不确定非线性离散系统指令滤波事件触发控制

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摘要: 本文提出了考虑输入饱和的一类不确定非线性离散系统的事件触发指令滤波控制方法. 采用指令滤波控制技术解决了传统反步法存在的“因果矛盾”问题, 引入补偿机制提高了系统的控制精度; 利用事件触发机制能够避免自适应律和控制律的频繁更新, 降低了计算负担, 提高了资源利用率; 运用模糊逻辑系统逼近系统中未知的非线性函数; 结合李雅普诺夫稳定性理论, 验证了提出的控制方案能够保证跟踪误差收敛到原点小的邻域内以及闭环系统的所有信号有界. 仿真结果表明, 本文提出的控制方法具有较强的鲁棒性及较好的跟踪性能.

关键词: 指令滤波反步; 事件触发机制; 模糊逻辑系统; 输入饱和; 离散

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Command filtered event-triggered control for uncertain nonlinear discrete-time systems

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Abstract: In this paper, an event-triggered command filtered control method for a class of uncertain nonlinear discrete-time systems with input saturation is proposed. The command filtered control technology is used to solve the noncausal problem existing in the traditional backstepping method, and the compensation mechanism is used to improve the control accuracy of the system. Event-triggered mechanism can avoid frequent updates, including adaptive law and control law, which is conducive to reducing the computational burden and improving the utilization of resources. Unknown nonlinear functions are approximated by using fuzzy logic systems. The Lyapunov function is combined in the stability proof part. It is verified that the proposed control scheme can fully ensure that the tracking error converges to the neighborhood with small origin. And it can also ensure that the signal is bounded for closed-loop systems. The simulation results show that the proposed control method has strong robustness and good tracking performance.

Key words: command filtered backstepping; event-triggered mechanism; fuzzy logic system; input saturation; discrete-time

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1　引言

反步法被大量应用于高阶非线性系统的控制器设计中[1–2]. 同时, 反步法成为了普遍采用的控制方法, 能够与自适应控制、模糊控制以及神经网络控制等先进的控制方法结合. 文献[3]将模糊逻辑系统与自适应控制相结合, 应用于多智能体系统. 文献[4]利用传统反步法设计控制器时, 需要将虚拟控制函数反复求差分, 就会出现将来时刻的状态, 从而产生“因果矛盾”问题. 为了解决反步法的不足, 文献[5–6]采用动态面控制方法解决了虚拟控制函数反复求差分问题, 从而降低了计算复杂性. 然而, 动态面控制技术利用一阶滤波器会带来滤波误差, 影响了系统控制精度. 文献

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[7–8]利用指令滤波控制方法不仅克服了“因果矛盾”问题, 而且采用补偿信号消除了滤波器产生的滤波误差, 以提高系统的稳定性和控制精度. 为了处理滤波误差问题, 文献[9]提出了指令滤波控制方法. 但是, 上述方法的大部分研究成果集中在连续系统中, 对于离散系统的研究成果相对较少. 离散系统在稳定性和可实现性上优越于连续系统, 并且在计算机控制领域具有实际的应用价值[10–12]. 文献[13]研究异步电动机离散系统, 提出了指令滤波自适应模糊控制方法. 此外, 针对实际系统在运行中的安全性以及约束性等因素, 需要考虑系统的输入饱和问题. 如何采取合理的方式处理输入饱和, 对提高系统的控制性能具有重要的意义[14–16]. 针对执行器饱和的离散系统, 文献[17]提出了执行器饱和的鲁棒预测控制策略. 文献[18]考虑了离散系统的输入饱和问题, 设计了饱和控制器, 提高了系统响应速度. 文献[19]将执行器饱和分为3种情形, 设计了饱和离散控制器. 文献[20]在考虑输入饱和的前提下, 提出了一种新型的滑模控制方法, 明显改善了系统的控制性能. 如何在节约系统资源的同时保证控制效果仍然具有挑战性. 与传统采样控制方法相比, 事件触发机制通过设定触发条件来判断系统是否触发, 有利于减少触发次数, 并且系统的自适应律和控制律不会频繁更新, 在实际应用中既保证了系统跟踪性能, 又减少了资源浪费[21–27]. 针对有限的带宽, 文献[28]提出了事件触发控制方案. 文献[29]针对离散随机多智能体系统, 提出了事件触发一致性控制问题, 采用事件触发机制来降低计算负担. 针对一类具有执行器饱和的非线性离散系统, 文献[30]结合事件触发控制策略, 设计了自适应滑模控制器. 到目前为止, 将命令滤波控制方法和事件触发机制相结合, 应用于不确定非线性离散系统中的相关研究成果较少. 基于以上分析, 本文在考虑输入饱和的前提下, 设计了事件触发指令滤波控制方法, 并且采用Lyapunov函数证明闭环系统中的所有信号是有界的. 相比于现有的结果, 本文设计控制器的创新点如下:

1) 与文献[1–4]相比, 本文将指令滤波控制方法应用于离散系统中, 解决了传统反步法的“因果矛盾”问题;

2) 采用模糊逻辑系统能够逼近系统中存在的未知非线性函数, 避免了计算复杂性; 引入补偿机制, 提高了系统的控制精度;

3) 将事件触发机制与指令滤波控制方法结合, 不仅能够降低自适应律和控制律频繁更新, 同时, 运用指令滤波控制技术进一步减轻了计算负担, 节省了系统资源.

2 问题描述与准备

考虑不确定非线性系统如下:

$$\begin{cases} x_i(k+1)=f_i(\bar{x}_i(k))+g_ix_{i+1}(k),\\ \quad i=1,2,\cdots,n-1,\\ x_n(k+1)=f_n(\bar{x}_n(k))+g_nu(k)+d(k),\\ y=x_1(k), \end{cases} \tag{1}$$

其中: $\bar{x}_i(k)=[x_1(k)\ x_2(k)\ \cdots\ x_i(k)]\in\mathbb{R}_i$为系统的状态向量; $\bar{x}_n(k)=[x_1(k)\ x_2(k)\ \cdots\ x_n(k)]\in\mathbb{R}_n$为系统的状态向量; y为系统的输出信号; $f_i(\bar{x}_i(k))$和$f_n(\bar{x}_n(k))$为不确定非线性光滑函数; g_i和g_n为已知的常数; $d(k)$为有界的额外扰动; $u(k)$为具有饱和特性的控制输入信号.

2.1 输入饱和特性

考虑系统的输入饱和现象. 具有饱和特性的输入信号$u(k)$描述为

$$u(k)=\mathrm{sat}v(k)=\begin{cases} u_{\max},\ v(k)\geqslant u_{\max},\\ v(k),\ u_{\min}<v(k)<u_{\max},\\ u_{\min},\ v(k)\leqslant u_{\min}, \end{cases} \tag{2}$$

其中: $u_{\max}>0$和$u_{\max}<0$是未知的常数, $v(k)$为系统的实际输入信号.

利用分段的平滑函数近似饱和函数, 即

$$g(v(k))=\begin{cases} u_{\max}\times\tanh\left(v(k)/u_{\max}\right),\ v(k)\geqslant 0,\\ u_{\min}\times\tanh\left(v(k)/u_{\min}\right),\quad v(k)<0, \end{cases} \tag{3}$$

$\mathrm{sat}v(k)$可以表述为$u(k)=\mathrm{sat}(v(k))=g(v(k))+Y(v(k))$. 其中$|Y(v(k))|=|\mathrm{sat}v(k)-g(v(k))|\leqslant\max\{u_{\max}(1-\tan(1))u_{\min}(\tan(1)-1)\}=D$, D是一个常数. 根据中值定理可知, 存在$0<\lambda<1$, 使得$g(v(k))=g(v(0))+g_{v_\lambda(k)}(v(k)-v(0))$, $g_{v_\lambda(k)}=(g(v(k+1))-g(v(k)))|_{v(k)=v_\lambda(k)}$. 其中: $v_\lambda(k)=\lambda v(k)+(1-\lambda)v(0)$; $v(0)=0$; $g(v(0))=0$, 可得$g(v(k))=g_{v_\lambda(k)}v(k)$, 通过上述分析得到

$$u(k)=g_{v_\lambda(k)}v(k)+Y(v(k)). \tag{4}$$

假设 1 参考信号$y_\mathrm{d}(t)$为光滑有界且已知的函数[10].

引理 1 对于离散系统, 指令滤波器定义为[11]

$$\begin{cases} \varphi_{l,1}(k+1)=\omega_n\varphi_{l,2}\varDelta_T+\varphi_{l,1}(k),\\ \varphi_{l,2}(k+1)=(-2\zeta\omega_n\varphi_{l,2}(k)-\omega_n(\varphi_{l,1}(k))-\\ \qquad\qquad\alpha_l(k))\varDelta_T+\varphi_{l,2}(k), \end{cases} \tag{5}$$

其中: $0<\zeta\leqslant 1$; $\omega_n>0$. 对于所有$k\geqslant 1$, 输入信号$|\alpha_i(k+1)-\alpha_i(k)|\leqslant b_1$和$|\alpha_i(k+2)+\alpha_i(k)-2\alpha_i(k+1)|\leqslant b_2$, b_1和b_2是正常数, $\varphi_{i,2}(0)=0$, $\varphi_{i,1}(0)=\alpha_i(0)$. 对于任意的常数$\varepsilon>0$, 得到$|\varphi_i(k)-\alpha_i(k)|\leqslant\varepsilon$是有界的.

引理 2 存在模糊逻辑系统$W^\mathrm{T}S(Z(k))$, 使

得$f(k)=W^{\mathrm{T}}S(Z(k))+\tau$, 其中: $f(k)$为定义在紧集Ω_{Z}中的非线性函数; τ为逼近误差, 对于任意的常数$\varepsilon>0$, 使得$|\tau|\leqslant\varepsilon$; $W\in\mathbb{R}^N$为权重向量; $S(Z(k))=[S_1(Z_1(k))\ \ S_2(Z_2(k))\ \ \cdots\ \ S_n(Z_n(k))]^{\mathrm{T}}$为基函数向量; $Z(k)=[Z_1(k)\ \ Z_2(k)\ \ \cdots\ \ Z_n(k)]^{\mathrm{T}}\in\Omega_{\mathrm{Z}}$为模糊逻辑系统的输入信号. 对于基函数向量, 满足如下条件: $\lambda_{\max}[S_i(Z_i(k))^{\mathrm{T}}\ \ S_i(Z_i(k))]<l$.

假设 2 模糊逻辑系统中的基函数$S_i(Z_i(k))$满足局部利普希茨条件, $|S_i(Z_i(k_s))-S_i(Z_i(k))|\leqslant L_i|Z_i(k_s)-Z_i(k)|, i=1,2,\cdots,n$, 其中$L_i>0$.

2.2 事件触发机制

本文采用事件触发机制, 来判断自适应律是否更新. 当满足触发条件时, 传感器将采集到的信号传输到控制器, 自适应律更新. 相反, 不满足触发条件, 自适应律没有更新, 就会采用零阶保持器(zero-order holder, ZOH)把信号维持到上一个触发状态. 相比于文献[1–20], 本文采用的事件触发机制是通过设定触发机制, 从而判断自适应律是否更新, 减少了资源浪费. 设定系统的事件触发机制如下:

$$\mathrm{ET}^1\wedge\mathrm{ET}^2\wedge\ \ \cdots\ \ \wedge\mathrm{ET}^n, \tag{6}$$

其中$\mathrm{ET}^i=\|h_i(k)\|\leqslant\mu_i|v_i|, i=1,2,\cdots,n$.

模糊系统输入信号之间的差值$h_i(k)$描述为

$$h_i(k)=Z(k)-Z(k_s), \tag{7}$$

其中: $\mu_i=\sqrt{\prod_i l/L_i^2\|\hat{W}_i(k)\|^2}$; k_s为上一次触发时刻; k_{s+1}为下一次的触发时刻. 当$k=k_s$, 自适应律更新. 当$k_s<k\leqslant k_{s+1}$, 采用ZOH保持上次的触发状态. 根据文献[22], 设置系统的自适应律为

$$\begin{aligned}\hat{W}_i(k+1)=\hat{W}_i(k)+&\frac{\eta(k)\beta_iS_i(Z_i(k))v_i(k+1)}{1+\|S_i(Z_i(k))\|^2v_i^2(k+1)}-\\&\eta(k)\xi_i\hat{W}_i(k),\end{aligned} \tag{8}$$

其中: $\beta_i>0$; $\xi_i>0$; $\hat{W}_i(k)$为模糊逻辑系统权重向量的估计值, 并且$\hat{W}_i(k)=W(k)+\tilde{W}_i(k)$; $S_i(Z_i(k))$为基函数向量. 指示函数$\eta(k)$描述为

$$\eta(k)=\begin{cases}1,\ k=k_s,\\0,\ k_s<k\leqslant k_{s+1}.\end{cases} \tag{9}$$

设计事件触发机制的原理图如图1所示.

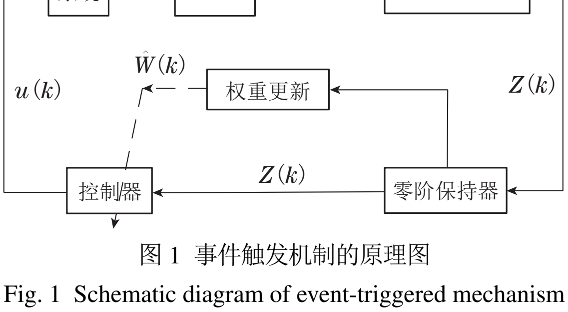

图 1 事件触发机制的原理图

Fig. 1 Schematic diagram of event-triggered mechanism

3 基于事件触发的指令滤波控制器设计

根据反步法原理, 定义系统误差为$e_1(k)=x_1(k)-y_{\mathrm{d}}(k)$, $e_i(k)=x_i(k)-x_{(i-1)\mathrm{d}}(k)$, 其中: $i=2,3,\cdots,n$; 补偿信号为$\zeta_j(k)=e_j(k)-v_j(k)$, 其中: $j=1,2,\cdots,n$; $y_{\mathrm{d}}(k)$为参考信号; $x_{(i-1)\mathrm{d}}(k)$为指令滤波器的输出信号.

步骤 1 根据式(1)的离散系统, 并结合$v_1(k)=e_1(k)-\zeta_1(k)$得

$$\begin{aligned}v_1(k+1)=f_1(\bar{x}_1(k))+g_1x_2(k)-\\y_{\mathrm{d}}(k+1)-\zeta_1(k+1).\end{aligned} \tag{10}$$

根据引理2, 存在模糊逻辑系统$W_1^{\mathrm{T}}S_1(Z_1(k))$, 使得

$$f_1(\bar{x}_1(k))=-(W_1^{\mathrm{T}}S_1(Z_1(k))+\tau_1), \tag{11}$$

其中: W_1^{T}为权重向量; $S_1(Z_1(k))$为基函数向量, $Z_1(k)=[x_1(k)\ \ x_2(k)\ \ y_{\mathrm{d}}(k)]^{\mathrm{T}}$; τ_1为逼近误差, 且对于任意的ε_1, 存在$|\tau_1|\leqslant\varepsilon_1$.

选取虚拟控制律$\alpha_1(k)$为

$$\alpha_1(k)=\frac{y_{\mathrm{d}}(k+1)+\hat{W}_1^{\mathrm{T}}(k)S_1(Z_1(k))}{g_1}+t_1\zeta_1(k). \tag{12}$$

选取补偿信号$\zeta_1(k+1)$为

$$\zeta_1(k+1)=g_1(\zeta_2(k)+x_{1\mathrm{d}}(k)-\alpha_1(k)+t_1\zeta_1(k)), \tag{13}$$

其中$|t_1|\leqslant 1$, 将式(11)–(13)代入式(10)得

$$\begin{aligned}v_1(k+1)=g_1v_2(k)+\hat{W}_1^{\mathrm{T}}(k)S_1(Z_1(k))-\varepsilon_1-\\W_1^{\mathrm{T}}S_1(Z_1(k)).\end{aligned} \tag{14}$$

构造Lyapunov函数$V_1(k)=\dfrac{1}{2}v_1^2(k)$, $V_1(k)$的一阶差分$\Delta V_1(k)$为

$$\begin{aligned}\Delta V_1(k)=2\tilde{W}_1^{\mathrm{T}}(k)\tilde{W}_1(k)\|S_1(Z_1(k))\|^2+\\2\varepsilon_1^2-\frac{1}{2}v_1^2(k)+g_1^2v_2^2(k).\end{aligned} \tag{15}$$

步骤 2 根据式(1)的离散系统, 并结合$v_i(k)=e_i(k)-\zeta_i(k), i=2,3,\cdots,n-1$得

$$\begin{aligned}v_i(k+1)=f_i(\bar{x}_i(k))-x_{(i-1)\mathrm{d}}(k+1)-\\\zeta_i(k+1)+g_ix_{i+1}(k).\end{aligned} \tag{16}$$

根据引理2, 存在模糊逻辑系统$W_i^{\mathrm{T}}S_i(Z_i(k))$, 使得

$$f_i(\bar{x}_i(k))=-(W_i^{\mathrm{T}}S_i(Z_i(k))+\tau_i). \tag{17}$$

选取虚拟控制律$\alpha_i(k)$为

$$\alpha_i(k)=\frac{x_{(i-1)\mathrm{d}}(k+1)-f_i(\bar{x}_i(k))}{g_i}+t_i\zeta_i(k). \tag{18}$$

选取补偿信号$\zeta_i(k+1)$为

$$\zeta_i(k+1)=g_i(\zeta_{i+1}(k)+x_{i\mathrm{d}}(k)-\alpha_i(k)+t_i\zeta_i(k)), \tag{19}$$

其中$|t_i|\leqslant 1$, 将式(17)–(19)代入式(16)得

$$\begin{aligned}v_i(k+1)=&g_iv_{i+1}(k)+\hat{W}_i^{\mathrm{T}}(k)S_i(Z_i(k))-\\&\varepsilon_i-W_i^{\mathrm{T}}S_i(Z_i(k)).\end{aligned} \tag{20}$$

构造Lyapunov函数 $V_i(k)=\dfrac{1}{2}v_i^2(k)+V_{i-1}(k)$, $V_i(k)$的一阶差分$\Delta V_i(k)$为

$$\begin{aligned}\Delta V_i(k)=&2\tilde{W}_i^{\mathrm{T}}(k)\tilde{W}_i(k)\|S_i(Z_i(k))\|^2+\\&g_i^2v_{i+1}^2(k)+2\varepsilon_i^2-\frac{1}{2}v_i^2(k)+\Delta V_{i-1}(k).\end{aligned} \tag{21}$$

步骤 3 令$\zeta_n(k)=0$. 根据式(1)的离散系统, 并结合$v_n(k)=e_n(k)$得

$$\begin{aligned}v_n(k+1)=&f_n(\bar{x}(k))+g_nu(k)+d(k)-\\&x_{(n-1)\mathrm{d}}(k).\end{aligned} \tag{22}$$

根据引理2, 存在模糊逻辑系统$W_n^{\mathrm{T}}S(Z_n(k))$, 使得

$$f_n(\bar{x}(k))-x_{(n-1)\mathrm{d}}(k)=-(W_n^{\mathrm{T}}S_n(Z_n(k))+\tau_n), \tag{23}$$

其中: $Z_n(k)=[Z_1(k)\ \ Z_2(k)\ \ \cdots\ \ Z_n(k)\ \ y_{\mathrm{d}}(k)]^{\mathrm{T}}$; W_n^{T}为权重向量; τ_n为逼近误差, 且$|\tau_n|\leqslant\varepsilon_n$.

根据式(4)可得, 具有饱和特性的控制输入信号为

$$u(k)=g_{v_\lambda(k)}v(k)+Y(v(k)), \tag{24}$$

把式(23)–(24)代入式(22)得

$$\begin{aligned}v_n(k+1)=&g_n(g_{v_\lambda(k)}v(k)+Y(v(k)))+\\&d(k)-W_n^{\mathrm{T}}S_n(Z_n(k))-\varepsilon_n,\end{aligned} \tag{25}$$

其中$g_{v_\lambda(k)}=1$, 事件触发自适应模糊控制器构造为

$$v(k)=\frac{1}{g_n}\hat{W}_n^{\mathrm{T}}(k)S_n(Z_n(k_s)). \tag{26}$$

将式(26)代入式(25)得

$$\begin{aligned}v_n(k+1)=&\hat{W}_n^{\mathrm{T}}(k)\,\|S_n(Z_n(k_s))-S_n(Z_n(k))\|+\\&\tilde{W}_n^{\mathrm{T}}(k)S_n(Z_n(k))+d(k)+g_nD-\varepsilon_n.\end{aligned} \tag{27}$$

4 权重误差值的有界性分析

依据文献[22], 权重误差值的有界性证明分为两部分, 分别为情形1和情形2. 结合式(8)以及$\hat{W}(k)=W(k)+\tilde{W}(k)$可得

$$\begin{aligned}\tilde{W}_i(k+1)=&\frac{\eta(k)\beta_iS_i(Z_i(k))v_1(k+1)}{1+\|S_i(Z_i(k))\|^2v_1^2(k+1)}+\\&\tilde{W}_i(k)-\eta(k)\xi_i\hat{W}_i(k).\end{aligned} \tag{28}$$

情形 1 在触发时刻, 自适应律能够更新, 并且$\eta(k)=1$. 构造Lyapunov函数$V_{\tilde{W}_i}(k)=\tilde{W}_i^{\mathrm{T}}(k)\tilde{W}_i(k)$, 可得$V_{\tilde{W}_i}(k)$的一阶差分为

$$\begin{aligned}\Delta V_{\tilde{W}_i}(k)=&\tilde{W}_i^{\mathrm{T}}(k+1)\tilde{W}_i(k+1)-\\&\tilde{W}_i^{\mathrm{T}}(k)\tilde{W}_i(k).\end{aligned} \tag{29}$$

把式(28)代入式(29)得

$$\begin{aligned}\Delta V_{\tilde{W}_i}(k)=&\beta_i^2v_1^2(k+1)\psi_s^{\mathrm{T}}(k)\psi_s(k)+\\&2\tilde{W}_i(k)\beta_iv_1(k+1)\psi_s(k)-\\&2\beta_iv_1(k+1)\psi_s(k)\xi_i\hat{W}_i(k)-\\&2\tilde{W}_i(k)\xi_i\hat{W}_i(k)+\xi_i^2\hat{W}_i^2(k),\end{aligned} \tag{30}$$

文献[22]处理$\psi_s(k)$的方式为

$$\psi_s(k)=\frac{S_i(Z_i(k))}{1+\|S_i(Z_i(k))\|^2v_1^2(k+1)},$$

且$\psi_s^{\mathrm{T}}(k)\psi_s(k)v_1^2(k+1)\leqslant\dfrac{1}{4}$. 因此, 结合不等式可得

$$\begin{cases}-2\hat{W}_i(k)\xi_iv_1(k+1)\psi_s(k)\beta_i\leqslant\\ \xi_i^2\|\hat{W}_i(k)\|^2+\dfrac{\beta_i^2}{4},\\ 2\beta_iv_1(k+1)\psi_s(k)\tilde{W}_i(k)\leqslant\\ \lambda\beta_i^2+\dfrac{\tilde{W}_i^{\mathrm{T}}(k)\tilde{W}_i(k)}{4\lambda},\end{cases} \tag{31}$$

其中: $\lambda>1$; $\beta_i>0$. 把式(31)代入式(30), 并利用公式$2\tilde{W}_i^{\mathrm{T}}(k)\hat{W}_i(k)=\|\hat{W}_i\|^2+\tilde{W}_i^{\mathrm{T}}(k)\tilde{W}_i(k)-\|W_i\|^2$, 进一步可以得到

$$\Delta V_{\tilde{W}_i}(k)\leqslant-(\xi_i-\frac{1}{4\lambda})\tilde{W}_i^{\mathrm{T}}(k)\tilde{W}_i(k)+\delta_{W_i}, \tag{32}$$

其中$\delta_{W_i}=(\dfrac{1}{2}+\lambda)\beta_i^2-\xi_i(1-2\xi_i)\|\hat{W}_i(k)\|^2+\xi_i\|W_i\|^2$, 且$\dfrac{1}{4\lambda}<\xi_i<\dfrac{1}{2}$.

令$\chi=\xi_i-\dfrac{1}{4\lambda}(0<\chi<\dfrac{1}{2})$, 为了保证权重误差值的有界性, 存在$-\chi\|\tilde{W}_i(k)\|^2+\delta_{W_i}\geqslant 0$, $\|\tilde{W}_i(k)\|$是有界的, 并且满足

$$\|\tilde{W}_i(k)\|^2\leqslant\frac{\delta_{W_i}}{\chi}:=\nu_{W_i^1}. \tag{33}$$

情形 2 在间隔时间$k_s<k\leqslant k_{s+1}$, 自适应律没有更新, $\hat{W}_i(k)$一直保持到下一次的触发状态, 并且控制输入信号保持不变, 因此, $\Delta V_{\tilde{W}_i}(k)=0$.

由于$\Delta V_{\tilde{W}_i}(k)=0$, 根据式(9)(28)–(29)可以得出, 权重估计误差$\tilde{W}_i(k)$在间隔时间内为常数, 从情形1中可得, $\tilde{W}_i(k_s)$在触发时刻为有界, 在情形2中需要证明$\tilde{W}_i(k_s+1)$是有界的, 根据式(32)得

$$\Delta V_{\tilde{W}_i}(k_s)\leqslant-\chi\tilde{W}_i^{\mathrm{T}}(k_s)\tilde{W}_i(k_s)+\delta_{W_i}. \tag{34}$$

把式(29)代入式(34)得到

$$\|\tilde{W}_i(k_s+1)\|^2 \leqslant (1-\chi)\|\tilde{W}_i(k_s)\|^2+\delta_{W_i}, \quad (35)$$

其中$\dfrac{1}{2}<1-\chi<1$, 将式(33)代入式(35)得

$$\|\tilde{W}_i(k_s+1)\|^2 \leqslant (1-\chi)\nu_{W_i^1}+\delta_{W_i}=\nu_{W_i^2}. \quad (36)$$

根据以上的分析可得, 在区间$k_s<k\leqslant k_{s+1}$, 权重估计误差$\tilde{W}_i(k)$是有界的.

5　稳定性分析

通过构造Lyapunov函数来验证闭环系统的稳定性, 并且在两种情况下进行证明.

情形 1　在触发时刻$k=k_s$. 构造Lyapunov函数$V_n(k)=\dfrac{1}{2}v_n^2(k)+V_{n-1}(k)$, 结合式(27)以及杨氏不等式, $V_n(k)$的一阶差分为

$$\begin{aligned}\Delta V_n(k)\leqslant & 2\tilde{W}_n^2(k)\|S_n(Z_n(k))\|^2+2g_n^2D^2+\\ & 2\varepsilon_n^2+2d^2(k)-\frac{1}{2}v_n^2(k)+\Delta V_{n-1}(k).\end{aligned} \quad (37)$$

令$d(k)=d_0$[22]. 同时, 选取Lyapunov函数$V(k)=\sum\limits_{i=1}^{n}V(k)+\theta V_{\tilde{W}_i}(k), \theta>1$. 结合式(15)(21)(27)(37), 及$\|S_i(Z_i(k))\|^2\leqslant l_i$, $V(k)$的一阶差分为

$$\begin{aligned}\Delta V(k)\leqslant & -\sum_{i=1}^{n}(\chi\theta-2l_i)\tilde{W}_i^{\mathrm{T}}(k)\tilde{W}_i(k)+\\ & \gamma_{v_1}-\frac{1}{2}\sum_{i=1}^{n}(1-2g_{i-1}^2)v_i^2(k),\end{aligned} \quad (38)$$

其中: $\gamma_{v_1}=2\sum\limits_{i=1}^{n}\varepsilon_i^2+2g_n^2D^2+2d_0^2+\theta\sum\limits_{i=1}^{n}\delta_{W_i}$; $g_0=0$.

式(38)可以简化为

$$\Delta V(k)\leqslant -\kappa_1 V(k)+\gamma_{v_1}, \quad (39)$$

其中$\kappa_1=\min\{\chi\theta-2l_i 1-2g_{i-1}^2\}$. 选择合适的参数$\xi_i, \beta_i, \lambda$, 及$\theta$, 使$\chi\theta-2l_i>0, l_i>0, 1-2g_{i-1}^2>0$, $i=1,2,\cdots,n$, 得到 $\sum\limits_{i=1}^{n}(1-2g_{i-1}^2)v_i^2(k)<2\gamma_{v_1}$. 进一步, 对于充分小的正数$a$, 存在 $\lim\limits_{k\to\infty}\|v_1(k)\|7\leqslant a$. 假设$x_{1\mathrm{d}}(k)-\alpha_1(k)$是有界的, $|t_1|\leqslant 1$, 可得$\zeta_1(k)$是有界的. 由于$\zeta_1(k)=e_1(k)-v_1(k)$, 得到$e_1(k)$是有界的. 同理, $e_2(k), e_3(k),\cdots,e_n(k)$是有界的. 因此, 在触发时刻, 闭环系统的所有信号是有界的.

情形 2　在间隔时间$k_s<k\leqslant k_{s+1}$. 权重估计值没有更新, $\Delta V_{\tilde{W}_i}(k)=0$. 利用利普希茨条件, 即

$$\begin{aligned}&|S_n(Z_n(k))-S_n(Z_n(k_s))|\leqslant\\ &L_n\sqrt{\prod_n/L_n^2\|\hat{W}_n(k)\|^2}|v_n|.\end{aligned} \quad (40)$$

构造Lyapunov函数$V_n(k)=\dfrac{1}{2}v_n^2(k)+V_{n-1}(k)$, 把式(40)代入式(27)可得$V_n(k)$的一阶差分为

$$\begin{aligned}\Delta V_n(k)\leqslant & 2\tilde{W}_n^{\mathrm{T}}(k)\tilde{W}_n(k)l_n+2\prod_u v_n^2(k)+\\ & 4\varepsilon_n^2+4g_n^2D^2+2d^2(k)-\frac{1}{2}v_n^2(k)+\\ & \Delta V_{n-1}(k).\end{aligned} \quad (41)$$

针对$d(k)=d_0$, 将式(15)(21)代入式(41)可得

$$\Delta V(k)\leqslant -\frac{1}{2}\sum_{i=1}^{n}(1-2g_{i-1}^2)v_i^2(k)+\gamma_{v_2}, \quad (42)$$

其中: $\gamma_{v_2}=4g_n^2D^2+2d_0^2+2\prod\limits_n v_n^2(k)+2\sum\limits_{i=1}^{n}\varepsilon_i^2+2\sum\limits_{i=1}^{n}\tilde{W}_i^T(k)\tilde{W}_i(k)l_i$; $g_0=0$.

选择合适的参数$\prod\limits_n$, 令$\Delta V(k)=-\kappa_2 V(k)+\gamma_{v_2}$, $\kappa_2=1-g_{i-1}^2>0$. 同理, 可得闭环系统的所有信号在$k_s<k\leqslant k_{s+1}$是有界的.

6　仿真结果分析

情况 1　为了验证方法的可行性, 选取系统如下:

$$\begin{cases}x_1(k+1)=0.01\dfrac{x_1^2(k)}{1+x_1^3(k)}+g_1x_2(k)+d_1(k),\\ x_2(k+1)=\dfrac{x_1(k)}{1+x_1^2(k)+x_2^2(k)}+g_2u(k)+d_2(k),\end{cases} \quad (43)$$

其中: $g_1=g_2=0.01$; 给定的参考信号为$y_{\mathrm{d}}(k)=0.8\sin k/9+0.07\cos k/9$; 系统初始条件为$x_1(0)=0, x_2(0)=0$, $d_1(k)=0.8\sin k/9$, 以及$d_2(k)=15\times\cos k/9$; 参数选取为$\xi_1=\xi_2=2$, $\beta_1=\beta_2=2.4$; 控制器参数为$\omega_n=250$, $\zeta=0.1$; 模糊逻辑系统$W_1^{\mathrm{T}}S_1(Z_1(k))$和$W_2^{\mathrm{T}}S_2(Z_2(k_s))$的隶属度函数选取为高斯函数$\mu_{F_1^i}=\exp(-(x_1+j)^2/2)$, 以及$\mu_{F_2^i}=\exp(-(x_2+j)^2/2)$, 其中: $j\in[-5\ \ 5]$; $i\in[1\ \ 9]$.

仿真结果如图2–7所示. 图2为状态变量$x_1(k)$能够较好的跟踪参考信号$y_{\mathrm{d}}(k)$. 图3为采用指令滤波控制方法的跟踪误差曲线图, 图4为采用动态面控制方法的跟踪误差曲线图, 对比两图可知, 本文提出控制方法的跟踪误差小于动态面方法, 具有较好的跟踪效果. 从图5可以看出, 本文在考虑输入饱和的情况下, 设计了饱和控制器, 同时能够把输入信号限制在一定的范围内, 避免输入信号过大或者过小影响系统正常运行. 图6为传统时间触发机制与事件触发机制的对比图. 从图中可以看出, 采用传统时间触发机制系统持续更新. 然而, 采用事件触发机制, 系统更新了54次, 在没有更新的情况下, 系统保持上一次的触发状态, 节省了系统资源. 图7为上一次触发时刻与当前触发时刻的时间间隔.

情况 2　本文采用与文献[10]相同的永磁同步电动机系统来验证提出方法的可行性. 该系统模型为

$$
\begin{cases}
x_1(k+1) = x_1(k) + \Delta_T x_2(k), \\
x_2(k+1) = (1+\Delta_T a_2)x_2(k) + a_1\Delta_T x_3(k) + \\
\qquad\qquad\quad a_3\Delta_T x_3(k)x_4(k) + a_4\Delta_T T_l, \\
x_3(k+1) = (1+b_1\Delta_T)x_3(k) + b_2\Delta_T x_2(k) + \\
\qquad\qquad\quad b_3\Delta_T x_2(k)x_4(k) + b_4\Delta_T u_{\mathrm{q}}(k), \\
x_4(k+1) = (1+c_1\Delta_T)x_4(k) + c_3\Delta_T u_{\mathrm{d}}(k) + \\
\qquad\qquad\quad c_2\Delta_T x_2(k)x_3(k),
\end{cases}
\tag{44}
$$

其中: $T_l = \begin{cases} 1.0\ \mathrm{N{\cdot}m},\ k < 4000 \\ 1.5\ \mathrm{N{\cdot}m},\ k \geqslant 4000 \end{cases}$ 为电机负载转矩; $u_{\mathrm{q}}, u_{\mathrm{d}}$为$q$轴和$d$轴电压; 参考信号为 $y_{\mathrm{d}}(k) = 1.3\times\cos\dfrac{k}{3} + \cos k$; 采样周期为$\Delta_T = 0.0025$ s.

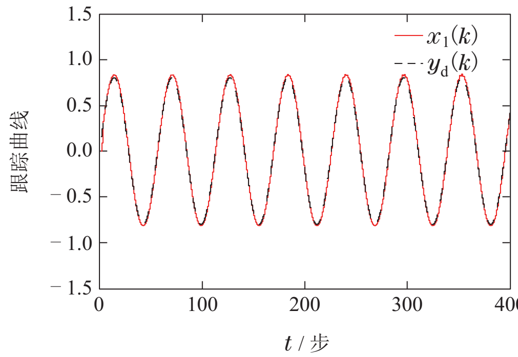

图 2 状态变量$x_1(k)$以及参考信号$y_{\mathrm{d}}(k)$

Fig. 2 State variable $x_1(k)$ and reference signal $y_{\mathrm{d}}(k)$

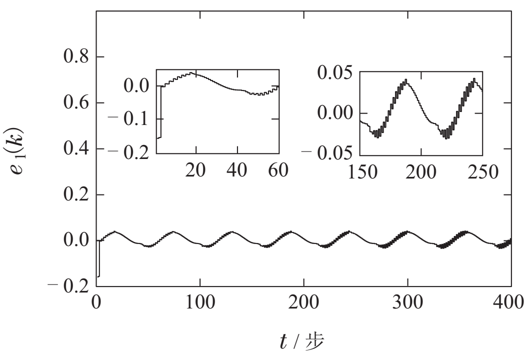

图 3 跟踪误差$e_1(k)$(指令滤波控制)

Fig. 3 Tracking error $e_1(k)$ (command filtered control)

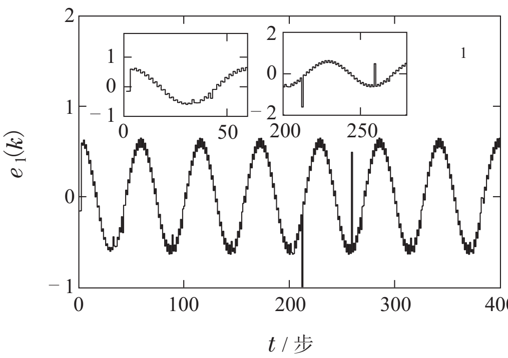

图 4 跟踪误差$e_1(k)$ (动态面控制)

Fig. 4 Tracking error $e_1(k)$ (dynamic surface control)

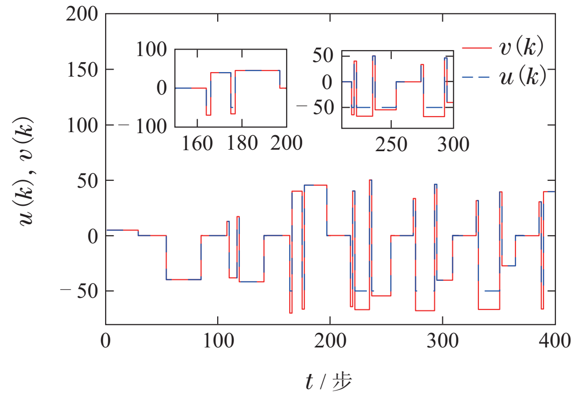

图 5 输入信号$u(k)$以及控制律$v(k)$

Fig. 5 Input signal $u(k)$ and control law $v(k)$

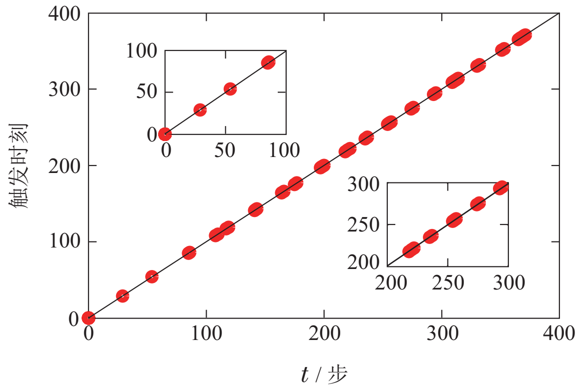

图 6 事件触发时刻

Fig. 6 Event-triggered instant

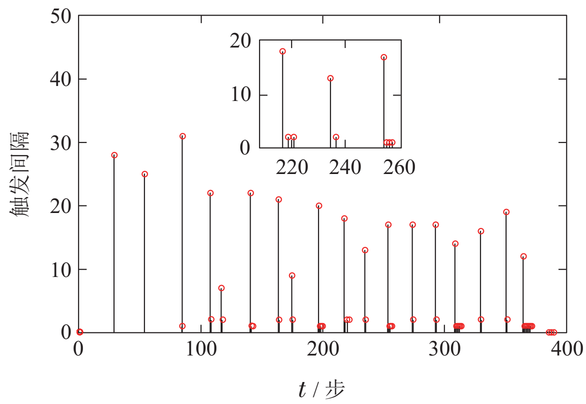

图 7 事件触发的时间间隔

Fig. 7 Event-triggered interval

仿真结果如图8–13所示. 图8为转子角位置跟踪曲线图. 从小图中可以看出, 在$k = 4000$时, 存在扰动, 经过较短的时间, 电机克服了扰动的影响. 图9为电动机的跟踪误差曲线. 图10为q轴电压曲线图, 明显可以看出, 利用饱和控制器能够把电压限制在一定的范围. 图11为d轴电压曲线图. 从图12可以看出, 相比于传统的时间触发机制, 本文采用事件触发机制, 系统触发了4173次. 图13为本文采用事件触发机制的时间间隔, 系统不会持续触发, 从而节省了系统资源.

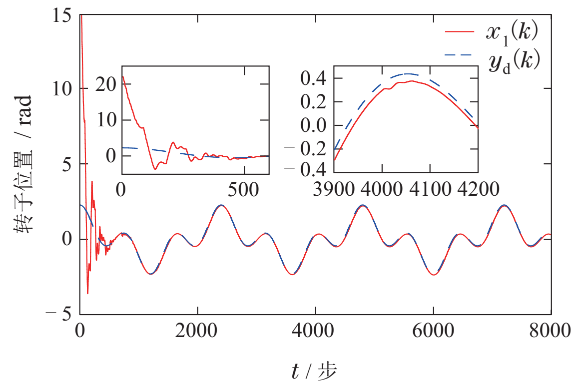

图 8 转子角位置$x_1(k)$以及参考信号$y_d(k)$

Fig. 8 Rotor angle position $x_1(k)$ and reference signal $y_d(k)$

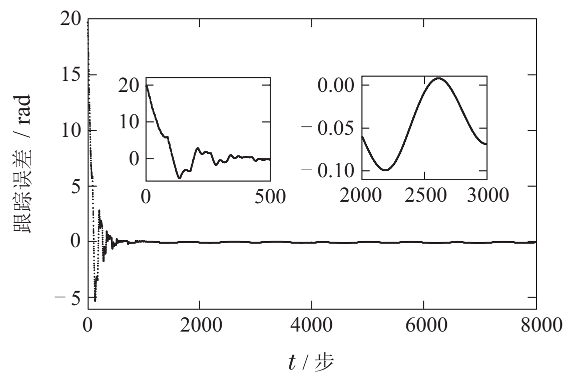

图 9 跟踪误差$e_1(k)$

Fig. 9 Tracking error $e_1(k)$

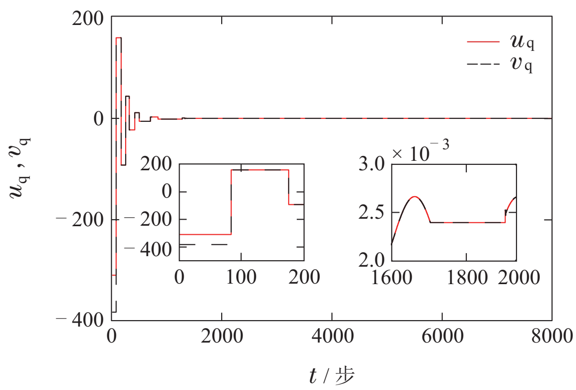

图 10 q 轴电压

Fig. 10 q axis voltage

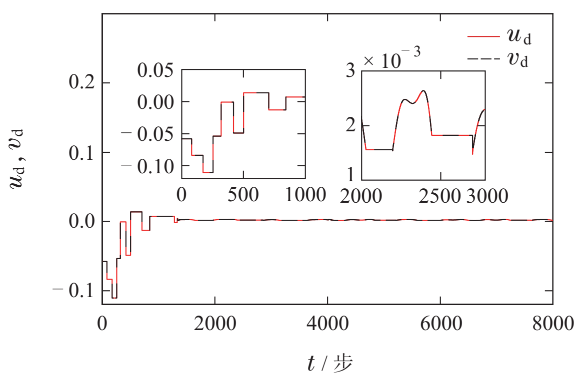

图 11 d轴电压

Fig. 11 d axis voltage

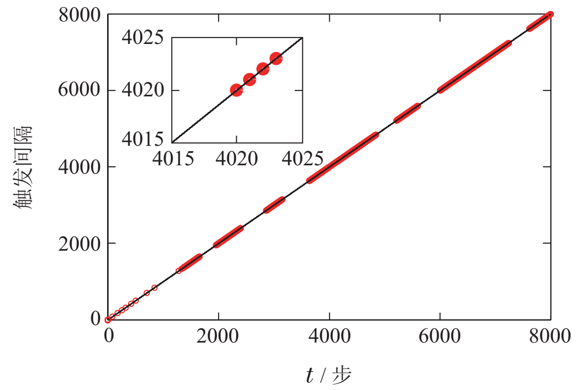

图 12 事件触发时刻

Fig. 12 Event-triggered instant

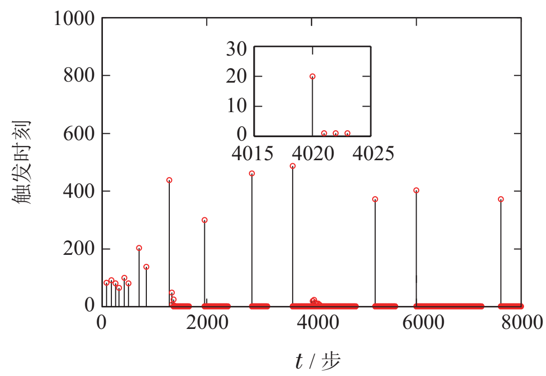

图 13 事件触发的时间间隔

Fig. 13 Event-triggered interval

7 结论

本文针对一类不确定非线性离散系统, 提出了事件触发指令滤波控制方法. 利用指令滤波控制技术不仅克服了反步法中存在的"因果矛盾"问题, 而且引进补偿信号消除了滤波误差, 提高了系统控制精度. 采用事件触发机制避免了自适应律频繁更新, 降低了计算负担, 同时, 解决了资源浪费问题. 考虑系统中的输入饱和现象, 明显改善了系统控制性能. 通过2组仿真结果验证了本文提出控制方法有较好的跟踪效果.

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