非线性时滞系统基于降阶观测器的反步镇定控制

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摘要: 针对一类状态未知的非线性严格反馈时滞系统, 本文提出了一种基于静态增益函数的输出反馈控制方案. 首先构造了降阶观测器以估计非线性系统的未知状态. 然后在Backstepping设计的每一步定义了具有控制增益函数的新型Lyapunov-Krasovskii泛函以补偿未知时变时滞, 定义新的选择不唯一的连续控制增益函数以补偿非匹配项以及Lyapunov-Krasovskii泛函补偿时滞时产生的非负项. 提出了一种无记忆输出反馈控制方案. 理论分析表明: 该控制方案消除了未知时滞的影响, 保证了闭环系统所有信号的有界性, 并使系统实现渐近稳定. 最后仿真结果验证了此控制方案的有效性.

关键词: 时滞系统; 输出反馈控制; 降阶观测器; 静态增益函数; 渐近稳定

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Reduced-order observer-based backstepping stabilization control for nonlinear time-delay systems

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Abstract: For a class of nonlinear strict-feedback time-delay systems with unknown system states, this paper proposes an output feedback control scheme based on a novel static gain function. Firstly, a reduced-order observer is constructed to estimate the unmeasured states of the nonlinear time-delay system. Then, a new Lyapunov-Krasovskii functional with an explicit control gain expression is defined in each step of the backstepping design process to compensate for the unknown time-varying delays, novel continuous control gain functions are defined to compensate for unmatched terms and the non-negative terms that are generated when the Lyapunov-Krasovskii function compensates for time delays. A memoryless output feedback control scheme is proposed. It shows that the control scheme eliminates the influence of unknown time delay, ensures the boundedness of all signals of the closed-loop system, and makes the system asymptotically stable. Finally, simulation results verify the effectiveness of the control scheme.

Key words: time-delay system; output feedback control; reduced-order observer; static gain function; asymptotic stability

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1 引言

时滞特性和非线性特性在实际系统中普遍存在, 譬如, 通讯系统、电力系统、网络传输系统等. 时滞的存在可使系统的动态性能变差甚至导致系统不稳定[1–2]. 文献[3]首次将Backstepping方法扩展到非线性时滞系统中且使用Lyapunov-Krasovskii泛函方法设计了无记忆状态反馈控制器. 随后文献[4]利用Backstepping方法设计了无记忆状态反馈控制器, 研究了一类具有多重时滞的不确定系统的镇定问题. 文献[5]通过低增益缩放和Backstepping方法, 解决了一类非线性时滞系统的全局自适应输出反馈调节问题.

状态反馈和输出反馈是系统控制设计中两种主要

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的反馈策略. 相对比而言, 状态反馈控制的控制率设计有更大的可选择范围, 闭环系统能达到更佳性能. 文献[6]引入了基于控制增益函数的具有积分型函数的新型Lyapunov-Krasovskii泛函, 解决了一类具有时滞和死区输入的非线性系统的状态反馈跟踪控制问题. 随后此控制技术被应用于文献 [7]中, 研究了一类具有互联时滞和多个死区输入的大型系统的模型跟踪控制问题. 文献[8]利用一种基于无明确表达形式的控制增益函数的状态反馈控制方法设计了无记忆状态反馈控制器, 使得所得到的闭环系统是渐近稳定的. 输出反馈所受限制条件较少且不依赖于状态信息, 更容易实现. 在实际系统中不易获得所有状态信息时, 输出反馈控制成为主要控制方法.

本文旨在解决一类具有状态时滞的非线性系统的输出反馈控制问题. 通过设计一种新的基于控制增益函数的具有积分型函数的新型Lyapunov-Krasovskii泛函处理时变时滞, 并给出控制增益函数的具体表达式. 通过Backstepping技术设计无记忆输出反馈控制器, 通过仿真结果验证了输出反馈控制的有效性.

为增加文章简洁性, 定义如下缩写:

$$\bar{x}_i=[x_1\ \ x_2\ \ \cdots\ \ x_i]^{\mathrm T},\ \bar{m}_i=[m_2\ \ m_3\ \ \cdots\ \ m_i]^{\mathrm T},$$
$$x_{id_i}=x_i(t-d_i(t)),\ \bar{x}_{id_i}=[x_{1d_1}\ \ \cdots\ \ x_{id_i}]^{\mathrm T},$$
$$\bar{m}_{id_i}=[m_{2d_2}\ \ \cdots\ \ m_{id_i}]^{\mathrm T}.$$

2 问题描述与观测器设计

2.1 问题描述

考虑如下严格反馈非线性时滞系统:

$$\begin{cases}\dot{x}_i(t)=\ell x_{i+1}(t)+g_i(\bar{x}_i(t))+f_i(\bar{x}_{id_i}),\\ \qquad\qquad i=1,2,\cdots,n-1,\\ \dot{x}_n(t)=u(t)+g_n(\bar{x}_n(t))+f_n(\bar{x}_{nd_n}),\\ y(t)=x_1(t),\end{cases}\tag{1}$$

其中: $x_i\in\mathbb{R}$和$y\in\mathbb{R}$分别是系统状态和系统输出, $u(t)\in\mathbb{R}$是控制输入; $\ell\in\mathbb{R}^+$; $\bar{x}_i(t)=[x_1(t)\ \ x_2(t)\ \ \cdots\ \ x_i(t)]^{\mathrm T}$; $g_i(\cdot)$是已知光滑函数, 满足$g_i(0)=0$; $f_i(\cdot)$是未知光滑非线性函数, 满足$f_i(0)=0$; $d_i(t)$是未知时变时滞, 且满足

$$0\leqslant d_i(t)\leqslant\tau_i,\ \dot{d}_i(t)\leqslant\bar{\tau}_i<1,\ i=1,2,\cdots,n,\tag{2}$$

定义$\tau=\max\{\bar{\tau}_1,\cdots,\bar{\tau}_n\}$, τ为正常数. $x(t)=\varphi(t)$, $t\in[-\tau,0]$为初始条件.

控制目标是设计无记忆输出反馈控制器使得闭环系统的所有信号有界, 同时使系统(1)的状态$x(t)$渐近趋于0.

2.2 观测器设计

由于输出状态x_1为唯一的可测状态, 可设计降阶观测器以估计不可测状态. 首先将系统(1)改写为如下形式:

$$\begin{cases}\dot{x}_1(t)=A_{11}x_1(t)+A_{12}m(t)+g_1(x_1(t))+\\ \qquad f_1(x_{1d_1}),\\ \dot{m}(t)=A_{21}x_1(t)+A_{22}m(t)+\bar{G}(x_1(t),\bar{m}(t))+\\ \qquad \bar{B}u(t)+\bar{F}(x_{1d_1},\bar{m}_{nd_n}),\end{cases}\tag{3}$$

其中:

$$A_{11}=0,\ A_{12}=[\ell\ \ 0\ \ \cdots\ \ 0]\in\mathbb{R}^{1\times(n-1)},$$
$$A_{21}=[0\ \ \cdots\ \ 0]^{\mathrm T}\in\mathbb{R}^{n-1},$$
$$\bar{B}=[0\ \ \cdots\ \ 0\ \ 1]^{\mathrm T}\in\mathbb{R}^{n-1},$$
$$A_{22}=\ell\begin{bmatrix}0 & & \\ 0 & I_{(n-2)\times(n-2)} & \\ \vdots & & \\ 0 & \cdots & 0\end{bmatrix},$$
$$\bar{G}(x_1,\bar{m})=$$
$$[g_2(x_1(t)\ \ m_2(t))\ \ \cdots\ \ g_n(x_1(t),\bar{m}(t))]^{\mathrm T}\in\mathbb{R}^{n-1},$$
$$\bar{F}(x_{1d_1},\bar{m}_{nd_n})=$$
$$[f_2(x_{1d_1},m_{2d_2})\ \ \cdots\ \ f_n(x_{1d_1},\bar{m}_{nd_n})]^{\mathrm T}\in\mathbb{R}^{n-1},$$
$$m(t)=[x_2(t)\ \ \cdots\ \ x_n(t)]^{\mathrm T}\triangleq[m_2(t)\ \ \cdots\ \ m_n(t)]^{\mathrm T}$$

为不可测状态变量. 定义

$$v(t)=A_{21}x_1(t)+\bar{B}u(t),\tag{4}$$
$$z(t)=\dot{x}_1(t)-A_{11}x_1(t)-g_1(x_1(t)),\tag{5}$$

将式(4)和式(5)代入到式(3), 得

$$\begin{cases}\dot{m}(t)=A_{22}m(t)+v(t)+\bar{G}(x_1(t),\bar{m}(t))+\\ \qquad \bar{F}(x_{1d_1},\bar{m}_{nd_n}),\\ z(t)=A_{12}m(t)+f_1(x_{1d_1}).\end{cases}\tag{6}$$

对于系统(6), 本文设计如下状态观测器

$$\dot{\hat{m}}(t)=(A_{22}+LA_{12})\hat{m}(t)+v(t)+$$
$$\hat{\bar{G}}(x_1(t),\hat{m}(t))-Lz(t),\tag{7}$$

其中$L=[l_2\ \ \cdots\ \ l_n]^{\mathrm T}$使得$A_{22}+LA_{12}$为Hurwitz矩阵. 将式(4)和式(5)代入到式(7), 可得

$$\dot{\hat{m}}(t)=E\hat{m}(t)+(A_{21}+LA_{11})y(t)+Lg_1(x_1(t))+$$
$$\bar{B}u(t)+\hat{\bar{G}}(x_1(t),\hat{m}(t))-L\dot{y}(t).\tag{8}$$

重新定义观测器的状态以避免上式中输出量y的微分

$$\omega(t)=\hat{m}(t)+Ly(t),\tag{9}$$

其导数为如下形式:

$$\dot{\omega}(t)=E\omega(t)+E'y(t)+\bar{B}u(t)+$$

$$\hat{\bar{G}}(x_1(t),\hat{\bar{m}}(t))+Lg_1(x_1(t)),\tag{10}$$

其中: $E=A_{22}+LA_{12}, E'=(A_{21}+LA_{11})-(A_{22}+LA_{12})L$具有如下形式:

$$E=\ell\begin{bmatrix}l_2 & & \\ l_3 & I_{(n-2)\times(n-2)} & \\ \vdots & & \\ l_n & \cdots & 0\end{bmatrix},$$

$$E'=\ell\begin{bmatrix}-l_2^2-l_3\\ -l_2l_3-l_4\\ \vdots\\ -l_2l_{n-1}-l_n\\ -l_2l_n\end{bmatrix}.$$

定义状态估计误差为

$$\varepsilon(t)=m(t)-\hat{m}(t),\tag{11}$$

则由式(6)(8)可推出$\varepsilon(t)$的导数为

$$\dot{\varepsilon}(t)=E\varepsilon(t)+\bar{G}(x_1(t),\bar{m}(t))-\hat{\bar{G}}(x_1(t),\hat{\bar{m}}(t))+Lf_1(x_{1d_1})+\bar{F}(x_{1d_1},\bar{m}_{n_{dn}}).\tag{12}$$

引入坐标变换

$$\begin{cases}z_1(t)=y(t),\\ z_i(t)=\omega_i(t)-\alpha_{i-1}(t),\ i=2,\cdots,n,\end{cases}\tag{13}$$

其中α_{i-1}为虚拟控制变量, 其定义如下:

$$\alpha_1=-\frac{1}{\ell}c_1z_1(t)-\frac{1}{\ell}g_1(z_1(t))+l_2z_1(t),\tag{14}$$

$$\alpha_2=-\frac{1}{\ell}c_2z_2(t)+l_3z_1(t)-\frac{1}{\ell}g_2(x_1(t),\hat{m}_2(t))-\frac{l_2}{\ell}g_1(z_1(t))-l_2\hat{m}_2(t)+\frac{\partial\alpha_1}{\partial y}(\hat{m}_2(t)+\frac{1}{\ell}g_1(z_1(t))),\tag{15}$$

$$\alpha_i=-\frac{1}{\ell}c_iz_i(t)+l_{i+1}z_1(t)-\frac{1}{\ell}g_i(x_1(t),\hat{\bar{m}}_i(t))-\frac{l_i}{\ell}g_1(z_1(t))-l_i\hat{m}_2(t)+\frac{\partial\alpha_{i-1}}{\partial y}(\hat{m}_2(t)+\frac{1}{\ell}g_1(z_1(t)))+\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{m}_j}\dot{\hat{m}}_j(t),\quad i=3,\cdots,n-1,\tag{16}$$

c_i为正设计参数. 在此坐标变换下, 系统转化为

$$\dot{\varepsilon}(t)=E\varepsilon(t)+\bar{G}(x_1(t),\bar{m}(t))-\hat{\bar{G}}(x_1(t),\hat{\bar{m}}(t))+Lf_1(x_{1d_1})+\bar{F}(x_{1d_1},\bar{m}_{n_{dn}}),\tag{17}$$

$$\dot{z}_1(t)=-c_1z_1(t)+\ell z_2(t)+\ell\varepsilon_2(t)+f_1(z_{1d_1})),\tag{18}$$

$$\dot{z}_i(t)=-c_iz_i(t)+\ell z_{i+1}(t)-\frac{\partial\alpha_{i-1}}{\partial y}(\ell\varepsilon_2+f_1(z_{1d_1}))),\ i=2,\cdots,n-1,\tag{19}$$

$$\dot{z}_n(t)=\ell l_n\hat{m}_2+u+g_n(x_1(t),\hat{\bar{m}}_n(t))+l_ng_1-\frac{\partial\alpha_{n-1}}{\partial y}\dot{y}-\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{m}_j}\dot{\hat{m}}_j(t).\tag{20}$$

为实现控制目标, 文章提出如下假设.

假设 1　存在矩阵D, 使得

$$G(\xi_1(t),\cdots,\xi_n(t))-\hat{\bar{G}}(\xi_1(t),\cdots,\hat{\xi}_n(t))=D(\xi(t)-\hat{\xi}(t)),\tag{21}$$

其中$\xi(t),\hat{\xi}(t)$是任意时变参数,

$$\bar{\xi}(t)=[\xi_2(t)\ \ \cdots\ \ \xi_n(t)]^{\mathrm{T}},$$
$$\hat{\bar{\xi}}(t)=[\hat{\xi}_2(t)\ \ \cdots\ \ \hat{\xi}_n(t)]^{\mathrm{T}},$$
$$\xi(t)=[\xi_1(t)\ \ \xi_2(t)\ \ \cdots\ \ \xi_n(t)]^{\mathrm{T}},$$
$$\hat{\xi}(t)=[\xi_1(t)\ \ \hat{\xi}_2(t)\ \ \cdots\ \ \hat{\xi}_n(t)]^{\mathrm{T}},$$
$$G(\xi_1(t),\cdots,\xi_n(t))=[g_1(\xi_1(t))\ \ g_2(\xi_1(t),\bar{\xi}_2(t))\ \ \cdots\ \ g_n(\xi_1(t),\bar{\xi}_n(t))]^{\mathrm{T}},$$
$$\hat{\bar{G}}(\xi_1(t),\cdots,\hat{\xi}_n(t))=[g_1(\xi_1(t))\ \ g_2(\xi_1(t),\hat{\bar{\xi}}_2(t))\ \ \cdots\ \ g_n(\xi_1(t),\hat{\bar{\xi}}_n(t))]^{\mathrm{T}}.$$

假设 2　存在非负不减的连续函数$h_{ijl}(\cdot)$和$p_k(\cdot)$满足$p_k(0)=0, h_{ijl}(0)=0$, 使得

$$f_i^2(x_{1d_1},x_{2d_2},\cdots,x_{id_i})\leqslant\sum_{j=1}^{i}\sum_{l=j}^{i}h_{ijl}(|z_{jd_l}|)+\sum_{k=2}^{i}p_k(\|\varepsilon_{d_k}\|).$$

假设 3　存在C^1函数$V_{01}(\cdot):\mathbb{R}^n\to\mathbb{R}^+$和$V_{q1}(\cdot):\mathbb{R}\to\mathbb{R}^+, q=1,\cdots,n$满足

$$\gamma_{01}(\|s\|)\leqslant V_{01}(s)\leqslant\gamma_{02}(\|s\|),\tag{22}$$

$$\gamma_{q1}(|s'|)\leqslant V_{q1}(s')\leqslant\gamma_{q2}(|s'|),\tag{23}$$

$$\nabla_s^{\mathrm{T}}V_{01}(s)(Es+Ds)\leqslant-\gamma_{03}(\|s\|),\tag{24}$$

$$\frac{\partial V_{q1}(s')}{\partial s'}(-c_qs')\leqslant-\gamma_{q3}(|s'|),\tag{25}$$

其中$s\in\mathbb{R}^{(n-1)\times1}$, $s'\in\mathbb{R}$为参数变量. $\gamma_{01}(\cdot),\gamma_{02}(\cdot),\gamma_{q1}(\cdot),\gamma_{q2}(\cdot)$为$K_\infty$类函数, $\gamma_{03}(\cdot),\gamma_{q3}(\cdot)$为$K$类函数.

假设 4　$\sum_{i=1}^{n}\sum_{k=2}^{i}p_k(\gamma_{01}^{-1}(\cdot))$ 和 $(\gamma_{01}^{-1}(\cdot))^2$ 分别与$\gamma_{03}(\gamma_{02}^{-1}(\cdot))$为同阶无穷小量, $(\gamma_{q1}^{-1}(\cdot))^2$和$\sum_{i=q}^{n}\sum_{l=q}^{i}h_{iql}\cdot(\gamma_{q1}^{-1}(\cdot))$分别与$\gamma_{q3}(\gamma_{q2}^{-1}(\cdot))$为同阶无穷小量. 因此, 存在常数$\varrho_{ij}$使以下结果成立:

$$\lim_{\chi\to0}\frac{\sum_{i=1}^{n}\sum_{k=2}^{i}p_k(\gamma_{01}^{-1}(\chi))}{\gamma_{03}(\gamma_{02}^{-1}(\chi))}=\varrho_{01},\tag{26}$$

$$\lim_{\chi\to0}\frac{(\gamma_{01}^{-1}(\chi))^2}{\gamma_{03}(\gamma_{02}^{-1}(\chi))}=\varrho_{02},\tag{27}$$

$$\lim_{\chi\to 0}\frac{\sum\limits_{i=q}^{n}\sum\limits_{l=q}^{i}h_{iql}(\gamma_{q1}^{-1}(\chi))}{\gamma_{q3}(\gamma_{q2}^{-1}(\chi))}=\varrho_{q1},\tag{28}$$

$$\lim_{\chi\to 0}\frac{(\gamma_{q1}^{-1}(\chi))^2}{\gamma_{q3}(\gamma_{q2}^{-1}(\chi))}=\varrho_{q2},\ q=2,\cdots,n.\tag{29}$$

3 控制器设计与稳定性分析

3.1 静态增益函数设计

连续函数$\varpi_q(\chi):[0,\infty)\to\mathbb{R}$定义如下: 当$\chi=0$时, $\varpi_q(\chi)=\varpi_q'$; 当$\chi>0$时,

$$\varpi_0(\chi)=[\frac{1}{\theta}\sum_{i=1}^{n}\sum_{k=2}^{i}p_k(\gamma_{01}^{-1}(\chi))+\frac{n(1-\tau)}{4\iota_2}\times (\gamma_{01}^{-1}(\chi))^2]/[(1-k_0)\gamma_{03}(\gamma_{02}^{-1}(\chi))],\tag{30}$$

$$\varpi_1(\chi)=[\frac{1}{\theta}\sum_{i=1}^{n}\sum_{l=1}^{i}h_{i1l}(\gamma_{11}^{-1}(\chi))+\frac{n}{4\iota_3}h_{111}(\gamma_{11}^{-1}(\chi))]/ [(1-k_1)\gamma_{13}(\gamma_{12}^{-1}(\chi))],\tag{31}$$

$$\varpi_q(\chi)=[\frac{1}{\theta}\sum_{i=q}^{n}\sum_{l=q}^{i}h_{iql}(\gamma_{q1}^{-1}(\chi))+\frac{1}{4\iota_1}(\gamma_{q1}^{-1}(\chi))^2]/ [(1-k_q)\gamma_{q3}(\gamma_{q2}^{-1}(\chi))],\ q=2,\cdots,n,\tag{32}$$

其中:

$$\varpi_0'=[\frac{1}{\theta}\varpi_{01}+\frac{n(1-\tau)}{4\iota_2}\varpi_{02}]/[1-k_0],$$
$$\varpi_1'=[\frac{n}{4\iota_3}\varpi_{12}+\frac{1}{\theta}\varpi_{11}]/[1-k_1],$$
$$\varpi_q'=[\frac{1}{4\iota_1}\varpi_{q2}+\frac{1}{\theta}\varpi_{q1}]/[1-k_q],$$

$0<k_q<1,\theta,\iota_1,\iota_2,\iota_3$是正常数. 控制增益函数$\omega_i:[0,\infty)\to R,i=0,\cdots,n$构造为

$$\omega_i(\chi)=\kappa_i+\varpi_i(\chi),\ \chi\geqslant 0,\tag{33}$$

其中$\kappa_i>0$是正常数.

假设 5 下列不等式成立:

$$k_0\gamma_{03}(\|s\|)-\frac{\theta(1-\tau)^{-1}}{4}\omega_0(V_{01}(\|s\|))\times (\|\nabla_s^{\mathrm T}V_{01}(\|s\|)\|^2+\|\nabla_s^{\mathrm T}V_{01}(\|s\|)L\|^2)>0,\tag{34}$$

$$k_q\gamma_{q3}(|s'|)-\iota_1\omega_q(V_{q1}(s'))(\ell\frac{\partial V_{q1}(s')}{\partial s'})^2- (1-\tau)^{-1}(\ell^2\iota_2+\iota_3)\omega_q(V_{q1}(s'))\times (\frac{\partial V_{q1}(s')}{\partial s'}\frac{\partial\alpha_{q-1}}{\partial y})^2>0,\tag{35}$$

$$k_n\gamma_{n3}(|s'|)-(1-\tau)^{-1}(\ell^2\iota_2+\iota_3)\times \omega_n(V_{n1}(s'))(\frac{\partial V_{n1}(s')}{\partial s'}\frac{\partial\alpha_{n-1}}{\partial y})^2>0,\tag{36}$$

其中$s\in\mathbb{R}^{(n-1)\times 1}$和$s'\in\mathbb{R}$为参数变量.

3.2 控制器设计

对于状态估计误差系统, 考虑以下Lyapunov候选泛函:

$$V_0=V_{0a}+V_{0b},\tag{37}$$

其中:

$$V_{0a}=\int_0^{V_{01}(\varepsilon)}\omega_0(\xi)\mathrm d\xi,\tag{38}$$

$$V_{0b}=\frac{1}{\theta}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i}\int_{t-d_l(t)}^{t}h_{ijl}(|z_j(\xi)|)\mathrm d\xi+ \frac{1}{\theta}\sum_{i=1}^{n}\sum_{k=2}^{i}\int_{t-d_k(t)}^{t}p_k(\|\varepsilon(\xi)\|)\mathrm d\xi.\tag{39}$$

V_0沿式(17)的导数为

$$\begin{aligned}&\dot V_0=\omega_0(V_{01}(\varepsilon))\dot V_{01}+\dot V_{0b}=\\&\omega_0(V_{01}(\varepsilon))(\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)(E\varepsilon(t)+\\&\bar G(x_1(t),\bar m(t))-\hat{\bar G}(x_1(t),\hat{\bar m}(t)))+\\&\omega_0(V_{01}(\varepsilon))\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)(\bar F+Lf_1(x_{1d_1}))+\\&\frac{1}{\theta}\sum_{i=1}^{n}\sum_{k=2}^{i}p_k(\|\varepsilon(t)\|)+\frac{1}{\theta}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i}h_{ijl}(|z_j(t)|)-\\&\frac{(1-\tau)}{\theta}(\sum_{i=1}^{n}\sum_{k=2}^{i}p_k(\|\varepsilon_{d_k}\|)+\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i}h_{ijl}(|z_{j_{d_l}}|)).\end{aligned}\tag{40}$$

借助Young's不等式, 并结合假设1–2, 式(22), 得

$$\begin{aligned}&\omega_0(V_{01}(\varepsilon))(\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)(E\varepsilon(t)+\\&\bar G(x_1(t),\bar m(t))-\hat{\bar G}(x_1(t),\hat{\bar m}(t))))=\\&\omega_0(V_{01}(\varepsilon))(\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon(t))(E\varepsilon(t)+D\varepsilon(t)))\leqslant\\&-\omega_0(V_{01}(\varepsilon))\gamma_{03}(\|\varepsilon(t)\|),\end{aligned}\tag{41}$$

$$\begin{aligned}&\omega_0(V_{01}(\varepsilon))\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)(Lf_1(x_{1d_1})+\\&\bar F(x_{1d_1},\bar m_{n_{dn}}))\leqslant\\&\frac{\theta(1-\tau)^{-1}}{4}\omega_0^2(V_{01}(\varepsilon))\|\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)\|^2+\\&\frac{\theta(1-\tau)^{-1}}{4}\omega_0^2(V_{01}(\varepsilon))\|\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)L\|^2+\\&\frac{(1-\tau)}{\theta}\|\bar F\|^2+\frac{(1-\tau)}{\theta}f_1^2(x_{1d_1})\leqslant\\&\frac{\theta(1-\tau)^{-1}}{4}\omega_0^2(V_{01}(\varepsilon))\|\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)\|^2+\\&\frac{\theta(1-\tau)^{-1}}{4}\omega_0^2(V_{01}(\varepsilon))\|\nabla_\varepsilon^{\mathrm T}V_{01}(\varepsilon)L\|^2+\\&\frac{(1-\tau)}{\theta}(\sum_{i=1}^{n}\sum_{k=2}^{i}p_k(\|\varepsilon_{d_k}\|)+\\&\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i}h_{ijl}(|z_{j_{d_l}}|)).\end{aligned}\tag{42}$$

将式(41)和式(42)代入式(40), 并利用假设5, 得

$$\dot V_0\leqslant-\omega_0(V_{01}(\varepsilon))[k_0\gamma_{03}(\|\varepsilon(t)\|)-\frac{\theta(1-\tau)^{-1}}{4}\times \omega_0(V_{01}(\|\varepsilon(t)\|))(\|\nabla_\varepsilon^{\mathrm T}V_{01}(\|\varepsilon(t)\|)\|^2+$$

$$\|\nabla_{\varepsilon}^{\mathrm{T}} V_{01}(\|\varepsilon(t)\|)L\|^2)] - (1-k_0)\omega_0(V_{01}(\varepsilon))\times \gamma_{03}(\|\varepsilon(t)\|) + \sum_{i=1}^{n}\sum_{k=2}^{i} p_k(\|\varepsilon_{d_k}\|) + \frac{1}{\theta}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i} h_{ijl}(|z_j(t)|) \leqslant -(1-k_0)\omega_0(V_{01}(\varepsilon))\gamma_{03}(\|\varepsilon(t)\|) + \frac{1}{\theta}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i} h_{ijl}(|z_j(t)|) + \sum_{i=1}^{n}\sum_{k=2}^{i} p_k(\|\varepsilon(t)\|), \tag{43}$$

由于$\gamma_{0i}, i=1,2$是K类函数, 故γ_{0i}^{-1}亦为K类函数. 所以由假设3可得

$$\|\varepsilon(t)\|^2 \leqslant (\gamma_{01}^{-1}(V_{01}(\varepsilon)))^2,$$
$$\gamma_{03}(\|\varepsilon(t)\|) \geqslant \gamma_{03}(\gamma_{02}^{-1}(V_{01}(\varepsilon))),$$
$$p_k(\|\varepsilon(t)\|) \leqslant p_k(\gamma_{01}^{-1}(V_{01}(\varepsilon))).$$

再结合控制增益函数$\omega_0(\chi)$的定义(33), 可得式(43)满足

$$\dot V_0 \leqslant -(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|) - (1-k_0)\times \varpi_0(V_{01}(\varepsilon))\gamma_{03}(\|\varepsilon(t)\|) + \sum_{i=1}^{n}\sum_{k=2}^{i} p_k(\|\varepsilon(t)\|) + \frac{1}{\theta}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i} h_{ijl}(|z_j(t)|) \leqslant -(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|) - \frac{n(1-\tau)}{4\iota_2}\times \|\varepsilon(t)\|^2 + \frac{1}{\theta}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=j}^{i} h_{ijl}(|z_j(t)|). \tag{44}$$

下面采用Backstepping方法设计控制率.

第1步 定义如下Lyapunov函数:

$$V_1 = V_0 + V_{1a} + V_{1b}, \tag{45}$$

其中:

$$V_{1a} = \int_0^{V_{11}(z_1(t))} \omega_1(\xi)\mathrm{d}\xi, \tag{46}$$

$$V_{1b} = \frac{1}{4\iota_3}\int_{t-d_1(t)}^{t} h_{111}(|z_1(\xi)|)\mathrm{d}\xi. \tag{47}$$

由式(18)和式(44), 可得V_1的导数满足

$$\dot V_1 = \dot V_0 + \dot V_{1b} + \dot V_{1a} = \dot V_0 + \dot V_{1b} + \omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1}(-c_1 z_1(t)) + \omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1}\ell z_2(t) + \omega_1(V_{11}(z_1))\times \frac{\partial V_{11}(z_1)}{\partial z_1}\ell\varepsilon_2(t) + \omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1}\times f_1(z_{1d_1}). \tag{48}$$

结合假设2, 式(25)及Young's不等式, 有

$$\omega_1(V_{11}(z_1))(\frac{\partial V_{11}(z_1)}{\partial z_1}(-c_1 z_1(t))) \leqslant -\omega_1(V_{11}(z_1))\gamma_{13}(|z_1(t)|), \tag{49}$$

$$\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1}\ell z_2 \leqslant \frac{1}{4\iota_1}z_2^2 + \iota_1(\ell\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1})^2, \tag{50}$$

$$\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1}\ell\varepsilon_2 \leqslant (1-\tau)^{-1}\iota_2(\ell\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1})^2 + \frac{(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2, \tag{51}$$

$$\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1}f_1(z_{1d_1})) \leqslant (1-\tau)^{-1}\iota_3(\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1})^2 + \frac{(1-\tau)}{4\iota_3}h_{111}(|z_{1d_1}|). \tag{52}$$

由假设3和函数$h_{i1l}(\cdot)$ $i=1,\cdots,n,\ l=1,\cdots,i$的性质, 可知

$$\gamma_{13}(|z_1(t)|) \geqslant \gamma_{13}(\gamma_{12}^{-1}(V_{11}(z_1))),$$
$$h_{i1l}(|z_1(t)|) \leqslant h_{i1l}(\gamma_{11}^{-1}(V_{11}(z_1))),$$

将式(49)–(52)代入式(48), 并结合上述不等式与控制增益函数$\omega_1(\chi)$的定义(33)及假设5, 可得

$$\dot V_1 \leqslant \dot V_0 - \omega_1(V_{11}(z_1))\gamma_{13}(|z_1(t)|) + (\iota_1 + \iota_2(1-\tau)^{-1})(\ell\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1})^2 + \iota_3(1-\tau)^{-1}(\omega_1(V_{11}(z_1))\frac{\partial V_{11}(z_1)}{\partial z_1})^2 + \frac{1}{4\iota_1}\times z_2^2(t) + \frac{(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2 + \frac{1}{4\iota_3}h_{111}(|z_1(t)|) + \frac{(1-\tau)}{4\iota_3}h_{111}(|z_{1d_1}|) - \frac{(1-\tau)}{4\iota_3}h_{111}(|z_{1d_1}|) \leqslant -(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|) - \omega_1(V_{11}(z_1))\times [k_1\gamma_{13}(|z_1(t)|) - \ell(\iota_1 + \iota_2(1-\tau)^{-1})\times \omega_1(V_{11}(z_1))(\frac{\partial V_{11}(z_1)}{\partial z_1})^2 - \iota_3(1-\tau)^{-1}\times \omega_1(V_{11}(z_1))(\frac{\partial V_{11}(z_1)}{\partial z_1})^2] - (1-k_1)\omega_1(V_{11}(z_1))\gamma_{13}(|z_1(t)|) + \frac{n}{4\iota_3}h_{111}(|z_1(t)|) - \frac{n-1}{4\iota_3}h_{111}(|z_1(t)|) +$$

$$\frac{1}{4\iota_1}z_2^2(t)+\frac{1}{\theta}\sum_{i=1}^{n}\sum_{l=1}^{i}h_{i1l}(|z_1(t)|)+ \frac{1}{\theta}\sum_{i=2}^{n}\sum_{j=2}^{i}\sum_{l=j}^{i}h_{ijl}(|z_j(t)|)- \frac{(n-1)(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2\leqslant$$

$$-(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)-(1-k_1)\kappa_1\times \gamma_{13}(|z_1(t)|)-(1-k_1)\varpi_1(V_{11}(z_1(t)))\times \gamma_{13}(\gamma_{12}^{-1}(V_{11}(z_1(t))))+\frac{1}{4\iota_1}z_2^2(t)+ \frac{1}{\theta}\sum_{i=1}^{n}\sum_{l=1}^{i}h_{i1l}(\gamma_{11}^{-1}(V_{11}(z_1(t))))+\frac{n}{4\iota_3}\times h_{111}(\gamma_{11}^{-1}(V_{11}(z_1(t))))-\frac{n-1}{4\iota_3}h_{111}(|z_1(t)|)+ \frac{1}{\theta}\sum_{i=2}^{n}\sum_{j=2}^{i}\sum_{l=j}^{i}h_{ijl}(|z_j(t)|)- \frac{(n-1)(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2\leqslant$$

$$-(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)-(1-k_1)\kappa_1\times \gamma_{13}(|z_1(t)|)-\frac{(n-1)(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2- \frac{n-1}{4\iota_3}h_{111}(|z_1(t)|)+\frac{1}{4\iota_1}z_2^2(t)+ \frac{1}{\theta}\sum_{i=2}^{n}\sum_{j=2}^{i}\sum_{l=j}^{i}h_{ijl}(|z_j(t)|). \tag{53}$$

第q步($q=2,\cdots,n-1$) 由上述迭代过程可以推断出

$$V_{q-1}=\int_0^{V_{q-1,1}(z_{q-1})}\omega_{q-1}(\xi)\mathrm{d}\xi+V_{1b}+V_{q-2}$$

的导数满足

$$\dot V_{q-1}\leqslant -(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)- \sum_{l=1}^{q-1}(1-k_l)\kappa_l\gamma_{l3}(|z_l(t)|)- \frac{n-q+1}{4\iota_3}h_{111}(|z_1(t)|)+\frac{1}{4\iota_1}z_q^2(t)- \frac{(n-q+1)(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2+ \frac{1}{\theta}\sum_{i=q}^{n}\sum_{j=q}^{i}\sum_{l=j}^{i}h_{ijl}(|z_j(t)|). \tag{54}$$

定义第q个Lyapunov候选函数为

$$V_q=V_{q-1}+V_{qa}+V_{1b}, \tag{55}$$

其中

$$V_{qa}=\int_0^{V_{q1}(z_q(t))}\omega_q(\xi)\mathrm{d}\xi. \tag{56}$$

结合式(19)和式(54), 利用Young's不等式, 可得V_q的时间导数满足

$$\dot V_q=\dot V_{q-1}+\dot V_{1b}+\dot V_{qa}= \dot V_{q-1}+\dot V_{1b}+\omega_q(V_{q1}(z_q))\times (\frac{\partial V_{q1}(z_q)}{\partial z_q}(-c_qz_q(t)))+ \omega_q(V_{q1}(z_q))\frac{\partial V_{q1}(z_q)}{\partial z_q}\ell z_{q+1}(t)- \omega_q(V_{q1}(z_q))\frac{\partial V_{q1}(z_q)}{\partial z_q}\frac{\partial\alpha_{q-1}}{\partial y}\ell\varepsilon_2(t)- \omega_q(V_{q1}(z_q))\frac{\partial V_{q1}(z_q)}{\partial z_q}\frac{\partial\alpha_{q-1}}{\partial y}f_1(z_{1d_1}))\leqslant \tag{57}$$

$$\dot V_{q-1}-\omega_q(V_{q1}(z_q))\gamma_{q3}(|z_q(t)|)+ \iota_1(\ell\omega_q(V_{q1}(z_q))\frac{\partial V_{q1}(z_q)}{\partial z_q})^2+(1-\tau)^{-1}\times (\ell^2\iota_2+\iota_3)(\omega_q(V_{q1}(z_q))\frac{\partial V_{q1}(z_q)}{\partial z_q}\frac{\partial\alpha_{q-1}}{\partial y})^2+ \frac{1}{4\iota_1}z_{q+1}^2(t)+\frac{(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2+ \frac{1}{4\iota_3}h_{111}(|z_1(t)|). \tag{58}$$

由假设3和函数$h_{iql}(\cdot), i=1,\cdots,n, l=1,\cdots,i$的性质, 可知

$$|z_q(t)|^2\leqslant(\gamma_{q1}^{-1}(V_{q1}(z_q)))^2,$$
$$\gamma_{q3}(|z_q(t)|)\geqslant\gamma_{q3}(\gamma_{q2}^{-1}(V_{q1}(z_q))),$$
$$h_{iql}(|z_q(t)|)\leqslant h_{iql}(\gamma_{q1}^{-1}(V_{q1}(z_q))),$$

结合上述不等式与控制增益函数$\omega_q(\chi)$的定义(33)及假设5, 可得

$$\dot V_q\leqslant -(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)-\sum_{l=1}^{q}(1-k_l)\kappa_l\times \gamma_{l3}(|z_l(t)|)-(1-k_q)\varpi_q(V_{q1})\times \gamma_{q3}(\gamma_{q2}^{-1}(V_{q1}(z_q(t))))-\frac{n-q}{4\iota_3}h_{111}(|z_1(t)|)+ \frac{1}{\theta}\sum_{i=q}^{n}\sum_{l=q}^{i}h_{iql}(\gamma_{q1}^{-1}(V_{q1}(z_q(t))))+ \frac{1}{4\iota_1}(\gamma_{q1}^{-1}(V_{q1}(z_q(t))))^2+\frac{1}{4\iota_1}z_{q+1}^2(t)+ \frac{1}{\theta}\sum_{i=q+1}^{n}\sum_{j=q+1}^{i}\sum_{l=j}^{i}h_{ijl}(|z_j(t)|)- \frac{(n-q)(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2\leqslant$$

$$-(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)-\sum_{l=1}^{q}(1-k_l)\kappa_l\times \gamma_{l3}(|z_l(t)|)+\frac{1}{\theta}\sum_{i=q+1}^{n}\sum_{j=q+1}^{i}\sum_{l=j}^{i}h_{ijl}(|z_j(t)|)-$$

$$\frac{n-q}{4\iota_3}h_{111}(z_1(t))-\frac{(n-q)(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2+\frac{1}{4\iota_1}z_{q+1}^2(t). \tag{59}$$

第n步　考虑Lyapunov候选函数

$$V_n=V_{n-1}+V_{na}+V_{1b}, \tag{60}$$

其中

$$V_{na}=\int_0^{V_{n1}(z_n(t))}\omega_n(\xi)\mathrm{d}\xi. \tag{61}$$

V_n沿着式(20)的导数为

$$\begin{aligned}&\dot V_n=\dot V_{n-1}+\dot V_{1b}+\dot V_{na}=\\&\dot V_{n-1}+\dot V_{1b}+\omega_n(V_{n1}(z_n))(\frac{\partial V_{n1}(z_n)}{\partial z_n}(u+\ell l_n\hat m_2+\\&g_n(x_1(t),\hat{\bar m}(t))+l_ng_1-\frac{\partial\alpha_{n-1}}{\partial y}(\ell\varepsilon_2+\ell\hat m_2+\\&g_1(z_1)+f_1(z_{1d_1})))-\sum_{i=2}^{n-1}\frac{\partial\alpha_{i-1}}{\partial\hat m_j}\dot{\hat m}_j(t))),\end{aligned} \tag{62}$$

控制器设计为

$$\begin{aligned}u(t)=&-c_nz_n-c'\omega_n(V_{n1}(z_n))\frac{\partial V_{n1}(z_n)}{\partial z_n}-\\&\ell l_n\hat m_2-g_n(x_1(t),\hat{\bar m}(t))-l_ng_1(z_1)+\\&\frac{\partial\alpha_{n-1}}{\partial y}(\ell\hat m_2+g_1(z_1)),\end{aligned} \tag{63}$$

其中c'是正设计常数. 在控制器(63)的作用下, 式(62)满足

$$\begin{aligned}&\dot V_n=\dot V_{n-1}+\dot V_{1b}+\dot V_{na}=\\&\dot V_{n-1}+\dot V_{1b}-c'\omega_n^2(V_{n1}(z_n))(\frac{\partial V_{n1}(z_n)}{\partial z_n})^2+\\&\omega_n(V_{n1}(z_n))(\frac{\partial V_{n1}(z_n)}{\partial z_n}(-c_nz_n(t)))-\\&\omega_n(V_{n1}(z_n))\frac{\partial V_{n1}(z_n)}{\partial z_n}\frac{\partial\alpha_{n-1}}{\partial y}f_1(z_{1d_1}))-\\&\omega_n(V_{n1}(z_n))\frac{\partial V_{n1}(z_n)}{\partial z_n}\frac{\partial\alpha_{n-1}}{\partial y}\ell\varepsilon_2(t).\end{aligned} \tag{64}$$

由假设3和函数$h_{inl}(\cdot),i=1,\cdots,n,l=1,\cdots,i$的性质可知,

$$\begin{aligned}&|z_n(t)|^2\leqslant(\gamma_{n1}^{-1}(V_{n1}(z_n)))^2,\\&\gamma_{n3}(|z_n(t)|)\geqslant\gamma_{n3}(\gamma_{n2}^{-1}(V_{n1}(z_n))),\\&h_{inl}(|z_n(t)|)\leqslant h_{inl}(\gamma_{n1}^{-1}(V_{n1}(z_n))).\end{aligned}$$

利用$\omega_n(\chi)$的定义(33), 假设2, Young's不等式及式(25), 得

$$\begin{aligned}&\dot V_n\leqslant\\&\dot V_{n-1}-c'\omega_n^2(V_{n1}(z_n))(\frac{\partial V_{n1}(z_n)}{\partial z_n})^2-\\&(1-k_n)\omega_n(V_{n1}(z_n))\gamma_{n3}(|z_n(t)|)-\\&\omega_n(V_{n1}(z_n))[k_n\gamma_{n3}(|z_n(t)|)-\\&(\ell^2\iota_2+\iota_3)(1-\tau)^{-1}\omega_n(V_{n1}(z_n))\times\\&(\frac{\partial V_{n1}(z_n)}{\partial z_n}\frac{\partial\alpha_{n-1}}{\partial y})^2]+\frac{(1-\tau)}{4\iota_2}\|\varepsilon(t)\|^2+\\&\frac{1}{4\iota_3}h_{111}(|z_1(t)|)\leqslant\\&-(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)+\frac{1}{4\iota_1}z_n^2(t)-\\&\sum_{i=1}^{n-1}(1-k_l)\kappa_l\gamma_{l3}(|z_l(t)|)+\\&\frac{1}{\theta}h_{nnn}(|z_n(t)|)-c'\omega_n^2(V_{n1}(z_n))\times\\&(\frac{\partial V_{n1}(z_n)}{\partial z_n})^2-(1-k_n)\omega_n(V_{n1}(z_n))\times\\&\gamma_{n3}(|z_n(t)|)\leqslant\\&-(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)-\sum_{l=1}^{n}(1-k_l)\times\\&\kappa_l\gamma_{l3}(|z_l(t)|)-c'\omega_n^2(V_{n1}(z_n))(\frac{\partial V_{n1}(z_n)}{\partial z_n})^2-\\&(1-k_n)\varpi_n\gamma_{n3}(\gamma_{n2}^{-1}(V_{n1}(z_n(t))))+\\&\frac{1}{\theta}h_{nnn}(\gamma_{n1}^{-1}(V_{n1}(z_n(t))))+\\&\frac{1}{4\iota_1}(\gamma_{n1}^{-1}(V_{n1}(z_n(t))))^2\leqslant\\&-c'\omega_n^2(V_{n1}(z_n))(\frac{\partial V_{n1}(z_n)}{\partial z_n})^2-\\&(1-k_0)\kappa_0\gamma_{03}(\|\varepsilon(t)\|)-\\&\sum_{l=1}^{n}(1-k_l)\kappa_l\gamma_{l3}(|z_l(t)|).\end{aligned} \tag{65}$$

3.3　稳定性分析

定理 1　由系统(1)、降阶状态观测器(9)、虚拟控制率(14)–(16)、控制器(63)构成的闭环系统在满足假设1–5的条件下, 所有闭环信号是有界的并且系统(1)的状态$x(t)$渐近趋于0.

证　对式(65)两端求t_0到t定积分, 可得

$$\begin{aligned}&V_n(t)+(1-k_0)\kappa_0\int_{t_0}^t\gamma_{03}(\|\varepsilon(s)\|)\mathrm{d}s+\\&\int_{t_0}^t c'\omega_n^2(V_{n1}(z_n(s)))(\frac{\partial V_{n1}(z_n(s))}{\partial z_n(s)})^2\mathrm{d}s+\\&\sum_{l=1}^{n}(1-k_l)\kappa_l\int_{t_0}^t\gamma_{l3}(|z_l(s)|)\mathrm{d}s\leqslant V_n(t_0).\end{aligned} \tag{66}$$

故$V_n(t)\in L_\infty,\gamma_{q3}\in L_1,q=0,\cdots,n$. 由式(60)可推出$V_{na}$有界. 由式(61)可知$c_nV_{n1}\leqslant V_{na}$, 所以$V_{n1}$是有界的, 因此$\omega_n(V_{n1})$有界. 当$q=n$时, 由式(60)得知$z_n$有界. 根据同样的方法可推出$z_q,q=1,\cdots,n$及$\varepsilon$有界. 观察式(14)可知$\alpha_1$有界, 由$z_2(t)=\omega_2-\alpha_1$以及$\omega_2=\hat m_2+l_2y$推断出$\hat m_2$有界, 再结合$\varepsilon$的有界性得到$m_2(t)$有界. 同样由式(15)可知$\alpha_2$是有界的, 结

合式(9)(11)(13), 得m_3是有界的. 以这种方式继续下去, 递归地得到$\alpha_3, m_4, \cdots, \alpha_{n-1}, m_n, u$的有界性. 所以, 所有闭环信号有界. 因此, $\dot{x}, \dot{\varepsilon}$有界. 由于$\gamma_{03}\cdot(\|\varepsilon\|) \in L_1, \gamma_{l3}(|z_l|) \in L_1, l = 1, \cdots, n$, 得到

$$\lim_{t\to\infty} \varepsilon = 0,\ \lim_{t\to\infty} z_l = 0.$$

因$y=z_1$, 有$\lim\limits_{t\to\infty} y=0$. 由于$g_i(\cdot)$光滑且满足$g_i(0) = 0$, 所以有$\lim\limits_{t\to\infty} g_1(z_1) = 0$.

由式(14)知$\lim\limits_{t\to\infty} \alpha_1 = 0$. 由$x_2 = m_2 = \varepsilon_2+z_2+\alpha_1$知$\lim\limits_{t\to\infty} x_2 = 0$. 最终递归地得到$\lim\limits_{t\to\infty} x_n = 0$. 故$\lim\limits_{t\to\infty} x(t) = 0$. 证毕.

4 仿真

考虑严格反馈非线性时滞系统如下:

$$\begin{cases} \dot{x}_1 = -x_1 + 0.2x_2 + 0.02\sin t x_{1d_1}, \\ \dot{x}_2 = -0.1x_2 + 0.02x_{1d_1} + u + 0.02\cos t x_{2d_2}, \\ y(t) = x_1(t), \end{cases} \tag{67}$$

系统(67)可表述为如下形式:

$$\begin{bmatrix} \dot{y} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y \\ m \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{B} \end{bmatrix} u + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix},$$

其中:

$$\begin{aligned} & A_{11} = 0,\ A_{12} = \ell = 0.2,\ A_{21} = 0,\ A_{22} = 0, \\ & \bar{B} = 1,\ g_1 = -y(t),\ g_2 = -0.1m(t), \\ & f_1 = 0.02\sin t x_{1d_1}, \\ & f_2 = 0.02x_{1d_1} + 0.01x_{2d_2} + 0.01\cos t x_{2d_2}, \\ & d_1(t) = 0.1(1+\sin t),\ d_2(t) = 0.1(1+\cos^2 t). \end{aligned}$$

选择参数$L=l_2=-0.5$, 则$E=-0.1$, $E'=-0.05$, 降阶观测器设计为

$$\begin{aligned} & \dot{\hat{m}}(t) = -0.2\hat{m}(t) + u(t) + 0.5y(t) + 0.5\dot{y}(t), \\ & \dot{\omega}(t) = -0.1\omega(t) + 0.45y(t) + u(t) - 0.1\hat{m}(t). \end{aligned}$$

对于此系统, 存在$D = -0.1$, 使得$g_2(m(t)) - \hat{g}_2(\hat{m}(t)) = D\varepsilon(t)$, 所以系统满足假设1. 选择$c_1 = 1$, $c_2 = 1$, 有

$$\begin{aligned} \alpha_1 = & -\frac{1}{\ell}c_1 z_1(t) - \frac{1}{\ell}g_1(z_1(t)) + l_2 z_1(t) = \\ & -0.5z_1(t), \end{aligned} \tag{68}$$

由于$f_1^2 = (0.02\sin t x_{1d_1})^2$, 存在函数$h_{111}(z_{1d_1}) = 0.0004(|z_{1d_1}|)^2$, 使得$f_1^2 \leqslant h_{111}(z_{1d_1})$. 存在函数

$$h_{211}(\cdot) = h_{212}(\cdot) = h_{222}(\cdot) = p(\cdot) = 0.0016(|\cdot|)^2,$$

使得

$$\begin{aligned} & f_2^2 = (0.02x_{1d_1} + 0.01x_{2d_2} + 0.01\cos t x_{2d_2})^2 = \\ & (0.02z_{1d_1} + 0.01(\varepsilon_{2d_2} + \hat{m}_{d_2}) + \\ & 0.01\cos t(\varepsilon_{2d_2} + \hat{m}_{d_2}))^2 = \\ & (0.02z_{1d_1} + 0.01(\varepsilon_{2d_2} + z_{2d_2} + \alpha_{1d_2} - l_2 z_{1d_2}) + \\ & 0.01\cos t(\varepsilon_{2d_2} + z_{2d_2} + \alpha_{1d_2} - l_2 z_{1d_2}))^2 = \\ & (0.02z_{1d_1} + (-0.01 - 0.01\cos t)z_{1d_2} + \\ & (0.01 + 0.01\cos t)z_{2d_2} + \\ & (0.01 + 0.01\cos t)\varepsilon_{2d_2})^2 \leqslant \\ & h_{211}(z_{1d_2}) + h_{212}(z_{1d_2}) + h_{222}(z_{2d_2}) + p(\varepsilon_{2d_2}). \end{aligned} \tag{69}$$

所以系统满足假设2. 设计如下函数:

$$\begin{aligned} & V_{01}(\varepsilon) = \varepsilon^2,\ V_{11}(z_1) = z_1^2,\ V_{21}(z_2) = z_2^2, \\ & \gamma_{01}(\chi) = \chi^2,\ \gamma_{02}(\chi) = \chi^2,\ \gamma_{03}(\chi) = 0.6\chi^2, \\ & \gamma_{11}(\chi) = \chi^2,\ \gamma_{12}(\chi) = \chi^2,\ \gamma_{13}(\chi) = 2\chi^2, \\ & \gamma_{21}(\chi) = \chi^2,\ \gamma_{22}(\chi) = \chi^2,\ \gamma_{23}(\chi) = 2\chi^2. \end{aligned}$$

由此可知系统满足假设3.

由以上结果可得

$$\begin{aligned} & \gamma_{01}^{-1}(\chi) = \gamma_{02}^{-1}(\chi) = \gamma_{11}^{-1}(\chi) = \gamma_{12}^{-1}(\chi) = \\ & \gamma_{21}^{-1}(\chi) = \gamma_{22}^{-1}(\chi) = \sqrt{\chi},\ \gamma_{03}(\gamma_{02}^{-1}(\chi)) = 0.6\chi, \\ & \gamma_{13}(\gamma_{12}^{-1}(\chi)) = 2\chi,\ \gamma_{23}(\gamma_{22}^{-1}(\chi)) = 2\chi, \\ & p(\gamma_{01}^{-1}(\chi)) = 0.0016\chi,\ h_{111}(\gamma_{11}^{-1}(\chi)) = 0.0004\chi, \\ & h_{211}(\gamma_{11}^{-1}(\chi)) = h_{212}(\gamma_{11}^{-1}(\chi)) = \\ & h_{222}(\gamma_{21}^{-1}(\chi)) = 0.0016\chi. \end{aligned}$$

由此可知系统满足假设4. 控制增益函数设计为

$$\begin{cases} \varpi_0(\chi) = \dfrac{0.001\cdot\dfrac{8}{3}\cdot\dfrac{1}{\theta} + \dfrac{5}{3}\cdot\dfrac{2(1-\tau)}{4\iota_2}}{1-k_0}, \\ \omega_0 = \kappa_0 + \varpi_0(\chi),\ \chi \geqslant 0, \end{cases} \tag{70}$$

$$\begin{cases} \varpi_1(\chi) = \dfrac{0.0002\dfrac{2}{4\iota_3} + 0.0018\dfrac{1}{\theta}}{1-k_1}, \\ \omega_1 = \kappa_1 + \varpi_1(\chi),\ \chi \geqslant 0, \end{cases} \tag{71}$$

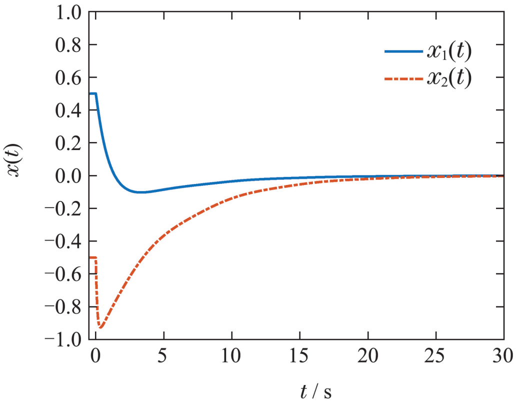
$$\begin{cases} \varpi_2(\chi) = \dfrac{0.5\dfrac{1}{4\iota_1} + 0.0008\dfrac{1}{\theta}}{1-k_2}, \\ \omega_2 = \kappa_2 + \varpi_2(\chi),\ \chi \geqslant 0. \end{cases} \tag{72}$$

设计参数为$\theta = 0.1$, $\tau = 0.1$, $k_0 = 0.5$, $k_1 = 0.5$, $k_2 = 0.5$, $\iota_1 = 6$, $\iota_2 = 7.4$, $\iota_3 = 0.5$, $\kappa_0 = 1$, $\kappa_1 = 0.04$, $\kappa_2 = 0.3$, 则系统满足假设5. 输出反馈控制器设计为

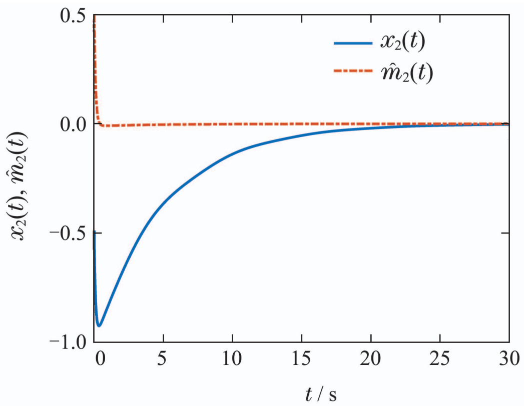
$$\begin{aligned} u(t) = & -c_2 z_2 - c'\omega_2(V_{21}(z_2))\frac{\partial V_{21}(z_2)}{\partial z_2} - \ell l_2 \hat{m}_2 - \\ & g_2(x_1(t), \hat{m}_2(t)) - l_2\, g_1(z_1) + \end{aligned}$$

$$\frac{\partial \alpha_1}{\partial y}(\ell \hat{m}_2 + g_1(z_1)), \tag{73}$$

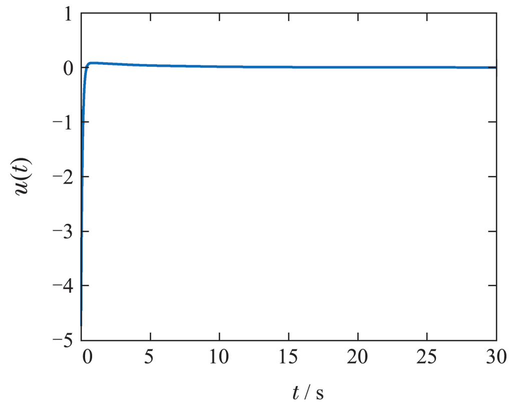
其中参数$c' = 12$. 初始值定为$x_1(t) = 0.5$, $x_2(t) = -0.5$, $t \in [-\tau, 0]$, $m(0) = 0.5$. 状态响应曲线如图1所示, 系统状态$x_1(t), x_2(t)$渐近收敛于0, 未知状态$x_2(t)$及其估计 $\hat{m}_2(t)$ 渐近收敛于 0, 图1(c)描述了控制输入$u(t)$. 仿真结果验证了控制器的有效性.



(a) 系统状态$x_1(t), x_2(t)$



(b) 未知状态$x_2(t)$及其状态估计$\hat{m}_2(t)$



(c) 系统控制输入$u(t)$

图 1　闭环系统响应

Fig. 1　Closed-loop system response

5　结论

本文研究了一类具有未知时滞的非线性系统的渐近稳定问题. 本文采用了降阶观测器估计不可测状态. 提出了新的Lyapunov-Krasovskii函数补偿未知时滞函数, 设计了控制增益函数处理非匹配项以及Lyapunov-Krasovskii函数补偿时滞时产生的非负项. 详细地证明了基于静态增益函数的的输出反馈控制方案可以实现所有闭环信号的有界性以及系统的渐近稳定性. 仿真结果验证了此控制方案的有效性. 如何放宽假设条件值得进一步研究.

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