while the system states converge to zero.

3 Event-triggered output-feedback controller

We first suitably choose positive parameters a_i and b_i, $i = 1, \cdots, n$ such that there exist symmetric positive definite matrices P and Q satisfying

$$\begin{cases} c_1 I \leqslant CP + PC \leqslant c_2 I,\ B^{\mathrm{T}}P + PB \leqslant -2I, \\ c_3 I \leqslant CQ + QC \leqslant c_4 I,\ A^{\mathrm{T}}Q + QA \leqslant -I, \end{cases} \tag{2}$$

where c_i's are positive constants, both $A = [-a\ \ [I_{n-1}\ \ 0_{n-1}]^{\mathrm{T}}]$ and $B = [[0_{n-1}\ \ I_{n-1}]^{\mathrm{T}}\ \ -b]^{\mathrm{T}}$ are Hurwitz with I_{n-1} being $(n-1)$-dimensional identity matrix and 0_{n-1} being $(n-1)$-dimensional column vector with all elements 0, $a = [a_1\ \ \cdots\ \ a_n]^{\mathrm{T}}$, $b = [b_1\ \ \cdots\ \ b_n]^{\mathrm{T}}$, $C = \operatorname{diag}\{\sigma, 1+\sigma, \cdots, n-1+\sigma\}$ with constant σ satisfying $0 < \sigma \leqslant \dfrac{1}{4p}$.

For system (1), we construct the following observer (**O**) and dynamic high gain (**G**) with $\gamma_1(0) \geqslant 1$ and $\gamma_2(0) \geqslant 1$:

$$\begin{cases} \textbf{O:} \begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + \gamma^i a_i(x_1 - \hat{x}_1), \\ \qquad i = 1, \cdots, n-1, \\ \dot{\hat{x}}_n = u + \gamma^n a_n(x_1 - \hat{x}_1), \end{cases} \\ \textbf{G:} \begin{cases} \gamma = \gamma(t) = \gamma_1(t)\gamma_2(t), \\ \dot{\gamma}_1 = m_1(1 + |x_1|^p)^2 - m_2(\gamma_1 - 1), \\ \dot{\gamma}_2 = \gamma_1^{1-2\sigma}\Big(\dfrac{x_1 - \hat{x}_1}{\gamma_2^{\sigma}}\Big)^2, \end{cases} \end{cases} \tag{3}$$

where m_1 and m_2 are positive constants satisfying $m_1 \geqslant \max\{\dfrac{1}{c_1}, \dfrac{1}{c_3}\}$ and $m_2 \leqslant \min\{\dfrac{1}{2c_2}, \dfrac{1}{2c_4}\}$, respectively. By (**G**), we know that $\gamma_2(t)$ is nondecreasing and $\gamma_1(t) \geqslant 1$, $\gamma_2(t) \geqslant 1$, $\forall t \geqslant 0$.

We then design the following event-triggered controller:

$$\begin{cases} u(t) = \omega((\gamma(t_k), \hat{x}(t_k)),\ t \in [t_k, t_{k+1}), \\ \omega(\gamma, \hat{x}) = -\sum\limits_{i=1}^{n} \gamma^{n-i+1} b_i \hat{x}_i, \end{cases} \tag{4}$$

and particularly the following dynamic event-triggering mechanism (to generate the execution times t_k's):

$$\begin{cases} t_{k+1} = \inf\{t > t_k | \varpi(t) \geqslant \chi(t)\}, \\ \varpi(u, \gamma, \hat{x}) = \dfrac{2\|PE_n\|^2 (u-\omega)^2}{\gamma^{2n-1+2\sigma}} - \dfrac{\|\hat{x}\|^2}{4\gamma^{2n-3+2\sigma}}, \\ \dot{\chi} = -d_1 \varpi(t) - d_2 \chi,\ \chi(0) \geqslant 0, \end{cases} \tag{5}$$

where $\varpi(t)$ denotes $\varpi(u(t), \gamma(t), \hat{x}(t))$ for brevity and $\varpi(0) \leqslant \chi(0)$, $E_n = [0\ \cdots\ 0\ 1]^{\mathrm{T}} \in \mathbb{R}^n$, constants d_1 and d_2 satisfy $0 < d_1 < 1$ and $d_2 \geqslant 1 - d_1$, respectively.

From (5) and noting $\varpi(0) \leqslant \chi(0)$, we have $\varpi(t) \leqslant \chi(t)$, $\forall t \geqslant 0$ and hence $\dot{\chi} \geqslant -(d_1 + d_2)\chi$, which implies $\chi(t) \geqslant \chi(0)\mathrm{e}^{-(d_1+d_2)t} \geqslant 0$, $\forall t \geqslant 0$.

4 Main results

Since $f_i(\cdot)$'s are locally Lipschitz, the right-hand sides of the entire dynamical system (1) and (3) are continuous in (t, u) and locally Lipschitz in $(x, \hat{x}, \gamma_1, \gamma_2)$. The existence and uniqueness theorem and continuation theorem, (see e.g., Theorem 3.1 on Page 18 and Theorem 2.1 on Page 17 of [19], respectively), together with the piecewise continuation in [11], suggest that for any given initial value $(x_0, \hat{x}_0, \gamma_1(0), \gamma_2(0))$, the resulting closed-loop system has a unique solution $(x(t), \hat{x}(t), \gamma_1(t), \gamma_2(t))$ on the maximum existence interval $[0, T_{\mathrm{m}})$, where $0 < T_{\mathrm{m}} \leqslant +\infty$. The case "$T_{\mathrm{m}} < +\infty$", as in [11], implies that

$$\lim_{t \to T_{\mathrm{m}}} (\|x(t)\| + \|\hat{x}(t)\| + \gamma_1(t) + \gamma_2(t)) = +\infty$$

or that Zeno occurs, i.e., $\lim\limits_{k \to +\infty} t_k = T_{\mathrm{m}}$.

For further development, we introduce the scaled coordinate transformation to $(x, \hat{x})$:

$$z_i = \frac{\hat{x}_i}{\gamma^{i-1+\sigma}},\ \varepsilon_i = \frac{x_i - \hat{x}_i}{\gamma^{i-1+\sigma}},\ i = 1, \cdots, n.$$

Then by (1) and (3), there are

$$\begin{cases} \dot{z} = -\dfrac{\dot{\gamma}}{\gamma} Cz + \gamma Bz + \gamma a \varepsilon_1 + \dfrac{E_n(u-\omega)}{\gamma^{n-1+\sigma}}, \\ \dot{\varepsilon} = -\dfrac{\dot{\gamma}}{\gamma} C\varepsilon + \gamma A \varepsilon + \tilde{f}, \end{cases} \tag{6}$$

where $z = [z_1\ \cdots\ z_n]^{\mathrm{T}}$, $\varepsilon = [\varepsilon_1\ \cdots\ \varepsilon_n]^{\mathrm{T}}$, and $\tilde{f} = [\dfrac{f_1}{\gamma^{\sigma}}\ \dfrac{f_2}{\gamma^{1+\sigma}}\ \cdots\ \dfrac{f_n}{\gamma^{n-1+\sigma}}]^{\mathrm{T}}$.

Remark that, once $\gamma(t)$ is bounded on $[0, +\infty)$, the stability of system (6) implies that of the original system $(x, \hat{x})$, and vice versa. Nevertheless, system (6) makes the stable mode explicit, which is more advantageous to the stability analysis.

We have the following propositions:

Proposition 1 Choose $V = z^{\mathrm{T}} P z + \bar{a} \varepsilon^{\mathrm{T}} Q \varepsilon + \chi$ with $\bar{a} = 4\|Pa\|^2 + 1$. Then along the trajectories of (6) and the dynamics χ in (5), there holds

$$D^+ V \leqslant (-\frac{\gamma_2}{4} + \varTheta)\gamma_1(\|z\|^2 + \|\varepsilon\|^2) - \bar{d}\chi, \tag{7}$$

where D^+V denotes the upper right-hand derivative of V, and $\varTheta$ and $\bar{d}$ are unknown positive and known nonnegative constants, respectively.

Proof Along the dynamics χ in (5) and the trajectories of (6), there holds

$$\begin{aligned} D^+V = & -\frac{\dot{\gamma}}{\gamma} z^{\mathrm{T}}(CP + PC)z + \gamma z^{\mathrm{T}}(B^{\mathrm{T}}P + PB)z + \\ & 2\gamma\varepsilon_1 z^{\mathrm{T}} Pa + 2\frac{u-\omega}{\gamma^{n-1+\sigma}} z^{\mathrm{T}} PE_n - \\ & \bar{a}\frac{\dot{\gamma}}{\gamma}\varepsilon^{\mathrm{T}}(CQ + QC)\varepsilon + \bar{a}\gamma\varepsilon^{\mathrm{T}}(A^{\mathrm{T}}Q + QA)\varepsilon + \\ & 2\bar{a}\varepsilon^{\mathrm{T}} Q\tilde{f} - d_1\varpi - d_2\chi. \end{aligned}$$

Then by (2) and the dynamics γ_1 in (3), and noting that $\dot{\gamma}_2 \geqslant 0$, $\gamma_2 \geqslant 1$, we have

$$\begin{aligned} D^+V \leqslant & -\frac{\dot{\gamma}_1}{\gamma_1} z^{\mathrm{T}}(CP + PC)z - 2\gamma\|z\|^2 + \\ & 2\gamma\|Pa\| \cdot \|z\| \cdot |\varepsilon_1| + 2\frac{\|PE_n\|}{\gamma^{n-1+\sigma}}|u-\omega| \cdot \\ & \|z\| - \bar{a}\frac{\dot{\gamma}_1}{\gamma_1}\varepsilon^{\mathrm{T}}(CQ + QC)\varepsilon - \bar{a}\gamma\|\varepsilon\|^2 + \\ & 2\bar{a}\|Q\| \cdot \|\varepsilon\| \cdot \|\tilde{f}\| - d_1\varpi - d_2\chi \leqslant \\ & -\frac{c_1 m_1}{\gamma_1}(1 + |x_1|^p)^2\|z\|^2 + c_2 m_2\|z\|^2 - \\ & \frac{c_1 m_2}{\gamma_1}\|z\|^2 - 2\gamma\|z\|^2 + 2\gamma\|Pa\| \cdot \|z\| \cdot \\ & |\varepsilon_1| + 2\frac{\|PE_n\|}{\gamma^{n-1+\sigma}}|u-\omega| \cdot \|z\| - \frac{\bar{a}c_3 m_1}{\gamma_1} \cdot \\ & (1 + |x_1|^p)^2 \cdot \|\varepsilon\|^2 + \bar{a}c_4 m_2\|\varepsilon\|^2 - \\ & \frac{\bar{a}c_3 m_2}{\gamma_1}\|\varepsilon\|^2 - \bar{a}\gamma\|\varepsilon\|^2 + 2\bar{a}\|Q\| \cdot \|\varepsilon\| \cdot \|\tilde{f}\| - \\ & d_1\varpi - d_2\chi. \end{aligned}$$

The positivity of c_1, c_3, m_2 and $\bar{a}$ and $\gamma_1 \geqslant 1$ imply

$$\begin{aligned} D^+V \leqslant & (c_2 m_2 - 2\gamma)\|z\|^2 - \frac{c_1 m_1}{\gamma_1}(1 + |x_1|^p)^2\|z\|^2 + \\ & 2\gamma\|Pa\| \cdot \|z\| \cdot |\varepsilon_1| + \frac{2\|PE_n\|}{\gamma^{n+\sigma-1}}|u-\omega| \cdot \|z\| + \\ & (\bar{a}c_4 m_2 - \bar{a}\gamma)\|\varepsilon\|^2 - \frac{\bar{a}c_3 m_1}{\gamma_1}(1 + |x_1|^p)^2 \cdot \\ & \|\varepsilon\|^2 + 2\bar{a}\|Q\| \cdot \|\varepsilon\| \cdot \|\tilde{f}\| - d_1\varpi - d_2\chi. \end{aligned} \tag{8}$$

By Assumption 1 and the completing squares technique, the following three terms satisfy:

$$\begin{cases} 2\gamma\|Pa\|\cdot\|z\|\cdot|\varepsilon_1| \leqslant \dfrac{\gamma}{2}\|z\|^2 + 2\gamma\|Pa\|^2\varepsilon_1^2, \\ \dfrac{2\|PE_n\|}{\gamma^{n-1+\sigma}}|u-\omega|\cdot\|z\| \leqslant \\ \dfrac{\gamma}{2}\|z\|^2 + \dfrac{2\|PE_n\|^2}{\gamma^{2n-1+2\sigma}}(u-\omega)^2, \\ 2\bar{a}\|Q\|\cdot\|\varepsilon\|\cdot\|\tilde{f}\| \leqslant \\ \dfrac{(1+|x_1|^p)^2}{\gamma_1}(\|z\|^2+\|\varepsilon\|^2)+\gamma_1(\bar{a}n\theta\|Q\|)^2\|\varepsilon\|^2. \end{cases} \tag{9}$$

Substituting this into (8), and noting the positivity of c_2, c_4, m_2 and $\bar{a}$ and $\gamma \geqslant 1$, we have

$$\begin{aligned} D^+V \leqslant & (c_2m_2-1)\gamma\|z\|^2 + \gamma_1\big(\bar{a}c_4m_2\gamma_2 - \bar{a}\gamma_2 + \\ & 2\|Pa\|^2\gamma_2 + 2(\bar{a}n\theta\|Q\|)^2\big)\|\varepsilon\|^2 + \\ & \frac{1-c_1m_1}{\gamma_1}(1+|x_1|^p)^2\|z\|^2 + \\ & \frac{1-\bar{a}c_3m_1}{\gamma_1}(1+|x_1|^p)^2\|\varepsilon\|^2 + \\ & \frac{2\|PE_n\|^2}{\gamma^{2n-1+2\sigma}}(u-\omega)^2 - d_1\varpi - d_2\chi. \end{aligned}$$

The choice of m_2, m_1 and $\bar{a}$, together with $\gamma_1 \geqslant 1$ and the expression of ϖ, implies that

$$\begin{aligned} D^+V \leqslant & -\frac{\gamma}{2}\|z\|^2 - \frac{\gamma_1}{2}(\gamma_2 - 4(\bar{a}n\theta\|Q\|)^2)\|\varepsilon\|^2 + \\ & \frac{2\|PE_n\|^2}{\gamma^{2n-1+2\sigma}}(u-\omega)^2 - d_1\varpi - d_2\chi \leqslant \\ & -\frac{\gamma}{4}\|z\|^2 - \frac{\gamma_1}{2}(\gamma_2 - 4(\bar{a}n\theta\|Q\|)^2)\|\varepsilon\|^2 + \\ & (1-d_1)\varpi - d_2\chi. \end{aligned}$$

From the event-triggering mechanism (5), it follows that $\varpi \leqslant \chi(t)$. Then we have

$$\begin{aligned} D^+V \leqslant & (-\frac{\gamma_2}{4} + 2(\bar{a}n\theta\|Q\|)^2)\gamma_1(\|z\|^2 + \|\varepsilon\|^2) + \\ & (1-d_1-d_2)\chi. \end{aligned} \tag{10}$$

Since $d_2 \geqslant 1 - d_1$, we immediately derive (7) with $\varTheta = 2(\bar{a}n\theta\|Q\|)^2$ and $\bar{d} = d_1 + d_2 - 1 \geqslant 0$.

In what follows, we denote $V(z(t), \varepsilon(t), \chi(t))$ by $V(t)$ (or directly V) for convenience.

Proposition 2 If $\gamma_2(t)$ is bounded on $[0, T_{\mathrm{m}})$, then $z(t)$ and $\varepsilon(t)$ are bounded on $[0, T_{\mathrm{m}})$, and moreover,

$$\lim_{t\to T_{\mathrm{m}}}\int_0^t(\gamma_1(\tau)\|z(\tau)\|^2 + \|\varepsilon(\tau)\|^2)\mathrm{d}\tau < +\infty.$$

Proof We first show the boundedness and square integrability of $z(t)$ on $[0, T_{\mathrm{m}})$. On the one hand, choose the Lyapunov function candidate $V_{\mathrm{z}} = z^{\mathrm{T}}Pz + \chi$. Along the dynamics χ and z in (5) and (6), and noting (2) and $\dot{\gamma}_2 \geqslant 0$, $\gamma_2 \geqslant 1$, there holds

$$\begin{aligned} D^+V_{\mathrm{z}} = & -\frac{\dot{\gamma}}{\gamma}z^{\mathrm{T}}(CP+PC)z + \gamma z^{\mathrm{T}}(B^{\mathrm{T}}P+PB)z + \\ & 2\gamma\varepsilon_1 z^{\mathrm{T}}Pa + 2\frac{u-\omega}{\gamma^{n-1+\sigma}}z^{\mathrm{T}}PE_n - \\ & d_1\varpi - d_2\chi \leqslant \\ & -\frac{\dot{\gamma}_1}{\gamma_1}z^{\mathrm{T}}(CP+PC)z - 2\gamma\|z\|^2 + 2\gamma\|Pa\|\cdot \\ & \|z\|\cdot|\varepsilon_1| + 2\gamma\frac{\|PE_n\|}{\gamma^{n+\sigma}}|u-\omega|\cdot\|z\| - \\ & d_1\varpi - d_2\chi. \end{aligned}$$

Then by (2) and the dynamics of γ_1 in (3), and noting the positivity of c_1, c_2, m_1, m_2, γ_1 and the first two inequalities in (9), we have

$$\begin{aligned} D^+V_{\mathrm{z}} \leqslant & -\frac{c_1m_1}{\gamma_1}(1+|x_1|^p)^2\|z\|^2 - \frac{c_1m_2}{\gamma_1}\|z\|^2 + \\ & c_2m_2\|z\|^2 - \gamma\|z\|^2 + 2\gamma\|Pa\|^2\varepsilon_1^2 + \\ & \frac{2\|PE_n\|^2}{\gamma^{2n-1+2\sigma}}(u-\omega)^2 - d_1\varpi - d_2\chi \leqslant \\ & (c_2m_2-1)\gamma\|z\|^2 + 2\gamma\|Pa\|^2\varepsilon_1^2 + \\ & \frac{2\|PE_n\|^2}{\gamma^{2n-1+2\sigma}}(u-\omega)^2 - d_1\varpi - d_2\chi \leqslant \\ & -\frac{\gamma}{2}\|z\|^2 + 2\gamma\|Pa\|^2\varepsilon_1^2 + \\ & \frac{2\|PE_n\|^2}{\gamma^{2n-1+2\sigma}}(u-\omega)^2 - d_1\varpi - d_2\chi. \end{aligned}$$

By the dynamics of γ_2 in (3) and similar to the derivation of (10), we get

$$\begin{aligned} D^+V_{\mathrm{z}} \leqslant & -\frac{\gamma}{4}\|z\|^2 + 2\|Pa\|^2\gamma_2\dot{\gamma}_2 + \\ & (1-d_1-d_2)\chi. \end{aligned} \tag{11}$$

Since $\gamma_2(t)$ is bounded and nondecreasing on $[0, T_{\mathrm{m}})$, we know that there exists a positive constant $\bar{\gamma}_2$ such that $\bar{\gamma}_2 = \lim\limits_{t\to T_{\mathrm{m}}}\gamma_2(t)$. Denote

$$\lambda_1 := \min\{\frac{1}{4l_{\max}(P)}, d_2 - d_1 - 1\}.$$

Then we have

$$D^+V_{\mathrm{z}} \leqslant -\lambda_1 V_{\mathrm{z}} + 2\|Pa\|^2\bar{\gamma}_2\dot{\gamma}_2,$$

from which it follows

$$D^+(\mathrm{e}^{\lambda_1 t}V_{\mathrm{z}}) \leqslant 2\|Pa\|^2\bar{\gamma}_2\mathrm{e}^{\lambda_1 t}\dot{\gamma}_2.$$

Integrating the above inequality from 0 to t, it holds

$$\begin{aligned} V_{\mathrm{z}} \leqslant & \frac{V_{\mathrm{z}}(0)}{\mathrm{e}^{\lambda_1 t}} + 2\|Pa\|^2\bar{\gamma}_2\int_0^t \mathrm{e}^{\lambda_1(\tau - t)}\mathrm{d}\gamma_2(\tau) \leqslant \\ & V_{\mathrm{z}}(0) + 2\|Pa\|^2\bar{\gamma}_2^2, \end{aligned}$$

which means the boundedness of $z(t)$ on $[0, T_{\mathrm{m}})$.

On the other hand, integrating (11) from 0 to t, we can derive

$$V_{\mathrm{z}}(t) - V_{\mathrm{z}}(0) \leqslant -\frac{1}{4}\int_0^t \gamma_1(\tau)\|z(\tau)\|^2\mathrm{d}\tau +$$

$$2\|Pa\|^2\bar{\gamma}_2\int_0^t\dot{\gamma}_2(\tau)\mathrm{d}\tau+$$
$$(1-d_1-d_2)\int_0^t\chi(\tau)\mathrm{d}\tau.$$

Then by $0<d_1\leqslant 1$ and $d_2\geqslant 1-d_1$, we have

$$\frac{1}{4}\int_0^t\gamma_1(\tau)\|z(\tau)\|^2\mathrm{d}\tau\leqslant V_{\mathrm{z}}(0)+2\|Pa\|^2\bar{\gamma}_2^2,$$

which implies that $\int_0^t\gamma_1(\tau)\|z(\tau)\|^2\mathrm{d}\tau<+\infty$.

We next show the boundedness and square integrability of $\varepsilon(t)$ on $[0,T_{\mathrm{m}})$. For this, we introduce the following scaling transformation:

$$\eta_i=\frac{x_i-\hat{x}_i}{(\gamma_1\gamma_2^*)^{i-1+\sigma}}, \tag{12}$$

where γ_2^* is a constant satisfying $\gamma_2^*\geqslant\max\{\bar{\gamma}_2,8n^2\times\theta^2\|Q\|^2+1\}$. Then by (1) and (3), we have

$$\dot{\eta}=-\frac{\dot{\gamma}_1}{\gamma_1}C\eta+\gamma_1\gamma_2^*A\eta+\gamma_1\gamma_2^*a\eta_1-\gamma\Gamma a\eta_1+\tilde{f}^*, \tag{13}$$

where

$$\Gamma=\mathrm{diag}\{1,\frac{\gamma_2}{\gamma_2^*},\cdots,(\frac{\gamma_2}{\gamma_2^*})^{n-1}\}$$

and $\tilde{f}^*=[\frac{f_1}{(\gamma_1\gamma_2^*)^{\sigma}}\ \cdots\ \frac{f_n}{(\gamma_1\gamma_2^*)^{n-1+\sigma}}]^{\mathrm{T}}$.

On one hand, choose the Lyapunov function candidate $V_\eta=\eta^{\mathrm{T}}Q\eta$. Then by (2) and the dynamics γ_1 in (3) and along the trajectories of (13), there holds

$$\begin{aligned}D^+V_\eta=&-\frac{\dot{\gamma}_1}{\gamma_1}\eta^{\mathrm{T}}(CQ+QC)\eta+\gamma_1\gamma_2^*\eta^{\mathrm{T}}(A^{\mathrm{T}}Q+\\&QA)\eta+2\gamma_1\gamma_2^*\eta_1\eta^{\mathrm{T}}Qa-2\gamma\eta_1\eta^{\mathrm{T}}Q\Gamma a+\\&2\eta^{\mathrm{T}}Q\tilde{f}^*\leqslant\\&-\frac{c_3m_1(1+|x_1|^p)^2}{\gamma_1}\|\eta\|^2+c_4m_2\|\eta\|^2-\\&\frac{c_3m_2}{\gamma_1}\|\eta\|^2-\gamma_1\gamma_2^*\|\eta\|^2+2\gamma_1\gamma_2^*|\eta_1|\cdot\|\eta\|\cdot\\&\|Qa\|+2\gamma|\eta_1|\cdot\|\eta\|\cdot\|Q\Gamma a\|+\\&2\|\eta\|\cdot\|Q\|\cdot\|\tilde{f}^*\|.\end{aligned} \tag{14}$$

By Assumption 1 and the method of completing squares, the last three terms satisfy

$$\begin{cases}2\gamma_1\gamma_2^*|\eta_1|\cdot\|\eta\|\cdot\|Qa\|\leqslant\\ \frac{\gamma_1\gamma_2^*}{8}\|\eta\|^2+\gamma_1\gamma_2^*\|Qa\|^2\eta_1^2,\\ 2\gamma|\eta_1|\cdot\|\eta\|\cdot\|Q\Gamma a\|\leqslant\frac{\gamma}{8}\|\eta\|^2+8\gamma\|Q\Gamma a\|^2\eta_1^2,\\ 2\|\eta\|\cdot\|Q\|\cdot\|\tilde{f}^*\|\leqslant\\ \frac{(1+|x_1|^p)^2}{\gamma_1}\|\eta\|^2+2n^2\theta^2\|Q\|^2\gamma_1(\|z\|^2+\|\eta\|^2).\end{cases}$$

Substituting the above inequalities into (14) and noting that $c_3m_1>1$ and $c_3<c_4$, we have

$$\begin{aligned}D^+V_\eta\leqslant&-\frac{(c_3m_1-1)(1+|x_1|^p)^2}{\gamma_1}\|\eta\|^2+\\&c_4m_2\|\eta\|^2-\frac{c_3m_2}{\gamma_1}\|\eta\|^2-\frac{3\gamma_1\gamma_2^*}{4}\|\eta\|^2+\\&8\gamma_1\gamma_2^*\|Qa\|^2\eta_1^2+8\gamma\|Q\Gamma a\|^2\eta_1^2+\\&2n^2\theta^2\|Q\|^2\gamma_1(\|z\|^2+\|\eta\|^2)\leqslant\\&-\frac{\gamma_1}{4}(\gamma_2^*-8n^2\theta^2\|Q\|^2)\|\eta\|^2+\\&8\gamma_1\gamma_2^*\|Qa\|^2\eta_1^2+8\gamma\|Q\Gamma a\|^2\eta_1^2+\\&2n^2\theta^2\|Q\|^2\gamma_1\|z\|^2.\end{aligned}$$

By $\gamma_2^*\geqslant\max\{\bar{\gamma}_2,8n^2\theta^2\|Q\|^2+1\}$ and $\gamma_1\geqslant 1$, we have

$$\eta_1^2=(\frac{x_1-\hat{x}_1}{(\gamma_1\gamma_2^*)^{\sigma}})^2=(\frac{x_1-\hat{x}_1}{\gamma^{\sigma}})^2(\frac{\gamma_2}{\gamma_2^*})^{2\sigma}\leqslant\varepsilon_1^2.$$

Then

$$\begin{aligned}D^+V_\eta\leqslant&-\frac{\gamma_1}{4}\|\eta\|^2+(8\gamma_2^*\|Qa\|^2+8\bar{\gamma}_2\|Q\Gamma a\|^2)\cdot\\&\gamma_1\varepsilon_1^2+2n^2\theta^2\|Q\|^2\gamma_1\|z\|^2\leqslant\\&-\lambda_2V_\eta+(8\gamma_2^*\|Qa\|^2+8\bar{\gamma}_2\|Q\Gamma a\|^2)\gamma_1\varepsilon_1^2+\\&2n^2\theta^2\|Q\|^2\gamma_1\|z\|^2,\end{aligned} \tag{15}$$

where $\lambda_2=\frac{1}{4l_{\max}(Q)}$. This can immediately derive

$$\begin{aligned}D^+(\mathrm{e}^{\lambda_2t}V_\eta)\leqslant&(8\gamma_2^*\|Qa\|^2+8\bar{\gamma}_2\|Q\Gamma a\|^2)\mathrm{e}^{\lambda_2t}\gamma_1\varepsilon_1^2+\\&2n^2\theta^2\|Q\|^2\mathrm{e}^{\lambda_2t}\gamma_1\|z\|^2,\end{aligned}$$

from which it follows

$$\begin{aligned}V_\eta\leqslant&\frac{V_\eta(0)}{\mathrm{e}^{\lambda_2t}}+\int_0^t(8\gamma_2^*\|Qa\|^2+8\bar{\gamma}_2\|Q\Gamma a\|^2)\mathrm{e}^{\lambda_2(\tau-t)}\times\\&\gamma_1(\tau)\varepsilon_1^2(\tau)\mathrm{d}\tau+\int_0^t2n^2\theta^2\|Q\|^2\mathrm{e}^{\lambda_2(\tau-t)}\gamma_1(\tau)\times\\&\|z(\tau)\|^2\mathrm{d}\tau\leqslant\\&V_\eta(0)+(8\gamma_2^*\|Qa\|^2+8\bar{\gamma}_2\|Q\Gamma a\|^2)\int_0^t\gamma_1(\tau)\times\\&\varepsilon_1^2(\tau)\mathrm{d}\tau+2n^2\theta^2\|Q\|^2\int_0^t\gamma_1(\tau)\|z(\tau)\|^2\mathrm{d}\tau.\end{aligned}$$

By (3) and the hypothesis, we know

$$\begin{aligned}\gamma_2(t)-\gamma_2(0)=&\int_0^t\dot{\gamma}_2(\tau)\mathrm{d}\tau=\\&\int_0^t\gamma_1(\tau)\varepsilon_1^2(\tau)\mathrm{d}\tau<+\infty,\end{aligned} \tag{16}$$

which, together with $\int_0^t\gamma_1(\tau)\|z(\tau)\|^2\mathrm{d}\tau<+\infty$, implies that $\eta(t)$ is bounded on $[0,T_{\mathrm{m}})$.

On the other hand, integrating the first inequality in (15) from 0 to t, we immediately have

$$\begin{aligned}&\frac{1}{4}\int_0^t\gamma_1(\tau)\|\eta(\tau)\|^2\mathrm{d}\tau\leqslant\\&V_\eta(0)+(8\gamma_2^*\|Qa\|^2+8\bar{\gamma}_2\|Q\Gamma a\|^2)\int_0^t\gamma_1(\tau)\varepsilon_1^2(\tau)\mathrm{d}\tau+\\&2n^2\theta^2\|Q\|^2\int_0^t\gamma_1(\tau)\|z(\tau)\|^2\mathrm{d}\tau.\end{aligned}$$

Noting (16) and $\int_0^t\gamma_1(\tau)\|z(\tau)\|^2\mathrm{d}\tau<+\infty$, we can derive $\int_0^t\gamma_1(\tau)\|\eta(\tau)\|^2\mathrm{d}\tau<+\infty$. Then by (12) and noting that $\gamma_2 \geqslant 1$ and $\gamma_1 \geqslant 1$, we know that $\int_0^t \|\varepsilon(\tau)\|^2 \mathrm{d}\tau < +\infty$. QED.

Proposition 3 If all the signals of the resulting closed-loop system are bounded on $[0, T_{\mathrm{m}})$, then $T_{\mathrm{m}} = +\infty$ and Zeno doesn't occur, and moreover, $\lim\limits_{t\to+\infty}(x(t), \hat{x}(t), u(t)) = 0$.

Proof Noting that the internal dynamic variable $\chi(t)$ satisfies $\chi(t) \geqslant \chi(0)\mathrm{e}^{-(d_1+d_2)t} \geqslant 0$, $\forall t \geqslant 0$, and $\gamma \geqslant 1$, the dynamic event-triggering mechanism (5) in this paper can be degenerated to the following time-varying one:

$$t_{k+1} = \inf\{t > t_k \,|\, 2\|PE_n\|^2(u-\omega)^2 \geqslant \chi(0)\mathrm{e}^{-(d_1+d_2)t}\}. \tag{17}$$

By using the existing analysis of eliminating Zeno phenomenon under time-varying event-triggered scheme (e.g. [15]) and integrating the compensation mechanism in this paper, a time-varying event-triggered scheme can be designed to achieve the global boundedness and ultimately convergence of system (1) under Assumption 1 while no Zeno occurs. The detailed derivation of the effectiveness of time-varying event-triggered scheme is omitted here due to the page limitation.

For a given system state, denote t_{k+1}^d and t_{k+1}^t be the next execution time of a dynamic event-triggering mechanism and time-varying event-triggering mechanism, respectively. We show that $t_{k+1}^d \geqslant t_{k+1}^t$. In fact, suppose that $t_{k+1}^d < t_{k+1}^t$. Then by (17), we have

$$2\|PE_n\|^2(u(t_{k+1}^d) - \omega(t_{k+1}^d))^2 < \chi(0)\mathrm{e}^{-(d_1+d_2)t_{k+1}^d}. \tag{18}$$

On the other hand, from the event-triggering mechanism (5), it follows that $\varpi(t_{k+1}^d) = \chi(t_{k+1}^d)$. By this and $\gamma(t) \geqslant 1$, we immediately derive

$$2\|PE_n\|^2(u(t_{k+1}^d) - \omega(t_{k+1}^d))^2 \geqslant \chi(t_{k+1}^d) \geqslant \chi(0)\mathrm{e}^{-(d_1+d_2)t_{k+1}^d},$$

which is a contradiction with (18). Thus we have $t_{k+1}^d \geqslant t_{k+1}^t$, which means that the minimum inter-execution time of the dynamic event-triggering mechanism cannot be smaller than that of a time-varying one. Then the fact that no Zeno occurs for time-varying event-triggering mechanism (just analysed) implies that no Zeno occurs for a dynamic one in (5).

Now suppose that $T_{\mathrm{m}} < +\infty$, then by the boundedness of all the signals of the resulting closed-loop system, there must exist Zeno phenomenon. Similar to the proof of Lemma 4.1 in [15], we know that, for any finite T, there holds $\inf\{t_{k+1}^t - t_k^t | [t_k^t, t_{k+1}^t) \subset [0, T)\} > 0$. Since $t_{k+1}^d \geqslant t_{k+1}^t$ (just proved), we know that $\lim\limits_{k\to+\infty} t_k^d = +\infty$, which results in a contradiction. Thus $T_{\mathrm{m}} = +\infty$ and no Zeno occurs.

We next prove the convergence of the system states. In fact, by the hypothesis and (6), both $\dot{z}(t)$ and $\dot{\varepsilon}(t)$ are bounded on $[0, +\infty)$. In addition, Proposition 2 indicates that $\lim\limits_{t\to T_{\mathrm{m}}} \int_0^t \|z(\tau)\|^2 \mathrm{d}\tau < +\infty$, and $\lim\limits_{t\to T_{\mathrm{m}}} \int_0^t \|\varepsilon(\tau)\|^2 \mathrm{d}\tau < +\infty$. Then by Barbălat Lemma in [20], we have

$$\lim_{t\to+\infty} z(t) = 0, \ \lim_{t\to+\infty} \varepsilon(t) = 0.$$

Then by coordinate transformations $z_i = \dfrac{\hat{x}_i}{\gamma^{i-1+\sigma}}$ and $\varepsilon_i = \dfrac{x_i - \hat{x}_i}{\gamma^{i-1+\sigma}}$, $i = 1, \cdots, n$ and noting the boundedness of γ, we have

$$\lim_{t\to+\infty} x(t) = 0, \ \lim_{t\to+\infty} \hat{x}(t) = 0.$$

Proposition 3 is thus proved. QED.

Theorem 1 Consider system (1) under Assumption 1. The event-triggered output-feedback controller (4) with dynamic event-triggering mechanism (5), based on the observer (**O**) and dynamic high gain (**G**), guarantees that, for any given initial value $(x_0, \hat{x}_0, \gamma_1(0), \gamma_2(0))$, all the resulting closed-loop system signals are well-defined and bounded on $[0, +\infty)$, and furthermore, $\lim\limits_{t\to+\infty}(x(t), \hat{x}(t), u(t)) = 0$ while no Zeno occurs.

Proof As discussed earlier, for any given initial value $(x_0, \hat{x}_0, \gamma_1(0), \gamma_2(0))$, the resulting closed-loop system (consisting of (1), (3) and (4) together with (5) has a unique solution $(x(t), \hat{x}(t), \gamma_1(t), \gamma_2(t))$ on $[0, T_{\mathrm{m}})$.

In view of Propositions 2 and 3, it suffices to prove the boundedness of $\gamma_1(t)$ and $\gamma_2(t)$ on $[0, T_{\mathrm{m}})$. In fact, from the boundedness of $\gamma_1(t)$ and $\gamma_2(t)$ and Proposition 2, it follows that all the signals of the resulting closed-loop system are bounded, which satisfies the hypothesis of Proposition 3. We first prove that $\gamma_2(t)$ is bounded. Suppose for contradiction that it is unbounded on $[0, T_{\mathrm{m}})$. The nondecreasing property of $\gamma_2(t)$ implies that $\lim\limits_{t\to T_{\mathrm{m}}} \gamma_2(t) = +\infty$, i.e., there is a $T_0 \in [0, T_{\mathrm{m}})$ such that, for any $t \in [T_0, T_{\mathrm{m}})$, $\gamma_2(t) \geqslant 4\Theta + 2$. This, together with (7), $\gamma_1(t) \geqslant 1$ and $\chi(t) \geqslant 0$, implies that $D^+V \leqslant -\dfrac{\gamma_1}{2}\|\varepsilon\|^2$. Then by integrating over $[T_0, t)$ for any $t \in [T_0, T_{\mathrm{m}})$, we have

$$\lim_{t\to T_{\mathrm{m}}} \frac{1}{2}\int_{T_0}^t \gamma_1(\tau)\|\varepsilon(\tau)\|^2 \mathrm{d}\tau \leqslant V(T_0).$$

By this and the dynamics of γ_2 in (3), we get

$$\begin{aligned} &+\infty = \\ &\lim_{t\to T_{\mathrm{m}}} \gamma_2(t) - \gamma_2(T_0) = \lim_{t\to T_{\mathrm{m}}} \int_{T_0}^t \dot{\gamma}_2(\tau)\mathrm{d}\tau \leqslant \\ &\lim_{t\to T_{\mathrm{m}}} \int_{T_0}^t \gamma_1(\tau)\|\varepsilon(\tau)\|^2 \mathrm{d}\tau \leqslant 2V(T_0) < +\infty, \end{aligned}$$

a contradiction. Thus, $\gamma_2(t)$ is bounded on $[0, T_{\rm m})$.

We next prove that $\gamma_1(t)$ is bounded on $[0, T_{\rm m})$. For this, we claim that $|x_1|/\gamma_1^\sigma$ is bounded. In fact,

$$\frac{|x_1|}{\gamma_1^\sigma} \leqslant \gamma_2^\sigma\Big(\frac{|\hat{x}_1|}{\gamma^\sigma} + \frac{|x_1 - \hat{x}_1|}{\gamma^\sigma}\Big) = \gamma_2^\sigma(|z_1| + |\varepsilon_1|),$$

which together with the boundedness of $\gamma_2(t)$ and Proposition 2, indicates that there is a positive constant $\bar{b}$ such that $|x_1| \leqslant \bar{b}\gamma_1^\sigma$. Then noting the dynamics of γ_1 in (3) and $0 < \sigma < \dfrac{1}{4p}$, we have

$$\begin{aligned}\dot{\gamma}_1 = {} & m_1(1 + |x_1|^p)^2 - m_2(\gamma_1 - 1) \leqslant \\ & 2m_1 + 2m_1\bar{b}^{2p}\gamma_1^{2\sigma p} - m_2\gamma_1 + m_2 \leqslant \\ & -\frac{m_2}{2}\gamma_1 + \frac{2m_1^2\bar{b}^{4p}}{m_2} + 2m_1 + m_2.\end{aligned}$$

This implies that $\gamma_1(t)$ is bounded on $[0, T_{\rm m})$. The proof is thus completed. QED.

Remark 1 The dynamic event-triggering mechanism proposed in this paper could possibly achieve further saving of computation/communication resources than a time-varying one. This can be seen from the proof of Proposition 3, which indicates that the minimum inter-execution time of the dynamic event-triggering mechanism cannot be smaller than that of a time-varying one.

5 A simulation example

In this section, a simulation example will be given to illustrate the effectiveness of the proposed event-triggered output-feedback controller of the following second-order controlled pendulum system:

$$ml\ddot{\xi} + kl\dot{\xi} + mg\sin\xi = u, \tag{19}$$

where ξ is the angle between the pendulum and the vertical direction, m and l are the mass of the bob and the length of the rod, respectively, k, which represents the friction coefficient, is an unknown constant and g is the acceleration of gravity.

Let $x_1 = ml\xi$ and $x_2 = ml\dot{\xi}$. Then system (19) becomes

$$\begin{cases}\dot{x}_1 = x_2, \\ \dot{x}_2 = u - mg\sin\dfrac{x_1}{ml} - \dfrac{k}{m}x_2, \\ y = x_1.\end{cases} \tag{20}$$

Choose $a_1 = 2$, $a_2 = 4$ and $\gamma_1 \equiv 4$. Then the observer is designed as

$$\begin{cases}\dot{\hat{x}}_1 = \hat{x}_2 + 8\gamma_2(x_1 - \hat{x}_1), \\ \dot{\hat{x}}_2 = u + 64\gamma_2^2(x_1 - \hat{x}_1),\end{cases}$$

where $\dot{\gamma}_2 = \dfrac{2(x_1 - \hat{x}_2)^2}{\gamma_2^{\frac{1}{2}}}$.

We can verify that system (20) satisfies Assumption (1) by setting $\theta = \max\{\dfrac{g}{l}, \dfrac{k}{m}\}$. Then by the previous design procedure, we design the event-triggered controller in (4) as ($t \in [t_k, t_{k+1})$):

$$u(t) = -6.4\gamma_2^2(t_k)\hat{x}_1(t_k) - 1.6\gamma_2(t_k)\hat{x}_2(t_k)$$

with $b_1 = 0.4$ and $b_2 = 0.4$, and introduce the following event-triggering mechanism and the inter-dynamic variable (with $\sigma = \dfrac{1}{4}$):

$$\begin{aligned}t_{k+1} = \inf\{t > t_k \mid & \frac{2\|PE_n\|^2(u - \omega)^2}{\gamma_2^{\frac{7}{2}}} - \\ & \frac{\|\hat{x}\|^2}{4\gamma_2^{\frac{3}{2}}} \geqslant \chi\},\end{aligned}$$

$$\dot{\chi} = \frac{\|\hat{x}\|^2}{4\gamma_2^{\frac{3}{2}}} - \frac{2\|PE_n\|^2(u - \omega)^2}{\gamma_2^{\frac{7}{2}}} - 5\chi$$

with $\chi(0) = 4$. The globally stabilization of system (19) can be achieved by the designed event-triggered output-feedback controller. Notably, owning to the unknown k, the event-triggered schemes in [7–8, 16–18] are not valid any more. In addition, the convergence of the system states can not be guaranteed by the event-triggered strategies proposed in [13].

Let $m = 0.25$, $l = 4$, $k = 0.25$ and g $= 10$ and the initial value is selected as $[\xi(0), \dot{\xi}(0), \hat{x}_1(0), \hat{x}_2(0), \chi(0)]^{\rm T} = [-2, 2, -1.5, 3, 1]^{\rm T}$. Then Figs. 1–4 are obtained to exhibit the trajectories of the resulting closed-loop system. From the figure we can see that (ξ, $\dot{\xi}$, $\hat{x}_1$, $\hat{x}_2$, γ_2, χ, u) are globally bounded, and furthermore, (ξ, $\dot{\xi}$, $\hat{x}_1$, $\hat{x}_2$, u) ultimately converge to zero.

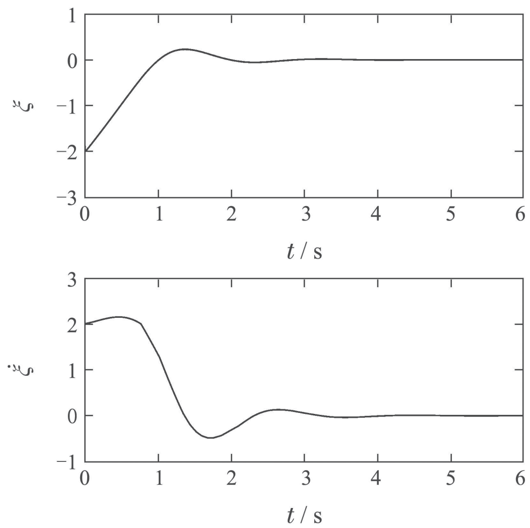
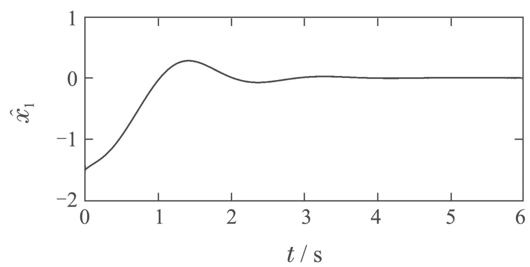


Fig. 1 The trajectories of ξ and $\dot{\xi}$



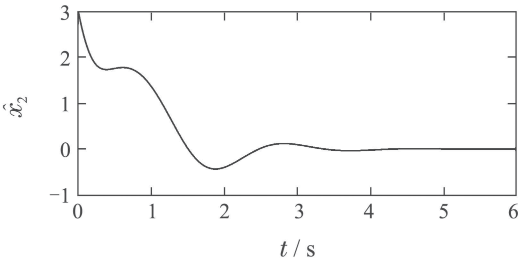


Fig. 2 The trajectories of $\hat{x}_1$ and $\hat{x}_2$

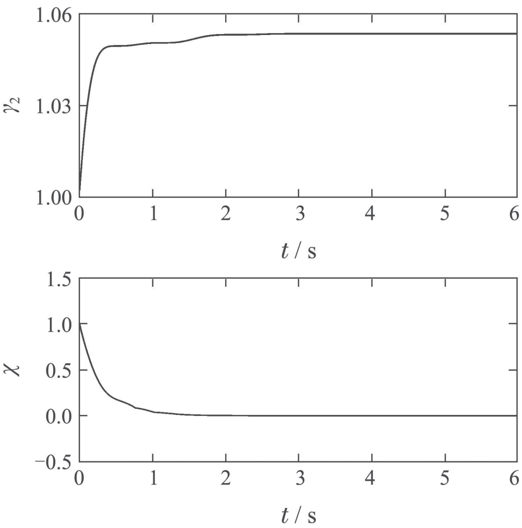


Fig. 3 The trajectories of γ_2 and χ

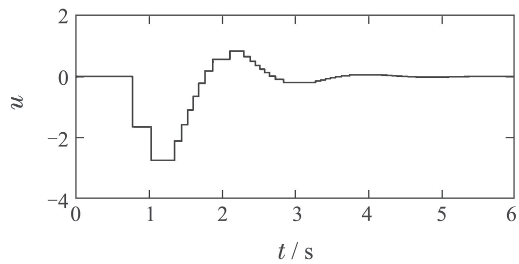


Fig. 4 The trajectory of u

6 Conclusions

In this paper, a dynamic event-triggered output-feedback scheme has been proposed for uncertain nonlinear systems. Detailedly, an observer with dynamic high gain is introduced to reconstruct the unmeasured system states, and meanwhile the unknown growth rate is counteracted by the dynamic high gain. Then, an event-triggered output-feedback controller with a dynamic event-triggering mechanism rather than a static one is successfully designed to achieve the global boundedness and convergence of the system states. It is worth pointing out that the dynamic event-triggered scheme could possibly possess more potential ability of further saving of computation/communication resources than a time-varying one.

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