具有执行器故障的不确定多智能体系统自适应动态面控制

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摘要: 本文研究了一类具有输入量化、未建模动态和执行器故障的非线性多智能体系统的一致跟踪问题. 引入一个可量测的动态信号消除未建模动态对系统的影响. 利用Young's不等式和高斯函数的性质, 有效地处理了多智能体邻居节点在设计的第一步中对子系统的耦合作用. 通过将滞回量化器表示为具有有界系数和有界扰动的输入线性函数, 并利用动态面控制方法, 提出一种自适应神经网络动态面控制方案, 简化了控制器的设计, 保证了闭环系统的所有信号都是半全局一致终结有界的, 所有跟随者都能实现期望的一致性. 最后, 仿真结果验证了所提出的自适应控制策略的有效性.

关键词: 多智能体系统; 执行器故障; 输入量化; 动态面控制; 未建模动态

引用格式: 林曼菲, 张天平. 具有执行器故障的不确定多智能体系统自适应动态面控制. 控制理论与应用, 2021, 38(9): 1452 – 1465

DOI: 10.7641/CTA.2021.00512

Adaptive dynamic surface control for uncertain multi-agent systems with actuator failures

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Abstract: In this paper, the problem of consensus tracking is studied for a class of nonlinear multi-agent systems with input quantization and unmodeled dynamics as well as actuator failures. A measurable dynamic signal is introduced to eliminate influence of unmodeled dynamics on the system. With the help of Young's inequality and the characteristic of Gaussian function, the interconnection of neighbors of multi-agent at the first step of controller design is effectively handled. Using the linear function of input with bounded coefficient and bounded disturbance for the hysteresis quantizer and dynamic surface control method, an adaptive dynamic surface control scheme is proposed, and the design of the controller is simplified. It guarantees that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded (UUB), and all the followers can accomplish the desired consensus results. Finally, the simulation results verify the availability of the proposed adaptive control strategy.

Key words: multi-agent systems; actuator failure; input quantification; dynamic surface control; unmodeled dynamics

Citation: LIN Manfei, ZHANG Tianping. Adaptive dynamic surface control for uncertain multi-agent systems with actuator failures. *Control Theory & Applications*, 2021, 38(9): 1452 – 1465

1 引言

多智能体系统是由多个智能体组成的分布式独立系统, 它能够解决单一智能体无法解决的问题, 并提高其效率及鲁棒性. 特别是分布式多智能体系统的一致性控制在传感器网络、随机多智能体系统、多机器人、机器人编队等领域得到了广泛的研究. 多智能体系统的模型可分为无头节点模型和带有头节点的模型[1]. 对于多智能体系统的分布式控制, 多智能体的一致性控制已成为一个主要的研究方向, 它的目的是设计分布式算法, 使得一组智能体达到给定的协议. 文献[2]利用强化学习控制方法(reinforcement learning control, RLC) 研究了多智能体系统 (multi-agent systems, MAS)中的一致性问题, 并给出了一种可视的分布式量化协议来更新动态系统. 文献[3]研究了不可测量状态非线性多智能体系统的一致量化控制设计问题. 文献[4]研究了具有固定拓扑结构的无向图上多

收稿日期: 2020−08−05; 录用日期: 2021−03−04.
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本文责任编委: 张承慧.
国家自然科学基金项目(62073283), 江苏省自然科学基金项目(BK20181218), 扬州大学高端人才支持计划项目(2016)资助.
Supported by the National Natural Science Foundation of China (62073283), the Natural Science Foundation of Jiangsu Province (BK20181218) and the Yangzhou University Top-level Talents Support Program (2016).

智能体系统的分布式一致跟踪问题, 并将命令滤波器后推技术推广到一致性跟踪控制问题中, 避免了后推设计中存在的“复杂性爆炸”问题. 文献[5]研究了严格反馈形式下的多智能体系统的一致性问题. 文献[6]考虑了一组高阶非线性严格反馈多智能体系统在定向通信拓扑上跟踪期望轨迹的输出一致问题.

在实际工业应用中, 当执行器将设计的控制信号输入到实际系统中时, 它们面临两个问题: 信号量化或执行器故障. 量化控制技术广泛应用于现代生产过程、电力系统、智能交通和需要通信的智能设备等领域. 量化是连续信号离散化的过程, 必然会给系统带来额外的非线性. 文献[7]引入了一种滞回量化器, 并设计了一种基于动态面控制的控制律方法, 去除了早期文献中量化系统的许多假设. 文献[8]讨论了一类具有输入量化、未建模动态和输出约束的非线性系统的自适应输出反馈跟踪控制问题. 引入了一类具有滞回和均匀量化的新的量化器来处理输入信号. 文献[9–10]提出了一种有限时间输入量化自适应控制方法. 文献[11–12]提出了一种由均匀量化器和对数量化器的组合的量化器. 另一个问题是执行器故障, 执行器故障分为部分失效和完全失效, 执行器完全失效时, 输出不再受执行器的影响[13–14]. 文献[15]引入光滑函数来处理量化以及执行器故障对于系统性能的影响, 并将量化以及执行器故障推广到多智能体系统中. 文献[16]针对未知执行器故障和系统不确定性, 提出了一种自适应控制方案, 在执行器故障的情况下实现期望的输出跟踪和闭环稳定.

将自适应动态面方法和神经网络相结合有效地解决了一类自适应非线性系统的鲁棒自适应跟踪控制问题. 自文献[17]提出后推设计方法以来, 后推设计被广泛应用于非线性系统的控制器设计. 但是由于在递归设计的每一步都需要对虚拟控制律进行求导, 造成了复杂性爆炸问题. 为了解决这一问题[18]在后推设计的每一步中引入一阶滤波器, 提出了动态面控制. 文献[19]研究了具有动态不确定性的自适应动态面控制. 文献[20]研究了一类带有未知死区的纯反馈非线性系统的自适应动态面控制. 文献[21]针对一类具有全状态约束和动态不确定性的纯反馈非线性系统, 利用径向基函数神经网络 (radial basis function neural networks, RBFNNs)建立了自适应神经网络动态面控制(dynamic surface control, DSC). 文献[22]将动态面控制方法应用于随机系统中. 在实际系统中存在许多不确定性, 如未建模动态、死区和状态约束, 这可能会降低系统的性能, 甚至影响系统的稳定性. 未建模动态是不确定非线性系统研究中考虑的一种不确定性. 文献[23–24]引入动态信号来处理未建模动态. 文献[25]用李雅普诺夫函数来处理未建模动态.

本文在已有文献的基础上, 主要解决了一类具有输入量化、未建模动态和执行器故障的非线性多智能体系统的一致跟踪问题. 当多智能体系统同时存在输入量化、未建模动态和执行器故障时, 每一种不确定性都会对系统稳定性产生影响, 如何处理这些不确定项对系统的影响, 以及如何将输入量化以及执行器故障模型线性化表示并设计合适的控制器是本文面临的挑战. 本文的主要贡献如下: 1) 利用Young’s不等式和高斯函数的性质, 有效地处理了多智能体邻居节点在设计的第1步中对子系统的耦合作用, 减少了所设计的虚拟控制中神经网络基向量的变量个数, 简化了稳定性分析; 2) 与文献[3]相比, 本文考虑了多智能体系统中存在未建模动态以及执行器故障的问题, 克服了一阶滤波器常数无法求解的问题, 并在设计过程中简化了自适应律的设计; 3) 与文献[15]相比, 本文对执行器故障模型和输入量化模型进行线性化处理, 避免设计中间控制器, 并且简化了自适应律的设计, 同时采用自适应动态面方法, 最后证明了领导者和跟随者之间达成期望一致, 同时保持误差一致有界.

2　问题描述与预备知识

2.1　问题描述

考虑由1个领导者和N个跟随者组成的具有滞回量化器和执行器故障的不确定多智能体系统, 第i个跟随者系统和滞回量化器输出$Q_i(u_{i,f})$描述如下:

$$\begin{cases}\dot{z}_i = q_i(z_i, \bar{x}_{i,n}, t),\\ \dot{x}_{i,1} = x_{i,2} + f_{i,1}(x_{i,1}) + \varDelta_{i,1}(z_i, \bar{x}_{i,n}, t),\\ \dot{x}_{i,j} = x_{i,j+1} + f_{i,j}(\bar{x}_{i,j}) + \varDelta_{i,j}(z_i, \bar{x}_{i,n}, t),\\ \dot{x}_{i,n} = \sum\limits_{f=1}^{p}[k_{i,f,h}Q_i(u_{i,f}(t)) + u_{i,sf,h}(t)] +\\ \qquad f_{i,n}(\bar{x}_{i,n}) + \varDelta_{i,n}(z_i, \bar{x}_{i,n}, t),\\ y_i = x_{i,1},\end{cases} \tag{1}$$

$$Q_i(u_{i,f}) = \begin{cases} u_{i,f,s}\,\mathrm{sgn}\,u_{i,f},\\ \dfrac{u_{i,f,s}}{1+\delta_{i,f}} < |u_{i,f}| \leqslant u_{i,f,s},\ \dot{u}_{i,f} < 0,\ \text{或}\\ u_{i,f,s} < |u_{i,f}| \leqslant \dfrac{u_{i,f,s}}{1-\delta_{i,f}},\ \dot{u}_{i,f} > 0;\\ u_{i,f,s}\,\mathrm{sgn}\,u_{i,f}(1+\delta_{i,f}),\\ u_{i,f,s} < |u_{i,f}| \leqslant \dfrac{u_{i,f,s}}{1-\delta_{i,f}},\ \dot{u}_{i,f} < 0,\ \text{或}\\ \dfrac{u_{i,f,s}}{1-\delta_{i,f}} < |u_{i,f}| \leqslant \dfrac{u_{i,f,s}(1+\delta_{i,f})}{1-\delta_{i,f}},\ \dot{u}_{i,f} > 0;\\ 0, 0 \leqslant |u_{i,f}| < \dfrac{u_{i,f\min}}{1+\delta_{i,f}},\ \dot{u}_{i,f} < 0,\ \text{或}\\ \dfrac{u_{i,f\min}}{1+\delta_{i,f}} \leqslant |u_{i,f}| \leqslant u_{i,f\min},\ \dot{u}_{i,f} > 0;\\ Q_i(u_{i,f}(t^-)),\ \dot{u}_{i,f} = 0,\end{cases} \tag{2}$$

其中: $j=2,\cdots,n-1$; $i=1,\cdots,N$; $\bar{x}_{i,k}=[x_{i,1}\ x_{i,2}\ \cdots\ x_{i,k}]^{\mathrm{T}}\in\mathbb{R}^k$, $k=1,\cdots,n$; $\bar{x}_{i,n}$是状态向量; $y_i\in\mathbb{R}$表示系统输出; $f_{i,j}(\bar{x}_{i,j})$为未知非线性光滑函数; $\varDelta_{i,j}(z_i,\bar{x}_{i,j},t)(j=1,\cdots,n)$是未知非线性动态扰动; $z_i\in\mathbb{R}$是未建模动态; $q_i(z_i,\bar{x}_{i,j},t)\in\mathbb{R}$是满足Lipschitz条件的未知函数向量; $u_{i,f}\in\mathbb{R}$ $(f=1,\cdots,p)$为第i个跟随者第f个执行器控制输入; $Q_i(u_{i,f}(t))$为滞回量化器的输出; $k_{i,f,h}\in[0,1]$是效率系数; $u_{i,sf,h}$是第i个智能体的第f个执行器故障函数, 它是一个未知函数, 同时满足分段连续有界条件; $u_{i,f,s}=\rho_{i,f}^{1-s}u_{i,f,\min}$, $s=1,2,\cdots$, $u_{i,f,\min}>0$与$0<\rho_{i,f}<1$决定了$Q_i(u_{i,f})$死区的大小; $\delta_{i,f}=\dfrac{1-\rho_{i,f}}{1+\rho_{i,f}}$. $Q_i(u_{i,f})$在集合$U=\{0,\pm u_{i,f,s},\pm u_{i,f}(1+\delta_{i,f,s})\}$内取值, $\rho_{i,f}$是量化密度. 令

$$q_{i,f,1}(u_{i,f})=\begin{cases}\dfrac{Q_i(u_{i,f})}{u_{i,f}}, & |u_{i,f}|>u_{i,f,\min},\\ 1, & |u_{i,f}|\leqslant u_{i,f,\min},\end{cases}\tag{3}$$

$$q_{i,f,2}(u_{i,f})=\begin{cases}0, & |u_{i,f}|>u_{i,f,\min},\\ -u_{i,f}, & |u_{i,f}|\leqslant u_{i,f,\min},\end{cases}\tag{4}$$

由式(2)得$1-\delta_{i,f}\leqslant q_{i,f,1}(u_{i,f})\leqslant 1+\delta_{i,f}$, $|q_{i,f,2}(u_{i,f})|\leqslant u_{i,f,\min}$. 将上式表示为

$$Q_i(u_{i,f})=q_{i,f,1}(u_{i,f})u_{i,f}+q_{i,f,2}(u_{i,f}).\tag{5}$$

第f个执行器在$t^{\mathrm{s}}_{i,f,h}$到$t^{\mathrm{e}}_{i,f,h}$时间段内发生第h次故障, 故障模型描述如下:

$$\begin{aligned}&k_{i,f,h}(q_{i,f,1}(t)u_{i,f}+q_{i,f,2}(t))+u_{i,sf,h}(t),\\&t\in[t^{\mathrm{s}}_{i,f,h},t^{\mathrm{e}}_{i,f,h}],\end{aligned}\tag{6}$$

$$k_{i,f,h}u_{i,sf,h}=0,\ h=1,2,3,\cdots,\tag{7}$$

其中: $k_{i,f,h}\in[0,1]$是效率系数; $u_{i,sf,h}$是第i个智能体第f个执行器故障函数, 它是一个未知函数, 同时满足分段连续有界条件. 在一段时间$[t^{\mathrm{s}}_{i,f},t^{\mathrm{e}}_{i,f}]$内, 执行器故障可能多次出现, 其中$t^{\mathrm{s}}_{i,f}$和$t^{\mathrm{e}}_{i,f}$为未知常数且满足$0\leqslant t^{\mathrm{s}}_{i,f}\leqslant t^{\mathrm{s}}_{i,f,1}\leqslant t^{\mathrm{e}}_{i,f,1}\leqslant t^{\mathrm{s}}_{i,f,2}\leqslant t^{\mathrm{e}}_{i,f,2}\leqslant\cdots\leqslant t^{\mathrm{s}}_{i,f,h}\leqslant t^{\mathrm{e}}_{i,f,h}\leqslant t^{\mathrm{e}}_{i,f}$. 例如$[t^{\mathrm{s}}_{i,f,1},t^{\mathrm{s}}_{i,f,1}]$表示第$f$个执行器第1次故障.

故障模型包含如下4种情况:

1) 当$k_{i,f,h}=1$, $u_{i,sf,h}=0$的情况下, 执行器未发生故障, 系统输入为量化信号;

2) 当$k_{i,f,h}\in(0,1)$, $u_{i,sf,h}=0$的情况下, $0<\underline{k}_{i,f,h}\leqslant k_{i,f,h}<1$, 执行器部分失效, 其中$\underline{k}_{i,f,h}$为已知常数;

3) 当$k_{i,f,h}=1$, $u_{i,sf,h}\neq 0$, 执行器未发生故障但对系统产生了一个干扰信号;

4) 当$k_{i,f,h}=0$, $u_{i,sf,h}\neq 0$, 执行器完全失效且对系统产生了一个干扰信号.

本文主要考虑情况1–3, 即系统在$[t^{\mathrm{s}}_{i,f},t^{\mathrm{e}}_{i,f}]$时间内, 第$f$个执行器发生$h$次故障, 在其余时间内, 第$f$个执行器未发生故障, 系统输入为量化信号.

系统的控制目标是考虑在执行器部分失效情况下, 在执行器发生故障的时间段内设计系统正常控制输入$u_{i,f}$, 使闭环系统中所有信号都是半全局一致终结有界的, 此外, 使领导者和追随者之间达成期望一致, 同时保持误差一致有界.

2.2 基本假设

假设 1[3] 相对于跟随者, 领导者的信号$y_{\mathrm{r}}\in\mathbb{R}$是可以获得的, 且满足

$$\begin{aligned}&[y_{\mathrm{r}}\ \dot{y}_{\mathrm{r}}\ \ddot{y}_{\mathrm{r}}]^{\mathrm{T}}\in\varPi_0=\\&\{[y_{\mathrm{r}}(t)\ \dot{y}_{\mathrm{r}}(t)\ \ddot{y}_{\mathrm{r}}(t)]^{\mathrm{T}}:y_{\mathrm{r}}^2(t)+\dot{y}_{\mathrm{r}}^2(t)+\ddot{y}_{\mathrm{r}}^2(t)\leqslant B_0\},\end{aligned}\tag{8}$$

其中$B_0>0$为常数.

假设 2[7] 未知动态扰动$\varDelta_{i,j}(z_i,\bar{x}_{i,n},t)$ $(i=1,\cdots,N;j=1,\cdots,n)$满足下列不等式

$$|\varDelta_{i,j}(z_i,\bar{x}_{i,n},t)|\leqslant\phi_{ij1}(\|\bar{x}_{i,j}\|)+\phi_{ij2}(\|z_i\|),\tag{9}$$

其中: $\phi_{ij1}(\cdot)$是未知光滑函数, $\phi_{ij2}(\cdot)$是未知非负单调递增连续函数.

假设 3[15] 函数$u_{i,sf,h}$有界, 且上界是未知正数, 即存在正常数$\bar{u}_{i,sf,h}$, 使得$|u_{i,sf,h}(t)|\leqslant\bar{u}_{i,sf,h}$, $\forall t\geqslant 0$.

假设 4[15] 所有执行机构不会同时失效且每个执行器发生故障的次数为有限次, 如果执行器故障小于p, 则闭环系统仍与其余执行器良好工作.

假设 5[24] 未建模动态z_i是指数输入状态实用稳定的, 对于系统$\dot{z}_i=q_i(z_i,\bar{x}_{i,n_i},t)$, 存在Lyapunov函数$V_{i0}(z_i)$, 使得

$$\bar{\alpha}_{i,1}(\|z_i\|)\leqslant V_{i,0}(z_i)\leqslant\bar{\alpha}_{i,2}(\|z_i\|),\tag{10}$$

$$\begin{aligned}&\frac{\partial V_{i,0}(z_i)}{\partial z_i}q_i(z_i,\bar{x}_{i,n_i},t)\leqslant\\&-c_iV_{i,0}(z_i)+\gamma_i(|x_{i,1}|)+\iota_i,\end{aligned}\tag{11}$$

其中: $\bar{\alpha}_{i,1}(\cdot),\bar{\alpha}_{i,2}(\cdot)$是$K_\infty$类函数, $\gamma_i(\cdot)$是已知K_∞类函数, c_i和ι_i是已知正常数.

假设 6[3] 多智能体存在根节点, 根节点与其他跟随者之间有连接且根节点能够获取来自于领导者的信息.

引理 1[24] 如果$V_{i,0}$是系统$\dot{z}_i=q_i(z_i,\bar{x}_{i,n},t)$的一个指数输入状态实用稳定的Lyapunov函数, 当式(10)–(11)成立, 对任意常数$c_{i0}\in(0,c_i)$, 任意初始时刻$t_{i0}>0$, 任意初始条件$z_{i0}=z_i(t_{i0})$, $v_{i0}>0$, 对任意

连续K_∞类函数$\bar{\gamma}_i(|x_{i,1}|)$, 满足$\bar{\gamma}_i(|x_{i,1}|)\geqslant\gamma_i(|x_{i,1}|)$, 则存在一个有限时间$T_{i0}=\max\{0,\ln[\dfrac{V_{i,0}(z_{i0})}{v_{i0}}]/(c_i-c_{i0})\}\geqslant 0$, 一个非负函数$D_i(t_{i0},t)$, 对于所有$t\geqslant t_{i0}$, 信号可以描述为

$$\dot{v}_i=-c_{i0}v_i+\bar{\gamma}_i(|x_{i,1}|)+\iota_i,\ v_i(t_{i0})=v_{i0}>0, \tag{12}$$

因此, 当$t\geqslant t_{i0}+T_{i0}$时, $D_i(t_{i,0},t)=0$, 且$V_{i,0}(z_i)\leqslant v_i(t)+D_i(t_{i0},t)$, 其中$D_i(t_{i0},t)=\max\{0,\mathrm{e}^{-c_i(t-t_{i0})}V_{i,0}(z_{i0})-\mathrm{e}^{-c_{i0}(t-t_{i0})}v_{i0}\}$.

2.3　图理论

令$G=(\varGamma,\zeta)$作为一个有向图来模拟N个跟随者之间的信息交换; 其中$\varGamma=\{n_1,\cdots,n_N\}$表示节点集合, $\zeta=\{(n_i,n_j)\}\in\varGamma\times\varGamma$表示边集合. 定义邻接矩阵$A=[a_{ij}]\in\mathbb{R}^{N\times N}$, $a_{ij}\geqslant 0$. 对于有向图, 若$(j,i)\in\zeta$, $i\leftarrow j$表明第i个智能体可以获取来自第j个智能体的信息, 此时$a_{ij}=1$, 否则$a_{ij}=0$. 定义度矩阵以$D=\mathrm{diag}\{d_1,\cdots,d_N\}$以及Laplacian矩阵$L$, $L=D-A$, 其中$d_i=\sum\limits_{j\in N_i}a_{ij}$, N_i为智能体i的邻居的集合, 定义为$N_i=\{j\in\varGamma:(j,i)\in\zeta\}$. 从智能体$i$到智能体$j$的有向图是$\{(i,l),(l,m),\cdots,(k,j)\}$的连续边缘序列形式. 若存在一个称为根节点的智能体i, 从i到其他智能体都有一条有向通路, 则有向图G被称为生成树. 此外, 定义$A_0=\mathrm{diag}\{a_{10},\cdots,a_{N0}\}$是领导者的邻接矩阵. $a_{i0}=1$表示第i个跟随者可以获取来自领导者的信息, 否则$a_{i0}=0$, 令$H=L+A_0$.

引理 2[3]　如果生成树存在于有向图G中, 根跟随者有机会获得领导者的信息, 那么矩阵H所有特征值均具有正实部.

2.4　神经网络逼近

在给定的紧集$\varOmega_{\mathrm{x}}\subset\mathbb{R}^n$, 利用径向基函数神经网络逼近未知连续函数$f_i(x)$, 有

$$f_i(x)=\theta_i^{*\mathrm{T}}\phi_i(x)+\delta_i(x), \tag{13}$$

其中: $\delta_i(x)$为逼近误差, $\phi_i(x)=[\phi_{i1}(x)\cdots\phi_{iM_i}(x)]^{\mathrm{T}}\in\mathbb{R}^{M_i}$为基向量, 其分量基函数$\phi_{ij}(x)$取为下述高斯函数:

$$\phi_{i1}(x)=\exp[-\frac{(x-\mu_{ij})^2}{b_{ij}^2}], \tag{14}$$

其中: $1\leqslant i\leqslant n$, $1\leqslant j\leqslant M_i$, μ_{ij}和b_{ij}分别为高斯函数的中心和宽度, M_i为第i个神经网络的节点数, 理想权重θ_i^*定义为

$$\theta_i^*=\arg\min_{\theta_i\in\mathbb{R}^{M_i}}[\sup_{x\in\varOmega_{\mathrm{x}}}|\theta_i^{\mathrm{T}}\phi_i(x)-f_i(x)|]. \tag{15}$$

引理 3[26]　若$Z=[\chi_1\ \ \cdots\ \ \chi_N]^{\mathrm{T}}\in\mathbb{R}^N$, $X=[\chi_{i_1}\ \ \cdots\ \ \chi_{i_k}]^{\mathrm{T}}\in\mathbb{R}^k$, $\{i_1,\cdots,i_k\}$是$\{1,\cdots,N\}$的一个子序列, $\mu=[\mu_1\ \ \cdots\ \ \mu_N]^{\mathrm{T}}$, $\sigma=[\sigma_1\ \ \cdots\ \ \sigma_N]^{\mathrm{T}}$是两个常数向量,

$$\sigma_1,\cdots,\sigma_N>0,$$
$$\varphi(Z)=\prod_{j=1}^N\exp\{-\frac{(\chi_j-\mu_j)^2}{2\sigma_j^2}\},$$
$$\varphi(X)=\prod_{j=1}^k\exp\{-\frac{(\chi_{i_j}-\mu_{i_j})^2}{2\sigma_{i_j}^2}\},$$

则$\varphi(Z)\leqslant\varphi(X)$, 即

$$\prod_{j=1}^N\exp\{-\frac{(\chi_j-\mu_j)^2}{2\sigma_j^2}\}\leqslant\prod_{j=1}^k\exp\{-\frac{(\chi_{i_j}-\mu_{i_j})^2}{2\sigma_{i_j}^2}\}.$$

3　自适应动态面控制器设计

为每个跟随者设计一种自适应神经网络动态面控制, 根据跟随者之间的关系定义坐标变换如下:

$$\begin{cases}s_{i,1}=\sum\limits_{j\in N_i}a_{ij}(y_i-y_j)+a_{i0}(y_i-y_{\mathrm{r}}(t)),\\ s_{i,j}=x_{i,j}-\omega_{i,j},\\ y_{i,j}=\omega_{i,j}-\alpha_{i,j},\end{cases} \tag{16}$$

其中: $j=2,\cdots,n$; $\omega_{i,j},\alpha_{i,j}$稍后给出.

为了叙述方便, 定义一些符号如下:

$$\underline{s}_{N,k}=[s_{1,k}\ \ \cdots\ \ s_{N,k}]^{\mathrm{T}},\ \bar{\underline{s}}_{N,k}=[\underline{s}_{N,1}^{\mathrm{T}}\ \ \cdots\ \ \underline{s}_{N,k}^{\mathrm{T}}]^{\mathrm{T}},$$
$$k=1,\cdots,n,\ \bar{y}_{i,j}=[y_{i,2}\ \ \cdots\ \ y_{i,j}]^{\mathrm{T}},$$
$$\underline{y}_{N,k}=[y_{1,k}\ \ \cdots\ \ y_{N,k}]^{\mathrm{T}},\ k=2,\cdots,n,$$
$$\bar{\underline{y}}_{N,k}=[\underline{y}_{N,2}^{\mathrm{T}}\ \ \cdots\ \ \underline{y}_{N,k}^{\mathrm{T}}]^{\mathrm{T}},\ k=2,\cdots,n,$$
$$y=[y_1\ \ y_2\ \ \cdots\ \ y_N]^{\mathrm{T}},\ \bar{v}_{\mathrm{N}}=\{v_1,\cdots,v_N\},$$
$$1_N=[1\ \ \cdots\ \ 1]^{\mathrm{T}}\in\mathbb{R}^N,\ e_{i,1}=y_i-y_{\mathrm{r}},$$
$$e_1=[e_{1,1}\ \ e_{2,1}\ \ \cdots\ \ e_{N,1}]^{\mathrm{T}},$$
$$\underline{x}_{N,1}=[x_{1,1}\ \ \cdots\ \ x_{N,1}]^{\mathrm{T}},\ \underline{x}_{N,2}=[x_{1,2}\ \ \cdots\ \ x_{N,2}]^{\mathrm{T}}.$$

由$s_{i,1}=\sum\limits_{j\in N_i}a_{ij}(y_i-y_j)+a_{i0}(y_i-y_{\mathrm{r}}(t))$可得

$$\underline{s}_{N,1}=\begin{bmatrix}\sum\limits_{j=1}^N a_{1j} & -a_{12} & \cdots & -a_{1N}\\ -a_{21} & \sum\limits_{j=1}^N a_{2j} & \cdots & -a_{2N}\\ \vdots & \vdots & & \vdots\\ -a_{N1} & -a_{N2} & \cdots & \sum\limits_{j=1}^N a_{Nj}\end{bmatrix}e_1+\begin{bmatrix}a_{10} & 0 & \cdots & 0\\ 0 & a_{20} & \cdots & 0\\ \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & a_{N0}\end{bmatrix}e_1=(L+A_0)e_1=H(y-1_Ny_{\mathrm{r}}(t)).$$

根据引理2可知, H可逆. 因此, 有

$$y = H^{-1}\underline{s}_{N,1} + 1_N y_{\mathrm{r}}(t). \tag{17}$$

步骤 1 由于第i个跟随者的同步误差为

$$s_{i,1} = \sum_{j\in N_i} a_{ij}(y_i - y_j) + a_{i0}(y_i - y_{\mathrm{r}}(t)).$$

将$s_{i,1}$对时间t求导, 可得

$$\begin{aligned}
&\dot{s}_{i,1} = \\
&(\sum_{j\in N_i} a_{ij} + a_{i0})\dot{y}_i - \sum_{j\in N_i} a_{ij}\dot{y}_j - a_{i0}\dot{y}_{\mathrm{r}}(t) = \\
&(\sum_{j\in N_i} a_{ij} + a_{i0})(x_{i,2} + f_{i,1}(x_{i,1}) + \\
&\varDelta_{i,1}(z_i, \bar{x}_{i,n}, t)) - a_{i0}\dot{y}_{\mathrm{r}}(t) - \\
&\sum_{j\in N_i} a_{ij}(x_{j,2} + f_{j,1}(x_{j,1}) + \varDelta_{j,1}(z_j, \bar{x}_{j,n}, t)) = \\
&(\sum_{j\in N_i} a_{ij} + a_{i0})(s_{i,2} + y_{i,2} + \alpha_{i,2} + \\
&f_{i,1}(x_{i,1}) + \varDelta_{i,1}(z_i, \bar{x}_{i,n}, t)) - \\
&\sum_{j\in N_i} a_{ij}(x_{j,2} + \varDelta_{j,1}(z_j, \bar{x}_{j,n}, t) + \\
&f_{j,1}(x_{j,1})) - a_{i0}\dot{y}_{\mathrm{r}}(t).
\end{aligned} \tag{18}$$

定义Lyapunov函数为

$$V_1 = \frac{1}{2}\sum_{i=1}^{N}[s_{i,1}^2 + \frac{1}{\eta_{i,1}}\tilde{\lambda}_{i,1}^2]. \tag{19}$$

将V_1关于时间t求导得

$$\begin{aligned}
&\dot{V}_1 = \\
&\sum_{i=1}^{N}\{s_{i,1}[(\sum_{j\in N_i} a_{ij} + a_{i0})(s_{i,2} + y_{i,2} + \alpha_{i,2} + \\
&f_{i,1}(x_{i,1}) + \varDelta_{i,1}(z_i, \bar{x}_{i,n}, t)) - \\
&a_{i0}\dot{y}_{\mathrm{r}}(t) - \sum_{j\in N_i} a_{ij}(x_{j,2} + f_{j,1}(x_{j,1}) + \\
&\varDelta_{j,1}(z_i, \bar{x}_{i,n}, t))] - \frac{1}{\eta_{i,1}}\tilde{\lambda}_{i,1}\dot{\hat{\lambda}}_{i,1}\} = \\
&\sum_{i=1}^{N}\{s_{i,1}[(d_i + a_{i0})(s_{i,2} + y_{i,2} + \alpha_{i,2} + \\
&f_{i,1}(x_{i,1}) + \varDelta_{i,1}(z_i, \bar{x}_{i,n}, t)) - a_{i0}\dot{y}_{\mathrm{r}}(t) - \\
&\sum_{j\in N_i} a_{ij}(x_{j,2} + f_{j,1}(x_{j,1}) + \\
&\varDelta_{j,1}(z_j, \bar{x}_{j,n}, t))] - \frac{\tilde{\lambda}_{i,1}\dot{\hat{\lambda}}_{i,1}}{\eta_{i,1}}\},
\end{aligned} \tag{20}$$

其中: $x_{i,2} = s_{i,2} + y_{i,2} + \alpha_{i,2}$, $d_i = \sum_{j\in N_i} a_{ij}$.

由Young's不等式得

$$\begin{aligned}
&s_{i,1}(d_i + a_{i0})(s_{i,2} + y_{i,2}) \leqslant \\
&s_{i,1}^2(d_i + a_{i0})^2 + \frac{1}{2}s_{i,2}^2 + \frac{1}{2}y_{i,2}^2,
\end{aligned} \tag{21}$$

$$\begin{aligned}
&s_{i,1}(d_i + a_{i0})\varDelta_{i,1}(z_i, \bar{x}_{i,n}, t) \leqslant \\
&|s_{i,1}|(d_i + a_{i0})[\phi_{i11}(|x_{i,1}|) + \phi_{i12}(\|z_i\|)] \leqslant \\
&s_{i,1}^2(d_i + a_{i0})^2[\phi_{i11}(|x_{i,1}|) + \\
&\phi_{i12}(\bar{\alpha}_{i,1}^{-1}(v_i + D_{i0}))]^2/2 + \frac{1}{2},
\end{aligned} \tag{22}$$

$$\begin{aligned}
&s_{i,1}\sum_{j\in N_i} a_{ij}[x_{j,2} + f_{j,1}(x_{j,1}) + \varDelta_{j,1}(z_j, \bar{x}_{j,n}, t)] \leqslant \\
&|s_{i,1}|\sum_{j\in N_i} a_{ij}[|x_{j,2}| + |f_{j,1}(x_{j,1})| + \\
&\phi_{j11}(|x_{j,1}|) + \phi_{j12}(\|z_j\|)] \leqslant \\
&s_{i,1}^2 d_i^2 \sum_{j\in N_i}[|x_{j,2}| + |f_{j,1}(x_{j,1})| + \phi_{j11}(|x_{j,1}|) + \\
&\phi_{j12}(\bar{\alpha}_{j,1}^{-1}(v_j + D_{j0}))]^2/2 + \frac{d_i}{2}.
\end{aligned} \tag{23}$$

令

$$\begin{aligned}
&H_{i,1}(X_{i,1}) = \\
&s_{i,1}(d_i + a_{i0})^2[\phi_{i11}(|x_{i,1}|) + \\
&\phi_{i12}(\bar{\alpha}_{i1}^{-1}(v_i + D_{i0}))]^2/2 + \\
&(d_i + a_{i0})f_{i,1}(x_{i,1}) + s_{i,1}d_i^2 \cdot \\
&\sum_{j\in N_i}[|x_{j2}| + |f_{j,1}(x_{j,1})| + \phi_{j11}(|x_{j,1}|) + \\
&\phi_{j12}(\bar{\alpha}_{j,1}^{-1}(v_j + D_{j0}))]^2/2,
\end{aligned} \tag{24}$$

其中: $X_{i,1} = [x_{j_1,1}\ x_{j_2,1}\ \cdots\ x_{j_{k_i},1}\ x_{j_1,2}\ x_{j_2,2}\ \cdots\ x_{j_{k_i},2}\ x_{i,1}\ s_{i,1}\ v_i\ v_{j_1}\ \cdots\ v_{j_{k_i}}]^{\mathrm{T}}$, $N_i = \{j_1, \cdots, j_{k_i}\}$.

用RBFNNs逼近未知函数$H_{i,1}(X_{i,1})$, 即$H_{i,1}(X_{i,1}) = \theta_{i,1}^{*\mathrm{T}}\varsigma_{i,1}(X_{i,1}) + \delta_{i,1}(X_{i,1})$, $\theta_{i,1}^*$是理想权向量, $\varsigma_{i,1}(X_{i,1})$是径向基函数向量, $\delta_{i,1}(X_{i,1})$是逼近误差.

$$\begin{aligned}
&\dot{V}_1 \leqslant \\
&\sum_{i=1}^{N}\{s_{i,1}[(d_i + a_{i0})\alpha_{i,2} + \theta_{i,1}^{*\mathrm{T}}\varsigma_{i,1}(X_{i,1}) + \\
&\delta_{i,1}(X_{i,1}) - a_{i0}\dot{y}_{\mathrm{r}}(t)] + s_{i,1}^2(d_i + a_{i0})^2 + \\
&\frac{1}{2}y_{i,2}^2 + \frac{a_{i1}^2}{2} - \frac{1}{\eta_{i,1}}\tilde{\lambda}_{i,1}\dot{\hat{\lambda}}_{i,1}\} + \sum_{i=1}^{N}\frac{d_i + 1}{2}.
\end{aligned} \tag{25}$$

根据Young'不等式和引理3得

$$\begin{aligned}
&s_{i,1}\theta_{i,1}^{*\mathrm{T}}\varsigma_{i,1}(X_{i,1}) \leqslant \\
&\frac{s_{i,1}^2\lambda_{i,1}\|\varsigma_{i,1}(X_{i,1})\|^2}{2b_{i1}^2} + \frac{b_{i1}^2}{2} \leqslant \\
&\frac{s_{i,1}^2\lambda_{i,1}\|\varsigma_{i1}(Z_{i,1})\|^2}{2b_{i1}^2} + \frac{b_{i1}^2}{2},
\end{aligned} \tag{26}$$

$$s_{i,1}\delta_{i,1}(X_{i,1}) \leqslant s_{i,1}^2 + \frac{1}{4}\delta_{i,1}^2(X_{i,1}), \tag{27}$$

其中$Z_{i,1} = [x_{i,1}\ \ s_{i,1}\ \ v_i]^{\mathrm{T}} \in \mathbb{R}^3$. 令$\lambda_{i,1} = \|\theta_{i,1}^*\|^2$, $\tilde{\lambda}_{i,1} = \lambda_{i,1} - \hat{\lambda}_{i,1}$, 则存在非负连续函数$B_{i,1}(X_{i,1})$, 使得

$$|\delta_{i,1}(X_{i,1})| \leqslant B_{i,1}(X_{i,1}). \tag{28}$$

故式(27)可重新写为

$$s_{i,1}\delta_{i,1}(X_{i,1}) \leqslant s_{i,1}^2 + \frac{1}{4}B_{i,1}^2. \tag{29}$$

根据式(26)(29)代入式(25)得

$$\begin{aligned}&\dot{V}_1 \leqslant \\ &\sum_{i=1}^{N}\{s_{i,1}[(d_i+a_{i0})\alpha_{i,2} + \frac{s_{i,1}\lambda_{i,1}\|\varsigma_{i,1}(Z_{i,1})\|^2}{2b_{i1}^2} - \\ &a_{i0}\dot{y}_{\rm r}(t)] + s_{i,1}^2(d_i+a_{i0})^2 + \frac{1}{2}s_{i,2}^2 + \frac{1}{2}y_{i,2}^2 + \frac{b_{i1}^2}{2} + \\ &\frac{1}{4}B_{i,1}^2 + s_{i,1}^2 - \frac{1}{\eta_{i,1}}\tilde{\lambda}_{i,1}\dot{\hat{\lambda}}_{i,1}\} + \sum_{i=1}^{N}\frac{1+d_i}{2}.\end{aligned} \tag{30}$$

设计虚拟控制律$\alpha_{i,2}$和估计参数$\hat{\lambda}_{i,1}$的自适应律如下:

$$\begin{aligned}&\alpha_{i,2} = \\ &\frac{1}{(d_i+a_{i0})}(-k_{i,1}s_{i,1} + a_{i0}\dot{y}_{\rm r}(t) - \\ &\frac{s_{i,1}\hat{\lambda}_{i,1}\|\varsigma_{i,1}(Z_{i,1})\|^2}{2b_{i1}^2}),\end{aligned} \tag{31}$$

$$\dot{\hat{\lambda}}_{i,1} = \eta_{i,1}(\frac{s_{i,1}^2\|\varsigma_{i,1}(Z_{i,1})\|^2}{2b_{i1}^2} - \sigma_{i,1}\hat{\lambda}_{i,1}), \tag{32}$$

其中: $k_{i,1} > 0,\ \eta_{i,1} > 0,\ \sigma_{i,1} > 0$是设计参数.

注 1 根据假设6可知系统存在根节点, 不妨设第i_0个智体是根节点. 由于根节点能从头节点获取信息, 因此有$a_{i_0 0} = 1$. 当$i \neq i_0$时, 存在根节点到多智能体i的通路, 即存在$j_0 \neq i,\ j_0 \in N_i$, 使得$a_{ij_0} = 1$. 故$d_i = \sum_{j\in N_i} a_{ij} \geqslant 1$. 进一步得$d_i + a_{i0} \geqslant 1,\ i = 1, \cdots, N$.

注 2 引理3提供了处理非严格反馈系统中未知函数变量不满足下三角条件的一种方法. 根据上述推导可知, 通过利用引理3和Young's不等式, 用于逼近未知函数的神经网络回归向量中全部状态变量可用部分满足下三角条件的变量代替, 从而简化了虚拟控制器和稍后的控制器设计以及稳定性分析.

将式(31)–(32)代入式(30)得

$$\begin{aligned}&\dot{V}_1 \leqslant \\ &\sum_{i=1}^{N}\{-(k_{i,1} - 1 - (d_i+a_{i0})^2)s_{i,1}^2 + \frac{1}{2}s_{i,2}^2 + \frac{1}{2}y_{i,2}^2 + \\ &\frac{b_{i1}^2}{2} + \frac{1}{4}B_{i,1}^2 + \sigma_{i,1}\tilde{\lambda}_{i,1}\hat{\lambda}_{i,1}\} + \sum_{i=1}^{N}\frac{1+d_i}{2}.\end{aligned} \tag{33}$$

引入一阶滤波器, 定义$\omega_{i,2}$如下:

$$\tau_{i,2}\dot{\omega}_{i,2} + \omega_{i,2} = \alpha_{i,2},\ \omega_{i,2}(0) = \alpha_{i,2}(0), \tag{34}$$

其中: $\tau_{i,2} > 0$是一个正常数, $\alpha_{i,2}$是一阶滤波器输入, $\omega_{i,2}$是一阶滤波器输出. 令

$$y_{i,2} = \omega_{i,2} - \alpha_{i,2}, \tag{35}$$

对式(35)关于时间t求导得

$$\dot{y}_{i,2} = -\frac{y_{i,2}}{\tau_{i,2}} - \dot{\alpha}_{i,2}. \tag{36}$$

进一步得

$$\begin{aligned}y_{i,2}\dot{y}_{i,2} \leqslant -\frac{y_{i,2}^2}{\tau_{i,2}} + |y_{i,2}|\kappa_{i,2}(&\underline{s}_{N,1}, \underline{s}_{N,2}, \\ &\underline{y}_{N,2}, \underline{\hat{\lambda}}_{N,1}, \bar{v}_N, y_{\rm r}, \dot{y}_{\rm r}, \ddot{y}_{\rm r}),\end{aligned} \tag{37}$$

其中: $\kappa_{i,2}(\underline{s}_{N,1}, \underline{s}_{N,2}, \underline{y}_{N,2}, \underline{\hat{\lambda}}_{N,1}, \bar{v}_N, y_{\rm r}, \dot{y}_{\rm r}, \ddot{y}_{\rm r})$是非负连续函数, $\underline{\hat{\lambda}}_{N,1} = [\hat{\lambda}_{1,1}\ \cdots\ \hat{\lambda}_{N,1}]^{\rm T}$.

利用Young's不等式得

$$y_{i,2}\dot{y}_{i,2} \leqslant -\frac{y_{i,2}^2}{\tau_{i,2}} + y_{i,2}^2 + \frac{1}{4}\kappa_{i,2}^2. \tag{38}$$

步骤 2 定义误差面如下:

$$s_{i,2} = x_{i,2} - \omega_{i,2}. \tag{39}$$

将$s_{i,2}$关于时间t求导得

$$\begin{aligned}\dot{s}_{i,2} = x_{i,3} + f_{i,2}(\bar{x}_{i,2}) + \varDelta_{i,2}(z_i, \bar{x}_{i,n}, t) - \dot{\omega}_{i,2} = \\ s_{i,3} + y_{i,3} + \alpha_{i,3} + f_{i,2}(\bar{x}_{i,2}) + \\ \varDelta_{i,2}(z_i, \bar{x}_{i,n}, t) - \dot{\omega}_{i,2}.\end{aligned} \tag{40}$$

定义Lyapunov函数如下:

$$V_2 = \frac{1}{2}\sum_{i=1}^{N}\{s_{i,2}^2 + \frac{1}{\eta_{i,2}}\tilde{\lambda}_{i,2}^2 + y_{i,2}^2\}. \tag{41}$$

将V_2关于时间t求导得

$$\begin{aligned}&\dot{V}_2 = \\ &\sum_{i=1}^{N}\{s_{i,2}\dot{s}_{i,2} - \frac{1}{\eta_{i,2}}\tilde{\lambda}_{i,2}\dot{\hat{\lambda}}_{i,2} + y_{i,2}\dot{y}_{i,2}\} = \\ &\sum_{i=1}^{N}\{s_{i,2}[s_{i,3} + y_{i,3} + \alpha_{i,3} + f_{i,2}(\bar{x}_{i,2}) + \\ &\varDelta_{i,2}(z_i, \bar{x}_{i,n}, t) - \dot{\omega}_{i,2}] - \frac{1}{\eta_{i,2}}\tilde{\lambda}_{i,2}\dot{\hat{\lambda}}_{i,2} + y_{i,2}\dot{y}_{i,2}\}.\end{aligned} \tag{42}$$

由Young's不等式得

$$s_{i,2}(s_{i,3} + y_{i,3}) \leqslant s_{i,2}^2 + \frac{1}{2}s_{i,3}^2 + \frac{1}{2}y_{i,3}^2, \tag{43}$$

$$\begin{aligned}&s_{i,2}\varDelta_{i,2}(z_i, \bar{x}_{i,n}, t) \leqslant \\ &s_{i,2}[\phi_{i21}(\|\bar{x}_{i,2}\|) + \phi_{i22}(\|z_i\|)] \leqslant \\ &s_{i,2}^2\frac{[\phi_{i21}(\|\bar{x}_{i,2}\|) + \phi_{i22}(\bar{\alpha}_{i,1}^{-1}(v_i + D_{i0}))]^2}{2} + \frac{1}{2},\end{aligned} \tag{44}$$

$$\begin{aligned}&\dot{V}_2 \leqslant \\ &\sum_{i=1}^{N}\{s_{i,2}[\alpha_{i,3} + f_{i,2} - \dot{\omega}_{i,2} + s_{i,2}(\bar{x}_{i,2})\cdot \\ &\frac{[\phi_{i21}(\|\bar{x}_{i,2}\|) + \phi_{i22}(\bar{\alpha}_{i,1}^{-1}(v_i + D_{i0}))]^2}{2}] + \\ &s_{i,2}^2 + \frac{1}{2}s_{i,3}^2 + \frac{1}{2}y_{i,3}^2 + \frac{1}{2} - \frac{1}{\eta_{i,2}}\tilde{\lambda}_{i,2}\dot{\hat{\lambda}}_{i,2} + y_{i,2}\dot{y}_{i,2}\}.\end{aligned} \tag{45}$$

令

$$H_{i,2}(X_{i,2}) = f_{i,2}(\bar{x}_{i,2}) - \dot{\omega}_{i,2} + s_{i,2}[\phi_{i21}(\|\bar{x}_{i,2}\|) + \phi_{i22}(\bar{\alpha}_{i,1}^{-1}(v_i + D_{i0}))]^2/2, \tag{46}$$

其中$X_{i,2} = [\bar{x}_{i,2}^{\mathrm{T}}\ \ s_{i,2}\ \ v_i\ \ \dot{\omega}_{i,2}]^{\mathrm{T}} \in \mathbb{R}^5$.

用RBFNNs逼近未知函数$H_{i,2}(X_{i,2})$, 即$H_{i,2}(X_{i,2}) = \theta_{i,2}^{*\mathrm{T}}\varsigma_{i,2}(X_{i,2}) + \delta_{i,2}(X_{i,2})$, θ_{i2}^*是理想权向量, $\varsigma_{i,2}(X_{i,2})$是径向基函数向量, $\delta_{i,2}(X_{i,2})$是逼近误差.

$$\dot{V}_2 \leqslant \sum_{i=1}^N \{s_{i,2}[\alpha_{i,3} + \theta_{i,2}^{*\mathrm{T}}\varsigma_{i,2}(X_{i,2}) + \delta_{i,2}(X_{i,2})] + s_{i,2}^2 + \frac{1}{2}s_{i,3}^2 + \frac{1}{2}y_{i,3}^2 + \frac{1}{2} - \frac{1}{\eta_{i,2}}\tilde{\lambda}_{i,2}\dot{\hat{\lambda}}_{i,2} + y_{i,2}\dot{y}_{i,2}\}. \tag{47}$$

由Young's不等式得

$$s_{i,2}\theta_{i,2}^{*\mathrm{T}}\varsigma_{i,2}(X_{i,2}) \leqslant \frac{s_{i,2}^2\lambda_{i,2}\|\varsigma_{i,2}(X_{i,2})\|^2}{2b_{i2}^2} + \frac{b_{i2}^2}{2}, \tag{48}$$

$$s_{i,2}\delta_{i,2}(X_{i,2}) \leqslant s_{i,2}^2 + \frac{1}{4}\delta_{i,2}^2(X_{i,2}). \tag{49}$$

令$\lambda_{i,2} = \|\theta_{i2}^*\|^2$, 且定义$\tilde{\lambda}_{i,2} = \lambda_{i,2} - \hat{\lambda}_{i,2}$. 存在非负连续函数$B_{i,2}(\bar{x}_{i,2}, s_{i,2}, v_i, \dot{\omega}_{i,2})$, 使得

$$|\delta_{i,2}(X_{i,2})| \leqslant B_{i,2}(\bar{x}_{i,2}, s_{i,2}, v_i, \dot{\omega}_{i,2}), \tag{50}$$

其中$\bar{x}_{i,2} = [x_{i,1}\ \ x_{i,2}]^{\mathrm{T}}$. 故式(49)可重新写为

$$s_{i,2}\delta_{i,2}(X_{i,2}) \leqslant s_{i,2}^2 + \frac{1}{4}B_{i,2}^2, \tag{51}$$

$$\dot{V}_2 \leqslant \sum_{i=1}^N \{s_{i,2}[\alpha_{i,3} + \frac{s_{i,2}\lambda_{i,2}\|\varsigma_{i,2}(X_{i,2})\|^2}{2b_{i2}^2}] + 2s_{i,2}^2 + \frac{1}{2}s_{i,3}^2 + \frac{1}{2}y_{i,3}^2 + \frac{1}{2} - \frac{1}{\eta_{i,2}}\tilde{\lambda}_{i,2}\dot{\hat{\lambda}}_{i,2} - \frac{y_{i,2}^2}{\tau_{i,2}} + y_{i,2}^2 + \frac{1}{4}\kappa_{i,2}^2 + \frac{1}{4}B_{i,2}^2 + \frac{b_{i2}^2}{2}\}. \tag{52}$$

设计虚拟控制律$\alpha_{i,3}$和估计参数$\hat{\lambda}_{i,2}$的自适应律如下:

$$\alpha_{i,3} = -k_{i,2}s_{i,2} - \frac{s_{i,2}\hat{\lambda}_{i,2}\|\varsigma_{i,2}(X_{i,2})\|^2}{2b_{i2}^2} - \frac{1}{2}s_{i,2}, \tag{53}$$

$$\dot{\hat{\lambda}}_{i,2} = \eta_{i,2}(\frac{s_{i,2}^2\|\varsigma_{i,2}(X_{i,2})\|^2}{2b_{i2}^2} - \sigma_{i,2}\hat{\lambda}_{i,2}), \tag{54}$$

其中$k_{i,2} > 0,\ \eta_{i,2} > 0,\ \sigma_{i,2} > 0$是设计参数.

将式(53)–(54)代入式(52)得

$$\dot{V}_2 \leqslant \sum_{i=1}^N \{-(k_{i,2} - 2)s_{i,2}^2 + \frac{1}{2}s_{i,3}^2 + \frac{1}{2}y_{i,3}^2 - \frac{1}{2}s_{i,2}^2 + \frac{1}{2} + (1 - \frac{1}{\tau_{i,2}})y_{i,2}^2 + \frac{1}{4}\kappa_{i,2}^2 + \frac{1}{4}B_{i,2}^2 + \frac{b_{i2}^2}{2} + \sigma_{i,2}\tilde{\lambda}_{i,2}\hat{\lambda}_{i,2}\}. \tag{55}$$

引入一阶滤波器, 定义$\omega_{i,3}$如下:

$$\tau_{i,3}\dot{\omega}_{i,3} + \omega_{i,3} = \alpha_{i,3},\ \omega_{i,3}(0) = \alpha_{i,3}(0). \tag{56}$$

其中: $\tau_{i,3} > 0$是一个正常数, $\alpha_{i,3}$是一阶滤波器输入, $\omega_{i,3}$是一阶滤波器输出. 令

$$y_{i,3} = \omega_{i,3} - \alpha_{i,3}. \tag{57}$$

对式(57)关于时间t求导得

$$\dot{y}_{i,3} = -\frac{y_{i,3}}{\tau_{i,3}} - \dot{\alpha}_{i,3}. \tag{58}$$

进一步得

$$y_{i,3}\dot{y}_{i,3} \leqslant -\frac{y_{i,3}^2}{\tau_{i,3}} + |y_{i,3}|\kappa_{i,3}(\bar{\underline{s}}_{N,3}, \bar{\underline{y}}_{N,3}, \hat{\underline{\lambda}}_{N,1}, \hat{\underline{\lambda}}_{N,2}, \bar{v}_N, y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}, \ddot{y}_{\mathrm{r}}), \tag{59}$$

其中: $\kappa_{i,3}(\bar{\underline{s}}_{N,3}, \bar{\underline{y}}_{N,3}, \hat{\underline{\lambda}}_{N,1}, \hat{\underline{\lambda}}_{N,2}, \bar{v}_N, y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}, \ddot{y}_{\mathrm{r}})$是非负连续函数, $\hat{\underline{\lambda}}_{N,2} = [\hat{\lambda}_{1,2}\ \ \cdots\ \ \hat{\lambda}_{N,2}]^{\mathrm{T}}$.

利用Young's不等式得

$$y_{i,3}\dot{y}_{i,3} \leqslant -\frac{y_{i,3}^2}{\tau_{i,3}} + y_{i,3}^2 + \frac{1}{4}\kappa_{i,3}^2. \tag{60}$$

步骤m ($3 \leqslant m \leqslant n-1$) 定义误差面如下:

$$s_{i,m} = x_{i,m} - \omega_{i,m}. \tag{61}$$

将$s_{i,m}$关于时间t求导得

$$\dot{s}_{i,m} = x_{i,m+1} + f_{i,m}(\bar{x}_{i,m}) + \varDelta_{i,m}(z_i, \bar{x}_{i,n}, t) - \dot{\omega}_{i,m} = s_{i,m+1} + y_{i,m+1} + \alpha_{i,m+1} + f_{i,m}(\bar{x}_{i,m}) + \varDelta_{i,m}(z_i, \bar{x}_{i,n}, t) - \dot{\omega}_{i,m}. \tag{62}$$

定义Lyapunov函数如下:

$$V_m = \frac{1}{2}\sum_{i=1}^N \{s_{i,m}^2 + \frac{1}{\eta_{i,m}}\tilde{\lambda}_{i,m}^2 + y_{i,m}^2\}. \tag{63}$$

将V_m关于时间t求导得

$$\dot{V}_m = \sum_{i=1}^N \{s_{i,m}\dot{s}_{i,m} - \frac{1}{\eta_{i,m}}\tilde{\lambda}_{i,m}\dot{\hat{\lambda}}_{i,m} + y_{i,m}\dot{y}_{i,m}\} = \sum_{i=1}^N \{s_{i,m}[s_{i,m+1} + y_{i,m+1} + \alpha_{i,m+1} + f_{i,m}(\bar{x}_{i,m}) + \varDelta_{i,m}(z_i, \bar{x}_{i,n}, t) - \dot{\omega}_{i,m}] - \frac{1}{\eta_{i,m}}\tilde{\lambda}_{i,m}\dot{\hat{\lambda}}_{i,m} + y_{i,m}\dot{y}_{i,m}\}. \tag{64}$$

由Young's不等式得

$$s_{i,m}(s_{i,m+1}+y_{i,m+1}) \leqslant s_{i,m}^2+\frac{1}{2}s_{i,m+1}^2+\frac{1}{2}y_{i,m+1}^2. \tag{65}$$

$$\begin{aligned}&s_{i,m}\varDelta_{i,m}(z_i,\bar{x}_{i,n},t) \leqslant\\&s_{i,m}[\phi_{im1}(\|\bar{x}_{i,m}\|)+\phi_{im2}(\|z_i\|)] \leqslant\\&s_{i,m}^2\frac{[\phi_{im1}(\|\bar{x}_{i,m}\|)+\phi_{im2}(\bar{\alpha}_{i,1}^{-1}(v_i+D_0))]^2}{2}+\frac{1}{2},\end{aligned} \tag{66}$$

$$\begin{aligned}&\dot{V}_m \leqslant\\&\sum_{i=1}^N\{s_{i,m}[\alpha_{i,m+1}+f_{i,m}(\bar{x}_{i,m})+\\&s_{i,m}\frac{[\phi_{im1}(\|\bar{x}_{i,m}\|)+\phi_{im2}(\bar{\alpha}_{i1}^{-1}(v_i+D_{i0}))]^2}{2}-\\&\dot{\omega}_{i,m}]+s_{i,m}^2+\frac{1}{2}s_{i,m+1}^2+\frac{1}{2}y_{i,m+1}^2+\frac{1}{2}-\\&\frac{1}{\eta_{i,m}}\tilde{\lambda}_{i,m}\dot{\hat{\lambda}}_{i,m}+y_{i,m}\dot{y}_{i,m}\}.\end{aligned} \tag{67}$$

令

$$\begin{aligned}&H_{i,m}(X_{i,m})=\\&f_{i,m}(\bar{x}_{i,m})-\dot{\omega}_{i,m}+s_{i,m}[\phi_{im1}(\|\bar{x}_{i,m}\|)+\\&\phi_{im2}(\bar{\alpha}_{i,1}^{-1}(v_i+D_{i0}))]^2/2,\end{aligned} \tag{68}$$

其中$X_{i,m}=[\bar{x}_{i,m}^{\mathrm{T}}\ \ s_{i,m}\ \ v_i\ \ \dot{\omega}_{i,m}]^{\mathrm{T}}\in\mathbb{R}^{m+3}$.

用 RBFNNs 逼近未知函数 $H_{i,m}(X_{i,m})$, 即 $H_{i,m}(X_{im})=\theta_{i,m}^{*\mathrm{T}}\varsigma_{i,m}(X_{i,m})+\delta_{i,m}(X_{i,m})$, $\theta_{i,m}^*$是理想权向量, $\varsigma_{i,m}(X_{i,m})$是径向基函数向量, $\delta_{i,m}(X_{i,m})$是逼近误差.

$$\begin{aligned}&\dot{V}_m \leqslant\\&\sum_{i=1}^N\{s_{i,m}[\alpha_{i,m+1}+\theta_{i,m}^{*\mathrm{T}}\varsigma_{i,m}(X_{i,m})+\delta_{i,m}(X_{i,m})]+\\&s_{i,m}^2+\frac{1}{2}s_{i,m}^2+\frac{1}{2}y_{i,m}^2+\frac{1}{2}-\\&\frac{1}{\eta_{i,m}}\tilde{\lambda}_{i,m}\dot{\hat{\lambda}}_{i,m}+y_{i,m}\dot{y}_{i,m}\}.\end{aligned} \tag{69}$$

由Young's不等式得

$$s_{i,m}\theta_{i,m}^{*\mathrm{T}}\varsigma_{i,m}(X_{i,m}) \leqslant \frac{s_{i,m}^2\lambda_{i,m}\|\varsigma_{i,m}(X_{i,m})\|^2}{2b_{im}^2}+\frac{b_{im}^2}{2}, \tag{70}$$

$$s_{i,m}\delta_{i,m}(X_{i,m}) \leqslant s_{i,m}^2+\frac{1}{4}\delta_{i,m}^2(X_{i,m}). \tag{71}$$

令 $\lambda_{i,m}=\|\theta_{i,m}^*\|^2$, 且定义 $\tilde{\lambda}_{i,m}=\lambda_{i,m}-\hat{\lambda}_{i,m}$. 存在非负连续函数$B_{i,m}(\bar{x}_{i,m},s_{i,m},v_i,\dot{\omega}_{i,m})$, 使得

$$|\delta_{i,m}(X_{i,m})| \leqslant B_{i,m}(\bar{x}_{i,m},s_{i,m},v_i,\dot{\omega}_{i,m}), \tag{72}$$

其中$\bar{x}_{i,m}=[x_{i,1}\ \ x_{i,2}\ \ \cdots\ \ x_{i,m}]^{\mathrm{T}}$. 故式(71)可重新写为

$$s_{i,m}\delta_{i,m}(X_{i,m}) \leqslant s_{i,m}^2+\frac{1}{4}B_{i,m}^2, \tag{73}$$

$$\begin{aligned}&\dot{V}_m \leqslant\\&\sum_{i=1}^N[s_{i,m}[\alpha_{i,m+1}+\frac{s_{i,m}\lambda_{i,m}\|\varsigma_{i,m}(X_{i,m})\|^2}{2b_{im}^2}]+\\&2s_{i,m}^2+\frac{1}{2}s_{i,m+1}^2+\frac{1}{2}y_{i,m+1}^2+\frac{1}{2}-\frac{1}{\eta_{i,m}}\tilde{\lambda}_{i,m}\dot{\hat{\lambda}}_{i,m}-\\&\frac{y_{i,m}^2}{\tau_{i,m}}+y_{i,m}^2+\frac{1}{4}\kappa_{i,m}^2+\frac{1}{4}B_{i,m}^2+\frac{b_{im}^2}{2}].\end{aligned} \tag{74}$$

设计虚拟控制律$\alpha_{i,m+1}$和参数$\hat{\lambda}_{i,m}$的自适应律如下:

$$\begin{aligned}\alpha_{i,m+1}=&-k_{i,m}s_{i,m}-\frac{s_{i,m}\hat{\lambda}_{i,m}\|\varsigma_{i,m}(X_{i,m})\|^2}{2b_{im}^2}-\\&\frac{1}{2}s_{i,m},\end{aligned} \tag{75}$$

$$\dot{\hat{\lambda}}_{i,m}=\eta_{i,m}(\frac{s_{i,m}^2\|\varsigma_{i,m}(X_{i,m})\|^2}{2b_{im}^2}-\sigma_{i,m}\hat{\lambda}_{i,m}), \tag{76}$$

其中$k_{i,m}>0$, $\eta_{i,m}>0$, $\sigma_{i,m}>0$是设计参数.

将式(75)–(76)代入式(74)得

$$\begin{aligned}&\dot{V}_m \leqslant\\&\sum_{i=1}^N\{-(k_{i,m}-2)s_{i,m}^2+\frac{1}{2}s_{i,m+1}^2+\frac{1}{2}y_{i,m+1}^2+\\&\frac{1}{2}+(1-\frac{1}{\tau_{i,m}})y_{i,m}^2-\frac{1}{2}s_{i,m}^2+\frac{1}{4}\kappa_{i,m}^2+\\&\frac{1}{4}B_{i,m}^2+\frac{b_{im}^2}{2}+\sigma_{i,m}\tilde{\lambda}_{i,m}\hat{\lambda}_{i,m}\}.\end{aligned} \tag{77}$$

定义$\omega_{i,m+1}$如下:

$$\tau_{i,m+1}\dot{\omega}_{i,m+1}+\omega_{i,m+1}=\alpha_{i,m+1}, \tag{78}$$

其中: $\omega_{i,m+1}(0)=\alpha_{i,m+1}(0)$, $\tau_{i,m+1}>0$是一个正常数, $\alpha_{i,m+1}$是一阶滤波器输入, $\omega_{i,m+1}$是一阶滤波器输出. 令

$$y_{i,m+1}=\omega_{i,m+1}-\alpha_{i,m+1}, \tag{79}$$

对式(79)关于时间t求导得

$$\dot{y}_{i,m+1}=-\frac{y_{i,m+1}}{\tau_{i,m+1}}-\dot{\alpha}_{i,m+1}. \tag{80}$$

进一步得

$$\begin{aligned}&y_{i,m+1}\dot{y}_{i,m+1} \leqslant\\&-\frac{y_{i,m+1}^2}{\tau_{i,m+1}}+|y_{i,m+1}|\kappa_{i,m+1}(\underline{\bar{s}}_{N,m+1},\\&\underline{\bar{y}}_{N,m+1},\underline{\bar{\hat{\lambda}}}_{N,m},\bar{v}_N,y_{\mathrm{r}},\dot{y}_{\mathrm{r}},\ddot{y}_{\mathrm{r}}),\end{aligned} \tag{81}$$

其中: $\kappa_{i,m+1}(\underline{\bar{s}}_{N,m+1},\underline{\bar{y}}_{N,m+1},\underline{\bar{\hat{\lambda}}}_{N,m},\bar{v}_N,y_{\mathrm{r}},\dot{y}_{\mathrm{r}},\ddot{y}_{\mathrm{r}})$是非负连续函数, $\underline{\bar{\hat{\lambda}}}_{N,m}=[\hat{\lambda}_{1,1}\ \ \cdots\ \ \hat{\lambda}_{N,1}\ \ \cdots\ \ \hat{\lambda}_{1,m}\ \ \cdots\ \ \hat{\lambda}_{N,m}]^{\mathrm{T}}$.

利用Young's不等式得

$$y_{i,m+1}\dot{y}_{i,m+1} \leqslant -\frac{y_{i,m+1}^2}{\tau_{i,m+1}} + y_{i,m+1}^2 + \frac{1}{4}\kappa_{i,m+1}^2. \tag{82}$$

步骤n 定义误差面如下:

$$s_{i,n} = x_{i,n} - \omega_{i,n}. \tag{83}$$

将$s_{i,n}$关于时间t求导得

$$\dot{s}_{i,n} = \sum_{f=1}^{p}[k_{i,f,h}q_{i,f,1}u_{i,f} + k_{i,f,h}q_{i,f,2} + u_{i,sf,h}] + f_{i,n}(\bar{x}_{i,n}) + \Delta_{i,n}(z_i, \bar{x}_{i,n}, t) - \dot{\omega}_{i,n}. \tag{84}$$

定义Lyapunov函数:

$$V_n = \frac{1}{2}\sum_{i=1}^{N}(s_{i,n}^2 + \frac{1}{\eta_{i,n}}\tilde{\lambda}_{i,n}^2 + y_{i,n}^2). \tag{85}$$

将V_n关于时间t求导得

$$\dot{V}_n = \sum_{i=1}^{N}\{\sum_{f=1}^{p}[s_{i,n}(k_{i,f,h}q_{i,f,1}u_{i,f} + k_{i,f,h}q_{i,f,2} + u_{i,sf,h} + f_{i,n}(\bar{x}_{i,n}) + \Delta_{i,n}(z_i, \bar{x}_{i,n}, t) - \dot{w}_{i,n})] - \frac{1}{\eta_{i,n}}\tilde{\lambda}_{i,n}\dot{\hat{\lambda}}_{i,n} + y_{i,n}\dot{y}_{i,n}\}. \tag{86}$$

根据Young's不等式得

$$s_{i,n}\Delta_{i,n}(z_i, \bar{x}_{i,n}, t) \leqslant s_{i,n}[\phi_{in1}(\|\bar{x}_{i,n}\|) + \phi_{in2}(\|z_i\|)] \leqslant s_{i,n}^2\frac{[\phi_{in1}(\|\bar{x}_{i,n}\|) + \phi_{in2}(\bar{\alpha}_{i,1}^{-1}(v_i + D_{i0}))]^2}{2} + \frac{1}{2}, \tag{87}$$

$$\dot{V}_n \leqslant \sum_{i=1}^{N}\{\sum_{f=1}^{p} s_{i,n}[k_{i,f,h}q_{i,f,1}u_{i,f} + k_{i,f,h}q_{i,f,2} + u_{i,sf,h} + f_{i,n}(\bar{x}_{i,n}) + s_{i,n}[\phi_{in1}(\|\bar{x}_{i,n}\|) + \phi_{in2}(\bar{\alpha}_{i,1}^{-1}(v_i + D_{i0}))]^2/2 - \dot{\omega}_{i,n}] - \frac{1}{\eta_{i,n}}\tilde{\lambda}_{i,n}\dot{\hat{\lambda}}_{i,n} + y_{i,n}\dot{y}_{i,n} + \frac{1}{2}\}. \tag{88}$$

令

$$H_{i,n}(X_{i,n}) = f_{i,n}(\bar{x}_{i,n}) - \dot{\omega}_{i,n} + s_{i,n}\frac{[\phi_{in1}(\|\bar{x}_{i,n}\|) + \phi_{in2}(\bar{\alpha}_{i,1}^{-1}(v_i + D_{i0}))]^2}{2a_{in}^2},$$

其中$X_{i,n} = [\bar{x}_{i,n}^{\mathrm{T}}\ \ s_{i,n}\ \ v_i\ \ \dot{\omega}_{i,n}]^{\mathrm{T}} \in \mathbb{R}^{n+3}$.

用RBFNNs逼近未知函数$H_{i,n}(X_{i,n})$, 即$H_{i,n}(X_{i,n}) = \theta_{i,n}^{*\mathrm{T}}\varsigma_{i,n}(X_{i,n}) + \delta_{i,n}(X_{i,n})$, $\theta_{i,n}^{*}$是理想权向量, $\varsigma_{i,n}(X_{i,n})$是径向基函数向量, $\delta_{i,n}(X_{i,n})$是逼近误差.

$$\dot{V}_n \leqslant \sum_{i=1}^{N}\sum_{f=1}^{p}\{s_{i,n}[k_{i,f,h}q_{i,f,1}u_{i,f} + k_{i,f,h}q_{i,f,2} + u_{i,sf,h} + \theta_{i,n}^{*\mathrm{T}}\varsigma_{i,n}(X_{i,n}) + \delta_{i,n}(X_{i,n})] - \frac{1}{\eta_{i,n}}\tilde{\lambda}_{i,n}\dot{\hat{\lambda}}_{i,n} + y_{i,n}\dot{y}_{i,n} + \frac{1}{2}\}. \tag{89}$$

由Young's不等式得

$$s_{i,n}\theta_{i,n}^{*\mathrm{T}}\varsigma_{i,n}(X_{i,n}) \leqslant \frac{s_{i,n}^2\lambda_{i,n}\|\varsigma_{i,n}(X_{i,n})\|^2}{2b_{in}^2} + \frac{b_{in}^2}{2}, \tag{90}$$

$$s_{i,n}\delta_{i,n}(X_{i,n}) \leqslant s_{i,n}^2 + \frac{1}{4}\delta_{i,n}^2(X_{i,n}). \tag{91}$$

令$\lambda_{i,n} = \|\theta_{in}^{*}\|^2$, $\tilde{\lambda}_{i,n} = \lambda_{i,n} - \hat{\lambda}_{i,n}$. 存在非负连续函数$B_{i,n}(\bar{x}_{i,n}, s_{i,n}, v_i, \dot{\omega}_{i,n})$, 使得

$$|\delta_{i,n}(X_{i,n})| \leqslant B_{i,n}(\bar{x}_{i,n}, s_{i,n}, v_i, \dot{\omega}_{i,n}), \tag{92}$$

其中: $\bar{x}_{i,n} = [x_{i,1}\ \ \cdots\ \ x_{i,n}]^{\mathrm{T}}$. 故式(91)可重新写为

$$s_{i,n}\delta_{i,n}(X_{i,n}) \leqslant s_{i,n}^2 + \frac{1}{4}B_{i,n}^2, \tag{93}$$

$$\dot{V}_n \leqslant \sum_{i=1}^{N}\{\sum_{f=1}^{p}[s_{i,n}(k_{i,f,h}q_{i,f,1}u_{i,f} + k_{i,f,h}q_{i,f,2} + u_{i,sf,h} + \frac{s_{i,n}\lambda_{i,n}\|\varsigma_{i,n}(X_{i,n})\|^2}{2b_{in}^2})] - \frac{1}{\eta_{i,n}}\tilde{\lambda}_{i,n}\dot{\hat{\lambda}}_{i,n} + y_{i,n}\dot{y}_{i,n} + \frac{b_{in}^2}{2} + \frac{1}{4}B_{i,n}^2 + s_{i,n}^2 + \frac{1}{2}\}. \tag{94}$$

由假设3–4以及式(3)–(4)得

$$\min_{1\leqslant f\leqslant p}\{\min_{t\geqslant 0}(k_{i,f,h}(t)q_{i,f,1}(t))\} \geqslant \min_{1\leqslant f\leqslant p}\{\underline{k}_{i,f,h}(1-\delta_{i,f})\} > 0.$$

令$r_i = \min\limits_{1\leqslant f\leqslant p}\{\underline{k}_{i,f,h}(1-\delta_{i,f})\}$, $\mu_i = \sum\limits_{f=1}^{p}(u_{i,f\min} + \bar{u}_{i,sf,h})$. 在执行器部分失效情况下, $r_i > 0$为已知常数.

控制律$u_{i,f}$以及估计参数$\hat{\lambda}_{i,n}$的自适应律设计如下:

$$u_{i,f} = -\frac{1}{r_i}(k_{i,n}s_{i,n} + \frac{1}{2}s_{i,n} + \frac{s_{i,n}\hat{\lambda}_{i,n}\|\varsigma_{i,n}(X_{i,n})\|^2}{2b_{in}^2}), \tag{95}$$

$$\dot{\hat{\lambda}}_{i,n} = \eta_{i,n}(\frac{s_{i,n}^2\|\varsigma_{i,n}(X_{i,n})\|^2}{2b_{in}^2} - \sigma_{i,n}\hat{\lambda}_{i,n}), \tag{96}$$

其中: $k_{i,n} > 0$, $\eta_{i,n} > 0$, $\sigma_{i,n} > 0$是设计参数. 将式(95)–(96)代入式(94)得

$$\dot{V}_n \leqslant$$

$$\sum_{i=1}^{N}\{-(k_{i,n}-1)s_{i,n}^2+|s_{i,n}|\mu_i+\sigma_{i,n}\tilde{\lambda}_{i,n}\hat{\lambda}_{i,n}-\frac{1}{2}s_{i,n}^2+(1-\frac{1}{\tau_{i,n}})y_{i,n}^2+\frac{1}{4}\kappa_{i,n}^2+\frac{b_{in}^2}{2}+\frac{1}{4}B_{i,n}^2+\frac{1}{2}\}. \tag{97}$$

由Young's不等式得

$$s_{i,n}\mu_i\leqslant s_{i,n}^2+\frac{1}{4}\mu_i^2, \tag{98}$$

$$\dot{V}_n\leqslant \sum_{i=1}^{N}\{-(k_{i,n}-2)s_{i,n}^2+\sigma_{i,n}\tilde{\lambda}_{i,n}\hat{\lambda}_{i,n}-\frac{1}{2}s_{i,n}^2+(1-\frac{1}{\tau_{i,n}})y_{i,n}^2+\frac{1}{4}\kappa_{i,n}^2+\frac{b_{in}^2}{2}+\frac{1}{4}B_{i,n}^2+\frac{1}{2}+\frac{1}{4}\mu_i^2\}. \tag{99}$$

4　稳定性分析

定义总的Lyapunov函数和紧集如下:

$$\begin{cases}V=\sum\limits_{j=1}^{n}V_j=\\ \dfrac{1}{2}\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{n}[s_{i,j}^2+\dfrac{1}{\eta_{i,j}}\tilde{\lambda}_{i,j}^{\mathrm{T}}\tilde{\lambda}_{i,j}+y_{i,j}^2],\\ \varPi_1=\{[\bar{\underline{s}}_{N,n}^{\mathrm{T}}\ \ \bar{\underline{y}}_{N,n}^{\mathrm{T}}\ \ \bar{\underline{\hat{\lambda}}}_{N,n}^{\mathrm{T}}]^{\mathrm{T}}:V\leqslant P\}\subset\mathbb{R}^{p_n},\end{cases} \tag{100}$$

其中: $y_{i,1}=0$, B_0和P是给定的正常数, $p_n=(3n-1)N$, $\bar{\underline{\lambda}}_{N,n}=[\lambda_{1,1}\ \cdots\ \lambda_{N,1}\ \cdots\ \lambda_{1,n}\ \lambda_{N,n}]^{\mathrm{T}}$. 当$y_i\in L_\infty$时, 有$v_i\in L_\infty$. 由式(17)得$\underline{x}_{N,1}=y=H^{-1}\underline{s}_{N,1}+1_N y_{\mathrm{r}}$. 因为$\underline{x}_{N,2}=\underline{s}_{N,2}+\underline{y}_{N,2}+[\alpha_{1,2}\ \cdots\ \alpha_{N,2}]^{\mathrm{T}}$, 所以易得在紧集$\varPi_0\times\varPi_1\subset\mathbb{R}^{3\times p_n}$上, 连续函数$B_{i,j}(\cdot)$有最大值$N_{i,j}(j=1,\cdots,n)$, $\kappa_{i,j}(\cdot)$有最大值$M_{i,j}$ $(j=2,\cdots,n)$.

定理 1　考虑具有控制器(95), 虚拟控制器(31)(53)(75), 以及自适应律(32)(54)(76)(96)的多智能体系统(1), 若假设1–6成立, 则对任意正常数P及有界初始条件$V(0)\leqslant P$, 存在$k_{i,1}, k_{i,j}(j=2,\cdots,n)$, $\tau_{i,j}$, $\eta_{i,j}$, $\sigma_{i,j}$满足式(101), 使得当系统具有量化输入、执行器故障及未建模动态的情况下, 闭环多智能体系统所有信号一致终结有界, 各跟随误差收敛到原点的一个小领域内, 其中

$$\begin{cases}k_{i,1}\geqslant 1+(d_i+a_{i0})^2+\dfrac{\alpha_0}{2},\\ k_{i,j}\geqslant 2+\dfrac{\alpha_0}{2},\ j=2,\cdots,n,\\ \dfrac{1}{\tau_{i,j}}\geqslant\dfrac{3}{2}+\dfrac{\alpha_0}{2},\\ \alpha_0\leqslant\min\limits_{1\leqslant i\leqslant N,1\leqslant j\leqslant n}\{\eta_{i,j}\sigma_{i,j}\}.\end{cases} \tag{101}$$

证　如果$V\leqslant P$, 可得$s_{i,1},\cdots,s_{i,n}$, $y_{i,2},\cdots,y_{i,n}$, $\tilde{\lambda}_{i,j}$有界, 故自适应律的估计值$\hat{\lambda}_{i,j}\in L_\infty$ $(j=1,\cdots,n)$. 因为$y_{\mathrm{r}}\in L_\infty$, $s_{i,1}\in L_\infty$, 所以y_i有界. 由$y_i=x_{i,1}$, 可得$x_{i,1}, v_i\in L_\infty$. 进一步得$\|z_i\|\leqslant\alpha_{i1}^{-1}(v_i+D_{i0})\in L_\infty$. 因为$y_{i,2}\in L_\infty$, 由$y_{i,2}=w_{i,2}-\alpha_{i,2}$可得$w_{i,2}, \alpha_{i,2}\in L_\infty$. 由$x_{i,2}=s_{i,2}+y_{i,2}+\alpha_{i,2}$可得$x_{i,2}\in L_\infty$. 同理可得$w_{i,j}, \alpha_{i,j}\in L_\infty(j=2,\cdots,n)$. 由$x_{i,j}=s_{i,j}+y_{i,j}+\alpha_{i,j}$可得$x_{i,j}\in L_\infty$. 由$s_{i,n}\in L_\infty$, $\hat{\lambda}_{i,n}\in L_\infty$, 以及

$$u_{i,f}=-\frac{1}{r_i}(k_{i,n}s_{i,n}+\frac{1}{2}s_{i,n}+\frac{s_{i,n}\hat{\lambda}_{i,n}\|\varsigma_{in}(X_{in})\|^2}{2b_{in}^2}),$$

可得$u_{i,f}\in L_\infty$. 若$V\leqslant P$, 连续函数$B_{i,j}(\cdot)$有最大值$N_{i,j}$, $\kappa_{i,j}(\cdot)$有最大值$M_{i,j}$. 利用式(33)(55)(77)(99)得

$$\begin{aligned}\dot{V}\leqslant&\\ &\sum_{i=1}^{N}-[k_{i,1}-1-(d_i+a_{i0})^2]s_{i,1}^2+\\ &\sum_{i=1}^{N}\sum_{j=2}^{n}-(k_{i,j}-2)s_{i,j}^2-\\ &\sum_{i=1}^{N}\sum_{j=2}^{n}(\frac{1}{\tau_{i,j}}-\frac{3}{2})y_{i,j}^2+\sum_{i=1}^{N}\sum_{j=1}^{n}\sigma_{i,j}\tilde{\lambda}_{i,j}\hat{\lambda}_{i,j}+\\ &\sum_{i=1}^{N}\frac{1+d_i}{2}+\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{b_{ij}^2}{2}+\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{B_{i,j}^2}{4}+\\ &\sum_{i=1}^{N}\sum_{j=2}^{n}\frac{\kappa_{i,j}^2}{4}+\sum_{i=1}^{N}\frac{1}{4}\mu_i^2+\sum_{j=2}^{n}\frac{1}{2}\leqslant\\ &\sum_{i=1}^{N}\sum_{j=1}^{n}-[k_{i,1}-1-(d_i+a_{i0})^2]s_{i,1}^2+\\ &\sum_{i=1}^{N}\sum_{j=2}^{n}-(k_{i,j}-2)s_{i,j}^2-\sum_{i=1}^{N}\sum_{j=2}^{n}(\frac{1}{\tau_{i,j}}-\frac{3}{2})y_{i,j}^2+\\ &\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{\sigma_{i,j}}{2}\lambda_{i,j}^2-\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{\sigma_{i,j}}{2}\tilde{\lambda}_{i,j}^2+\\ &\sum_{i=1}^{N}\frac{1+d_i}{2}+\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{b_{ij}^2}{2}+\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{N_{i,j}^2}{4}+\\ &\sum_{i=1}^{N}\sum_{j=2}^{n}\frac{M_{i,j}^2}{4}+\sum_{j=2}^{n}\frac{1}{2}+\sum_{i=1}^{N}\frac{1}{4}\mu_i^2,\end{aligned} \tag{102}$$

其中

$$\begin{aligned}D_0=&\\ &\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{\sigma_{i,j}}{2}\lambda_{i,j}^2+\sum_{i=1}^{N}\frac{1+d_i}{2}+\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{b_{ij}^2}{2}+\\ &\sum_{i=1}^{N}\frac{1}{4}\mu_i^2+\sum_{j=2}^{n}\frac{1}{2}+\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{N_{i,j}^2}{4}+\sum_{i=1}^{N}\sum_{j=2}^{n}\frac{M_j^2}{4}.\end{aligned} \tag{103}$$

所以

$$\dot{V}\leqslant-\alpha_0 V+D_0. \tag{104}$$

若$V=P$, $\alpha_0\geqslant\dfrac{D_0}{P}$, 则有$\dot{V}\leqslant 0$. 根据$V(0)\leqslant P$, $\alpha_0\geqslant\dfrac{D_0}{P}$得$V(t)\leqslant P$, $\forall t\geqslant 0$, 在式(104)两边同乘

$e^{\alpha_0 t}$得

$$\frac{\mathrm{d}}{\mathrm{d}t}(V(t)e^{\alpha_0 t}) \leqslant e^{\alpha_0 t}D_0. \tag{105}$$

将上式两边积分得

$$0 \leqslant V(t) \leqslant \frac{D_0}{\alpha_0} + (V(0) - \frac{D_0}{\alpha_0})e^{-\alpha_0 t}. \tag{106}$$

因此, 闭环系统的所有信号都为半全局一致终结有界. 因为$\frac{1}{2}\|\underline{s}_{N,1}\|^2 \leqslant V(t)$, 根据引理2得

$$\|e_1\| = \|y - 1_N y_{\mathrm{r}}\| \leqslant \frac{1}{\sigma(H)}\sqrt{\frac{2D_0}{\alpha_0} + 2(V(0) - \frac{D_0}{\alpha_0})e^{-\alpha_0 t}}, \tag{107}$$

其中: $\sigma(H)$是矩阵H的最小奇异值. 对于给定的设计参数σ_{ij}, 选取充分大的设计参数γ_{ij}. 由式(101)可知, 常数α_0可充分大. 根据式(104)可知, 常数D_0与所有的设计参数γ_{ij}无关. 因此, 对于给定的设计参数σ_{ij}和正常数P, 通过选取充分大的设计参数γ_{ij}使常数α_0充分大, 从而使$\frac{D_0}{\alpha_0}$充分小. 根据式(107)可知, 随着时间t的不断增大, 跟踪误差能够变得足够小.

5 仿真算例

下面给出一个算例验证所设计控制器的有效性.

例 1 考虑如下多智能体系统:

$$\begin{cases}\dot{z}_i = q(z_i, \bar{x}_2, t),\\ \dot{x}_{i,1} = x_{i,2} + f_{i,1}(x_{i,1}) + \varDelta_{i,1}(z_i, \bar{x}_2, t),\\ \dot{x}_{i,2} = \sum\limits_{f=1}^{3} W_{i,f}(t) + f_{i,2}(\bar{x}_{i,2}) + \varDelta_{i,2}(z_i, \bar{x}_2, t),\\ y_{i,1} = x_{i,1},\end{cases}$$

其中: $i = 1, 2, 3, 4$. 故障模型描述如下:

$$W_{i,1}(t) = \begin{cases}k_{i,1,1}Q_i(u_{i,1}(t)) + u_{i,s1,1}(t), & t \in [0, 3),\\ Q_i(u_{i,1}(t)), & t \in [3, 5),\\ k_{i,1,2}Q_i(u_{i,1}(t)) + u_{i,s1,2}(t), & t \in [5, +\infty),\end{cases}$$

$$k_{i,1,1} = k_{i,1,2} = 0.3,\ u_{i,s1,1} = u_{i,s1,2} = 0.1\sin t,$$

$$W_{i,2}(t) = \begin{cases}k_{i,2,1}Q_i(u_{i,1}(t)) + u_{i,s2,1}(t), & t \in [0, 5),\\ Q_i(u_{i,2}(t)), & t \in [5, 8),\\ k_{i,2,2}Q_i(u_{i,1}(t)) + u_{i,s2,2}(t), & t \in [8, +\infty),\end{cases}$$

$$k_{i,2,1} = k_{i,2,2} = 0.5,\ u_{i,s2,1} = u_{i,s2,2} = 0.5\cos t,$$

$$W_{i,3}(t) = Q_i(u_{i,3}(t)),\ t \in [0, +\infty).$$

未建模动态为$\dot{z}_i = -z_i + |y_i|^2 + 0.5$, 动态信号为$\dot{v}_i = -0.6v_i + 1.5|y_i|^4 + 1.5$, 期望的跟踪轨迹$y_{\mathrm{r}} = 0.5[\sin t + \sin(0.5t)]$.

考虑具有一个领导者和4个跟随者的多智能体系统, 4个跟随者通过如图1所示的有向通信拓扑图连接. 在图1中, 领导者有指向所有跟随者“1”, “2”, “3”, “4”的路径. 由拓扑图可知,

$$a_{11} = a_{13} = a_{14} = 0,\ a_{12} = 1,$$
$$a_{21} = a_{22} = a_{23} = a_{24} = 0,$$
$$a_{31} = a_{33} = a_{34} = 0,\ a_{32} = 1,$$
$$a_{41} = a_{42} = a_{44} = 0, a_{43} = 1.$$

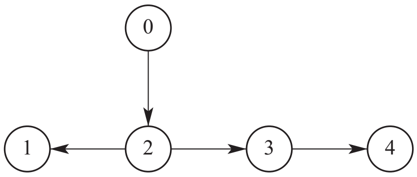


图 1 多智能体的通讯拓扑

Fig. 1 Communication topology of multi-agents

故邻接矩阵A为

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}.$$

由拓扑图可知, 第2个跟随者能够获取来自领导者的信息, 即$a_{20} = 1$, A_0为对角矩阵, 表示为$A_0 = \mathrm{diag}\{0, 1, 0, 0\}$.

选取

$$f_{1,1} = \frac{x_{1,1}}{1 + x_{1,1}^4},\ f_{1,2} = -0.3\cos x_{1,1}e^{-(x_{1,2}^4)},$$
$$f_{2,1} = \frac{x_{2,1}}{1 + x_{2,1}^4},\ f_{2,2} = 0.1\cos x_{2,1}e^{-(x_{2,2}^4)},$$
$$f_{3,1} = \frac{x_{3,1}}{1 + x_{3,1}^4},\ f_{3,2} = 0.1\cos x_{3,1}e^{-(x_{3,2}^4)},$$
$$f_{4,1} = \frac{x_{4,1}}{1 + x_{4,1}^4},\ f_{4,2} = 0.1\cos x_{4,1}e^{-(x_{4,2}^4)}.$$

误差面定义如下:

$$\begin{cases}s_{i,1} = \sum\limits_{j \in N_i} a_{ij}(y_i - y_j) + a_{i0}(y_i - y_{\mathrm{r}}(t)),\\ s_{i,2} = x_{i,2} - \omega_{i,2},\end{cases}$$

虚拟控制律设计如下:

$$\alpha_{i,2} = \frac{1}{(d_i + a_{i0})}(-k_{i,1}s_{i,1} + a_{i0}\dot{y}_{\mathrm{r}}(t) - \frac{s_{i,1}^2\hat{\lambda}_{i,1}\|\varsigma_{i,1}(Z_{i,1})\|^2}{2b_{i1}^2}),$$

自适应律设计如下:

$$\dot{\hat{\lambda}}_{i,1} = \eta_{i,1}(\frac{s_{i,1}^2\|\varsigma_{i,1}(Z_{i,1})\|^2}{2b_{i1}^2} - \sigma_{i,1}\hat{\lambda}_{i,1}),$$

$$\dot{\hat{\lambda}}_{i,2} = \eta_{i,2}(\frac{s_{i,2}^2\|\varsigma_{i,2}(X_{i,2})\|^2}{2b_{i2}^2} - \sigma_{i,2}\hat{\lambda}_{i,2}),$$

控制律设计如下:

$$u_{i,f} = -\frac{1}{r_i}(k_{i,4}s_{i,2} + \frac{1}{2}s_{i,2} + \frac{s_{i,2}\hat{\lambda}_{i,2}\|\varsigma_{i,2}(X_{i,2})\|^2}{2b_{i2}^2}),$$

其中: $\tau_{i,2}\dot{\omega}_{i,2} + \omega_{i,2} = \alpha_{i,2}$.

选取量化器$Q_i(u_{i,f}(t))$的设计参数如下:

$$\rho_{i,f} = 0.24,\ \delta_{i,f} = \frac{1-\rho_{i,f}}{1+\rho_{i,f}},\ u_{i,f,\min} = 0.1,$$

$$u_{i,f} = \rho_{i,f}^{k-1}u_{i,f,\min},\ k = 1, 2, \cdots, \infty.$$

选取设计参数如下:

$$\kappa_{1,1} = 40,\ \kappa_{1,2} = 50,\ \kappa_{1,3} = 80,\ \kappa_{1,4} = 90,$$

$$\kappa_{2,1} = 50,\ \kappa_{2,2} = 80,\ \kappa_{2,3} = 80,\ \kappa_{2,4} = 90,$$

$$\eta_{i,j} = 0.2,\ \sigma_{i,j} = 0.6,\ i = 1, 2, 3, 4,\ j = 1, 2,$$

$$b_{11} = b_{12} = b_{13} = b_{14} = 25,$$

$$b_{21} = b_{22} = b_{23} = b_{24} = 25,$$

$$r_1 = 0.3,\ r_2 = 0.3,\ r_3 = 0.3,\ r_4 = 0.3.$$

选取初值如下:

$$x_{1,1}(0) = 0.16,\ x_{1,2}(0) = 0.01,\ x_{1,3}(0) = -0.08,$$

$$x_{1,4}(0) = -0.055,\ x_{2,1}(0) = 0,\ x_{2,2}(0) = 0,$$

$$x_{2,3}(0) = 0,\ x_{2,4}(0) = 0,$$

$$\hat{\lambda}_{1,1}(0) = 0.4,\ \hat{\lambda}_{2,1}(0) = 0.35,\ \hat{\lambda}_{3,1}(0) = 0.2,$$

$$\hat{\lambda}_{4,1}(0) = 0.1,\ \hat{\lambda}_{1,2}(0) = 0.5,\ \hat{\lambda}_{2,2}(0) = 0.4,$$

$$\hat{\lambda}_{3,2}(0) = 0.3,\ \hat{\lambda}_{4,2}(0) = 0.1.$$

仿真结果如图2–9所示. 图2表明各跟随智能体对头智能体具有良好的一致跟踪性; 图4表明各执行器输入信号是有界的; 图5–8表明不同智能体的故障信号是有界的; 图9表明估计参数随着时间的增大趋于稳定.

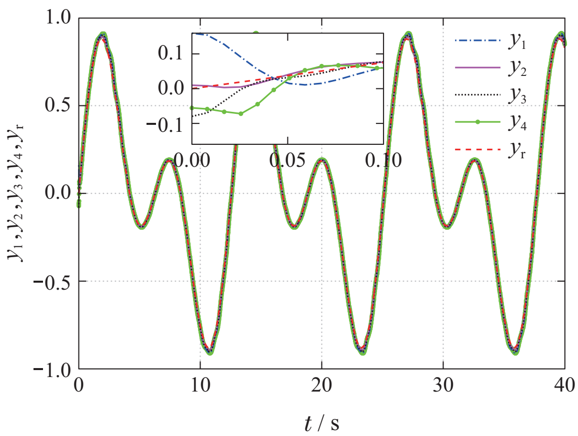
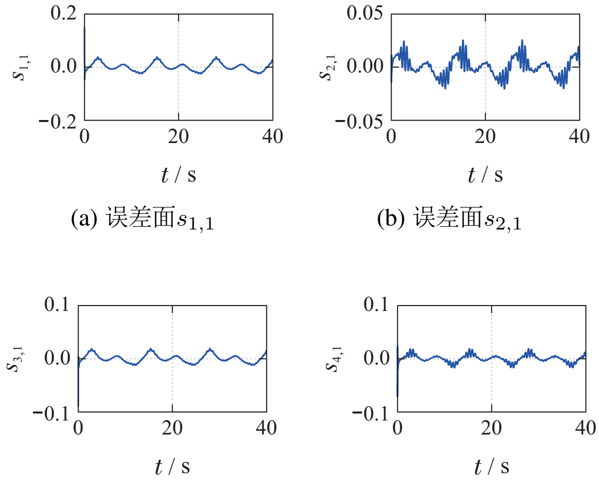


图 2　输出$y_i(i=1,2,3,4)$ 和给定的期望轨迹y_r

Fig. 2　Outputs $y_i(i=1,2,3,4)$ and desired trajectory y_r

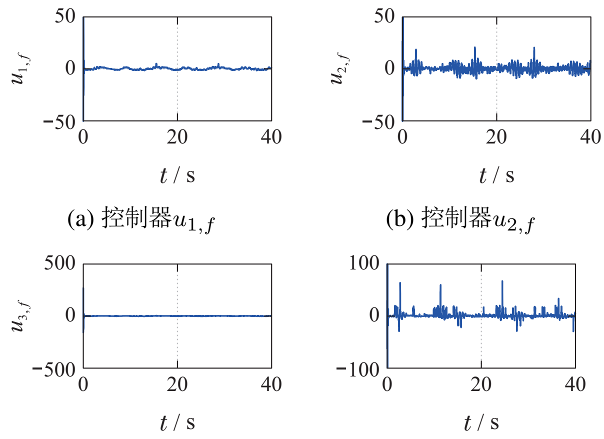


(a) 误差面$s_{1,1}$　(b) 误差面$s_{2,1}$

(c) 误差面$s_{3,1}$　(d) 误差面$s_{4,1}$

图 3　跟踪误差曲线

Fig. 3　Tracking error curves

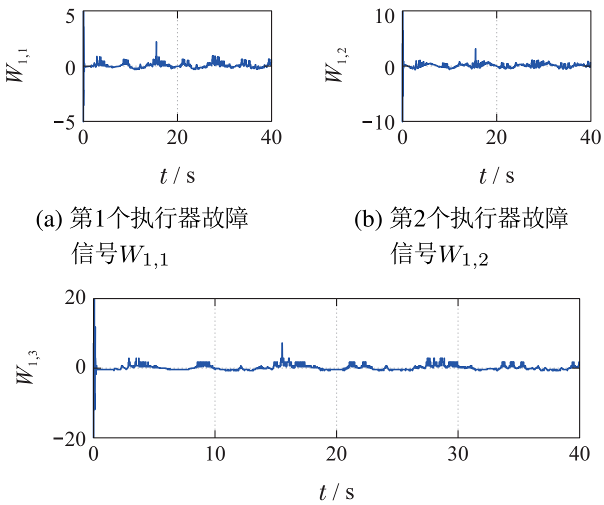


(a) 控制器$u_{1,f}$　(b) 控制器$u_{2,f}$

(c) 控制器$u_{3,f}$　(d) 控制器$u_{4,f}$

图 4　控制信号

Fig. 4　Control signals

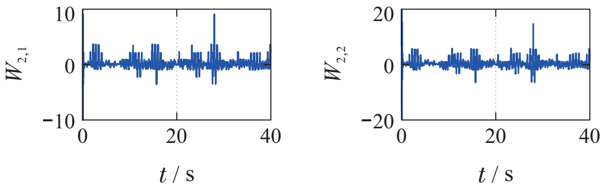


(a) 第1个执行器故障信号$W_{1,1}$　(b) 第2个执行器故障信号$W_{1,2}$

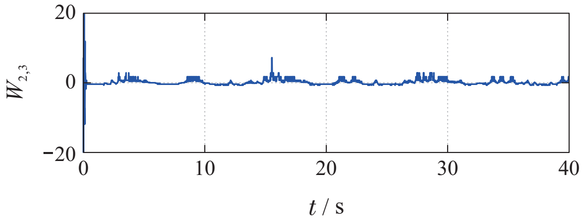
(c) 第3个执行器故障信号$W_{1,3}$

图 5　第1个跟随者的执行器故障信号

Fig. 5　The actuator fault signals of the first follower



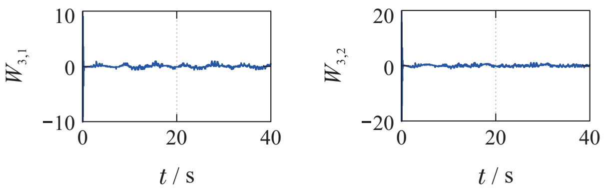
(a) 第1个执行器故障信号$W_{2,1}$　(b) 第2个执行器故障信号$W_{2,2}$



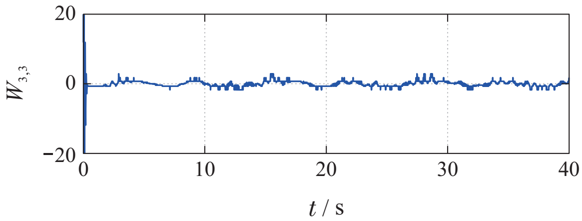
(c) 第3个执行器故障信号$W_{2,3}$

图 6 第2个跟随者的执行器故障信号

Fig. 6 The actuator fault signals of the second follower



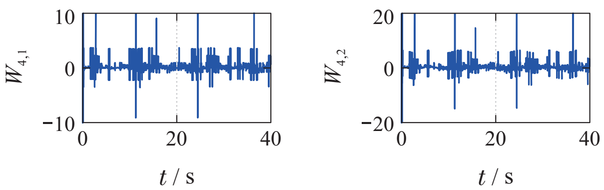
(a) 第1个执行器故障信号$W_{3,1}$　(b) 第2个执行器故障信号$W_{3,2}$



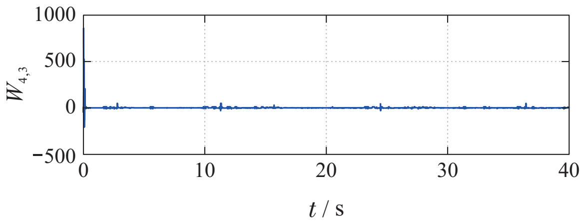
(c) 第3个执行器故障信号$W_{3,3}$

图 7 第3个跟随者的执行器故障信号

Fig. 7 The actuator fault signals of the third follower



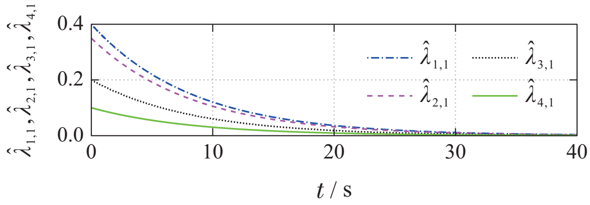
(a) 第1个执行器故障信号$W_{4,1}$　(b) 第2个执行器故障信号$W_{4,2}$



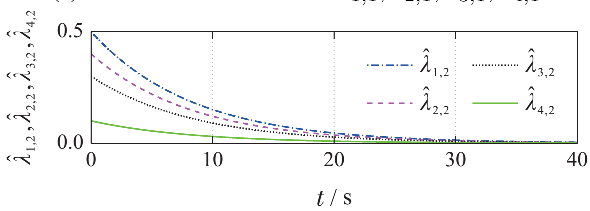
(c) 第3个执行器故障信号$W_{4,3}$

图 8 第4个跟随者的执行器故障信号

Fig. 8 The actuator fault signals of the fourth follower



(a) 第1步的自适应调节参数$\hat{\lambda}_{1,1}, \hat{\lambda}_{2,1}, \hat{\lambda}_{3,1}, \hat{\lambda}_{4,1}$



(b) 第2步的自适应调节参数$\hat{\lambda}_{1,2}, \hat{\lambda}_{2,2}, \hat{\lambda}_{3,2}, \hat{\lambda}_{4,2}$

图 9 自适应调节参数

Fig. 9 Adaptive adjustment parameters

6 结论

本文针对一类具有输入量化、未建模动态和执行器故障的非线性多智能体系统, 提出了一种自适应动态面控制方法. 通过理论分析证明了闭环控制系统是半全局一致终结有界的, 所有跟随者都能实现期望的一致性. 仿真结果证明了所提出方案的有效性.

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