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基于仿射Bessel-Legendre不等式和不确定转移率的神经网络稳定性

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摘要: 针对具有时变时滞和不确定转移率的马尔科夫神经网络系统, 充分考虑马尔科夫转移率的不确定特性, 利用基于松弛变量的有效技术代替传统不等式来约束转移速率中的不确定项, 从而减少了决策变量的个数并降低了计算复杂度. 通过建立时滞依赖的增广Lyapunov-Krasovskii泛函, 并基于仿射Bessel-Legendre(B-L)不等式, 给出依赖于时滞和时滞导数上下界的具有较小保守性的神经网络系统稳定条件. 最后, 通过两个数值例子说明了理论结果的有效性.

关键词: 马尔科夫神经网络系统; 不确定转移率; 仿射Bessel-Legendre(B-L)不等式; 增广Lyapunov-Krasovskii泛函

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Stability for neural networks based on affine Bessel-Legendre inequality and uncertain transition rates

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Abstract: For Markovian neural network with time-varying delays and uncertain transition rates, the effective relaxation variable technique instead of the traditional inequality is adopted to restrain the uncertain terms of the transition rates by fully considering the uncertain characteristic of Markovian transition rates, which reduces the number of decision variables and the computational complexity. By applying the delayed-dependent augmented Lyapunov-Krasovskii functional, and affine Bessel-Legendre (B-L) inequality, the less conservative stability conditions that are dependent on upper and lower bounds of delay and delay derivative are proposed. Finally, two numerical examples are presented to illustrate the effectiveness of the theoretical results.

Key words: Markovian neural networks system; uncertain transition rates; affine Bessel-Legendre (B-L) inequality; augmented Lyapunov-Krasovskii functional

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1 引言

神经网络已经在图像加密、联想记忆、组合优化、模式识别、故障诊断等领域得到广泛应用, 并取得了许多研究成果[1–7]. 在神经元处理信息和信号传递过程中往往会产生时滞, 从而导致系统震荡、分岔、不稳定或降低系统性能[7–10]. 因此, 时滞神经网络的稳定性分析引起了人们的广泛关注. 文献[8]研究了四元数值时滞神经网络的全局指数稳定问题, 并给出相应的稳定性准则. 文献[9]基于广义的自由权矩阵方法研究了时滞神经网络的稳定性问题. 基于文献[9], 文献

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[10]利用改进的广义自由权矩阵不等式和三重积分函数, 研究了时滞神经网络的稳定性问题, 并给出了具有较小保守性的稳定性条件.

由于系统中元件的故障或外部扰动的影响, 神经网络系统会具有有限个模态, 并且模态之间的切换满足马尔科夫跳变[8–15], 很多学者研究了马尔科夫神经网络的稳定性问题. 文献[12]基于模态依赖的时变时滞和增广Lyapunov-Krasovskii泛函, 研究了马尔科夫耦合神经网络的采样数据同步控制问题. 文献[13]研究了马尔科夫惯性神经网络的同步控制问题, 并在图像加密领域给出了相应的应用. 文献[14]研究了半马尔科夫离散时间神经网络的事件驱动状态估计问题, 并给出了相应的事件驱动控制器. 文献[15]研究了一类具有离散和分布时变时滞的马尔科夫神经网络的稳定性问题. 由于设备限制以及不确定因素的影响, 很难得到转移率的确切值[16–20]. 文献[16]研究了具有部分未知转移概率的时滞神经网络稳定性和同步控制问题, 并分别给出了相应的稳定性条件. 文献[17]基于变量变换法, 将二阶微分方程转化为一阶微分方程, 研究了具有广义马尔科夫跳变的时变时滞惯性神经网络同步问题. 文献[18]研究了具有不确定转移率和时变时滞的高阶马尔科夫跳变反应扩散Hopfield神经网络的均方指数稳定性. 大多数文献在处理转移率中的不确定项时, 利用传统的不等式方法, 从而增加了计算复杂度. 因此, 对于含有时变时滞和不确定转移率的神经网络稳定性, 还需进一步研究和探讨.

基于以上分析, 本文研究不确定转移率的时滞神经网络的稳定性问题, 通过松弛变量约束转移率的不确定元素, 从而将不确定元素转化为有界问题, 构造含有增广向量和时滞依赖的Lyapunov-Krasovskii泛函, 使得Lyapunov-Krasovskii泛函包含更多的矩阵块和系统信息, 并给出了具有较小保守性的稳定性判据. 所得结果与以往文献相比, 更具有实用性和广泛性. 此外, 通过两个仿真算例验证了本文方法的有效性.

全文沿用如下记号: $\mathbb{R}$表示实数集合; $\mathbb{R}^n$和$\mathbb{R}^{n\times n}$分别表示n维实数向量集合及$n\times n$维实数矩阵集合; X^{T}表示矩阵X的转置; $X>0$表示X为对称正定矩阵; $X<0$表示X为对称负定矩阵; $\mathrm{Sym}\{X\}=X+X^{\mathrm{T}}$; I_n表示n维单位矩阵; $\begin{pmatrix} X & Y \\ * & Z \end{pmatrix}$表示$\begin{pmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{pmatrix}$; $\mathcal{E}$表示数学期望; $\mathcal{L}$表示弱无穷小算子;

$$e_\kappa=[0_{n\times(\kappa-1)n}\ \ I_n\ \ 0_{n\times(12-\kappa)n}],\ \kappa=1,2,\cdots,12.$$

2 模型描述和预备知识

考虑如下具有时变时滞的Markovin跳变神经网络模型:

$$\dot{x}(t)=-C(\eta_t)x(t)+A(\eta_t)f(x(t))+ B(\eta_t)f(x(t-\tau(t))), \tag{1}$$

其中: $x(\cdot)=[x_1(\cdot)\ \ x_2(\cdot)\ \ \cdots\ \ x_n(\cdot)]^{\mathrm{T}}\in\mathbb{R}^n$为状态向量,

$$C(\eta_t)=\mathrm{diag}\{c_1(\eta_t),c_2(\eta_t),\cdots,c_n(\eta_t)\}\in\mathbb{R}^{n\times n}$$

为正定对角矩阵,

$$A(\eta_t)=(a_{ij}(\eta_t))_{n\times n}\in\mathbb{R}^{n\times n},$$
$$B(\eta_t)=(b_{ij}(\eta_t))_{n\times n}\in\mathbb{R}^{n\times n}$$

为连接权重矩阵,

$$f(x(\cdot))=[f_1(x_1(\cdot))\ f_2(x_2(\cdot))\ \cdots\ f_n(x_n(\cdot))]^{\mathrm{T}}\in\mathbb{R}^n$$

为激活函数. 时变时滞$\tau(t)$满足$0\leqslant\tau(t)\leqslant\tau$, $\mu_1\leqslant\dot{\tau}(t)\leqslant\mu_2$, 此处$\tau,\mu_1,\mu_2$为常数, $\bar{\tau}(t)=1-\dot{\tau}(t)$. 系统模态信号$\{\eta_t,\ t\geqslant0\}$是一个在有限集合$\mathcal{N}=\{1,2,\cdots,s\}$内取值的右连续Markovian跳变过程, 具有如下转移率:

$$\mathrm{P}\{\eta_{t+\Delta t}=j|\eta_t=i\}=\begin{cases}\hat{\pi}_{ij}\Delta t+\mathrm{o}(\Delta t), & i\neq j\\ 1+\hat{\pi}_{ij}\Delta t+\mathrm{o}(\Delta t), & i=j,\end{cases}$$

其中$\hat{\pi}_{ij}\geqslant0(i\neq j)$为$t$时刻的模态$i$到$t+\Delta t$时刻的模态$j$的转移速率, 且

$$\hat{\pi}_{ii}=-\sum_{j=1,j\neq i}^{s}\hat{\pi}_{ij},\ \ \lim_{\Delta t\to0}(\frac{\mathrm{o}(\Delta t)}{\Delta t})=0,\ \Delta t>0.$$

由于马尔科夫过程的转移率是不确定的, 有

$$D_\pi=\{\hat{\mathit{\Pi}}=\mathit{\Pi}+\Delta\mathit{\Pi}:|\Delta\pi_{ij}|\leqslant\delta_{ij},\delta_{ij}\geqslant0,\ \forall i,j\in\mathcal{N},i\neq j\},$$

其中: $\mathit{\Pi}=(\pi_{ij})$, $i,j\in\mathcal{N}$为已知的常数矩阵, $\Delta\mathit{\Pi}=(\Delta\pi_{ij})$表示模态转移率矩阵中的不确定项. 对于所有的$i,j\in\mathcal{N},i\neq j,\pi_{ij}>0$表示$\hat{\pi}_{ij}$的估计值. $\Delta\pi_{ij}=\hat{\pi}_{ij}-\pi_{ij}$表示估计误差, 且属于区间$[-\delta_{ij},\delta_{ij}]$. 对于任意的$i\in\mathcal{N}$,

$$\pi_{ii}=-\sum_{j=1,j\neq i}^{s}\pi_{ij},\ \Delta\pi_{ii}=-\sum_{j=1,j\neq i}^{s}\Delta\pi_{ij}$$

成立, $\underline{\pi}_{ij}=\pi_{ij}-\delta_{ij}$定义为$\hat{\pi}_{ij}$的下界.

令$\eta_t=i$, $A(\eta_t),B(\eta_t),C(\eta_t)$分别表示为$A_i,B_i,C_i$, 则系统(1)可表示为

$$\dot{x}(t)=-C_ix(t)+A_if(x(t))+B_if(x(t-\tau(t))). \tag{2}$$

首先, 给出如下假设和引理:

假设 1[21] 对于任意的$x_1,x_2\in\mathbb{R}$, 存在常数e_r^-, e_r^+满足下面的式子

$$e_r^-\leqslant\frac{f_r(x_1)-f_r(x_2)}{x_1-x_2}\leqslant e_r^+,\ r=1,2,\cdots,n.$$

记

$$E_1=\mathrm{diag}\{e_1^+e_1^-,\cdots,e_n^+e_n^-\},$$

$$E_2=\mathrm{diag}\{\frac{e_1^+ + e_1^-}{2},\cdots,\frac{e_n^+ + e_n^-}{2}\}.$$

引理 1[22](仿射B-L不等式)　对于在区间$[a,b]\to\mathbb{R}^n$内任意连续可微函数$x(\cdot)$和所有整数$N\in\mathbb{R}$, 给定一个矩阵$R=R^{\mathrm T}>0$, 存在一个矩阵X使如下不等式成立:

$$-\int_a^b \dot x^{\mathrm T}(s)R\dot x(s)\mathrm ds\leqslant-\xi_N^{\mathrm T}(t)\Theta(X)\xi_N(t),$$

其中:

$$\Theta(X)=XH_N+(XH_N)^{\mathrm T}-(b-a)X\tilde RX^{\mathrm T},$$

$$H_N=[\varGamma_N^{\mathrm T}(0)\ \ \varGamma_N^{\mathrm T}(1)\ \ \cdots\ \ \varGamma_N^{\mathrm T}(N)]^{\mathrm T},$$

$$\tilde R=\mathrm{diag}\{R^{-1},\frac13R^{-1},\cdots,\frac{1}{2N+1}R^{-1}\},$$

$$\xi_N(t)=\begin{cases}[x^{\mathrm T}(b)\ \ x^{\mathrm T}(a)]^{\mathrm T},\ N=0,\\ [x^{\mathrm T}(b)\ x^{\mathrm T}(a)\ \ \frac{1}{b-a}\varOmega_0^{\mathrm T}\ \cdots\ \frac{1}{b-a}\varOmega_{N-1}^{\mathrm T}]^{\mathrm T},\\ \qquad N>0,\end{cases}$$

$$\varGamma_N(k)=\begin{cases}[I\ \ -I],\ N=0,\\ [I\ \ (-1)^{k+1}I\ \ \gamma_{Nk}^0 I\ \ \cdots\ \ \gamma_{Nk}^{N-1}I],\\ \qquad N>0,\end{cases}$$

$$\varOmega_k=\int_a^b L_k(s)x(s)\mathrm ds,$$

$$\gamma_{Nk}^i=\begin{cases}-(2i+1)(1-(-1)^{k+i}),\ \ i\leqslant k,\\ 0,\qquad i\geqslant k+1,\end{cases}$$

$$L_k(u)=(-1)^k\sum_{l=0}^{k}[(-1)^l\begin{pmatrix}k\\ l\end{pmatrix}\begin{pmatrix}k+l\\ l\end{pmatrix}](\frac{u-a}{b-a})^l.$$

3　主要结果

定理 1　对于给定的常数τ,μ_1,μ_2, 如果存在有对称正定矩阵$[P]_{7n\times7n}$, $[Q]_{4n\times4n}$, $[S]_{4n\times4n}$, $[R]_{n\times n}$, $[D_1]_{2n\times2n}$, $[D_2]_{2n\times2n}$, $[D_3]_{n\times n}$, $[U_1]_{2n\times2n}$, $[U_2]_{n\times n}$, $[U_3]_{n\times n}$, $[Z_i]_{n\times n}$, $[W_{ij}]_{n\times n}$, $i,j\in\mathcal N$, 正定对角矩阵$[R_1]_{n\times n}$, $[R_2]_{n\times n}$, $[R_3]_{n\times n}$, 并且任意矩阵$[X_1]_{4n\times3n}$, $[X_2]_{4n\times3n}$, 使得对于$\forall i\in\mathcal N$, 有如下线性矩阵不等式成立:

$$\begin{bmatrix}\tilde\varPhi_i(\tau,\mu_k) & \tau\varXi_l^{\mathrm T}X_l\\ \tau X_l^{\mathrm T}\varXi_l & -\tau R_N\end{bmatrix}<0,\ k,l=1,2,\tag{3}$$

$$Z_j-Z_i-W_{ij}\leqslant0,\ \ \forall j\in\mathcal N,\ j\neq i,\tag{4}$$

则具有时变时滞和不确定转移率的Markovian神经网络系统(2)是均方渐近稳定的. 为了简化不等式(3), 定义如下表达式:

$$\begin{aligned}\tilde\varPhi_i(\tau,\mu_k)=&\varPhi(\tau(t),\dot\tau(t))+\mathrm{Sym}\{e_1^{\mathrm T}Z_ie_0\}+\\ &e_1^{\mathrm T}(\sum_{j=1,j\neq i}^{s}\underline{\pi}_{ij}(Z_j-Z_i)+2\delta_{ij}W_{ij})e_1-\\ &\varXi_1^{\mathrm T}(X_1M+M^{\mathrm T}X_1^{\mathrm T})\varXi_1-\\ &\varXi_2^{\mathrm T}(X_2M+M^{\mathrm T}X_2^{\mathrm T})\varXi_2,\end{aligned}$$

$$\begin{aligned}&\varPhi(\tau(t),\dot\tau(t))=\\ &\tau e_0^{\mathrm T}Re_0+G_2^{\mathrm T}QG_2-\bar\tau(t)G_3^{\mathrm T}(Q-S)G_3-\\ &G_4^{\mathrm T}SG_4+\mathrm{Sym}\{G_0^{\mathrm T}(\dot\tau(t))PG_1(\tau(t))\}+\varOmega(\tau(t))+\\ &\mathrm{Sym}\{(\tau-\tau(t))e_7^{\mathrm T}D_3(e_1-e_3)\}-\dot\tau(t)e_7^{\mathrm T}U_2e_7+\\ &\mathrm{Sym}\{e_7^{\mathrm T}U_2[\bar\tau(t)e_2-e_3+\dot\tau(t)e_7]\}+\dot\tau(t)e_6^{\mathrm T}U_3e_6+\\ &\mathrm{Sym}\{e_6^{\mathrm T}U_3[e_1-\bar\tau(t)e_2-\dot\tau(t)e_6]\}+\\ &\mathrm{Sym}\{\tau(t)e_6^{\mathrm T}D_3(e_1-e_3)\}+\\ &[e_1^{\mathrm T}\ \ e_{10}^{\mathrm T}]D_1\begin{bmatrix}e_1\\ e_{10}\end{bmatrix}-\bar\tau(t)[e_2^{\mathrm T}\ \ e_{11}^{\mathrm T}]D_1\begin{bmatrix}e_2\\ e_{11}\end{bmatrix}+\\ &[e_1^{\mathrm T}\ \ e_{10}^{\mathrm T}]D_2\begin{bmatrix}e_1\\ e_{10}\end{bmatrix}-[e_3^{\mathrm T}\ \ e_{12}^{\mathrm T}]D_2\begin{bmatrix}e_3\\ e_{12}\end{bmatrix}+\\ &\mathrm{Sym}\{\begin{bmatrix}\tau(t)e_6\\ (\tau-\tau(t))e_7\end{bmatrix}^{\mathrm T}U_1\begin{bmatrix}e_1-\bar\tau(t)e_2\\ \bar\tau(t)e_2-e_3\end{bmatrix}\}+\\ &\begin{bmatrix}e_1\\ e_{10}\end{bmatrix}^{\mathrm T}\begin{bmatrix}-R_1E_1 & R_1E_2\\ * & -R_1\end{bmatrix}\begin{bmatrix}e_1\\ e_{10}\end{bmatrix}+\\ &\begin{bmatrix}e_2\\ e_{11}\end{bmatrix}^{\mathrm T}\begin{bmatrix}-R_2E_1 & R_2E_2\\ * & -R_2\end{bmatrix}\begin{bmatrix}e_2\\ e_{11}\end{bmatrix}+\\ &\begin{bmatrix}e_3\\ e_{12}\end{bmatrix}^{\mathrm T}\begin{bmatrix}-R_3E_1 & R_3E_2\\ * & -R_3\end{bmatrix}\begin{bmatrix}e_3\\ e_{12}\end{bmatrix},\end{aligned}$$

$$\varXi_1=[e_1^{\mathrm T}\ \ e_2^{\mathrm T}\ \ e_6^{\mathrm T}\ \ e_8^{\mathrm T}]^{\mathrm T},$$

$$\varXi_2=[e_2^{\mathrm T}\ \ e_3^{\mathrm T}\ \ e_7^{\mathrm T}\ \ e_9^{\mathrm T}]^{\mathrm T},$$

$$R_N=\mathrm{diag}\{R,3R,5R\},$$

$$\begin{aligned}&G_0(\dot\tau(t))=\\ &[e_0^{\mathrm T}\ \ \bar\tau(t)e_4^{\mathrm T}\ \ e_5^{\mathrm T}\ \ (e_1-\bar\tau(t)e_2)^{\mathrm T}\ \ (\bar\tau(t)e_2-e_3)^{\mathrm T}\\ &(e_1-\bar\tau(t)e_6-\dot\tau(t)e_8)^{\mathrm T}\ \ (\bar\tau(t)e_2-e_7+\dot\tau(t)e_9)^{\mathrm T}]^{\mathrm T},\end{aligned}$$

$$\begin{aligned}G_1(\tau(t))=&[e_1^{\mathrm T}\ \ e_2^{\mathrm T}\ \ e_3^{\mathrm T}\ \ \tau(t)e_6^{\mathrm T}\ \ (\tau-\tau(t))e_7^{\mathrm T}\\ &\tau(t)e_8^{\mathrm T}\ \ (\tau-\tau(t))e_9^{\mathrm T}]^{\mathrm T},\end{aligned}$$

$$G_2=[e_1^{\mathrm T}\ \ e_0^{\mathrm T}\ \ 0_{12n\times n}\ \ (e_1-e_3)^{\mathrm T}]^{\mathrm T},$$

$$G_3=[e_2^{\mathrm T}\ \ e_4^{\mathrm T}\ \ (e_1-e_2)^{\mathrm T}\ \ (e_2-e_3)^{\mathrm T}]^{\mathrm T},$$

$$G_4=[e_3^{\mathrm T}\ \ e_5^{\mathrm T}\ \ (e_1-e_3)^{\mathrm T}\ \ 0_{12n\times n}]^{\mathrm T},$$

$$\begin{aligned}\varOmega(\tau(t))=&\mathrm{Sym}\{\tau(t)e_6^{\mathrm T}(Q_{13}e_0-Q_{14}e_5)+\\ &(e_1-e_2)^{\mathrm T}(Q_{23}e_0-Q_{24}e_5)+\\ &\tau(t)(e_1-e_6)^{\mathrm T}(Q_{33}e_0-Q_{34}e_5)+\\ &\tau(t)(e_6-e_3)^{\mathrm T}(Q_{43}e_0-Q_{44}e_5)+\\ &(\tau-\tau(t))e_7^{\mathrm T}(S_{13}e_0-S_{14}e_5)+\\ &(e_2-e_3)^{\mathrm T}(S_{23}e_0-S_{24}e_5)+\\ &(\tau-\tau(t))(e_1-e_7)^{\mathrm T}(S_{33}e_0-S_{34}e_5)+\\ &(\tau-\tau(t))(e_7-e_3)^{\mathrm T}(S_{43}e_0-S_{44}e_5)\},\end{aligned}$$

$$M=\begin{bmatrix} I_n & -I_n & 0_n & 0_n \\ I_n & I_n & -2I_n & 0_n \\ I_n & -I_n & 6I_n & -12I_n \end{bmatrix},$$

$e_0=-C_ie_1+A_ie_{10}+B_ie_{11}$.

证 对于系统(2), 以及任意$i\in\mathcal{N}$, 考虑如下Lyapunov-Krasovskii泛函:

$$V(x(t),t,i)=\sum_{q=1}^{6}V_q(x(t),t,i). \tag{5}$$

令

$$\eta(t)=\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix},\ \upsilon(s)=\begin{bmatrix} \int_{t-\tau(t)}^{t}x(s)\mathrm{d}s \\ \int_{t-\tau}^{t-\tau(t)}x(s)\mathrm{d}s \end{bmatrix},$$

$$\begin{aligned}\eta_1(t)=[&x^{\mathrm T}(t)\ \ x^{\mathrm T}(t-\tau((t))\ \ x^{\mathrm T}(t-\tau)\\ &\int_{t-\tau(t)}^{t}x^{\mathrm T}(s)\mathrm{d}s\ \ \int_{t-\tau}^{t-\tau(t)}x^{\mathrm T}(s)\mathrm{d}s\\ &\frac{1}{\tau(t)}\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}x^{\mathrm T}(s)\mathrm{d}s\mathrm{d}\theta\\ &\frac{1}{\tau-\tau(t)}\int_{-\tau}^{-\tau(t)}\int_{t+\theta}^{t-\tau(t)}x^{\mathrm T}(s)\mathrm{d}s\mathrm{d}\theta]^{\mathrm T},\end{aligned}$$

$$\begin{aligned}\eta_2(s)=[&x^{\mathrm T}(s)\ \ \dot x^{\mathrm T}(s)\ \ \int_{s}^{t}\dot x^{\mathrm T}(\alpha)\mathrm{d}\alpha\\ &\int_{t-\tau}^{s}\dot x^{\mathrm T}(\alpha)\mathrm{d}\alpha]^{\mathrm T},\end{aligned}$$

$$\begin{aligned}\xi(t)=[&x^{\mathrm T}(t)\ \ x^{\mathrm T}(t-\tau(t))\ \ x^{\mathrm T}(t-\tau)\\ &\dot x^{\mathrm T}(t-\tau(t))\ \ \dot x^{\mathrm T}(t-\tau)\ \ \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}x^{\mathrm T}(s)\mathrm{d}s\\ &\frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)}x^{\mathrm T}(s)\mathrm{d}s\\ &\frac{1}{\tau(t)^2}\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}x^{\mathrm T}(s)\mathrm{d}s\mathrm{d}\theta\\ &\frac{1}{(\tau-\tau(t))^2}\int_{-\tau}^{-\tau(t)}\int_{t+\theta}^{t-\tau(t)}x^{\mathrm T}(s)\mathrm{d}s\mathrm{d}\theta\\ &f^{\mathrm T}(x(t))\ \ f^{\mathrm T}(x(t-\tau(t)))\ \ f^{\mathrm T}(x(t-\tau))]^{\mathrm T}.\end{aligned}$$

构建增广Lyapunov-Krasovskii泛函如下:

$$V_1(x(t),t,i)=x^{\mathrm T}(t)Z_ix(t), \tag{6}$$

$$\begin{aligned}&V_2(x(t),t,i)=\\ &\eta_1^{\mathrm T}(t)P\eta_1(t)+\int_{t-\tau(t)}^{t}\eta_2^{\mathrm T}(s)Q\eta_2(s)\mathrm{d}s+\\ &\int_{t-\tau}^{t-\tau(t)}\eta_2^{\mathrm T}(s)S\eta_2(s)\mathrm{d}s+\\ &\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot x^{\mathrm T}(s)R\dot x(s)\mathrm{d}s\mathrm{d}\theta,\end{aligned} \tag{7}$$

$$\begin{aligned}V_3(x(t),t,i)=&\int_{t-\tau(t)}^{t}\eta^{\mathrm T}(s)D_1\eta(s)\mathrm{d}s+\\ &\int_{t-\tau}^{t}\eta^{\mathrm T}(s)D_2\eta(s)\mathrm{d}s,\end{aligned} \tag{8}$$

$$V_4(x(t),t,i)=\int_{t-\tau}^{t}x^{\mathrm T}(s)\mathrm{d}sD_3\int_{t-\tau}^{t}x(s)\mathrm{d}s, \tag{9}$$

$$V_5(x(t),t,i)=\upsilon^{\mathrm T}(s)U_1\upsilon(s), \tag{10}$$

$$\begin{aligned}&V_6(x(t),t,i)=\\ &\frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)}x^{\mathrm T}(s)\mathrm{d}sU_2\int_{t-\tau}^{t-\tau(t)}x(s)\mathrm{d}s+\\ &\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}x^{\mathrm T}(s)\mathrm{d}sU_3\int_{t-\tau(t)}^{t}x(s)\mathrm{d}s.\end{aligned} \tag{11}$$

利用弱无穷小算子$\mathcal{L}$, 计算$V(x(t),t,i)$的导数, 可得到

$$\begin{aligned}&\mathcal{L}V_1(x(t),t,i)=\\ &\xi^{\mathrm T}(t)\{\mathrm{Sym}\{e_1^{\mathrm T}Z_ie_0\}+\\ &e_1^{\mathrm T}\sum_{j=1}^{s}(\pi_{ij}+\Delta\pi_{ij})Z_je_1\}\xi(t)=\\ &\xi^{\mathrm T}(t)\{\mathrm{Sym}\{e_1^{\mathrm T}Z_ie_0\}+\\ &e_1^{\mathrm T}\sum_{j=1,j\neq i}^{s}(\pi_{ij}-\delta_{ij}+\delta_{ij}+\Delta\pi_{ij})(Z_j-Z_i)e_1\}\xi(t)=\\ &\xi^{\mathrm T}(t)\{\mathrm{Sym}\{e_1^{\mathrm T}Z_ie_0\}+e_1^{\mathrm T}\sum_{j=1,j\neq i}^{s}(\underline{\pi}_{ij}(Z_j-Z_i)+\\ &(\delta_{ij}+\Delta\pi_{ij})(Z_j-Z_i))e_1\}\xi(t),\end{aligned} \tag{12}$$

$$\begin{aligned}&\mathcal{L}V_2(x(t),t,i)=\\ &\xi^{\mathrm T}(t)\{\mathrm{Sym}\{G_0^{\mathrm T}(\dot\tau(t))PG_1(\tau(t))\}+G_2^{\mathrm T}QG_2-\\ &\bar\tau(t)G_3^{\mathrm T}(Q-S)G_3-G_4^{\mathrm T}SG_4+\tau e_0^{\mathrm T}Re_0+\\ &\Omega(\tau(t))\}\xi(t)-\int_{t-\tau}^{t}\dot x^{\mathrm T}(s)R\dot x(s)\mathrm{d}s,\end{aligned} \tag{13}$$

$$\begin{aligned}&\mathcal{L}V_3(x(t),t,i)=\\ &\xi^{\mathrm T}(t)\{[e_1^{\mathrm T}\ \ e_{10}^{\mathrm T}]D_1\begin{bmatrix} e_1 \\ e_{10} \end{bmatrix}-\bar\tau(t)[e_2^{\mathrm T}\ \ e_{11}^{\mathrm T}]D_1\begin{bmatrix} e_2 \\ e_{11} \end{bmatrix}+\\ &[e_1^{\mathrm T}\ \ e_{10}^{\mathrm T}]D_2\begin{bmatrix} e_1 \\ e_{10} \end{bmatrix}-[e_3^{\mathrm T}\ \ e_{12}^{\mathrm T}]D_2\begin{bmatrix} e_3 \\ e_{12} \end{bmatrix}\}\xi(t),\end{aligned} \tag{14}$$

$$\begin{aligned}&\mathcal{L}V_4(x(t),t,i)=\\ &\xi^{\mathrm T}(t)\{\mathrm{Sym}\{\tau(t)e_6^{\mathrm T}D_3(e_1-e_3)+(\tau-\tau(t))e_7^{\mathrm T}\times\\ &D_3(e_1-e_3)\}\}\xi(t),\end{aligned} \tag{15}$$

$$\begin{aligned}&\mathcal{L}V_5(x(t),t,i)=\\ &\xi^{\mathrm T}(t)\{\mathrm{Sym}\{\begin{bmatrix} \tau(t)e_6 \\ (\tau-\tau(t))e_7 \end{bmatrix}^{\mathrm T}U_1\times\\ &\begin{bmatrix} e_1-\bar\tau(t)e_2 \\ \bar\tau(t)e_2-e_3 \end{bmatrix}\}\}\xi(t),\end{aligned} \tag{16}$$

$$\begin{aligned}&\mathcal{L}V_6(x(t),t,i)=\\ &\xi^{\mathrm T}(t)\{-\dot\tau(t)e_7^{\mathrm T}U_2e_7+\\ &\mathrm{Sym}\{e_7^{\mathrm T}U_2[\bar\tau(t)e_2-e_3+\dot\tau(t)e_7]\}+\dot\tau(t)e_6^{\mathrm T}U_3e_6+\end{aligned}$$

$$\mathrm{Sym}\{e_6^{\mathrm{T}}U_3[e_1-\bar{\tau}(t)e_2-\dot{\tau}(t)e_6]\}\}\xi(t). \tag{17}$$

根据式(13)、引理1及文献[23], 当$N=2$时可得

$$-\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s\leqslant$$
$$-\xi^{\mathrm{T}}(t)\{\varXi_1^{\mathrm{T}}(X_1M+M^{\mathrm{T}}X_1^{\mathrm{T}}-\tau(t)X_1R_N^{-1}X_1^{\mathrm{T}})\varXi_1+\varXi_2^{\mathrm{T}}(X_2M+M^{\mathrm{T}}X_2^{\mathrm{T}}-(\tau-\tau(t))X_2R_N^{-1}X_2^{\mathrm{T}})\varXi_2\}\xi(t). \tag{18}$$

根据假设1, 可得

$$\eta^{\mathrm{T}}(t)\begin{bmatrix}-R_1E_1 & R_1E_2\\ * & -R_1\end{bmatrix}\eta(t)\geqslant 0, \tag{19}$$

$$\eta^{\mathrm{T}}(t-\tau(t))\begin{bmatrix}-R_2E_1 & R_2E_2\\ * & -R_2\end{bmatrix}\eta(t-\tau(t))\geqslant 0, \tag{20}$$

$$\eta^{\mathrm{T}}(t-\tau)\begin{bmatrix}-R_3E_1 & R_3E_2\\ * & -R_3\end{bmatrix}\eta(t-\tau)\geqslant 0. \tag{21}$$

根据式(4)和$W_{ij}, \Delta\pi_{ij}\in[-\delta_{ij},\delta_{ij}]$, $i,j\in\mathcal{N}, i\neq j$可得

$$(\delta_{ij}+\Delta\pi_{ij})(Z_j-Z_i)\leqslant(\delta_{ij}+\Delta\pi_{ij})W_{ij}\leqslant 2\delta_{ij}W_{ij}.$$

根据式(12), 可得

$$\mathcal{L}V_1(x(t),t,i)\leqslant\xi^{\mathrm{T}}(t)\{\mathrm{Sym}\{e_1^{\mathrm{T}}Z_ie_0\}+e_1^{\mathrm{T}}\sum_{j=1,j\neq i}^{s}(\underline{\pi}_{ij}(Z_j-Z_i)+2\delta_{ij}W_{ij})e_1\}\xi(t). \tag{22}$$

由式(6)–(22), 可得

$$\mathcal{E}\{\mathcal{L}V(x(t),t,i)\}\leqslant\mathcal{E}\{\xi^{\mathrm{T}}(t)\varPsi(\tau(t),\dot{\tau}(t))\xi(t)\}, \tag{23}$$

其中:

$$\varPsi(\tau(t),\dot{\tau}(t))=\varXi_1^{\mathrm{T}}(\tau(t)X_1R_N^{-1}X_1^{\mathrm{T}})\varXi_1+\varXi_2^{\mathrm{T}}((\tau-\tau(t))X_2R_N^{-1}X_2^{\mathrm{T}})\varXi_2+\tilde{\varPhi}_i(\tau(t),\dot{\tau}(t)).$$

根据文献[22], 如果$\varPsi(\tau(t),\dot{\tau}(t))$在区间$[0,\tau]\times[\mu_1,\mu_2]$顶点处满足$\varPsi(\tau(t),\dot{\tau}(t))<0$, 则在区间内也满足$\varPsi(\tau(t),\dot{\tau}(t))<0$. 因此, 根据Schur补定理, 线性矩阵不等式(3)–(4), 可得$\varPsi(\tau(t),\dot{\tau}(t))<0$.　证毕.

注 1　在处理不确定转移率时, 文献[20]采用了两组松弛变量G_{ij}和T_{ij}, 从而导致了决策变量和计算复杂度的增加. 在本文中, 只采用了松弛变量W_{ij}来处理不确定转移率, 从而减少了决策变量的个数.

注 2　根据文献[22], 在定理1中, 采用了高维数的Lyapunov-Krasovskii矩阵$[P]_{7n\times 7n}$, $[Q]_{4n\times 4n}$, $[S]_{4n\times 4n}$, 增加了矩阵块变量. 此外, 采用了含有时滞依赖的Lyapunov-Krasovskii泛函$V_6(x(t),t,i)$, 从而使得导数项含有更多的时滞和时滞导数信息, 有利于得到具有较小保守性的稳定性条件.

注 3　对于积分不等式(18), 本文采用具有较小保守性的B–L不等式[22], 当N取值更大整数时, 会得到更紧的不等式下界, 但同时也会增加计算的复杂度, 因此本文中$N=2$.

当不考虑Markovian跳变时, 系统模型(1)简化为式(24)形式, 可得推论1

$$\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-\tau(t))). \tag{24}$$

推论 1　对于给定的常数τ, μ_1, μ_2, 如果存在对称正定矩阵$[P]_{7n\times 7n}$, $[Q]_{4n\times 4n}$, $[S]_{4n\times 4n}$, $[R]_{n\times n}$, $[D_1]_{2n\times 2n}$, $[D_2]_{2n\times 2n}$, $[D_3]_{n\times n}$, $[U_1]_{2n\times 2n}$, $[U_2]_{n\times n}$, $[U_3]_{n\times n}$, 正定对角矩阵$[R_1]_{n\times n}$, $[R_2]_{n\times n}$, $[R_3]_{n\times n}$, 任意矩阵$[X_1]_{4n\times 3n}$, $[X_2]_{4n\times 3n}$, 有如下线性矩阵不等式成立:

$$\begin{bmatrix}\tilde{\varPhi}(\tau,\mu_k) & \tau\varXi_l^{\mathrm{T}}X_l\\ \tau X_l^{\mathrm{T}}\varXi_l & -\tau R_N\end{bmatrix}<0,\ k,l=1,2, \tag{25}$$

则时变时滞神经网络(24)为全局渐近稳定的. 其中:

$$\hat{e}_0=-Ce_1+Ae_{10}+Be_{11},$$
$$\tilde{\varPhi}(\tau,\mu_k)=\hat{\varPhi}(\tau(t),\dot{\tau}(t))-\varXi_1^{\mathrm{T}}(X_1M+M^{\mathrm{T}}X_1^{\mathrm{T}})\varXi_1-\varXi_2^{\mathrm{T}}(X_2M+M^{\mathrm{T}}X_2^{\mathrm{T}})\varXi_2.$$

在$\hat{\varPhi}(\tau(t),\dot{\tau}(t))$中, $\hat{e}_0$与定理1中$\varPhi(\tau(t),\dot{\tau}(t))$的$e_0$不同, 其余项与$\varPhi(\tau(t),\dot{\tau}(t))$各项相同.

证　对于系统(24), 考虑如下Lyapunov-Krasovskii泛函:

$$V(t)=\sum_{q=1}^{5}V_q(t), \tag{26}$$

其中:

$$V_1(t)=\eta_1^{\mathrm{T}}(t)P\eta_1(t)+\int_{t-\tau(t)}^{t}\eta_2^{\mathrm{T}}(s)Q\eta_2(s)\mathrm{d}s+\int_{t-\tau}^{t-\tau(t)}\eta_2^{\mathrm{T}}(s)S\eta_2(s)\mathrm{d}s+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{27}$$

$$V_2(t)=\int_{t-\tau(t)}^{t}\eta^{\mathrm{T}}(s)D_1\eta(s)\mathrm{d}s+\int_{t-\tau}^{t}\eta^{\mathrm{T}}(s)D_2\eta(s)\mathrm{d}s, \tag{28}$$

$$V_3(t)=\int_{t-\tau}^{t}x^{\mathrm{T}}(s)\mathrm{d}sD_3\int_{t-\tau}^{t}x(s)\mathrm{d}s, \tag{29}$$

$$V_4(t)=\upsilon^{\mathrm{T}}(s)U_1\upsilon(s), \tag{30}$$

$$V_5(t)=\frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)}x^{\mathrm{T}}(s)\mathrm{d}sU_2\times\int_{t-\tau}^{t-\tau(t)}x(s)\mathrm{d}s+\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}x^{\mathrm{T}}(s)\mathrm{d}s\times U_3\int_{t-\tau(t)}^{t}x(s)\mathrm{d}s. \tag{31}$$

然后对(26)求导, 其余的证明过程与定理1相似, 此处省略.

4 数值仿真

例子 1 考虑具有3个模态Markovian跳变时变时滞神经网络系统(2), 系统参数如下:

$$C_1=\begin{bmatrix}2.03 & 0\\ 0 & 2.03\end{bmatrix},\ A_1=\begin{bmatrix}1.01 & 1.01\\ -1.01 & -1.01\end{bmatrix},$$

$$B_1=\begin{bmatrix}0.87 & 1.02\\ 1.02 & 1.02\end{bmatrix},\ C_2=\begin{bmatrix}2.1 & 0\\ 0 & 2.6\end{bmatrix},$$

$$A_2=\begin{bmatrix}-1.1 & 0.6\\ 0.6 & -1.1\end{bmatrix},\ B_2=\begin{bmatrix}-0.6 & 0.6\\ 0.6 & 0.6\end{bmatrix},$$

$$C_3=\begin{bmatrix}2.3 & 0\\ 0 & 2.8\end{bmatrix},\ A_3=\begin{bmatrix}-1.2 & 0.5\\ 0.5 & -1.2\end{bmatrix},$$

$$B_3=\begin{bmatrix}-0.4 & 0.4\\ 0.4 & 0.4\end{bmatrix}.$$

不确定转移概率矩阵为

$$\hat{\Pi}=\begin{bmatrix}-0.5+\Delta\pi_{11} & 0.2+\Delta\pi_{12} & 0.3+\Delta\pi_{13}\\ 0.2+\Delta\pi_{21} & -0.3+\Delta\pi_{22} & 0.1+\Delta\pi_{23}\\ 0.1+\Delta\pi_{31} & 0.1+\Delta\pi_{32} & -0.2+\Delta\pi_{33}\end{bmatrix},$$

$|\Delta\pi_{ij}| \leqslant 0.5\pi_{ij},\ i,j\in\mathcal{N},\ j\neq i.$

激活函数为$f(x(t))=\tan(x(t))$, 假设初始值

$$x_0=\begin{bmatrix}-1\\ 1\end{bmatrix},\ \tau(t)=0.2+0.2\sin(2t).$$

根据假设1, 可得到$E_1=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$, $E_2=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}$.

例子1的状态轨迹如图1所示, 可知系统(2)收敛到原点, 系统(2)的3个跳变模态如图2所示.

当时滞导数μ分别为0.4, 0.45, 0.5, 0.55时, 根据定理1, 得最大时滞上界τ分别为0.5106, 0.4903, 0.4733和0.4606, 与文献[4]对比如表1所示, 时滞上界的改进率分别为65.8%, 66.1%, 66.9%, 68.8%.

例子 2 对于时变时滞神经网络系统(24), 参考文献[24]中Example 1的系统参数如下:

$$C=\begin{bmatrix}1.5 & 0\\ 0 & 0.7\end{bmatrix},\ A=\begin{bmatrix}0.0503 & 0.0454\\ 0.0987 & 0.2075\end{bmatrix},$$

$$B=\begin{bmatrix}0.2381 & 0.9320\\ 0.0388 & 0.5602\end{bmatrix},\ E_1=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},$$

$$E_2=\begin{bmatrix}0.15 & 0\\ 0 & 0.4\end{bmatrix}.$$

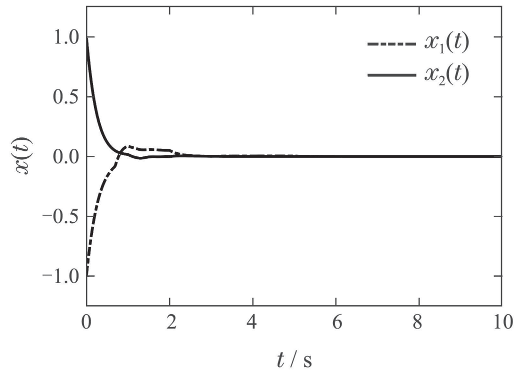

图 1 系统(2)的状态轨迹

Fig. 1 State trajectories of system (2)

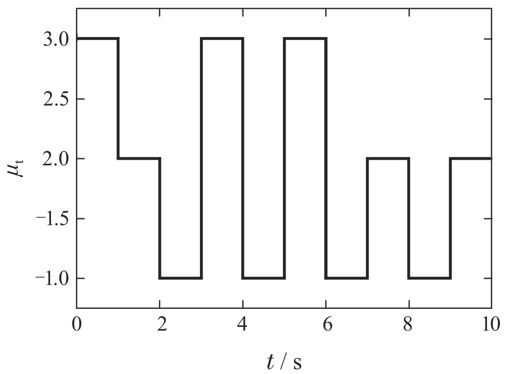

图 2 系统(2)的3个模态跳变

Fig. 2 Three jumping modes of system (2)

表 1 例子1在不同的μ下所允许的上界τ

Table 1 Allowable upper bound τ for various μ of Example 1

方法	条件$\dot{\tau}(t)$	μ			
		0.4	0.45	0.5	0.55
定理1[4]$(D=0)$	$\dot{\tau}(t)<\mu$	0.3080	0.2952	0.2836	0.2729
定理1	$-\mu\leqslant\dot{\tau}(t)\leqslant\mu$	0.5106	0.4903	0.4733	0.4606

表 2 例子2在不同的μ下所允许的上界τ

Table 2 Allowable upper bound τ for various μ of Example 2

方法	条件$\dot{\tau}(t)$	μ			
		0.4	0.45	0.5	0.55
定理1[25]$(N=4)$	$-\mu\leqslant\dot{\tau}(t)\leqslant\mu$	5.2761	4.4527	4.1208	4.0026
定理2[24]	$-\mu\leqslant\dot{\tau}(t)\leqslant\mu$	5.3079	4.5267	4.2924	4.1903
推论1	$-\mu\leqslant\dot{\tau}(t)\leqslant\mu$	5.9454	5.5149	5.2550	5.0620

当时滞导数μ分别为0.4, 0.45, 0.5, 0.55时, 根据推论1, 可得本文方法最大时滞上界τ分别为5.9454, 5.5149, 5.2550, 5.0620. 与其他文献对比如表2所示, 本文方法的最大时滞上界与文献[25]相比, 改进率分别为12.7%, 23.8%, 27.5%, 26.5%, 与文献[24]相比, 改进率分别为12.0%, 21.8%, 22.4%, 20.8%.

5　结论

本文研究了时变时滞和不确定转移率的马尔科夫神经网络的稳定性问题. 通过构造时滞依赖的增广Lyapunov-Krasovskii泛函和采用仿射B-L不等式, 得到了具有较小保守性稳定性准则. 最后通过两个数值仿真例子说明了本文理论分析的有效性.

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