连续时间无穷维正则状态信号系统的最优控制

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摘要: 本文研究连续时间线性无穷维正则状态信号(s/s)系统的最优问题—–线性二次调节器(LQR)最优控制问题和卡尔曼滤波问题. 正则s/s系统的最优问题可解与正则s/s系统的某个正则i/s/o表示的最优问题可解是等价的. 在正则s/s系统有一个预解集非空的正则i/s/o表示的前提下, 建立了系统本身的未来最优花费与系统表示的未来最优花费之间的联系, 并给出了相应的例子.

关键词: 正则状态信号(s/s)系统; 正则i/s/o表示; 线性二次调节器(LQR)最优控制问题; 卡尔曼滤波问题; 广义稳定轨迹

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Optimal control on infinite-dimensional continuous-time regular state signal systems

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Abstract: This paper considers the linear quadratic regulator (LQR) optimal control problem and Kalman filtering problem for a regular state signal (s/s) system. The solvability of the optimal control problems for the regular s/s system is equivalent to that for some regular i/s/o representation of the regular s/s system. The connection on optimal future costs between the regular s/s system and some regular i/s/o representation with a nonempty resolvent set is proposed. Two examples are given to illustrate the results.

Key words: regular s/s system; regular i/s/o representation; LQR optimal control problem; Kalman filtering problem; generalized stable trajectory

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1 Introduction

The optimal control problems are important subjects in control theory. Kalman[1] has used the Hamilton -Jacobi theory to arrive at RDE and to deduce optimality of the linear quadratic (LQ) control gain for time-varying systems. Lions[2] has examined the optimal control problems for deterministic distributed parameter systems by exploiting the properties of the partial differential equations. Many researchers have considered the optimal control problems by using the semigroup approach[3–4]. The linear quadratic regulator (LQR) problem for a regular i/s/o system is to minimize the future quadratic cost function. It is shown that the finite future cost condition for a discrete-time i/s/o system holds if and only if the control Riccati equation has a classical solution. In this setting, the solvability of the LQR problem for the discrete-time i/s/o system is equivalent to the existence of a right factorization of its transfer function[5]. Opmeer and Staffans[6] have considered the LQR problem for the discrete-time i/s/o system by defining the finite future incremental cost condition and rewriting the control Riccati equation in terms of sesquilinear forms. These foundations are used in [7] to study the optimal control problems for the continuous-time regular i/s/o system. The control Riccati equation has been extended to the generalized control Riccati equation which consists of unbounded operators, and the finite future cost condition has been reduced to the input finite future cost condition. The Kalman filtering problem for a regular i/s/o system is

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to find a control such that the past quadratic cost function is minimal. The Kalman filtering problem is related to the filter Riccati equation and the factorization theory. It is shown in [8] that if the output coercive past cost condition holds for a discrete-time linear system, then it is equivalent to the filter Riccati equation having a solution. Opmeer and Staffans[7] have defined the output coercive past cost condition for the continuous-time regular i/s/o system. If this condition holds, then it is equivalent to the generalized filter Riccati equation having a solution, and another equivalent condition is that the transfer function of the regular i/s/o system has a weakly coprime left H_∞-factorization.

Arov and Staffans have put forward the regular s/s system, which does not distinguish the input u from the output y. The state space X shows the internal properties of the system and the signal space W describes interactions with the surrounding world. The s/s system is a more generalized system, it is necessary to consider the optimal control problems for regular s/s systems. Arov and Staffans[9] have shown that the optimal signal of the LQR problem for a discrete-time s/s system is in the form of state feedback. In addition, the optimal signal of the Kalman filtering problem for a discrete-time s/s system is in the form of signal injection.

The generalized stable trajectory theory and multi-valued operator theory have not been considered to solve the optimal problem for regular s/s systems yet. Following Opmeer[5–8, 10] and Staffans[11–12], this paper considers the optimal control problems for continuous-time regular s/s systems. The generalized stable future (past) trajectories of the regular s/s system are defined to give its optimal future (past) cost. The equivalence of the solvability of the optimal problem for the regular s/s system and the solvability of the optimal problems for some regular i/s/o representations is obtained. In the case that the regular s/s system allows a regular i/s/o representation with a nonempty resolvent set, the relationship between the optimal future cost of the regular s/s system and that of the regular i/s/o representation is given.

This paper is structured as follows: Section 2 introduces some preliminaries; Section 3 and Section 4 show the main results on the optimal problems; Section 5 gives two examples; Section 6 concludes the paper.

2 Preliminaries

The symbols $\mathbb{C}$ and $\mathbb{C}^+$ denote the complex plane and the right plane of the complex plane, respectively. $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{R}^- = (-\infty, 0]$ and $\mathcal{I} = \mathbb{R}^+$ or $\mathbb{R}^-$.

A linear continuous-time regular i/s/o system[7] is defined by the equations

$$\varSigma_{\text{i/s/o}} : \begin{bmatrix}\dot{x}(t)\\ y(t)\end{bmatrix} = S\begin{bmatrix}x(t)\\ u(t)\end{bmatrix},\ x(0) = x_0,\ t \in \mathcal{I}, \tag{1}$$

on a triple of Hilbert spaces, namely, the input space U, the state space X and the output space Y, where $\dot{x}(t), x(t) \in X,\ u(t) \in U,\ y(t) \in Y,\ S = \begin{bmatrix}A\,\&\,B\\ C\,\&\,D\end{bmatrix}$: $\mathrm{dom}(S) \subset \begin{bmatrix}X\\ U\end{bmatrix} \to \begin{bmatrix}X\\ Y\end{bmatrix}$ is a closed linear operator with dense domain. A is the generator of a C_0-semigroup in X, $B : U \to X$ is the control operator, $C : X \to Y$ is the observation operator, and $D : U \to Y$ is the feedthrough operator. The regular i/s/o system (1) is denoted by $\varSigma_{\text{i/s/o}} = (S; X, U, Y)$. The transfer function of the system $\varSigma_{\text{i/s/o}}$ (1) is the operator-valued function $\hat{\mathfrak{D}} : \mathbb{C} \to B(U; Y)$ with $\hat{\mathfrak{D}}(\lambda) = C\,\&\,D\begin{bmatrix}(\lambda - A|_X)^{-1}B\\ 1_{\mathrm{U}}\end{bmatrix}$, $\lambda \in \rho(A)$, where $\rho(A)$ denotes the resolvent set of A.

A linear continuous-time regular s/s system[13] is defined by the equations

$$\varSigma_{\text{s/s}} : \dot{x}(t) = F\begin{bmatrix}x(t)\\ w(t)\end{bmatrix},\ t \in \mathcal{I},\ x(0) = x_0, \tag{2}$$

where the initial (final) state $x_0 \in X,\ x(t) \in X,\ w(t) \in W$, X is the state space, W is the signal space, X and W are Hilbert spaces. $F : \mathrm{dom}(F) \subset \begin{bmatrix}X\\ W\end{bmatrix} \to X$ is a closed linear operator with dense domain. Replace $\mathrm{gph}(F)$ with V, the graph form of the regular s/s system $\varSigma_{\text{s/s}}$ (2) is

$$\varSigma_{\text{s/s}} : \begin{bmatrix}\dot{x}(t)\\ x(t)\\ w(t)\end{bmatrix} \in V,\ x(0) = x_0,\ t \in \mathcal{I}, \tag{3}$$

where the generating subspace V is a closed subspace of $\begin{bmatrix}X\\ X\\ W\end{bmatrix}$, and the subspace X_0 consisting of the second elements of V is dense in X. This regular s/s system is denoted by $\varSigma_{\text{s/s}} = (V; X, W)$.

Definition 1[10] A multi-valued operator $T : X \to Y$ is a subspace V_{T} of $\begin{bmatrix}Y\\ X\end{bmatrix}$. The operator T is closed if V_{T} is closed. The domain, kernel, range, and multi-valued part of T are given by

$$\mathrm{dom}(T) = \{x \in X | \begin{bmatrix}y\\ x\end{bmatrix} \in V_{\mathrm{T}} \text{ for some } y \in Y\};$$

$$\ker(T) = \{x \in X | \begin{bmatrix}0\\ x\end{bmatrix} \in V_{\mathrm{T}}\};$$

$$\mathrm{ran}(T) = \{y \in Y | \begin{bmatrix}y\\ x\end{bmatrix} \in V_{\mathrm{T}} \text{ for some } x \in X\};$$

$$\mathrm{mul}(T) = \{y \in Y | \begin{bmatrix}y\\ 0\end{bmatrix} \in V_{\mathrm{T}}\}.$$

V_{T} is the graph of T denoted by $\mathrm{gph}(T)$. The inverse of T is $T^{-1} : Y \to X$ whose graph is given by

$\mathrm{gph}(T^{-1}) = \{\begin{bmatrix}x\\y\end{bmatrix} \in \begin{bmatrix}X\\Y\end{bmatrix} \mid \begin{bmatrix}y\\x\end{bmatrix} \in \mathrm{gph}(T)\}$. T is single-valued if $\mathrm{mul}(T)=0$. Let Z be a closed subspace of X, P_{Z} denotes the projection from X onto Z. $T_{\mathrm{s}} = P_{\mathrm{mul}(T)^{\perp}}T$ is a single-valued operator which is called the operator part of T.

Definition 2[12] Let W be a Hilbert space.

i) A vector bundle is given by a family of subspaces $\mathfrak{Q} = \{\mathfrak{Q}(\lambda)\}_{\lambda\in\mathrm{dom}(\mathfrak{Q})}$ of W parameterized by a complex parameter $\lambda \in \mathrm{dom}(\mathfrak{Q}) \subset \mathbb{C}$. The subspace $\mathfrak{Q}(\lambda)$ of W is called the fiber of $\mathfrak{Q}$.

ii) The vector bundle $\mathfrak{Q}$ is analytic at a point $\lambda_0 \in \mathrm{dom}(\mathfrak{Q})$ if there exists a neighborhood $O(\lambda_0)$ of λ_0 and some direct sum decomposition $W = U \dot{+} Y$ such that the restriction of $\mathfrak{Q}$ to $O(\lambda_0)$ is the graph of an analytic $B(U;Y)$–valued function in $O(\lambda_0)$.

3 The LQR problem for the regular s/s system

The LQR problem for the system $\Sigma_{\mathrm{s/s}}$ (3) is to minimize the cost function $J_{\mathrm{fut}}(x_0,w) = \int_0^{+\infty}\|w(t)\|_W^2\mathrm{d}t$. In this section, we first find out the optimal signal w^{opt} of the LQR problem for the system $\Sigma_{\mathrm{s/s}}$ (3). Then, we prove that the solvability of the LQR problem for the system $\Sigma_{\mathrm{s/s}}$ (3) and that for its regular i/s/o representations are equivalent. An element $x_0 \in X$ has a finite future cost if there exists a signal $w \in L^2(\mathbb{R}^+;W)$ such that the system $\Sigma_{\mathrm{s/s}}$ (3) holds. The set of finite future cost states is denoted by Ξ_+. The system $\Sigma_{\mathrm{s/s}}$ (3) is said to satisfy the finite future cost condition if $\Xi_+ = X$. For the existence of an optimal signal, the finite future cost condition holds, i.e., for every $x_0 \in X$, there exists a signal w such that $J_{\mathrm{fut}}(x_0,w) = \int_0^{+\infty}\|w(t)\|_W^2\mathrm{d}t < \infty$. The characteristic node bundle of the system $\Sigma_{\mathrm{s/s}}$ (3) is the family of subspaces $\hat{\mathfrak{E}} = \{\hat{\mathfrak{E}}(\lambda)\}_{\lambda\in\mathbb{C}}$, where

$$\hat{\mathfrak{E}}(\lambda) = \{\begin{bmatrix}x_0\\\hat{x}(\lambda)\\\hat{w}(\lambda)\end{bmatrix} \mid \begin{bmatrix}\lambda\hat{x}(\lambda)-x_0\\\hat{x}(\lambda)\\\hat{w}(\lambda)\end{bmatrix} \in V\},\ \lambda\in\mathbb{C},$$

and x_0 is the initial state, $\hat{x}$ and $\hat{w}$ are the Laplace transforms of x and w, respectively. The characteristic signal bundle of the system $\Sigma_{\mathrm{s/s}}$ (3) is the family of subspaces $\hat{\mathfrak{F}} = \{\hat{\mathfrak{F}}(\lambda)\}_{\lambda\in\mathbb{C}}$ of the signal space W, where

$$\hat{\mathfrak{F}}(\lambda) = \begin{bmatrix}0 & 0 & 1_{\mathrm{W}}\end{bmatrix}(\hat{\mathfrak{E}}(\lambda)\cap\begin{bmatrix}0\\X\\W\end{bmatrix}),\ \lambda\in\mathbb{C}.$$

The characteristic node bundle $\hat{\mathfrak{E}}$ of the system $\Sigma_{\mathrm{s/s}}$ (3) is analytic in $\mathbb{C}$. Each fiber of an analytic vector bundle is closed.

The set of the resolvent set of the regular s/s system $\Sigma_{\mathrm{s/s}}$ is denoted by $\rho(\Sigma_{\mathrm{s/s}})$, more details, see [13].

Definition 3 Given an open subset Ω in $\rho(\Sigma_{\mathrm{s/s}}) \cap \mathbb{C}^+$.

i) The set of generalized stable future trajectories of the system $\Sigma_{\mathrm{s/s}}$ (3) denoted by $\mathfrak{M}_+$ is all pairs $\begin{bmatrix}x_0\\w\end{bmatrix} \in \begin{bmatrix}X\\L^2(\mathbb{R}^+;W)\end{bmatrix}$ which satisfy $\begin{bmatrix}x_0\\\hat{x}(\lambda)\\\hat{w}(\lambda)\end{bmatrix} \in \hat{\mathfrak{E}}(\lambda)$ for some $\hat{x}(\lambda)\in X,\ \lambda\in\Omega$.

ii) The set of the stable future behavior of the system $\Sigma_{\mathrm{s/s}}$ (3) denoted by $\mathfrak{M}_+^0$ is all elements $w \in L^2(\mathbb{R}^+;W)$ which satisfy $\begin{bmatrix}0\\\hat{x}(\lambda)\\\hat{w}(\lambda)\end{bmatrix} \in \hat{\mathfrak{E}}(\lambda)$ for some $\hat{x}(\lambda)\in X,\ \lambda\in\Omega$.

Remark 1 Throughout this paper assume that $\rho(\Sigma_{\mathrm{s/s}}) \cap \mathbb{C}^+$ is connected and nonempty. In this case, Definition 3 is independent of the choice of Ω.

Lemma 1[13] Let $\Sigma_{\mathrm{s/s}} = (V;X,W)$ be a regular s/s system. The following statements are equivalent:

i) $\lambda \in \rho(\Sigma_{\mathrm{s/s}})$.

ii) There exists a continuous linear operator $\mathfrak{L}(\lambda)$: $\mathrm{dom}(\mathfrak{L}(\lambda)) \subset \begin{bmatrix}X\\W\end{bmatrix} \to X$ with closed domain such that $\hat{x}(\lambda) = \mathfrak{L}(\lambda)\begin{bmatrix}x_0\\\hat{w}(\lambda)\end{bmatrix}$, where $\begin{bmatrix}x_0\\\hat{x}(\lambda)\\\hat{w}(\lambda)\end{bmatrix} \in \hat{\mathfrak{E}}(\lambda)$.

Lemma 2 $\mathfrak{M}_+$ is closed .

Proof Fix a $\lambda \in \Omega$. If $\begin{bmatrix}x_0^n\\w^n\end{bmatrix} \in \mathfrak{M}_+$ converges to $\begin{bmatrix}x_0\\w\end{bmatrix}$ as $n\to\infty$, then there exist $\hat{x}(\lambda)^n$ such that $\begin{bmatrix}x_0^n\\\hat{x}(\lambda)^n\\\hat{w}(\lambda)^n\end{bmatrix} \in \hat{\mathfrak{E}}(\lambda)$. According to Lemma 1,

$$\hat{x}(\lambda)^n = \mathfrak{L}(\lambda)\begin{bmatrix}x_0^n\\\hat{w}(\lambda)^n\end{bmatrix}.$$

It is clear that $\begin{bmatrix}x_0^n\\\hat{x}(\lambda)^n\\\hat{w}(\lambda)^n\end{bmatrix}$ converges to $\begin{bmatrix}x_0\\\mathfrak{L}(\lambda)\begin{bmatrix}x_0\\\hat{w}(\lambda)\end{bmatrix}\\\hat{w}(\lambda)\end{bmatrix}$. Since $\hat{\mathfrak{E}}(\lambda)$ is closed, $\begin{bmatrix}x_0\\\mathfrak{L}(\lambda)\begin{bmatrix}x_0\\\hat{w}(\lambda)\end{bmatrix}\\\hat{w}(\lambda)\end{bmatrix} \in \hat{\mathfrak{E}}(\lambda)$. Hence, $\mathfrak{M}_+$ is closed when λ takes over all Ω. QED.

Lemma 3[6] Let $\mathcal{H}$ be a Hilbert space and $\mathcal{K}$ a nonempty closed subspace of $\mathcal{H}$. For $h_0 \in \mathcal{H}$, define the affine set

$$\mathcal{K}(h_0) = \{h\in\mathcal{H} : h = h_0 + k \text{ for some } k\in\mathcal{K}\}.$$

Then there exists a unique $h_{\min} \in \mathcal{K}(h_0)$ such that

$$\|h_{\min}\| = \min_{h\in\mathcal{K}(h_0)}\|h\|.$$

The vector $h_{\min}$ is characterized by the fact that $\mathcal{K}(h_0) \cap \mathcal{K}^{\perp} = h_{\min}$.

Theorem 1 Let $\mathfrak{T}$ be a multi-valued operator from X to $L^2(\mathbb{R}^+; W)$ with $\mathrm{gph}(\mathfrak{T}^{-1}) = \mathfrak{M}_+$. A $x_0 \in X$ has a finite future cost if and only if $x_0 \in \mathrm{dom}(\mathfrak{T})$. The optimal future cost of x_0 is

$$J_{\mathrm{fut}}^{\min}(x_0, w) = \|P_{[\mathfrak{T}0]^{\perp}} \mathfrak{T} x_0\|^2_{L^2(\mathbb{R}^+;W)}.$$

Proof By Lemma 2, $\mathfrak{T}^{-1}$ is a closed operator and $\mathrm{mul}(\mathfrak{T}) = \mathfrak{M}_+^0$. If $x_0 \in \mathrm{dom}(\mathfrak{T})$. Let $\mathcal{H} = L^2(\mathbb{R}^+; W)$ and $\mathcal{K} = \mathfrak{M}_+^0$ in Lemma 3. For $w_0 \in \mathcal{H}$, it is clear that $\mathcal{K}(w_0) = \{\mathfrak{T}x_0 | x_0 \in \mathrm{dom}(\mathfrak{T})\}$, then there exists a unique $w_{\min} = P_{[\mathfrak{T}0]^{\perp}} \mathfrak{T} x_0 \in \mathcal{K}(w_0) \cap \mathcal{K}^{\perp}$. Hence, $J_{\mathrm{fut}}^{\min}(x_0, w) = \|P_{[\mathfrak{T}0]^{\perp}} \mathfrak{T} x_0\|^2_{L^2(\mathbb{R}^+;W)}$. If $x_0 \in X$ has a finite future cost. $x_0 \in \mathrm{dom}(\mathfrak{T})$ is obvious.

QED.

Remark 2 By Theorem 1, the system $\Sigma_{\text{s/s}}$ (3) satisfies the finite future cost condition if and only if the LQR problem for the system $\Sigma_{\text{s/s}}$ (3) has a solution.

In the following, we consider the relationship between the LQR problem for the system $\Sigma_{\text{s/s}}$ (3) and that for its regular i/s/o representations.

Definition 4[12–13] Let W be a Hilbert space.

i) By an i/o representation of W it means the ordered pair (U, Y) of two closed subspaces U and Y of W such that $W = U \dot{+} Y$ is an ordered direct sum decomposition of W.

ii) By the transition matrix Θ from (U_1, Y_1) to (U_2, Y_2) it means the bounded operator Θ defined by

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix} = \begin{bmatrix} P_{\mathrm{U}_2}^{\mathrm{Y}_2}|_{\mathrm{U}_1} & P_{\mathrm{U}_2}^{\mathrm{Y}_2}|_{\mathrm{Y}_1} \\ P_{\mathrm{Y}_2}^{\mathrm{U}_2}|_{\mathrm{U}_1} & P_{\mathrm{Y}_2}^{\mathrm{U}_2}|_{\mathrm{Y}_1} \end{bmatrix},$$

where $(U_1, Y_1), (U_2, Y_2)$ are two i/o representations of W, $P_{\mathrm{U}_2}^{\mathrm{Y}_2}$ is the projection to U_2 along Y_2, $P_{\mathrm{Y}_2}^{\mathrm{U}_2}$ is the projection to Y_2 along U_2.

iii) By a regular i/s/o representation of the regular s/s system $\Sigma_{\text{s/s}}$ it means a regular i/s/o system $\Sigma_{\text{i/s/o}} = (S; X, U, Y)$, where $U \dot{+} Y$ is a direct sum decomposition of W and V and S are connected to each other by

$$V = \left\{ \begin{bmatrix} z \\ x \\ w \end{bmatrix} \subset \begin{bmatrix} X \\ X \\ W \end{bmatrix} \;\middle|\; \begin{matrix} \begin{bmatrix} x \\ P_{\mathrm{U}}^{\mathrm{Y}} w \end{bmatrix} \in \mathrm{dom}(S) \\ \text{and } \begin{bmatrix} z \\ P_{\mathrm{Y}}^{\mathrm{U}} w \end{bmatrix} = S \begin{bmatrix} x \\ P_{\mathrm{U}}^{\mathrm{Y}} w \end{bmatrix} \end{matrix} \right\}.$$

Lemma 4[13] Let $\Sigma_{\text{i/s/o}}^i = (S_i; X, U_i, Y_i)$, $i = 1, 2$ be two regular i/s/o representations with the transition matrix Θ from (U_1, Y_1) to (U_2, Y_2). Then

$$\mathrm{gph}(S_2) = \begin{bmatrix} 1_{\mathrm{X}} & 0 & 0 & 0 \\ 0 & \Theta_{22} & 0 & \Theta_{21} \\ 0 & 0 & 1_{\mathrm{X}} & 0 \\ 0 & \Theta_{12} & 0 & \Theta_{11} \end{bmatrix} \mathrm{gph}(S_1).$$

Lemma 5 Let $\Sigma_{\text{i/s/o}}^i = (S_i; X, U_i, Y_i)$, $i = 1, 2$ be two regular i/s/o representations of the system $\Sigma_{\text{s/s}}$ (3). If the system $\Sigma_{\text{i/s/o}}^1$ satisfies the finite future cost condition, then the system $\Sigma_{\text{i/s/o}}^2$ also satisfies the finite future cost condition.

Proof By Lemma 4, $\begin{bmatrix} u_2(t) \\ y_2(t) \end{bmatrix} = \Theta \begin{bmatrix} u_1(t) \\ y_1(t) \end{bmatrix}$ for some Θ. For any $x_0 \in X$, there exists a control u_1 such that $J_1(x_0, u_1) = \int_0^{\infty} (\|u_1(t)\|^2_{U_1} + \|y_1(t)\|^2_{\mathrm{Y}_1}) \mathrm{d}t < \infty$. Then, $J_2(x_0, u_2) \leqslant \|\Theta\|^2 J_1(x_0, u_1) < \infty$.

QED.

Theorem 2 The following statements are equivalent:

i) The regular s/s system $\Sigma_{\text{s/s}}$ (3) satisfies the finite future cost condition.

ii) For some regular i/s/o representation of the system $\Sigma_{\text{s/s}}$ satisfies the finite future cost condition.

iii) Every regular i/s/o representation of the system $\Sigma_{\text{s/s}}$ satisfies the finite future cost condition. QED.

Proof i) $\Rightarrow$ ii). By [13, Theorem 2.2.18], there exists a regular i/s/o representation $\Sigma_{\text{i/s/o}} = (S; X, U, Y)$, where $U = \bar{W}_0$, $W_0 = \begin{bmatrix} 0 & 0 & 1_{\mathrm{W}} \end{bmatrix} \left(V \cap \begin{bmatrix} X \\ 0 \\ W \end{bmatrix}\right)$ and Y be an arbitrary direct complement to U.

Hence, $\begin{bmatrix} x(t) \\ P_{\mathrm{U}}^{\mathrm{Y}} w(t) \\ P_{\mathrm{Y}}^{\mathrm{U}} w(t) \end{bmatrix}$ is a trajectory of the i/s/o representation $\Sigma_{\text{i/s/o}}$ when $\begin{bmatrix} x(t) \\ w(t) \end{bmatrix}$ is a trajectory of the system $\Sigma_{\text{s/s}}$. Since $\begin{bmatrix} I_{\mathrm{U}} \; I_{\mathrm{Y}} \end{bmatrix} : \begin{bmatrix} U \\ Y \end{bmatrix} \rightarrow W$ is one to one and onto and $\| \begin{bmatrix} I_{\mathrm{U}} \; I_{\mathrm{Y}} \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix} \|^2 \leqslant 2 \| \begin{bmatrix} u \\ y \end{bmatrix} \|^2$, there exists a $m > 0$ such that $\| \begin{bmatrix} I_{\mathrm{U}} \; I_{\mathrm{Y}} \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix} \| \geqslant m \| \begin{bmatrix} u \\ y \end{bmatrix} \|$. For any $x_0 \in X$, there exists a w such that $\int_0^{\infty} \|w(t)\|^2_{\mathrm{W}} \mathrm{d}t < \infty$. Take $u(t) = P_{\mathrm{U}}^{\mathrm{Y}} w(t)$ and $y(t) = P_{\mathrm{Y}}^{\mathrm{U}} w(t)$, then $J(x_0, u) = \int_0^{\infty} (\|u(t)\|^2_{\mathrm{U}} + \|y(t)\|^2_{\mathrm{Y}}) \mathrm{d}t \leqslant \dfrac{1}{m} \int_0^{\infty} \|w(t)\|^2_{\mathrm{W}} \mathrm{d}t < \infty$.

ii) $\Leftrightarrow$ iii). It is obvious by Lemma 5.

ii) $\Rightarrow$ i). $\Sigma_{\text{i/s/o}} = (S; X, U, Y)$ is a regular i/s/o representation of the system $\Sigma_{\text{s/s}}$ (3). $\Sigma_{\text{s/s}}^1 = (V_1; X, W_1)$ denotes the regular s/s system induced by the i/s/o representation $\Sigma_{\text{i/s/o}}$, where $V_1 = \begin{bmatrix} 1_{\mathrm{X}} & 0 & 0 & 0 \\ 0 & 0 & 1_{\mathrm{X}} & 0 \\ 0 & \begin{bmatrix} 0 & I_{\mathrm{Y}} \end{bmatrix} & 0 & \begin{bmatrix} I_{\mathrm{U}} \\ 0 \end{bmatrix} \end{bmatrix} \mathrm{gph}(S)$. For any $x_0 \in X$, there exists a control u such that $\int_0^\infty \| \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \|^2 \mathrm{d}t < \infty$. Take $w_1(t) = \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}$, then $J_1(x_0, w_1) < \infty$. By [13, Proposition 2.2.15], $V_1 = \begin{bmatrix} 1_X & 0 & 0 \\ 0 & 1_X & 0 \\ 0 & 0 & \begin{bmatrix} P_U^Y \\ P_Y^U \end{bmatrix} \end{bmatrix} V$. Take $w(t) = \begin{bmatrix} I_U & I_Y \end{bmatrix} w_1(t)$, then $J(x_0, w) = \int_0^\infty \|w(t)\|_W^2 \mathrm{d}t \leqslant 2\int_0^\infty \|w_1(t)\|_W^2 \mathrm{d}t < \infty$.

QED.

The connection on optimal future costs between the system $\Sigma_{\text{s/s}}$ (3) and some regular i/s/o representation of the system $\Sigma_{\text{s/s}}$ (3) is given in the following. $\rho(\Sigma_{\text{i/s/o}})$ denotes the resolvent set of the regular i/s/o system $\Sigma_{\text{i/s/o}}$. Assume that $\rho(\Sigma_{\text{i/s/o}}) \cap \mathbb{C}^+$ is connected and nonempty. Ω_1 is an open subset of $\rho(\Sigma_{\text{i/s/o}}) \cap \mathbb{C}^+$.

Lemma 6[13] Let $\Sigma_{\text{i/s/o}} = (S; X, U, Y)$ be a regular i/s/o system. Then the following statements are equivalent:

i) $\lambda \in \rho(\Sigma_{\text{i/s/o}})$.

ii) There exists a bounded linear operators $\hat{\mathfrak{G}}(\lambda) = \begin{bmatrix} \hat{\mathfrak{U}}(\lambda) & \hat{\mathfrak{B}}(\lambda) \\ \hat{C}(\lambda) & \hat{\mathfrak{D}}(\lambda) \end{bmatrix} : \begin{bmatrix} X \\ U \end{bmatrix} \to \begin{bmatrix} X \\ Y \end{bmatrix}$ such that $\begin{bmatrix} \hat{x}(\lambda) \\ \hat{y}(\lambda) \end{bmatrix} = \hat{\mathfrak{G}}(\lambda) \begin{bmatrix} x_0 \\ \hat{u}(\lambda) \end{bmatrix}$, where $\hat{\mathfrak{U}}(\lambda) = (\lambda - A)^{-1}$, $\hat{C}(\lambda) = C(\lambda - A)^{-1}$.

Lemma 7[13] Let $\Sigma_{\text{s/s}}$ be a regular s/s system, then $\rho(\Sigma_{\text{s/s}})$ is the union of the resolvent sets $\rho(\Sigma_{\text{i/s/o}})$ over all i/s/o representations $\Sigma_{\text{i/s/o}}$ of the system $\Sigma_{\text{s/s}}$.

Theorem 3 The following statements are equivalent:

i) The LQR problem for the regular s/s system $\Sigma_{\text{s/s}}$ (3) has a solution.

ii) The LQR problem for some regular i/s/o representation of the system $\Sigma_{\text{s/s}}$ has a solution.

iii) The LQR problem for every regular i/s/o representation of the system $\Sigma_{\text{s/s}}$ has a solution.

Moreover, if the system $\Sigma_{\text{s/s}}$ has a regular i/s/o representation $\Sigma_{\text{i/s/o}}$ with a nonempty resolvent set, then $J_{\text{fut}}^{\min}(x_0, w) = \|P_{[\mathfrak{T}0]^\perp} \mathfrak{T} x_0\|_{L^2(\mathbb{R}^+;W)}^2$ whenever $J_{\text{fut}}^{\min}(x_0, u) = \|P_{[\mathfrak{T}'0]^\perp} \mathfrak{T}' x_0\|^2_{\begin{bmatrix} L^2(\mathbb{R}^+;U) \\ L^2(\mathbb{R}^+;Y) \end{bmatrix}}$, where $(\mathfrak{T}x_0)(t) = \begin{bmatrix} I_U & I_Y \end{bmatrix} (\mathfrak{T}' x_0)(t)$. QED.

Proof It is obvious that i) ii) and iii) are equivalent by Theorem 2 and Remark 2. According to [7, Definition 3.2], $\mathrm{gph}(\mathfrak{T}'^{-1})$ is the set of all triples $\begin{bmatrix} x_0 \\ u \\ y \end{bmatrix} \in \begin{bmatrix} X \\ L^2(\mathbb{R}^+;U) \\ L^2(\mathbb{R}^+;Y) \end{bmatrix}$ which satisfy $\hat{y}(\lambda) = C(\lambda - A)^{-1} x_0 + \hat{\mathfrak{D}}(\lambda) \hat{u}(\lambda)$, $\lambda \in \Omega_1$. It is clear that $(\mathfrak{T}x_0)(t) = \begin{bmatrix} I_U & I_Y \end{bmatrix} (\mathfrak{T}' x_0)(t)$. For any $\lambda \in \Omega_1$, it follows from Lemma 6 that

$$\hat{x}(\lambda) = [\hat{\mathfrak{U}}(\lambda) \ \ \hat{\mathfrak{B}}(\lambda)] \begin{bmatrix} 1_X & 0 \\ 0 & P_U^Y \end{bmatrix} \begin{bmatrix} x_0 \\ \hat{w}(\lambda) \end{bmatrix}. \tag{4}$$

By (4) and the system $\Sigma_{\text{s/s}}$ (3), $\begin{bmatrix} x_0 \\ \hat{x}(\lambda) \\ \hat{w}(\lambda) \end{bmatrix} \in \hat{\mathfrak{E}}(\lambda)$. By Lemma 7, $\Omega_1 \subset \rho(\Sigma_{\text{s/s}}) \cap \mathbb{C}^+$. By Definition 3, the set of $\begin{bmatrix} x_0 \\ w \end{bmatrix}$ denoted by $\mathrm{gph}(\mathfrak{T}^{-1})$ is generalized stable future trajectories of the system $\Sigma_{\text{s/s}}$, where $w(t) = \begin{bmatrix} I_U & I_Y \end{bmatrix} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}$ and $\begin{bmatrix} x_0 \\ \hat{x}(\lambda) \\ \hat{w}(\lambda) \end{bmatrix} \in \hat{\mathfrak{E}}(\lambda)$ for some $\hat{x}(\lambda) \in X$, $\lambda \in \Omega_1$. By Theorem 1, the optimal future cost $J_{\text{fut}}^{\min}(x_0, w)$ is $\|P_{[\mathfrak{T}0]^\perp} \mathfrak{T} x_0\|_{L^2(\mathbb{R}^+;W)}^2$. QED.

4 The Kalman filtering problem for the regular s/s system

In this section, we solve the Kalman filtering problem for the regular s/s system. For the convenience of defining generalized stable past trajectories of the regular s/s system, we consider the regular s/s system (2).

Definition 5 Let $e_\lambda : t \to \mathrm{e}^{\lambda t}$, $t \in \mathbb{R}^-$.

i) The set of generalized stable past trajectories of the system $\Sigma_{\text{s/s}}$ (2) denoted by $\mathfrak{N}_-$ is the closure of

$$\mathrm{span}\{ \begin{bmatrix} (\lambda - F \begin{bmatrix} I \\ 0 \end{bmatrix})^{-1} F \begin{bmatrix} 0 \\ I \end{bmatrix} w_0 \\ e_\lambda w_0 \end{bmatrix} \in \begin{bmatrix} X \\ L^2(\mathbb{R}^-; W) \end{bmatrix} \},$$

where $\lambda \in \Omega$ and $w_0 \in W$.

ii) The stable past behavior of the system $\Sigma_{\text{s/s}}$ (2) denoted by $\mathfrak{N}_-^0$ is the closure of

$$\mathrm{span}\{e_\lambda w_0 \in L^2(\mathbb{R}^-; W) | w_0 \in W, \ \lambda \in \Omega\}.$$

An elements $x_0 \in X$ has a finite past cost if there exists a signal $w \in L^2(\mathbb{R}^-; W)$ with $\|x_0\|_X \leqslant c\|w\|_{L^2(\mathbb{R}^-;W)}$ for some $c > 0$ such that the system $\Sigma_{\text{s/s}}$ (2) holds. The Kalman filtering problem for the system $\Sigma_{\text{s/s}}$ (2) is to minimize the cost function $J_{\text{past}}(x_0, w) = \int_{-\infty}^0 \|w(t)\|_W^2 \mathrm{d}t$. A necessary condition to the Kalman filtering problem for the system $\Sigma_{\text{s/s}}$ (2) is the coercive past cost condition, i.e., there exists a $c > 0$ such that $\|x_0\|_X \leqslant c\|w\|_{L^2(\mathbb{R}^-;W)}$ for every generalized stable past trajectory of the system $\Sigma_{\text{s/s}}$ (2).

Theorem 4 Let $\mathfrak{P}$ be a multi-valued operator from $L^2(\mathbb{R}^-; W)$ to X with $\mathrm{gph}(\mathfrak{P}) = \mathfrak{N}_-$. A $x_0 \in X$ has a finite past cost if and only if $x_0 \in \mathrm{ran}(\mathfrak{P})$. The optimal past cost of x_0 is

$$J_{\text{past}}^{\min}(x_0, w) = \|P_{[\mathfrak{P}^{-1}0]^\perp} \mathfrak{P}^{-1} x_0\|_{L^2(\mathbb{R}^-;W)}^2.$$

Proof It is similar to the proof of Theorem 1.

QED.

Theorem 5 The following statements are equivalent:

i) The regular s/s system $\varSigma_{\text{s/s}}$ (2) satisfies the coercive past cost condition.

ii) For some regular i/s/o representation with a nonempty resolvent set of the system $\varSigma_{\text{s/s}}$ satisfies the state coercive past cost condition.

Proof i) $\Rightarrow$ ii). Let $\varSigma_{\text{i/s/o}}$ be the regular i/s/o representation with a nonempty resolvent set of the system $\varSigma_{\text{s/s}}$. Given a $\lambda \in \varOmega$, by Lemma 7, $\lambda \in \rho(\varSigma_{\text{i/s/o}}) \cap \mathbb{C}^{+}$. By $\text{gph}(F) = \begin{bmatrix} 1_{\text{X}} & 0 & 0 & 0 \\ 0 & 0 & 1_{\text{X}} & 0 \\ 0 & I_{\text{Y}} & 0 & I_{\text{U}} \end{bmatrix} \text{gph}(S)$, $\begin{bmatrix} x_0 \\ P_{\text{U}}^{\text{Y}} e_\lambda w_0 \\ P_{\text{Y}}^{\text{U}} e_\lambda w_0 \end{bmatrix}$ is a generalized stable past trajectory of the i/s/o representation $\varSigma_{\text{i/s/o}}$ whenever $\begin{bmatrix} x_0 \\ e_\lambda w_0 \end{bmatrix}$ is a generalized stable past trajectory of the system $\varSigma_{\text{s/s}}$. Since there exists a $c > 0$ such that $\|x_0\|_{\text{X}} \leqslant c\|e_\lambda w_0\|_{L^2(\mathbb{R}^-;W)}$, then we have

$$\|x_0\|_{\text{X}} \leqslant \sqrt{2}c \left\| \begin{bmatrix} P_{\text{U}}^{\text{Y}} e_\lambda w_0 \\ P_{\text{Y}}^{\text{U}} e_\lambda w_0 \end{bmatrix} \right\|_{L^2(\mathbb{R}^-;\left[\begin{smallmatrix} U \\ Y \end{smallmatrix}\right])}.$$

ii) $\Rightarrow$ i). $\varSigma_{\text{i/s/o}}$ denotes the regular i/s/o representation with a nonempty resolvent set of the system $\varSigma_{\text{s/s}}$ (2), by [7, Definition 3.8], the closure of the set

$$\text{span}\{ \begin{bmatrix} x_0 \\ e_\lambda u_0 \\ e_\lambda \hat{\mathfrak{D}}(\lambda) u_0 \end{bmatrix} | \lambda \in \varOmega_1,\ u_0 \in U\}$$

is the generalized stable past trajectories of the i/s/o representation $\varSigma_{\text{i/s/o}}$. By Lemma 7, $\varOmega_1$ is a subset of $\rho(\varSigma_{\text{s/s}}) \cap \mathbb{C}^{+}$. According to Definition 5, the closure of the set $\text{span}\{ \begin{bmatrix} x_0 \\ e_\lambda w_0 \end{bmatrix} | w_0 = \begin{bmatrix} I_{\text{U}} & I_{\text{Y}} \end{bmatrix} \begin{bmatrix} u_0 \\ \hat{\mathfrak{D}}(\lambda) u_0 \end{bmatrix} \}$ is the generalized stable past trajectories of the system $\varSigma_{\text{s/s}}$, where $\lambda \in \varOmega_1,\ w_0 \in W$. Since there exists a $c' > 0$ such that

$$\|x_0\|_{\text{X}} \leqslant c' \left\| \begin{bmatrix} e_\lambda u_0 \\ e_\lambda \hat{\mathfrak{D}}(\lambda) u_0 \end{bmatrix} \right\|_{L^2(\mathbb{R}^-;\left[\begin{smallmatrix} U \\ Y \end{smallmatrix}\right])}.$$

Hence, $\|x_0\|_{\text{X}} \leqslant c' \| \begin{bmatrix} P_{\text{U}}^{\text{Y}} \\ P_{\text{Y}}^{\text{U}} \end{bmatrix} \| \|e_\lambda w_0\|_{L^2(\mathbb{R}^-;W)}$.

QED.

The following theorem holds by Theorem 5.

Theorem 6 The Kalman filtering problem for the system $\varSigma_{\text{s/s}}$ (2) has a solution if and only if the Kalman filtering problem for some regular i/s/o representation with a nonempty resolvent set of the system $\varSigma_{\text{s/s}}$ (2) has a solution.

5 Example

In this section, we give two examples to show the application of Theorem 2 and Theorem 3.

Example 1 Let X, W be Hilbert spaces. The second order differential equation with signal $w(t) \in W$ is given by

$$\ddot{z}(t) + \dot{z}(t) - Tz(t) = T_1 w(t),$$
$$w(t) = -2z(t) + \dot{z}(t),$$

where $T \in L(X)$, $T_1 : \text{dom}(T_1) \subset W \to W$ is a closed operator with closed range and dense domain, and $z(t)$, $\dot{z}(t)$, $\ddot{z}(t) \in X$. Take the state to be $x(t) := \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}$. This gives the system equation

$$\dot{x}(t) = \begin{bmatrix} 0 & I & 0 \\ T & -I & T_1 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \tag{5}$$

where $x(t) \in \begin{bmatrix} X \\ X \end{bmatrix}$ and $w(t) \in \text{dom}(T_1)$.

Take $U = \text{ran}(T_1)$ and Y as an arbitrary closed subspace of W such that $W = \text{ran}(T_1) \dot{+} Y$, then there exists a regular i/s/o system with $A = \begin{bmatrix} 0 & I \\ T & -I \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $C = \begin{bmatrix} -2I & I \end{bmatrix}$ and $D = -I$ in (1). By Definition 4 iii), the regular i/s/o system is a regular i/s/o representation of the regular s/s system (5).

Let $T = T_1 = 1$, then $A = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 1 \end{bmatrix}$ and $D = -1$. In this case, $U = W$ and $Y = \{0\}$. The control Riccati equation of the regular i/s/o system is $Q \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} Q - Q \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 2 & -1 \end{bmatrix} Q = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$. We get a nonnegative solution $Q = \begin{bmatrix} \sqrt{10}+1 & 2 \\ 2 & \sqrt{10}-1 \end{bmatrix}$. For any $x_0 = \begin{bmatrix} z(0) \\ \dot{z}(0) \end{bmatrix} \in \begin{bmatrix} X \\ X \end{bmatrix}$, the optimal cost of the regular i/s/o system is $\langle x_0, Qx_0 \rangle$. The optimal costs of the regular i/s/o system and the system (5) are the same. Hence, the optimal cost of the regular s/s system (5) is $(\sqrt{10} + 1)|z(0)|^2 + 4\text{Re}\langle z(0),\ \dot{z}(0)\rangle + (\sqrt{10} - 1)|\dot{z}(0)|^2$.

Example 2 Let $\varSigma_{\text{s/s}} = (V; \mathbb{R}^2, \mathbb{R}^2)$ be a regular s/s system with its signal bundle $\hat{\mathfrak{F}}(\lambda) = \text{ran} \begin{bmatrix} I \\ \begin{bmatrix} \frac{1}{\lambda+2} & 0 \\ 0 & \frac{1}{\lambda+3} \end{bmatrix} \end{bmatrix}$, then there exists a regular i/s/o representation $\varSigma_{\text{i/s/o}} = (\begin{bmatrix} A & B \\ C & D \end{bmatrix}; \mathbb{R}^2, U, Y)$ with the transfer function $\hat{\mathfrak{D}}(\lambda) = \begin{bmatrix} \frac{1}{\lambda+2} & 0 \\ 0 & \frac{1}{\lambda+3} \end{bmatrix}$

and $W = U\dot{+}Y$, where $A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$, $B = I$, $C = I$, $D = 0$. The control Riccati equation of the regular i/s/o representation $\Sigma_{\text{i/s/o}}$ is $Q\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} Q = Q^2 - I$. We get a nonnegative solution $Q = \begin{bmatrix} \sqrt{5}-2 & 0 \\ 0 & \sqrt{10}-3 \end{bmatrix}$. For any $x_0 = \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} \in \mathbb{R}^2$, the optimal input and the optimal output are $u^{\text{opt}}(t) = \begin{bmatrix} -(\sqrt{5}-2)\mathrm{e}^{-\sqrt{5}t}x_{01} \\ -(\sqrt{10}-3)\mathrm{e}^{-\sqrt{10}t}x_{02} \end{bmatrix}$ and $y^{\text{opt}}(t) = \begin{bmatrix} \mathrm{e}^{-\sqrt{5}t}x_{01} \\ \mathrm{e}^{-\sqrt{10}t}x_{02} \end{bmatrix}$, respectively. Hence, the optimal future cost of the regular i/s/o representation $\Sigma_{\text{i/s/o}}$ is $(\sqrt{5}-2)x_{01}^2 + (\sqrt{10}-3)x_{02}^2$. Then the regular i/s/o representation $\Sigma_{\text{i/s/o}}$ satisfies the finite future cost condition. By Theorem 2, the regular s/s system $\Sigma_{\text{s/s}}$ satisfies the finite future cost condition. By Remark 2 and Theorem 3, the LQR problem for the regular s/s system $\Sigma_{\text{s/s}}$ has a solution.

By Lemma 7 and the eigenvalues of A are 2 and 3, $\rho(\Sigma_{\text{i/s/o}}) = \rho(A) \neq \{\varnothing\}$. Then

$$w^{\text{opt}}(t) = \begin{bmatrix} I_{\text{U}} & I_{\text{Y}} \end{bmatrix} \begin{bmatrix} u^{\text{opt}}(t) \\ y^{\text{opt}}(t) \end{bmatrix} = \begin{bmatrix} (3-\sqrt{5})\mathrm{e}^{-\sqrt{5}t}x_{01} \\ (4-\sqrt{10})\mathrm{e}^{-\sqrt{10}t}x_{02} \end{bmatrix}.$$

Therefore, the optimal future cost of the regular s/s system $\Sigma_{\text{s/s}}$ is $\frac{1}{10}[(14\sqrt{5}-30)x_{01}^2 + (13\sqrt{10}-40)x_{02}^2]$.

6 Conclusion

This paper has dealt with the optimal control problems for infinite-dimensional continuous-time regular s/s systems. The optimal control problems for regular s/s systems are solved. It is shown that the solvability of the optimal control problems for the regular s/s system and that for some regular i/s/o representations of the regular s/s system are equivalent.

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