精确估计下的多智能体系统漏斗复合控制

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摘要: 针对一类具有时变扰动的非线性多智能体系统, 研究其在有向拓扑下的一致性跟踪问题, 提出一种基于精确估计的复合自适应预设有限时间(PFT)漏斗控制方法. 首先, 构建一种新的PFT漏斗控制, 使跟踪误差约束在PFT漏斗边界内. 其次, 采用神经网络(NN)逼近系统的未知非线性, 并利用NN逼近信息设计扰动观测器, 建立基于NN和扰动观测器的复合估计模型, 将得到的预测误差引入NN权值的复合更新律中, 实现对未知非线性和时变扰动的精确估计. 然后, 利用动态面技术和误差补偿机制, 在解决传统反步法"计算爆炸"问题的同时, 消除滤波器误差对系统的影响. 最后, 通过Lyapunov稳定性理论证明闭环系统所有信号均为有界的, 并通过仿真实验验证控制方法的有效性.

关键词: 多智能体系统; 神经网络; 扰动观测器; 复合估计模型

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Funnel composite control for multi-agent systems with accurate estimation

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Abstract: For a class of nonlinear multi-agent systems with time-varying disturbances, the problem of consensus tracking in directed topology is studied, and a composite adaptive preassigned finite-time (PFT) funnel control based on accurate estimation is proposed. Firstly, a new PFT funnel control is used to constrain the tracking error within the boundary of PFT funnel. Secondly, neural networks (NNs) are employed to approximate the unknown nonlinearities of the system, and the disturbance observer is constructed with NN approximation information. The composite estimation model with NN and the disturbance observer is built to obtain the prediction error. The prediction error is introduced into the NN weight updating composite law to accurately estimate the unknown nonlinearities and time-varying disturbances. Then, the dynamic surface control technique is utilized to solve the "explosion of complexity" problem caused by the traditional backstepping technique, and the error compensation method is utilized to solve the influence of filter errors. Finally, according to the Lyapunov stability theory, all signals in the closed-loop system are bounded, and the effectiveness of the proposed control algorithm is verified by simulation results.

Key words: multi-agent systems; neural network; disturbance observer; composite estimation model

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1 引言

近年来, 多智能体系统协同控制因其通信成本低、灵活性与鲁棒性高等特点, 在卫星集群、无人车跟踪、移动机器人等领域得到了广泛应用[1–4]. 作为协同控制的基本问题, 一致性跟踪控制受到了国内外专家学者的广泛关注, 并取得了大量研究成果[5–8]. 实际系统大多是本质非线性的, 而神经网络 (neural network, NN)作为一种处理非线性的有效手段, 广泛应用于控制器设计中[9–16]. 其中, 文献[12]采用NN逼近严格反馈系统的未知非线性函数, 并基于动态面技术设计了自适应NN控制器, 解决了传统反步法的"计算爆炸"问题. 文献[13]针对一类具有周期扰动的纯反馈非线性系统, 提出了自适应NN控制方法, 放宽了未知非仿射函数的有界条件, 并消除了非仿射函数可微的限制. 文献[14]研究了一类非严格反馈系统的自适应NN控制问题, 利用系统边界函数的单调性和径向基NN的结构特征, 克服了非严格反馈结构带来的设计困难. 文献[15]提出了一种自适应NN容错控制方法, 利用NN逼近大系统的非线性互联项, 并设计了扰动观测器对逼近误差、未知死区和外部扰动组成的复合扰动进行估计. 文献[16]将NN扩展到多智能体系统的研究中, 设计了NN状态观测器, 解决了高阶非线性多智能体系统的状态不可测问题. 然而, 文献[9–16]采用的传统NN不能准确逼近未知非线性. 为了得到更好的逼近效果和期望性能, 文献[17]提出了一种复合NN动态面控制方法, 在权值更新律的设计中引入预测误差, 实现了对未知非线性函数的精确逼近. 文献[18]采用文献[17]提出的思想, 设计了基于NN和扰动观测器的估计模型, 解决了具有时变扰动的未知非线性系统自适应控制问题. 文献[17–18]将复合自适应律的设计思想引入简单系统中, 得到了准确逼近的效果.

系统受物理器件和外界条件的限制, 在控制过程中往往需要对跟踪误差进行约束, 从而保证系统安全稳定运行. 漏斗控制作为一种有效的约束方法, 可以较好地调节闭环系统稳态和瞬态响应. 目前, 针对漏斗控制的研究已取得了丰硕的成果[19–24]. 其中, 文献[22]首次提出了漏斗控制设计思想, 并通过简单的误差反馈控制, 使跟踪误差满足规定约束. 文献[23]将漏斗误差变换与动态面技术相结合, 提高了严格反馈系统的输出性能. 文献[24]改进了漏斗误差变换, 解决了文献[23]中跟踪误差为零时存在的不可微问题. 需要指出的是, 上述研究仅能得到渐近稳定的结果, 而实际工程控制中往往需要闭环系统在有限时间内达到稳定状态. 为此, 文献[25]针对非仿射非线性系统, 设计了自适应有限时间漏斗控制器, 使闭环系统中的信号均为半全局实际有限时间有界的. 文献[26]针对具有未知输入约束的永磁同步电机系统, 提出了一种有限时间NN漏斗控制方法, 使跟踪误差有限时间稳定且约束在规定的漏斗边界内. 但文献[25–26]提出的有限时间都受限于系统初始条件和设计参数, 且设计过程相对复杂.

鉴于以上分析, 本文研究一类具有未知非线性和时变扰动的多智能体系统一致性跟踪问题, 设计基于精确估计的预设有限时间 (preassigned finite-time, PFT)漏斗复合控制器. 不同于文献[25–26]的结果, 本文提出的PFT漏斗控制方法保证跟踪误差收敛时间的独立性和灵活性, 并且设计过程简单. 此外, 尽管文献[18]在文献[17]的基础上处理了外界扰动对系统的影响, 但是它们的研究对象均为简单的非线性系统, 无法直接应用到复杂的多智能体系统中, 而本文结合智能体间的拓扑结构与权值复合更新律的设计思想, 建立复合估计模型来提高逼近精度, 进而增强多智能体系统的鲁棒性.

2 预备知识与问题阐述

2.1 代数图论

本文采用图论描述智能体间的通信拓扑, 定义有向图$\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$. 其中, $\mathcal{V}=(v_1,v_2,\cdots,v_N)$为$N$个智能体的非空集合, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$为智能体边的集合, $(v_i,v_j)\in\mathcal{E}$表示智能体i接收到智能体j的信息. 智能体i的邻接节点集合表示为$\mathcal{N}_i=\{v_j\in|(v_i,v_j)\in\mathcal{E}, i\neq j\}$. 定义权重邻接矩阵为$\mathcal{A}=[a_{i,j}]\in\mathbb{R}^{N\times N}$, 如果$(v_i,v_j)\in\mathcal{E}$, 那么$a_{i,j}>0$; 否则$a_{i,j}=0$. 假设拓扑图中不存在自环, 即$a_{i,i}=0, \forall i\in\mathcal{V}$. 定义节点$i$的入度矩阵为$b_i=\sum\limits_{j\in\mathcal{N}_i}a_{i,j}$, 入度对角矩阵$\mathcal{R}=\mathrm{diag}\{b_1,b_2,\cdots,b_N\}$, 则图$\mathcal{G}$的拉普拉斯矩阵为$\mathcal{L}=\mathcal{R}-\mathcal{A}$. 定义$\mathcal{H}=\mathrm{diag}\{a_{1,0},a_{2,0},\cdots,a_{N,0}\}$, 如果节点$i$能接收到领导者0的信息, 则$a_{i,0}>0$; 否则, $a_{i,0}=0$.

引理 1[5] 如果存在一条路径能够从根节点0到达所有其他节点, 那么称有向图$\mathcal{G}$具有一个生成树, 则矩阵$\mathcal{L}+\mathcal{H}$是非奇异的.

假设 1[6] 领导者的信号y_0只能被部分智能体直接获取. 且y_0已知, y_0及其导数均为连续有界的.

2.2 系统描述

本文研究的多智能体系统包含N个跟随者, 第i个跟随者可描述为一类具有未知非线性与时变扰动的n_i阶严格反馈系统

$$\begin{cases}\dot{x}_{i,m}=x_{i,m+1}+f_{i,m}(\bar{x}_{i,m})+d_{i,m}(t),\\ y_i=x_{i,1},\\ u_i=x_{i,n_i+1},\end{cases}\tag{1}$$

其中: $\bar{x}_{i,m}=[x_{i,1}\ \ x_{i,2}\ \ \cdots\ \ x_{i,m}]^{\mathrm{T}}\in\mathbb{R}^m$为第$i$个智能体的状态向量; $u_i\in\mathbb{R}$和$y_i\in\mathbb{R}$分别表示第i个智能体的控制输入和系统输出; $f_{i,m}(\bar{x}_{i,m})$为未知光滑的

非线性函数; $d_{i,m}(t)$为时变扰动, $i=1,2,\cdots,N, m=1,2,\cdots,n_i$.

假设 2[5] 存在正常数$d^0_{i,m}$和$d^1_{i,m}$, 使得时变扰动$d_{i,m}(t)$满足$|d_{i,m}(t)|\leqslant d^0_{i,m}$且$|\dot{d}_{i,m}(t)|\leqslant d^1_{i,m}$.

2.3 漏斗控制

漏斗控制通过引入时变的控制增益$\varrho(t)$达到控制目的, 文献[22]将漏斗控制的控制输入定义如下:

$$\xi(t)=\varrho(F_\psi(t),\|e(t)\|)e(t),$$

其中: $F_\psi(t)$为漏斗边界; $\|e(t)\|$为跟踪误差的欧几里得范数.

控制增益$\varrho(t)$可选择为

$$\varrho(t)=\frac{1}{d_v(t)}=\frac{1}{F_\psi(t)-\|e(t)\|},$$

其中$d_v(t)=F_\psi(t)-\|e(t)\|$为边界$F_\psi(t)$与$\|e(t)\|$的垂直距离.

文献[19–26]选择如下漏斗边界:

$$F_\psi=(\rho_0-\rho_\infty)\mathrm{e}^{-\beta t}+\rho_\infty,$$

其中: ρ_0为边界函数的初始值; β为指数函数的收敛速度; $\lim\limits_{t\to\infty}F_\psi(t)=\rho_\infty, \rho_0, \beta, \rho_\infty$为设计参数.

定义 1[27] 如果一个光滑函数$F_\psi(t)$满足以下条件: 1) $F_\psi(t)>0$; 2) $\dot{F}_\psi(t)\leqslant 0$; 3) $\lim\limits_{t\to T}F_\psi(t)=\rho_T>0, t\geqslant T$时, $F_\psi(t)=\rho_T$, 其中ρ_T和T分别为下界和收敛时间, 则称为PFT性能函数.

根据定义1, 设计如下PFT漏斗边界

$$F_{\psi,i}=\begin{cases}(\rho_{i,0}-\dfrac{t}{T_i})\mathrm{e}^{(1-\frac{T_i}{T_i-t})}+\rho_{i,T_i}, & t\in[0,T_i),\\ \rho_{i,T_i}, & t\in[T_i,+\infty),\end{cases}$$

其中: $i=1,2,\cdots,N, T_i>0, \rho_{i,0}>0, \rho_{i,T_i}>0$为设计参数.

选择的漏斗误差变换如下:

$$\xi_{i,1}=\frac{e_{i,1}}{\sqrt{F^2_{\psi,i}-e^2_{i,1}}}, \tag{2}$$

对式(2)两端求导, 得

$$\begin{aligned}\dot{\xi}_{i,1}=&\frac{F^2_{\psi,i}}{\sqrt{(F^2_{\psi,i}-e^2_{i,1})^3}}(\dot{e}_{i,1}-\frac{e_{i,1}\dot{F}_{\psi,i}}{F_{\psi,i}})=\\&\mathit{\Gamma}_{i,1}[(b_i+a_{i,0})(x_{i,2}+f_{i,1}(\bar{x}_{i,1})+d_{i,1})-\\&\sum_{j\in\mathcal{N}_i}a_{i,j}(x_{j,2}+f_{j,1}(\bar{x}_{j,1})+d_{j,1})-\\&a_{i,0}\dot{y}_0-\frac{e_{i,1}\dot{F}_{\psi,i}}{F_{\psi,i}}],\end{aligned} \tag{3}$$

其中: $\mathit{\Gamma}_{i,1}=\dfrac{F^2_{\psi,i}}{\sqrt{(F^2_{\psi,i}-e^2_{i,1})^3}}$; 跟踪误差$e_{i,1}=\sum\limits_{j\in\mathcal{N}_i}a_{i,j}(y_i-y_j)+a_{i,0}(y_i-y_0)$.

注 1 根据文献[19–26]中采用的漏斗边界可知, 其需在时间趋于无穷时收敛于ρ_∞. 相比之下, 本文所设计的PFT漏斗边界具有更好的暂态性能, 因其可在有限时间T_i收敛到提前设定的界内, 这也体现了设计方法的灵活性.

3 控制器设计和稳定性分析

3.1 预设有限时间漏斗复合控制器设计

本节设计了一种PFT漏斗复合控制器, 使跟踪误差约束在PFT漏斗内, 并保证闭环系统中的所有信号均为有界的. 基于NN的逼近特性, 系统(1)可转换为如下形式:

$$\begin{cases}\dot{x}_{i,m}=x_{i,m+1}+w^{*\mathrm{T}}_{i,m}\theta_{i,m}(\bar{x}_{i,m})+D_{i,m},\\ y_i=x_{i,1},\end{cases} \tag{4}$$

其中: $w^*_{i,m}$是NN最优权重向量; $\theta_{i,m}(\bar{x}_{i,m})$表示基函数向量; $D_{i,m}=\varepsilon_{i,m}+d_{i,m}(t)$; $\varepsilon_{i,m}$是NN逼近误差, 且$|\varepsilon_{i,m}|\leqslant\varepsilon_M$. 由假设2可知, $|D_{i,m}|\leqslant\epsilon_{i,m}, |\dot{D}_{i,m}|\leqslant\bar{\epsilon}_{i,m}$, 且$\epsilon_{i,m}, \bar{\epsilon}_{i,m}$是未知正常数.

接下来进行如下坐标变换:

$$e_{i,r}=x_{i,r}-\alpha^c_{i,r-1},\ r=2,3,\cdots,n_i, \tag{5}$$

其中$\alpha^c_{i,r-1}$为虚拟控制器$\alpha^d_{i,r-1}$通过一阶低通滤波器的输出. 本文选择如下的一阶滤波器:

$$\tau_{i,r-1}\dot{\alpha}^c_{i,r-1}+\alpha^c_{i,r-1}=\alpha^d_{i,r-1},$$

其中: $\alpha^c_{i,r-1}(0)=\alpha^d_{i,r-1}(0)$; $\tau_{i,r-1}>0$为设计参数.

本文针对滤波误差、NN逼近不精确和扰动问题, 分别定义如下误差

a) 定义补偿后的跟踪误差信号$v_{i,m}$为

$$\begin{aligned}v_{i,1}&=\xi_{i,1}-z_{i,1},\\ v_{i,r}&=e_{i,r}-z_{i,r},\end{aligned} \tag{6}$$

其中$z_{i,m}$为补偿信号.

b) 定义预测误差$\eta_{i,m}$为

$$\eta_{i,m}=x_{i,m}-\hat{x}_{i,m}, \tag{7}$$

其中$\hat{x}_{i,m}$为估计$w^{*\mathrm{T}}_{i,m}\theta_{i,m}(\bar{x}_{i,m})+D_{i,m}$得到的预测状态.

c) 定义扰动估计误差$\tilde{D}_{i,m}$为

$$\tilde{D}_{i,m}=D_{i,m}-\hat{D}_{i,m}, \tag{8}$$

其中$\hat{D}_{i,m}$为$D_{i,m}$估计值.

设计如下基于NN的扰动观测器:

$$\begin{cases}\hat{D}_{i,1}=s_{i,1}+\lambda_{i,1}x_{i,1},\\ \hat{D}_{i,r}=s_{i,r}+\lambda_{i,r}x_{i,r},\end{cases} \tag{9}$$

$$\begin{cases}\dot{s}_{i,1}=-\lambda_{i,1}(x_{i,2}+\hat{w}^{\mathrm{T}}_{i,1}\theta_{i,1}+\hat{D}_{i,1})+\\ \qquad\mathit{\Upsilon}_{i,1}v_{i,1}+\gamma_{\eta_{i,1}}\eta_{i,1},\\ \dot{s}_{i,r}=-\lambda_{i,r}(x_{i,r+1}+\hat{w}^{\mathrm{T}}_{i,r}\theta_{i,r}+\hat{D}_{i,r})+\\ \qquad v_{i,r}+\gamma_{\eta_{i,r}}\eta_{i,r},\end{cases} \tag{10}$$

其中: $s_{i,m}$为扰动观测器的辅助系统状态; $\hat{w}_{i,m}$是$w_{i,m}^*$的估计值; $\varUpsilon_{i,1}=\varGamma_{i,1}(b_i+a_{i,0})$, 且$\tilde{w}_{i,m}=w_{i,m}^*-\hat{w}_{i,m}$, $\gamma_{\eta_{i,m}}>0$, $\lambda_{i,m}>0$为设计参数.

建立基于NN和扰动观测器的复合估计模型$\hat{w}_{i,m}^{\mathrm{T}}\theta_{i,m}+\hat{D}_{i,m}$, 将$\dot{\hat{x}}_{i,m}$设计为

$$\dot{\hat{x}}_{i,m}=x_{i,m+1}+\hat{w}_{i,m}^{\mathrm{T}}\theta_{i,m}+\hat{D}_{i,m}+\beta_{i,m}\eta_{i,m}, \tag{11}$$

其中: $\beta_{i,m}>0$为设计参数; $\hat{x}_{i,m}(0)=x_{i,m}(0)$.

步骤 1 选择如下Lyapunov函数:

$$V_{i,1}=\frac{1}{2}v_{i,1}^2+\frac{1}{2}\tilde{D}_{i,1}^2+\frac{1}{2}\gamma_{\eta_{i,1}}\eta_{i,1}^2+\frac{1}{2\gamma_{i,1}}\tilde{w}_{i,1}^{\mathrm{T}}\tilde{w}_{i,1}, \tag{12}$$

其中$\gamma_{i,1}>0$为设计参数.

对式(12)两端求导, 并由式(3)–(11), 得

$$\begin{aligned}\dot{V}_{i,1}=&v_{i,1}[\varGamma_{i,1}((b_i+a_{i,0})(e_{i,2}+(\alpha_{i,1}^c-\alpha_{i,1}^d)+\\&\alpha_{i,1}^d+w_{i,1}^{*\mathrm{T}}\theta_{i,1}+D_{i,1})-\sum_{j\in\mathcal{N}_i}a_{i,j}(x_{j,2}+\\&w_{j,1}^{*\mathrm{T}}\theta_{j,1}+D_{j,1})-a_{i,0}\dot{y}_0-\frac{e_{i,1}\dot{F}_{\psi,i}}{F_{\psi,i}})-\\&\dot{z}_{i,1}]+\tilde{D}_{i,1}[\dot{D}_{i,1}-\lambda_{i,1}(\tilde{w}_{i,1}^{\mathrm{T}}\theta_{i,1}+\tilde{D}_{i,1})-\\&\varUpsilon_{i,1}v_{i,1}-\gamma_{\eta_{i,1}}\eta_{i,1}]+\gamma_{\eta_{i,1}}\eta_{i,1}(\tilde{w}_{i,1}^{\mathrm{T}}\theta_{i,1}+\\&\tilde{D}_{i,1}-\beta_{i,1}\eta_{i,1})-\frac{1}{\gamma_{i,1}}\tilde{w}_{i,1}^{\mathrm{T}}\dot{\hat{w}}_{i,1}.\end{aligned} \tag{13}$$

由Young's不等式, 得

$$-\tilde{D}_{i,1}\tilde{w}_{i,1}^{\mathrm{T}}\theta_{i,1}\leqslant\frac{1}{2}c_{i,1}\varsigma_{i,l}\tilde{D}_{i,1}^2+\frac{1}{2c_{i,1}}\tilde{w}_{i,1}^{\mathrm{T}}\tilde{w}_{i,1}, \tag{14}$$

$$\tilde{D}_{i,1}\dot{D}_{i,1}\leqslant\frac{1}{2}\tilde{D}_{i,1}^2+\frac{1}{2}\bar{\epsilon}_{i,1}^2, \tag{15}$$

$$\begin{aligned}\varXi\leqslant&\varGamma_{i,1}^2\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(1+\theta_{j,1}^{\mathrm{T}}\theta_{j,1})v_{i,1}^2+\\&\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2),\end{aligned} \tag{16}$$

其中: $\|\theta_{i,1}\|^2\leqslant\varsigma_{i,1}$, $\varsigma_{i,1}$为NN节点数; $|\dot{D}_{i,1}|\leqslant\bar{\epsilon}_{i,1}$, $\varXi=-\varGamma_{i,1}[\sum_{j\in\mathcal{N}_i}a_{i,j}(\tilde{w}_{j,1}^{\mathrm{T}}\theta_{j,1}+\tilde{D}_{j,1})]v_{i,1}$, $c_{i,1}>0$为设计参数.

设计虚拟控制器$\alpha_{i,1}^d$为

$$\begin{aligned}\alpha_{i,1}^d=&-\hat{w}_{i,1}^{\mathrm{T}}\theta_{i,1}-\hat{D}_{i,1}+\frac{1}{b_i+a_{i,0}}[\sum_{j\in\mathcal{N}_i}a_{i,j}\times\\&(x_{j,2}+\hat{w}_{j,1}^{\mathrm{T}}\theta_{j,1}+\hat{D}_{j,1}-\\&\frac{1}{2}\varGamma_{i,1}(1+\theta_{j,1}^{\mathrm{T}}\theta_{j,1})\xi_{i,1})+a_{i,0}\dot{y}_0+\\&\frac{e_{i,1}\dot{F}_{\psi,i}}{F_{\psi,i}}-k_{i,1}\varGamma_{i,1}\xi_{i,1}],\end{aligned} \tag{17}$$

其中$k_{i,1}>0$为设计参数.

将式(14)–(15)(17)代入式(13), 得

$$\begin{aligned}\dot{V}_{i,1}\leqslant&v_{i,1}[-k_{i,1}\varGamma_{i,1}^2\xi_{i,1}+\varUpsilon_{i,1}e_{i,2}+\varUpsilon_{i,1}(\alpha_{i,1}^c-\\&\alpha_{i,1}^d)-\varGamma_{i,1}^2\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(1+\theta_{j,1}^{\mathrm{T}}\theta_{j,1})\xi_{i,1}-\\&\dot{z}_{i,1}+\varUpsilon_{i,1}\tilde{w}_{i,1}^{\mathrm{T}}\theta_{i,1}-\varGamma_{i,1}\sum_{j\in\mathcal{N}_i}a_{i,j}(\tilde{w}_{j,1}^{\mathrm{T}}\theta_{j,1}+\\&\tilde{D}_{j,1})]-(\lambda_{i,1}-\frac{1}{2}c_{i,1}\lambda_{i,1}\varsigma_{i,1}-\frac{1}{2})\tilde{D}_{i,1}^2+\\&\frac{1}{2}\bar{\epsilon}_{i,1}^2+\frac{1}{2c_{i,1}}\lambda_{i,1}\tilde{w}_{i,1}^{\mathrm{T}}\tilde{w}_{i,1}-\gamma_{\eta_{i,1}}\beta_{i,1}\eta_{i,1}^2+\\&\tilde{w}_{i,1}^{\mathrm{T}}(\gamma_{\eta_{i,1}}\eta_{i,1}\theta_{i,1}-\frac{1}{\gamma_{i,1}}\dot{\hat{w}}_{i,1}).\end{aligned} \tag{18}$$

将补偿信号$z_{i,1}$的导数选择为

$$\begin{aligned}\dot{z}_{i,1}=&-k_{i,1}\varGamma_{i,1}^2z_{i,1}+\varUpsilon_{i,1}z_{i,2}+\varUpsilon_{i,1}(\alpha_{i,1}^c-\alpha_{i,1}^d)-\\&\varGamma_{i,1}^2\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(1+\theta_{j,1}^{\mathrm{T}}\theta_{j,1})z_{i,1},\end{aligned} \tag{19}$$

其中$z_{i,1}(0)=0$.

由式(16)(18)–(19), 得

$$\begin{aligned}\dot{V}_{i,1}\leqslant&-k_{i,1}\varGamma_{i,1}^2v_{i,1}^2+\varUpsilon_{i,1}v_{i,1}v_{i,2}-(\lambda_{i,1}-\\&\frac{1}{2}c_{i,1}\lambda_{i,1}\varsigma_{i,1}-\frac{1}{2})\tilde{D}_{i,1}^2+\frac{1}{2}\bar{\epsilon}_{i,1}^2+\\&\frac{1}{2c_{i,1}}\lambda_{i,1}\tilde{w}_{i,1}^{\mathrm{T}}\tilde{w}_{i,1}-\gamma_{\eta_{i,1}}\beta_{i,1}\eta_{i,1}^2+\\&\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2)+\tilde{w}_{i,1}^{\mathrm{T}}\times\\&[(\varUpsilon_{i,1}v_{i,1}+\gamma_{\eta_{i,1}}\eta_{i,1})\theta_{i,1}-\frac{1}{\gamma_{i,1}}\dot{\hat{w}}_{i,1}].\end{aligned} \tag{20}$$

设计NN权值$\hat{w}_{i,1}$的复合更新律为

$$\dot{\hat{w}}_{i,1}=\gamma_{i,1}[(\varUpsilon_{i,1}v_{i,1}+\gamma_{\eta_{i,1}}\eta_{i,1})\theta_{i,1}-\delta_{i,1}\hat{w}_{i,1}], \tag{21}$$

其中$\delta_{i,1}>0$为设计参数.

根据式(20)–(21), 得

$$\begin{aligned}\dot{V}_{i,1}\leqslant&-k_{i,1}\varGamma_{i,1}^2v_{i,1}^2+\varUpsilon_{i,1}v_{i,1}v_{i,2}-\\&(\lambda_{i,1}-\frac{1}{2}c_{i,1}\lambda_{i,1}\varsigma_{i,1}-\frac{1}{2})\tilde{D}_{i,1}^2+\\&\frac{1}{2}\bar{\epsilon}_{i,1}^2+\frac{1}{2c_{i,1}}\lambda_{i,1}\tilde{w}_{i,1}^{\mathrm{T}}\tilde{w}_{i,1}-\gamma_{\eta_{i,1}}\beta_{i,1}\eta_{i,1}^2+\\&\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2)+\delta_{i,1}\tilde{w}_{i,1}^{\mathrm{T}}\hat{w}_{i,1}.\end{aligned}$$

步骤 2 构造Lyapunov函数为

$$V_{i,2}=V_{i,1}+\frac{1}{2}v_{i,2}^2+\frac{1}{2}\tilde{D}_{i,2}^2+\frac{1}{2}\gamma_{\eta_{i,2}}\eta_{i,2}^2+\frac{1}{2\gamma_{i,2}}\tilde{w}_{i,2}^{\mathrm{T}}\tilde{w}_{i,2}, \tag{22}$$

其中$\gamma_{i,2}>0$为设计参数.

对式(22)两端求导, 并由式(4)–(11), 得

$$\begin{aligned}\dot{V}_{i,2}=&\dot{V}_{i,1}+v_{i,2}[w_{i,2}^{*\mathrm{T}}\theta_{i,2}+D_{i,2}+(\alpha_{i,2}^c-\alpha_{i,2}^d)+\\&\alpha_{i,2}^d-\dot{\alpha}_{i,1}^c+e_{i,3}-\dot{z}_{i,2}]+\tilde{D}_{i,2}[\dot{D}_{i,2}-\end{aligned}$$

$$\lambda_{i,2}(\tilde{w}_{i,2}^{\mathrm{T}}\theta_{i,2}+\tilde{D}_{i,2})-v_{i,2}-\gamma_{\eta_{i,2}}\eta_{i,2}]+ \\ \gamma_{\eta_{i,2}}\eta_{i,2}(\tilde{w}_{i,2}^{\mathrm{T}}\theta_{i,2}+\tilde{D}_{i,2}-\beta_{i,2}\eta_{i,2})- \\ \frac{1}{\gamma_{i,2}}\tilde{w}_{i,2}^{\mathrm{T}}\dot{\hat{w}}_{i,2}. \tag{23}$$

由Young's不等式, 得

$$-\tilde{D}_{i,2}\tilde{w}_{i,2}^{\mathrm{T}}\theta_{i,2}\leqslant\frac{1}{2}c_{i,2}\varsigma_{i,2}\tilde{D}_{i,2}^2+\frac{1}{2c_{i,2}}\tilde{w}_{i,2}^{\mathrm{T}}\tilde{w}_{i,2}, \tag{24}$$

$$\tilde{D}_{i,2}\dot{D}_{i,2}\leqslant\frac{1}{2}\tilde{D}_{i,2}^2+\frac{1}{2}\bar{\epsilon}_{i,2}^2, \tag{25}$$

其中: $\|\theta_{i,2}\|^2\leqslant\varsigma_{i,2}$, $\varsigma_{i,2}$为NN节点数; $|\dot{D}_{i,2}|\leqslant\bar{\epsilon}_{i,2}$, $c_{i,2}>0$为设计参数.

设计虚拟控制器$\alpha_{i,2}^d$为

$$\alpha_{i,2}^d=-\hat{w}_{i,2}^{\mathrm{T}}\theta_{i,2}-\hat{D}_{i,2}+\dot{\alpha}_{i,1}^c-\varUpsilon_{i,1}\xi_{i,1}- \\ k_{i,2}e_{i,2}, \tag{26}$$

其中$k_{i,2}>0$为设计参数.

将式(24)–(26)代入式(23), 得

$$\dot{V}_{i,2}=\dot{V}_{i,1}+v_{i,2}[-\varUpsilon_{i,1}\xi_{i,1}-k_{i,2}e_{i,2}+e_{i,3}+ \\ (\alpha_{i,2}^c-\alpha_{i,2}^d)-\dot{z}_{i,2}+\tilde{w}_{i,2}^{\mathrm{T}}\theta_{i,2}]- \\ (\lambda_{i,2}-\frac{1}{2}c_{i,2}\lambda_{i,2}\varsigma_{i,2}-\frac{1}{2})\tilde{D}_{i,2}^2- \\ \gamma_{\eta_{i,2}}\beta_{i,2}\eta_{i,2}^2+\frac{1}{2c_{i,2}}\lambda_{i,2}\tilde{w}_{i,2}^{\mathrm{T}}\tilde{w}_{i,2}+ \\ \frac{1}{2}\bar{\epsilon}_{i,2}^2+\tilde{w}_{i,2}^{T}(\gamma_{\eta_{i,2}}\eta_{i,2}\theta_{i,2}-\frac{1}{\gamma_{i,2}}\dot{\hat{w}}_{i,2}). \tag{27}$$

将补偿信号$z_{i,2}$的导数选择为

$$\dot{z}_{i,2}=-\varUpsilon_{i,1}z_{i,1}-k_{i,2}z_{i,2}+z_{i,3}+(\alpha_{i,2}^c-\alpha_{i,2}^d), \tag{28}$$

其中$z_{i,2}(0)=0$.

由式(27)–(28), 得

$$\dot{V}_{i,2}\leqslant-k_{i,1}\varGamma_{i,1}^2v_{i,1}^2-k_{i,2}v_{i,2}^2+v_{i,2}v_{i,3}+ \\ \sum_{l=1}^{2}[-(\lambda_{i,l}-\frac{1}{2}c_{i,l}\lambda_{i,l}\varsigma_{i,l}-\frac{1}{2})\tilde{D}_{i,l}^2+ \\ \frac{1}{2}\bar{\epsilon}_{i,l}^2+\frac{1}{2c_{i,l}}\lambda_{i,l}\tilde{w}_{i,l}^{\mathrm{T}}\tilde{w}_{i,l}-\gamma_{\eta_{i,l}}\beta_{i,l}\eta_{i,l}^2]+ \\ \sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2)+\delta_{i,1}\tilde{w}_{i,1}^{\mathrm{T}}\hat{w}_{i,1}+ \\ \tilde{w}_{i,2}^{\mathrm{T}}[(v_{i,2}+\gamma_{\eta_{i,2}}\eta_{i,2})\theta_{i,2}-\frac{1}{\gamma_{i,2}}\dot{\hat{w}}_{i,2}]. \tag{29}$$

设计NN权值$\hat{w}_{i,2}$的复合更新律为

$$\dot{\hat{w}}_{i,2}=\gamma_{i,2}[(v_{i,2}+\gamma_{\eta_{i,2}}\eta_{i,2})\theta_{i,2}-\delta_{i,2}\hat{w}_{i,2}], \tag{30}$$

其中$\delta_{i,2}>0$为设计参数.

根据式(29)–(30), 得

$$\dot{V}_{i,2}\leqslant-k_{i,1}\varGamma_{i,1}^2v_{i,1}^2-k_{i,2}v_{i,2}^2+v_{i,2}v_{i,3}+ \\ \sum_{l=1}^{2}[-(\lambda_{i,l}-\frac{1}{2}c_{i,l}\lambda_{i,l}\varsigma_{i,l}-\frac{1}{2})\tilde{D}_{i,l}^2+ \\ \frac{1}{2}\bar{\epsilon}_{i,l}^2+\frac{1}{2c_{i,l}}\lambda_{i,l}\tilde{w}_{i,l}^{\mathrm{T}}\tilde{w}_{i,l}-\gamma_{\eta_{i,l}}\beta_{i,l}\eta_{i,l}^2+ \\ \delta_{i,l}\tilde{w}_{i,l}^{\mathrm{T}}\hat{w}_{i,l}]+\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2).$$

步骤 $\boldsymbol{m}$ $(m=3,\cdots,n_i-1)$　选择Lyapunov函数为

$$V_{i,m}=V_{i,m-1}+\frac{1}{2}v_{i,m}^2+\frac{1}{2}\tilde{D}_{i,m}^2+\frac{1}{2}\gamma_{\eta_{i,m}}\eta_{i,m}^2+ \\ \frac{1}{2\gamma_{i,m}}\tilde{w}_{i,m}^{\mathrm{T}}\tilde{w}_{i,m}, \tag{31}$$

其中$\gamma_{i,m}>0$为设计参数.

与步骤2相似, 设计虚拟控制器$\alpha_{i,m}^d$为

$$\alpha_{i,m}^d=-\hat{w}_{i,m}^{\mathrm{T}}\theta_{i,m}-\hat{D}_{i,m}+\dot{\alpha}_{i,m-1}^c- \\ e_{i,m-1}-k_{i,m}e_{i,m}, \tag{32}$$

其中$k_{i,m}>0$为设计参数.

将补偿信号$z_{i,m}$的导数选择为

$$\dot{z}_{i,m}=-z_{i,m-1}-k_{i,m}z_{i,m}+ \\ z_{i,m+1}+(\alpha_{i,m}^c-\alpha_{i,m}^d), \tag{33}$$

其中$z_{i,m}(0)=0$.

设计NN权值$\hat{w}_{i,m}$的复合更新律为

$$\dot{\hat{w}}_{i,m}=\gamma_{i,m}[(v_{i,m}+\gamma_{\eta_{i,m}}\eta_{i,m})\theta_{i,m}- \\ \delta_{i,m}\hat{w}_{i,m}], \tag{34}$$

其中$\delta_{i,m}>0$为设计参数.

将式(4)–(11)和式(32)–(34)代入式(31), 得

$$\dot{V}_{i,m}\leqslant-k_{i,1}\varGamma_{i,1}^2v_{i,1}^2-\sum_{l=2}^{m}k_{i,l}v_{i,l}^2+v_{i,m}v_{i,m+1}+ \\ \sum_{l=1}^{m}[-(\lambda_{i,l}-\frac{1}{2}c_{i,l}\lambda_{i,l}\varsigma_{i,l}-\frac{1}{2})\tilde{D}_{i,l}^2+ \\ \frac{1}{2}\bar{\epsilon}_{i,l}^2+\frac{1}{2c_{i,l}}\lambda_{i,l}\tilde{w}_{i,l}^{\mathrm{T}}\tilde{w}_{i,l}-\gamma_{\eta_{i,l}}\beta_{i,l}\eta_{i,l}^2+ \\ \delta_{i,l}\tilde{w}_{i,l}^{\mathrm{T}}\hat{w}_{i,l}]+\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2).$$

步骤 $\boldsymbol{n}_i$　构造Lyapunov函数为

$$V_{i,n_i}=V_{i,n_i-1}+\frac{1}{2}v_{i,n_i}^2+\frac{1}{2}\tilde{D}_{i,n_i}^2+ \\ \frac{1}{2}\gamma_{\eta_{i,n_i}}\eta_{i,n_i}^2+\frac{1}{2\gamma_{i,n_i}}\tilde{w}_{i,n_i}^{\mathrm{T}}\tilde{w}_{i,n_i}, \tag{35}$$

其中$\gamma_{i,n_i}>0$为设计参数.

与步骤2相似, 设计控制器u_i为

$$u_i=-\hat{w}_{i,n_i}^{\mathrm{T}}\theta_{i,n_i}-\hat{D}_{i,n_i}+\dot{\alpha}_{i,n_i-1}^c- \\ e_{i,n_i-1}-k_{i,n_i}e_{i,n_i}, \tag{36}$$

其中$k_{i,n_i}>0$为设计参数.

将补偿信号z_{i,n_i}的导数选择为

$$\dot{z}_{i,n_i}=-z_{i,n_i-1}-k_{i,n_i}z_{i,n_i}, \tag{37}$$

其中$z_{i,n_i}(0)=0$.

设计NN权值$\hat{w}_{i,n_i}$的复合更新律为

$$\begin{aligned}\dot{\hat{w}}_{i,n_i}=&\gamma_{i,n_i}[(v_{i,n_i}+\gamma_{\eta_{i,n_i}}\eta_{i,n_i})\theta_{i,n_i}-\\&\delta_{i,n_i}\hat{w}_{i,n_i}],\end{aligned} \tag{38}$$

其中$\delta_{i,n_i}>0$为设计参数.

将式(4)–(11)和式(36)–(38)代入式(36), 得

$$\begin{aligned}\dot{V}_{i,n_i}\leqslant&-k_{i,1}\varGamma_{i,1}^2v_{i,1}^2-\sum_{l=2}^{n_i}k_{i,l}v_{i,l}^2+\sum_{l=1}^{n_i}[-(\lambda_{i,l}-\\&\frac{1}{2}c_{i,l}\lambda_{i,l}\varsigma_{i,l}-\frac{1}{2})\tilde{D}_{i,l}^2+\frac{1}{2}\bar{\epsilon}_{i,l}^2+\\&\frac{1}{2c_{i,l}}\lambda_{i,l}\tilde{w}_{i,l}^{\mathrm{T}}\tilde{w}_{i,l}-\gamma_{\eta_{i,l}}\beta_{i,l}\eta_{i,l}^2+\delta_{i,l}\tilde{w}_{i,l}^{\mathrm{T}}\hat{w}_{i,l}]+\\&\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2).\end{aligned} \tag{39}$$

3.2 稳定性分析

本节采用Lyapunov函数对多智能体系统进行稳定性分析.

定理 1 在假设1和假设2成立的条件下, 考虑虚拟控制信号(17)(26)(32), 实际控制器(36), 滤波补偿信号(19)(28)(33)(37), NN权值的复合更新律(21)(30)(34)(38), 如果跟踪误差$e_{i,1}(t)$的初始条件满足$e_{i,1}(0)<F_{\psi,i}(0)$, 则多智能体系统(1)的所有信号均为有界的, 且跟踪误差$e_{i,1}(t)$约束在规定的PFT漏斗边界内.

证 选取Lyapunov函数为

$$\begin{aligned}V=&\frac{1}{2}\sum_{i=1}^{N}\sum_{l=1}^{n_i}(v_{i,l}^2+\tilde{D}_{i,l}^2+\gamma_{\eta_{i,l}}\eta_{i,l}^2+\\&\frac{1}{\gamma_{i,l}}\tilde{w}_{i,l}^{\mathrm{T}}\tilde{w}_{i,l}+z_{i,l}^2).\end{aligned} \tag{40}$$

由文献[8]可知, 虚拟控制信号$\alpha_{i,r-1}^d$通过一阶滤波器后, 满足$\|\alpha_{i,r-1}^c-\alpha_{i,r-1}^d\|\leqslant\iota$, ι为未知正常数. 由于

$$\tilde{w}_{i,l}^{\mathrm{T}}\hat{w}_{i,l}\leqslant-\frac{1}{2}\tilde{w}_{i,l}^{\mathrm{T}}\tilde{w}_{i,l}+\frac{1}{2}\|w_{i,l}^*\|^2,$$

$$\varUpsilon_{i,1}(\alpha_{i,1}^c-\alpha_{i,1}^d)z_{i,1}\leqslant\frac{1}{2}(b_i+a_{i,0})^2\iota^2+\frac{1}{2}\varGamma_{i,1}^2z_{i,1}^2,$$

$$(\alpha_{i,l}^c-\alpha_{i,l}^d)z_{i,l}\leqslant\frac{1}{2}\iota^2+\frac{1}{2}z_{i,1}^2,$$

且

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\frac{a_{i,j}}{2}(\tilde{w}_{j,1}^{\mathrm{T}}\tilde{w}_{j,1}+\tilde{D}_{j,1}^2)=\\&\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{a_{j,i}}{2}(\tilde{w}_{i,1}^{\mathrm{T}}\tilde{w}_{i,1}+\tilde{D}_{i,1}^2)\end{aligned}$$

成立.

对式(40)两端求导, 得

$$\begin{aligned}\dot{V}\leqslant&-\sum_{i=1}^{N}k_{i,1}\varGamma_{i,1}^2v_{i,1}^2-\sum_{i=1}^{N}\sum_{l=2}^{n_i}k_{i,l}v_{i,l}^2-\\&\sum_{i=1}^{N}(\lambda_{i,1}-\frac{1}{2}c_{i,1}\lambda_{i,1}\varsigma_{i,1}-\frac{1}{2}-\sum_{j=1}^{N}\frac{a_{j,i}}{2})\times\\&\tilde{D}_{i,1}^2-\sum_{i=1}^{N}\sum_{l=2}^{n_i}(\lambda_{i,l}-\frac{1}{2}c_{i,l}\lambda_{i,l}\varsigma_{i,l}-\frac{1}{2})\tilde{D}_{i,l}^2-\\&\sum_{i=1}^{N}\sum_{l=1}^{n_i}\gamma_{\eta_{i,l}}\beta_{i,l}\eta_{i,l}^2-\sum_{i=1}^{N}(\frac{1}{2}\delta_{i,1}-\frac{1}{2c_{i,1}}\lambda_{i,1}-\\&\sum_{j=1}^{N}\frac{a_{j,i}}{2})\tilde{w}_{i,1}^{\mathrm{T}}\tilde{w}_{i,1}-\sum_{i=1}^{N}\sum_{l=2}^{n_i}(\frac{1}{2}\delta_{i,l}-\\&\frac{1}{2c_{i,l}}\lambda_{i,l})\tilde{w}_{i,l}^{\mathrm{T}}\tilde{w}_{i,l}-\sum_{i=1}^{N}(k_{i,1}-\frac{1}{2})\varGamma_{i,1}^2z_{i,1}^2-\\&\sum_{i=1}^{N}\sum_{l=2}^{n_i-1}(k_{i,l}-\frac{1}{2})z_{i,l}^2-\sum_{i=1}^{N}k_{i,n_i}z_{i,n_i}^2+\\&\sum_{i=1}^{N}\sum_{l=1}^{n_i}(\frac{1}{2}\delta_{i,l}\|w_{i,l}^*\|^2+\bar{\epsilon}_{i,l}^2+\\&\sum_{i=1}^{N}\frac{1}{2}(b_i+a_{i,0})^2\iota^2+\sum_{i=1}^{N}\sum_{l=2}^{n_i-1}\frac{1}{2}\iota^2).\end{aligned} \tag{41}$$

设计参数$\lambda_{i,1},\lambda_{i,l},c_{i,1},c_{i,l},\delta_{i,1},\delta_{i,l},k_{i,1},k_{i,l}$, 使

$$\lambda_{i,1}-\frac{1}{2}c_{i,1}\lambda_{i,1}\varsigma_{i,1}>\frac{1}{2}+\sum_{j=1}^{N}\frac{a_{j,i}}{2},$$

$$\lambda_{i,l}-\frac{1}{2}c_{i,l}\lambda_{i,l}\varsigma_{i,l}>\frac{1}{2},$$

$$\frac{1}{2}\delta_{i,1}-\frac{1}{2c_{i,1}}\lambda_{i,1}>\sum_{j=1}^{N}\frac{a_{j,i}}{2},$$

$$\frac{1}{2}\delta_{i,l}-\frac{1}{2c_{i,l}}\lambda_{i,l}>0,\ k_{i,1}>\frac{1}{2},\ k_{i,l}>\frac{1}{2}$$

成立. 令

$$\begin{aligned}\kappa=&\sum_{i=1}^{N}\sum_{l=1}^{n_i}(\frac{1}{2}\delta_{i,l}\|w_{i,l}^*\|^2+\bar{\epsilon}_{i,l}^2+\\&\sum_{i=1}^{N}\frac{1}{2}(b_i+a_{i,0})^2\iota^2+\sum_{i=1}^{N}\sum_{l=2}^{n_i-1}\frac{1}{2}\iota^2),\end{aligned}$$

$$\begin{aligned}\mu=\min\{&2(\lambda_{i,1}-\frac{1}{2}c_{i,1}\lambda_{i,1}\varsigma_{i,1}-\frac{1}{2}-\sum_{j=1}^{N}\frac{a_{j,i}}{2}),\\&2(\lambda_{i,l}-\frac{1}{2}c_{i,l}\lambda_{i,l}\varsigma_{i,l}-\frac{1}{2}),\\&2(\frac{1}{2}\delta_{i,1}-\frac{1}{2c_{i,1}}\lambda_{i,1}-\sum_{j=1}^{N}\frac{a_{j,i}}{2}),\\&2(\frac{1}{2}\delta_{i,l}-\frac{1}{2c_{i,l}}\lambda_{i,l}),\\&2(k_{i,1}-\frac{1}{2})\varGamma_{i,1}^2,2(k_{i,l-1}-\frac{1}{2}),\\&2k_{i,1}\varGamma_{i,1}^2,2k_{i,l},2\gamma_{\eta_{i,l}}\beta_{i,l}\}.\end{aligned} \tag{42}$$

根据式(41)–(42), 得

$$\dot{V}\leqslant-\mu V+\kappa. \tag{43}$$

由式(43), 得

$$0 \leqslant V \leqslant (V(0) - \frac{\kappa}{\mu})\mathrm{e}^{-\mu t} + \frac{\kappa}{\mu}. \tag{44}$$

从式(44)可知, 当$t \to \infty$时, $V \to (\kappa/\mu)$. 可以证明Lyapunov函数(40)中的所有信号均为有界的. 并由式(6)可得误差$|\xi_{i,1}|$和$|e_{i,r}|$是有界的, 令

$$|\xi_{i,1}| \leqslant \vartheta, \tag{45}$$

其中边界ϑ的大小与设计参数有关.

根据式(45), 得

$$\xi_{i,1}^2 \leqslant \vartheta^2. \tag{46}$$

将式(2)代入式(46), 得

$$\frac{e_{i,1}^2}{F_{\psi,i}^2 - e_{i,1}^2} \leqslant \vartheta^2. \tag{47}$$

由式(47)得

$$|e_{i,1}| \leqslant \sqrt{\frac{\vartheta^2}{1+\vartheta^2}}|F_{\psi,i}| < |F_{\psi,i}|.$$

则多智能体的跟踪误差约束在PFT漏斗边界内, 并且通过调整设计参数可以使跟踪误差任意减小.

证毕.

4　数值仿真

为验证所提出控制方法的有效性, 本节考虑的多智能体系统由4个跟随者和1个领导者组成, 每个跟随者为具有时变扰动的二阶严格反馈非线性系统

$$\begin{aligned}
&\dot{x}_{i,1} = x_{i,1} + f_{i,1}(\bar{x}_{i,1}) + d_{i,1}(t),\\
&\dot{x}_{i,2} = u_i + f_{i,2}(\bar{x}_{i,2}) + d_{i,2}(t),\\
&y_i = x_{i,1},\ i = 1,2,3,4,
\end{aligned}$$

其中: $f_{i,1}(\bar{x}_{i,1}) = 2x_{i,1}^2 \sin x_{i,1}$; $f_{i,2}(\bar{x}_{i,2}) = x_{i,1} + x_{i,2} \cdot \cos x_{i,1}$; $d_{i,1}(t) = \cos t$, $d_{i,2}(t) = \sin t$.

多智能体通信拓扑见图1. 其中领导者的输出信号为$y_0 = \sin t$. 假设所有边的权重都为1, 跟随者与领导者之间的连接权重为$\mathcal{H} = \mathrm{diag}\{1,0,1,0\}$, 选择邻接矩阵$\mathcal{A}$与拉普拉斯矩阵$\mathcal{L}$如下:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & 0 & 1 & 0 \end{bmatrix},\ \mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 0\\ -1 & 1 & 0 & 0\\ -1 & 0 & 1 & 0\\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

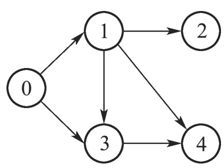


图 1　通信拓扑图

Fig. 1　Communication topology

在仿真中, 初值设置为$x_{i,1}(0) = \hat{x}_{i,1}(0) = [0.1\ \ 0\ \ 0.1\ \ 0]$, $x_{i,2}(0) = \hat{x}_{i,2}(0) = [0\ \ 0\ \ 0\ \ 0]$, $s_{i,1}(0) = [0.9\ \ 1\ \ 0.9\ \ 1]$, $s_{i,2}(0) = [0.1\ \ 0\ \ 0.1\ \ 0]$, $\alpha_{i,1}^c(0) = \alpha_{i,1}^d(0) = [0\ \ 0\ \ 0\ \ 0]$. 将NN初始权值设为0, 其中$x_{i,1}$, $x_{i,2}$的中心点在$[-1,1]$, $[-2,2]$均匀间隔, 对$f_{i,1}$选择NN节点数为$\varsigma_{i,1} = 7$, 对$f_{i,2}$选择NN节点数为$\varsigma_{i,2} = 25$. 选取设计参数的值为$\rho_{i,0} = 1$, $k_{i,1} = k_{i,2} = 10$, $\rho_{i,T_i} = 0.02$, $T_i = 0.1$, $\tau_{i,1} = 0.005$, $\beta_{i,1} = \beta_{i,2} = 1$, $\gamma_{i,1} = \gamma_{i,2} = 1$, $\gamma_{\eta_{i,1}} = \gamma_{\eta_{i,2}} = 50$, $\delta_{i,1} = \delta_{i,2} = 10$, $\lambda_{i,1} = \lambda_{i,2} = 10$.

仿真结果如图2–6所示. 图2为跟随者y_i的跟踪效果图, 在较短的时间实现了一致性跟踪. 图3为跟踪误差, 由图3(a)可知, 漏斗控制使跟踪误差约束在给定的漏斗边界内; 对比图3(a)和3(b)可看出, 不具有PFT漏斗控制的跟踪误差渐近稳定且稳态误差接近0.02, 而加入PFT漏斗控制后的跟踪误差在预设时间0.1 s前稳定且稳态误差小于0.0001, 因此, PFT漏斗控制显著减小了一致性跟踪的稳态误差, 并保证误差在预设时间内收敛. 图4–5给出了未知非线性和时变扰动的逼近效果, 由图4(a)和5(a)可知, 采用NN和扰动观测器的复合估计模型, 能精确估计多智能体系统的未知非线性和时变扰动; 与图4(b)和5(b)的传统NN逼近方法相比, 本文设计的复合更新率使估计模型具有更精确的逼近效果. 图6为控制器输入u_i, 系统稳定后控制输入均小于3, 表明该控制器能耗较低、性能良好.

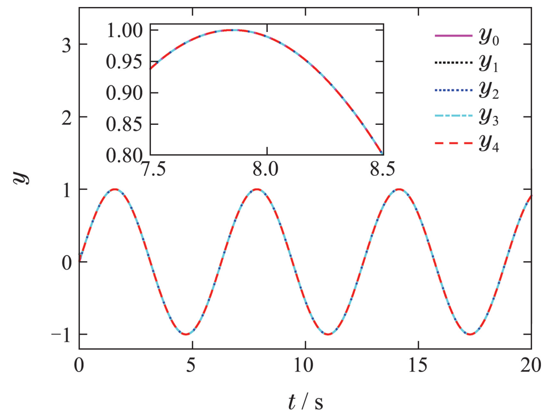
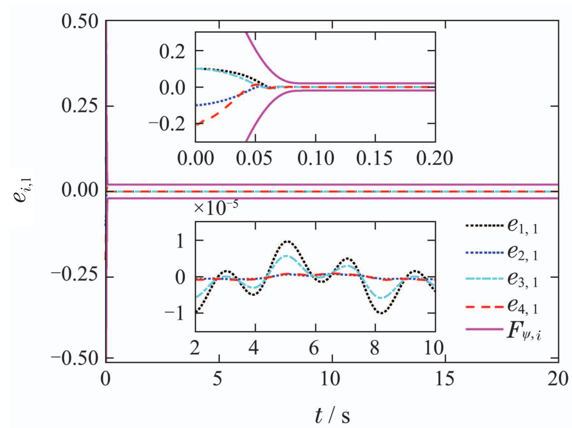
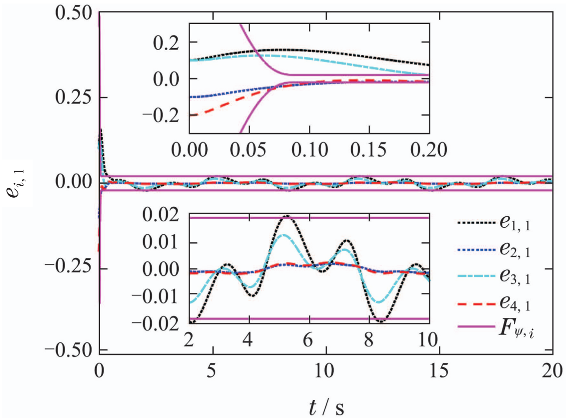


图 2　参考信号y_0和输出信号y_i

Fig. 2　Reference signal y_0 and output y_i



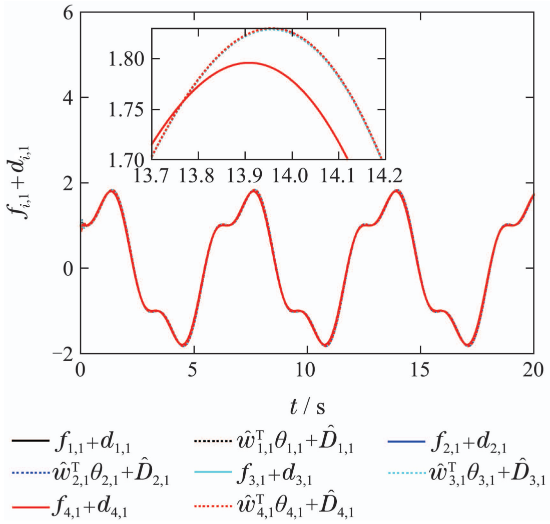
(a) 具有PFT漏斗控制的跟踪误差$e_{i,1}$



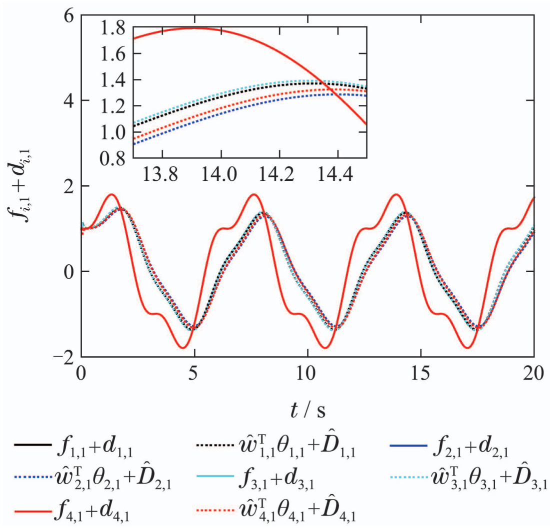
(b) 不具有PFT漏斗控制的跟踪误差$e_{i,1}$

图 3 跟踪误差$e_{i,1}$

Fig. 3 Tracking errors $e_{i,1}$



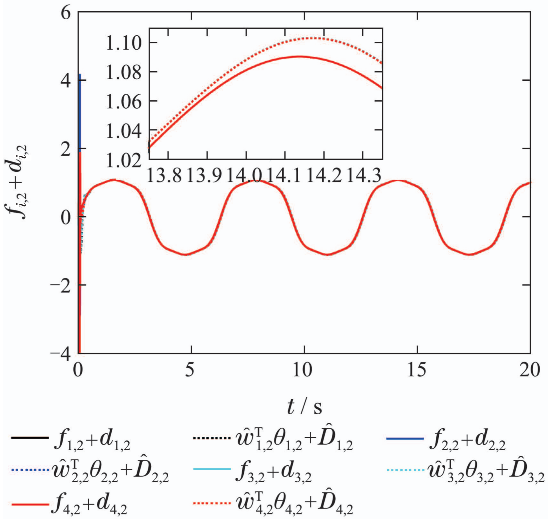
(a) 基于NN和扰动观测器的复合估计模型



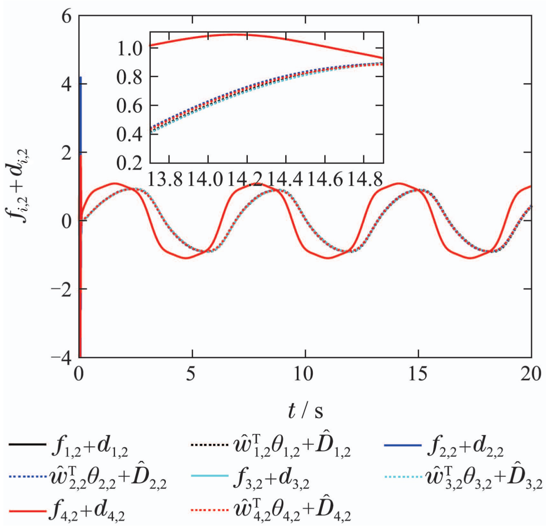
(b) 传统NN

图 4 两种估计$(f_{i,1}+d_{i,1})$的方法

Fig. 4 Two estimation methods of $(f_{i,1}+d_{i,1})$



(a) 基于NN和扰动观测器的复合估计模型



(b) 传统NN

图 5 两种估计$(f_{i,2}+d_{i,2})$的方法

Fig. 5 Two estimation methods of $(f_{i,2}+d_{i,2})$

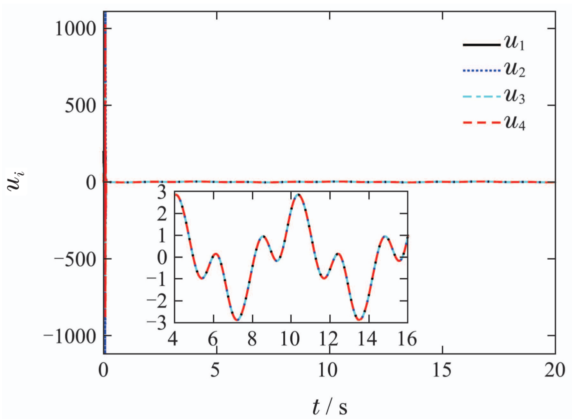


图 6 控制信号u_i

Fig. 6 Control signal u_i

5 结论

本文研究了具有未知非线性和时变扰动的多智能体系统一致性跟踪问题. 在控制器设计过程中, 引入了PFT漏斗控制方法, 使跟踪误差约束在PFT漏斗内, 达到了预设时间收敛的目标. 建立了基于NN和扰动观测器的复合估计模型, 精确估计了多智能体系统的未知非线性和时变扰动. 并结合动态面技术设计了基于精确估计的PFT漏斗复合控制器, 实现了对领导者的一致性跟踪. 通过仿真验证了该方法的有效性. 在未来研究中, 考虑将该方法应用于更复杂的环境中.

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