Article ID: 1000−8152(2010)02−0226−07

Synchronization of multi-chaotic systems via ring impulsive control

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Abstract: The ring control approach to multi-chaotic systems synchronization based on the impulsive control theory is presented in this article. The operator differential mid-value theorem and the matrix operations are applied to them. With the help of Gronwall Inequality, the controller is thus obtained according to the jumped impulsive response. The global synchronization of multi-chaotic systems via ring impulsive control is derived. Finally, the simulation results of a typical time-delay chaotic Hopfield neural networks and chaotic Lorenz system demonstrate that the proposed approach is effective and feasible, and has strong robust performance.

Key words: ring impulsive control; chaos synchronization; Gronwall Inequality; time-delay Hopfield neural networks; Lorenz system

CLC number: TP273 **Document code:** A

环状脉冲控制下的多个混沌系统同步

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摘要: 针对多个混沌系统同步问题, 提出了一种基于脉冲控制理论环状控制方法. 利用微分算子中值定理和矩阵运算, 通过Gronwall不等式和跳跃的脉冲响应设计控制器, 从而推导出了环状脉冲控制下多个混沌系统全局同步. 典型的时滞混沌Hopfield神经网络和Lorenz混沌系统仿真结果表明, 该方法有效、可靠, 且具有强鲁棒性.

关键词: 环状脉冲控制; 混沌同步; Gronwall不等式; 时滞Hopfield神经网络; Lorenz系统

1 Introduction

Since its introduction by Pecora and Carrol[1] in 1990, chaos synchronization of coupled systems is of great practical significance and has aroused great interest in recent years[2∼6]. However, most synchronization is realized between two chaotic systems. The problem for the chaotic synchronization control is proved to have many applications[7∼9]. In this paper, the synchronization problem for multi-chaotic systems will be considered by designing linear ring impulsive error control terms and using impulsive control theory with the help of the operator differential mid-value theorem and Gronwall Inequality.

This paper is organized as follows. Some preliminaries are given in Section 2. Section 3 deals with multi-chaos synchronization. The theoretical results and simulations are applied to typical time-delay chaotic Hopfield neural networks and Lorenz system. Finally, some concluding remarks are given in Section 4.

2 Preliminaries

First, we consider a class of recurrently delayed system, which is described by the following set of differential equations with delays[10∼15]:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})) + u_i,\ i=1,2,\cdots,n, \quad (1)$$

or, in a compact form:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau)) + U, \quad (2)$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of the neural networks, $C = \mathrm{diag}\{c_1, c_2, \cdots, c_n\}$ is a diagonal matrix, $A = (a_{ij})_{n\times n}$ is a

Received date: 2009−06−30; Revised date: 2009−09−09.

weight matrix, $B=(b_{ij})_{n\times n}$ is the delay weight matrix, $U=\mathrm{diag}\{u_1,u_2,\cdots,u_n\}^{\mathrm{T}}\in\mathbb{R}^n$ is the input vector function, $\tau(r)=(\tau_{ij})$ with the delays $\tau_{ij}>0(i,j=1,2,\cdots,n)$, and $f(x(t))=(f_1(x_1(t)), f_2(x_2(t)),\cdots,f_n(x_n(t)))^{\mathrm{T}}$. The initial conditions of (1) are given by $x_i(t)=\phi_i(t)\in C([-\rho,0],R)$ with $\rho=\max\limits_{1\leqslant i,j\leqslant n}\tau_{ij}$, $C([-\rho,0],R)$ denotes the set of all continuous functions from $[-\rho,0]$ to R.

The system with linear ring impulsive control terms is

$$\begin{cases}\dot{y}_1(t)=-Cy_1(t)+Af(y_1(t))+\\ \qquad Bf(y_1(t-\tau))+U,\\ \qquad t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta y_1(t)=B_{1k}(y_2(t)-y_1(t)),\ t=t_k,\\ y_1(t_0^+)=y_{10},\\ \dot{y}_2(t)=-Cy_2(t)+Af(y_2(t))+\\ \qquad Bf(y_2(t-\tau))+U,\\ \qquad t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta y_2(t)=B_{2k}(y_3(t)-y_2(t)),\ t=t_k,\\ y_2(t_0^+)=y_{20},\\ \qquad\vdots\\ \dot{y}_n(t)=-Cy_n(t)+Af(y_n(t))+\\ \qquad Bf(y_n(t-\tau))+U,\\ \qquad t\neq t_k,k=1,2,3,\cdots,\\ \Delta y_n(t)=B_{1k}(y_1(t)-y_n(t)),\ t=t_k,\\ y_n(t_0^+)=y_{n0},\end{cases}\tag{3}$$

where $y_i=(y_{i1},y_{i2},\cdots,y_{in})^{\mathrm{T}}\in\mathbb{R}^n$ is the state vector of node i, $f:\mathbb{R}^n\to\mathbb{R}^n$ is a nonlinear vector function and $f(0)=0$.

$$\begin{cases}\dot{e}_1(t)=-Ce_1(t)+A(f(y_2(t))-f(y_1(t)))+\\ \qquad B(f(y_2(t-\tau))-f(y_1(t-\tau))),\\ \qquad t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta e_1(t)=B_{2k}e_2(t)-B_{1k}e_1(t),\ t=t_k,\\ e_1(t_0^+)=e_{10},\\ \dot{e}_2(t)=-Ce_2(t)+A(f(y_3(t))-f(y_2(t)))+\\ \qquad B(f(y_3(t-\tau))-f(y_2(t-\tau))),\\ \qquad t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta e_2(t)=B_{3k}e_3(t)-B_{2k}e_2(t),\ t=t_k,\\ e_2(t_0^+)=e_{20},\\ \qquad\vdots\\ \dot{e}_n(t)=-Ce_n(t)+A(f(y_1(t))-f(y_n(t)))+\\ \qquad B(f(y_1(t-\tau))-f(y_n(t-\tau))),\\ \qquad t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta e_n(t)=B_{1k}e_1(t)-B_{nk}e_n(t),\ t=t_k,\\ e_n(t_0^+)=e_{n0}.\end{cases}$$

Where $e_1=y_2-y_1,\ e_2=y_3-y_2,\cdots,e_{n-1}=y_n-y_{n-1},\ e_n=y_1-y_n$.

Using the operator differential mid-value theorem[16,17], we have

$$\begin{cases}\dot{e}_1(t)=\\ -Ce_1(t)+\\ A\int_0^1\frac{\partial f(\beta y_2(t)+(1-\beta)y_1(t))}{\partial y_1(t)}\mathrm{d}\beta\cdot e_1(t)+\\ B\int_0^1\frac{\partial f(\beta y_2(t-\tau)+(1-\beta)y_1(t-\tau))}{\partial y_1(t-\tau)}\mathrm{d}\beta\cdot\\ e_1(t-\tau),\ t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta e_1(t)=B_{2k}e_2(t)-B_{1k}e_1(t),\ t=t_k,\\ e_1(t_0^+)=e_{10},\\ \dot{e}_2(t)=\\ -Ce_2(t)+\\ A\int_0^1\frac{\partial f(\beta y_3(t)+(1-\beta)y_2(t))}{\partial y_2(t)}\mathrm{d}\beta\cdot e_2(t)+\\ B\int_0^1\frac{\partial f(\beta y_3(t-\tau)+(1-\beta)y_2(t-\tau))}{\partial y_2(t-\tau)}\mathrm{d}\beta\cdot\\ e_2(t-\tau),\ t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta e_2(t)=B_{3k}e_3(t)-B_{2k}e_2(t),\ t=t_k,\\ e_2(t_0^+)=e_{20},\\ \qquad\vdots\\ \dot{e}_n(t)=\\ -Ce_n(t)+\\ A\int_0^1\frac{\partial f(\beta y_1(t)+(1-\beta)y_n(t))}{\partial y_n(t)}\mathrm{d}\beta\cdot e_n(t)+\\ B\int_0^1\frac{\partial f(\beta y_1(t-\tau)+(1-\beta)y_n(t-\tau))}{\partial y_n(t-\tau)}\mathrm{d}\beta\cdot\\ e_n(t-\tau),\ t\neq t_k,\ k=1,2,3,\cdots,\\ \Delta e_n(t)=B_{1k}e_1(t)-B_{nk}e_n(t),\ t=t_k,\\ e_n(t_0^+)=e_{n0},\end{cases}\tag{4}$$

where $\frac{\partial f(\beta y_2(t)+(1-\beta)y_1(t))}{\partial y_1(t)}$ is the value which $\frac{\partial f(x(t))}{\partial x(t)}$ is on $\beta y_2(t)+(1-\beta)y_1(t)$.

Note

$$P_1(t,\beta)=\frac{\partial f(\beta y_2(t)+(1-\beta)y_1(t))}{\partial y_1(t)},$$

$$P_2(t,\beta)=\frac{\partial f(\beta y_3(t)+(1-\beta)y_2(t))}{\partial y_2(t)},$$

$$\vdots$$

$$P_n(t,\beta)=\frac{\partial f(\beta y_1(t)+(1-\beta)y_n(t))}{\partial y_n(t)},$$

$$Q_1(t,\beta)=\frac{\partial f(\beta y_2(t-\tau)+(1-\beta)y_1(t-\tau))}{\partial y_1(t-\tau)},$$

$$Q_2(t,\beta)=\frac{\partial f(\beta y_3(t-\tau)+(1-\beta)y_2(t-\tau))}{\partial y_2(t-\tau)},$$
$$\vdots$$
$$Q_n(t,\beta)=\frac{\partial f(\beta y_1(t-\tau)+(1-\beta)y_n(t-\tau))}{\partial y_n(t-\tau)},$$

then the error system (4) is rewritten:

$$\begin{cases}\dot e_1(t)=\int_0^1(-C+AP_1(t,\beta))\mathrm{d}\beta\cdot e_1(t)+\\ \qquad\int_0^1 BQ_1(t,\beta)\mathrm{d}\beta\cdot e_1(t-\tau),\\ \qquad t\neq t_k, k=1,2,3,\cdots,\\ \Delta e_1(t)=B_{2k}e_2(t)-B_{1k}e_1(t),\ t=t_k,\\ e_1(t_0^+)=e_{10},\\ \quad\vdots\\ \dot e_n(t)=\int_0^1(-C+AP_n(t,\beta))\mathrm{d}\beta\cdot e_n(t)+\\ \qquad\int_0^1 BQ_n(t,\beta)\mathrm{d}\beta\cdot e_n(t-\tau),\\ \qquad t\neq t_k, k=1,2,3,\cdots,\\ \Delta e_n(t)=B_{1k}e_1(t)-B_{nk}e_n(t),\ t=t_k,\\ e_n(t_0^+)=e_{n0}.\end{cases}\tag{5}$$

Let $e(t)=(e_1^{\mathrm T}(t),e_2^{\mathrm T}(t),\cdots,e_n^{\mathrm T}(t))^{\mathrm T}$, and differentiating $\|e(t)\|^2$ with respect to time along the solution of (5), we have the following result:

$$\begin{aligned}&\frac{\mathrm{d}\|e(t)\|^2}{\mathrm{d}t}=\\&\sum_{i=1}^n e_i^{\mathrm T}(t)\int_0^1(-C+AP_i(t,\beta))\mathrm{d}\beta\cdot e_i(t)+\\&\sum_{i=1}^n e_i^{\mathrm T}(t)\int_0^1 BQ_i(t,\beta)\mathrm{d}\beta\cdot e_i(t-\tau)+\\&\sum_{i=1}^n e_i^{\mathrm T}(t)\int_0^1(-C+AP_i(t,\beta))^{\mathrm T}\mathrm{d}\beta\cdot e_i(t)+\\&\sum_{i=1}^n e_i^{\mathrm T}(t-\tau)\int_0^1(BQ_i(t,\beta))^{\mathrm T}\mathrm{d}\beta\cdot e_i(t)=\\&\int_0^1(e^{\mathrm T}(t)\ e^{\mathrm T}(t-\tau))P(t,\beta)(e^{\mathrm T}(t)\ e^{\mathrm T}(t-\tau))^{\mathrm T}\mathrm{d}\beta,\end{aligned}$$

where

$$P(t,\beta)=\mathrm{diag}\{D_1(t,\beta),\cdots,D_n(t,\beta)\},$$
$$D_i(t,\beta)=\begin{pmatrix}M_i(t,\beta) & BQ_i(t,\beta)\\ Q_i^{\mathrm T}(t,\beta)B^{\mathrm T} & 0\end{pmatrix},$$
$$M_i(t,\beta)=-C+AP_i(t,\beta)-C^{\mathrm T}+(AP_i(t,\beta))^{\mathrm T}.$$

Let $\lambda(t,\beta)$ be the largest eigenvalue of $P(t,\beta)$, and there is a positive constant α such that $\lambda(t,\beta)\leqslant\alpha$ for any $t\geqslant t_0$, then the conclusion is

$$\frac{\mathrm{d}\|e(t)\|^2}{\mathrm{d}t}\leqslant\int_0^1\begin{pmatrix}e(t)\\ e(t-\tau)\end{pmatrix}^{\mathrm T}\alpha\begin{pmatrix}e(t)\\ e(t-\tau)\end{pmatrix}\mathrm{d}\beta=$$
$$\alpha\|e(t)\|^2+\alpha\|e(t-\tau)\|^2.$$

For any $t\in(t_{k-1},t_k]$,

$$\|e(t)\|^2\leqslant\|e(t_{k-1}^+)\|^2\exp\{\alpha(t-t_{k-1})\}+\int_{t_{k-1}}^t\exp\{\alpha(t-s)\}\alpha\|e(s-\tau)\|^2\mathrm{d}s,$$

then

$$\begin{aligned}&\exp\{-\alpha(t-t_{k-1})\}\|e(t)\|^2\leqslant\\&\|e(t_{k-1}^+)\|^2+\int_{t_{k-1}}^t\exp\{-\alpha(s-t_{k-1})\}\alpha\|e(s-\tau)\|^2\mathrm{d}s=\\&\|e(t_{k-1}^+)\|^2+\int_{t_{k-1}-\tau}^{t-\tau}\exp\{-\alpha(s+\tau-t_{k-1})\}\alpha\|e(s)\|^2\mathrm{d}s.\end{aligned}$$

Using the Gronwall Inequality[18], we obtain

$$\begin{cases}\exp\{-\alpha(t-t_{k-1})\}\|e(t)\|^2\leqslant\\ \|e(t_{k-1}^+)\|^2\exp\{\alpha\exp\{-\alpha\tau\}(t-t_{k-1})\},\\ \|e(t)\|^2\leqslant\\ \|e(t_{k-1}^+)\|^2\exp\{\alpha(1+\exp\{-\alpha\tau\})(t-t_{k-1})\},\\ t\in(t_{k-1},t_k],\ k=1,2,\cdots.\end{cases}\tag{6}$$

3 Impulsive synchronization

Theorem 1 Suppose $0<\omega_k=t_k-t_{k-1}<\infty(k=1,2,\cdots)$, γ_k is the largest eigenvalue of $(I+B_k)^{\mathrm T}(I+B_k)$, the impulsive synchronization of Eq.(3) is achieved if there exists a constant $\theta>1$ such that

$$\ln(\theta\gamma_k)+\alpha(1+\exp\{-\alpha\tau\})\omega_k\leqslant 0,\ k=1,2,\cdots,\tag{7}$$

where

$$B_k=\begin{pmatrix}-B_{1k} & B_{2k} & 0 & \cdots & 0\\ 0 & -B_{2k} & B_{3k} & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ B_{1k} & 0 & 0 & \cdots & -B_{nk}\end{pmatrix}.$$

Proof we have known the conclusion (6). When $t=t_k$, we obtain

$$\|e(t_k^+)\|^2=e^{\mathrm T}(t_k)(I+B_k)^{\mathrm T}(I+B_k)e(t_k)\leqslant\gamma_k\|e(t_k)\|^2.\tag{8}$$

Considering the conditions (6)(7) and (8), we have

$$\begin{aligned}&\|e(t)\|^2\leqslant\\&\gamma_1\gamma_2\cdots\gamma_k\|e(t_0^+)\|^2\exp\{\mu(t-t_0)\}\leqslant\\&\frac{1}{\theta^k}\|e(t_0^+)\|^2\exp\{\mu(t-t_k)\},\ t\in(t_k,t_{k+1}],\end{aligned}$$

where $\mu=\alpha(1+\exp\{-\alpha\tau\})$. When $t\to\infty$, $\|e(t)\|^2\to 0$, say $\|e(t)\|\to 0$, and makes system (3) synchronization.

When $B=0$ in system (2), system (5) becomes the following:

$$\begin{cases} \dot{e}_1(t)=\int_0^1(-C+AP_1(t,\beta))\mathrm{d}\beta\cdot e_1(t), \\ \qquad t\neq t_k,\ k=1,2,3,\cdots, \\ \Delta e_1(t)=B_{2k}e_2(t)-B_{1k}e_1(t),\ t=t_k, \\ e_1(t_0^+)=e_{10}, \\ \dot{e}_2(t)=\int_0^1(-C+AP_2(t,\beta))\mathrm{d}\beta\cdot e_2(t), \\ \qquad t\neq t_k,\ k=1,2,3,\cdots, \\ \Delta e_2(t)=B_{3k}e_3(t)-B_{2k}e_2(t),\ t=t_k, \\ e_2(t_0^+)=e_{20}, \\ \qquad \vdots \\ \dot{e}_n(t)=\int_0^1(-C+AP_n(t,\beta))\mathrm{d}\beta\cdot e_n(t), \\ \qquad t\neq t_k, k=1,2,3,\cdots, \\ \Delta e_n(t)=B_{1k}e_1(t)-B_{nk}e_n(t),\ t=t_k, \\ e_n(t_0^+)=e_{n0}. \end{cases} \tag{9}$$

So the following Corollary is obtained.

Corollary 1 Suppose $0<\omega_k=t_k-t_{k-1}<\infty$ $(k=1,2,\cdots)$, γ_k is the largest eigenvalue of $(I+B_k)^{\mathrm{T}}(I+B_k)$, $\lambda(t,\beta)$ is the largest eigenvalue of $P(t,\beta)$, and $\lambda(t,\beta)\leqslant\alpha$, where α is a positive constant, the impulsive synchronization of Eq.(9) is achieved if there exists a constant $\theta>1$ such that

$$\ln(\theta\gamma_k)+\alpha\,\omega_k\leqslant 0,\ k=1,2,\cdots. \tag{10}$$

Example 1 Consider a typical delayed Hopfield neural networks[12,14] with two neurons:

$$\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-\tau)), \tag{11}$$

where

$$x(t)=(x_1(t),x_2(t))^{\mathrm{T}},$$
$$f(x(t))=(\tanh(x_1(t)),\tanh(x_2(t)))^{\mathrm{T}},$$
$$\tau=(1)\text{ and }C=\begin{pmatrix}1&0\\0&1\end{pmatrix},$$
$$A=\begin{pmatrix}2.0&-0.1\\-5.0&3.0\end{pmatrix},B=\begin{pmatrix}-1.5&-0.1\\-0.2&-2.5\end{pmatrix}.$$

It should be noted that the network is actually a chaotic delayed Hopfield neural network.

In system (3), suppose each $y_i(i=1,2,\cdots,n)$ is $x(t)$ as the system (11) stated. Choose

$$\omega_k=t_k-t_{k-1}=0.005,$$
$$B_{1k}=\mathrm{diag}\{-(1-0.58)^2,-(1-0.6)^2\}^{\mathrm{T}},$$
$$B_{2k}=\mathrm{diag}\{-(1-0.8),-(1-0.8)\}^{\mathrm{T}},$$
$$B_{3k}=\mathrm{diag}\{-(1-0.75)^2,-(1-0.56)\}^{\mathrm{T}},$$
$$B_{4k}=\mathrm{diag}\{-(1-0.58),-(1-0.6)\}^{\mathrm{T}},$$
$$B_{5k}=\mathrm{diag}\{-(1-0.65),-(1-0.48)\}^{\mathrm{T}},$$
$$B_{6k}=\mathrm{diag}\{-(1-0.57),-(1-0.65)\}^{\mathrm{T}},$$
$$B_{7k}=\mathrm{diag}\{-(1-0.8),-(1-0.6)\}^{\mathrm{T}},$$
$$B_{8k}=\mathrm{diag}\{-(1-0.7),-(1-0.7)\}^{\mathrm{T}},$$
$$B_{9k}=B_{10k}=B_{11k}=B_{12k}=B_{13k}=B_{14k}=\mathrm{diag}\{-(1-0.8),-(1-0.8)\}^{\mathrm{T}},$$
$$B_{15k}=B_{16k}=B_{17k}=B_{18k}=B_{19k}=B_{20k}=B_{21k}=B_{22k}=B_{23k}=\mathrm{diag}\{-(1-0.6),-(1-0.6)\}^{\mathrm{T}},$$
$$r_1=(34,4.3,17,12.8,27,49,1.2,2.4,0.8,9.6,11,16,21,33,8,3)^{\mathrm{T}},$$
$$r_2=(-12,-11,-12,28,27,26,15.4,9,11,0.1,0.3,0.6)^{\mathrm{T}},$$
$$r_3=(2,-1,-2,8,7,6,15.4,9,11,0.1,0.3,0.6,21,17,19,-1,-2,8)^{\mathrm{T}},$$

respectively. Let the initial conditions be r_1, $(r_1^{\mathrm{T}},r_2^{\mathrm{T}})^{\mathrm{T}}$, $(r_1^{\mathrm{T}},r_2^{\mathrm{T}},r_3^{\mathrm{T}})^{\mathrm{T}}$, then it can be clearly seen in Fig.1$\sim$Fig.3 that the impulsive synchronization is achieved when $n=8$, $n=14$, $n=23$, respectively.

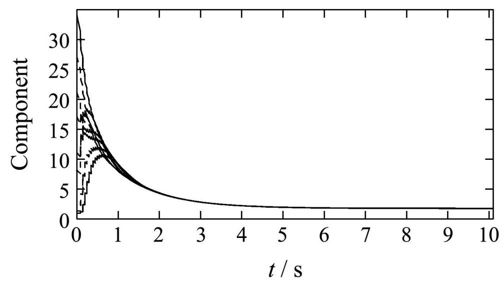

Fig. 1(a) Impulsive synchronization of one-component of each sub-system ($n=8$)

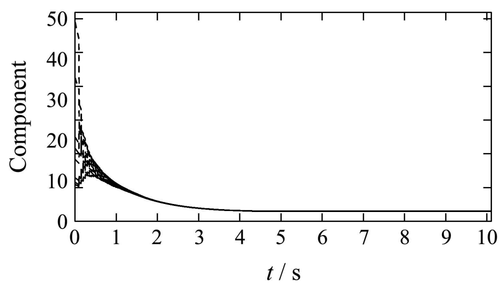

Fig. 1(b) Impulsive synchronization of two-component of each sub-system ($n=8$)

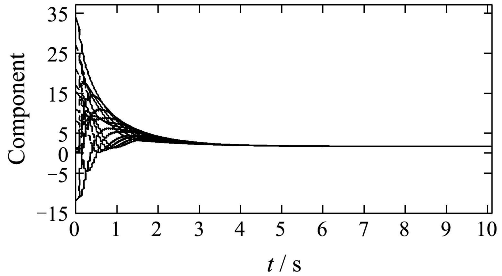

Fig. 2(a) Impulsive synchronization of one-component of each sub-system $(n = 14)$

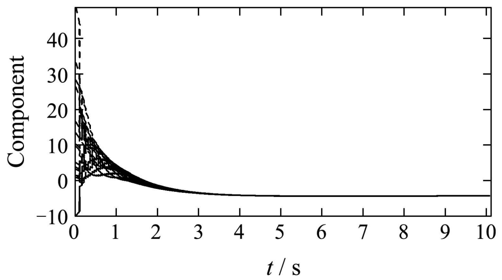

Fig. 2(b) Impulsive synchronization of two-component of each sub-system $(n = 14)$

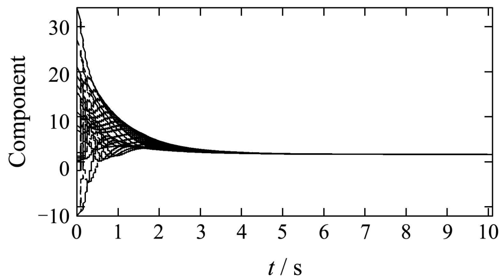

Fig. 3(a) Impulsive synchronization of one-component of each sub-system $(n = 23)$

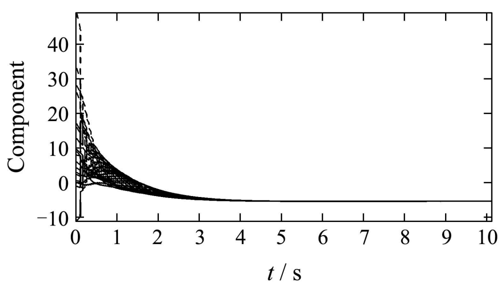

Fig. 3(b) Impulsive synchronization of two-component of each sub-system $(n = 23)$

Example 2 Consider chaotic Lorenz system:

$$\begin{cases}\dot{x} = -10(x - y),\\ \dot{y} = 28x - y - xz,\\ \dot{z} = xy - \dfrac{8}{3}z.\end{cases} \tag{12}$$

In system (3), suppose each $y_i(i = 1, 2, \cdots, n)$ is $(x(t), y(t), z(t))^{\mathrm{T}}$ as system (12) stated. Choose

$\omega_k = t_k - t_{k-1} = 0.005,$

$B_{1k} =$
$\mathrm{diag}\{-(1-0.58)^2, -(1-0.6)^2, -(1-0.75)^2\}^{\mathrm{T}},$

$B_{2k} =$
$\mathrm{diag}\{-(1 - 0.58), -(1 - 0.8), -(1 - 0.65)\}^{\mathrm{T}},$

$B_{3k} =$
$\mathrm{diag}\{-(1 - 0.48), -(1 - 0.58), -(1 - 0.58)\}^{\mathrm{T}},$

$B_{4k} =$
$\mathrm{diag}\{-(1 - 0.28), -(1 - 0.48)^2, -(1 - 0.5)\}^{\mathrm{T}},$

$B_{5k} =$
$\mathrm{diag}\{-(1 - 0.48)^2, -(1 - 0.78), -(1 - 0.61)\}^{\mathrm{T}},$

$B_{6k} =$
$\mathrm{diag}\{-(1 - 0.58), -(1 - 0.8), -(1 - 0.65)\}^{\mathrm{T}},$

$B_{7k} =$
$\mathrm{diag}(-(1 - 0.48), -(1 - 0.58), -(1 - 0.58)\}^{\mathrm{T}},$

$B_{8k} =$
$\mathrm{diag}\{-(1 - 0.28), -(1 - 0.38)^2, -(1 - 0.3)\}^{\mathrm{T}},$

$B_{9k} =$
$\mathrm{diag}\{-(1 - 0.68), -(1 - 0.8)^2, -(1 - 0.8)\}^{\mathrm{T}},$

$B_{10k} =$
$\mathrm{diag}\{-(1 - 0.78)^2, -(1 - 0.88)^2, -(1 - 0.5)^3\}^{\mathrm{T}},$

$B_{11k} =$
$\mathrm{diag}\{-(1 - 0.68)^3, -(1 - 0.96)^2, -(1 - 0.63)^3\}^{\mathrm{T}},$

$B_{12k} =$
$\mathrm{diag}\{-(1 - 0.58)^2, -(1 - 0.6)^2, -(1 - 0.75)^2\}^{\mathrm{T}},$

$B_{13k} =$
$\mathrm{diag}\{-(1 - 0.58), -(1 - 0.8), -(1 - 0.65)\}^{\mathrm{T}},$

$B_{14k} =$
$\mathrm{diag}\{-(1 - 0.48), -(1 - 0.58), -(1 - 0.58)\}^{\mathrm{T}},$

$B_{15k} =$
$\mathrm{diag}\{-(1 - 0.28), -(1 - 0.48)^2, -(1 - 0.5)\}^{\mathrm{T}},$

$B_{16k} =$
$\mathrm{diag}\{-(1 - 0.48)^2, -(1 - 0.78), -(1 - 0.6)\}^{\mathrm{T}},$

$r_1 = (2, -1, -2, 8, 7, 6, 15.4, 9, 11, 0.1, 0.3, 0.6, 21, 17, 19)^{\mathrm{T}},$

$r_2 = (-1, -2, 8, 1, 2, 3, 17, 19, 34, 12, 15, 20)^{\mathrm{T}},$

$r_3 = (27, 21, 3.04, 6, 3.97, -6, -1, -2, 8, 7, 6, 15.4, 9, 11, -2.1, 34.8, 12.6, 15, -2, -3, -4)^{\mathrm{T}},$

respectively. Let the initial conditions be $r_1, (r_1^{\mathrm{T}}, r_2^{\mathrm{T}})^{\mathrm{T}}$, $(r_1^{\mathrm{T}}, r_2^{\mathrm{T}}, r_3^{\mathrm{T}})^{\mathrm{T}}$, then it can be clearly seen in Fig.4$\sim$ Fig.6 that the impulsive synchronization is achieved when $n = 5, n = 9, n = 16$, respectively.

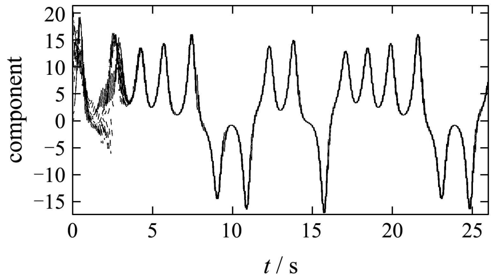

Fig. 4(a) Impulsive synchronization of one-component of each sub-system $(n = 5)$

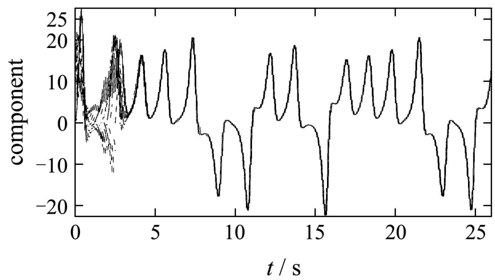

Fig. 4(b) Impulsive synchronization of two-component of each sub-system $(n = 5)$

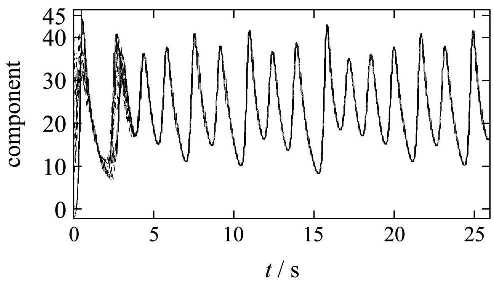

Fig. 4(c) Impulsive synchronization of three-component of each sub-system $(n = 5)$

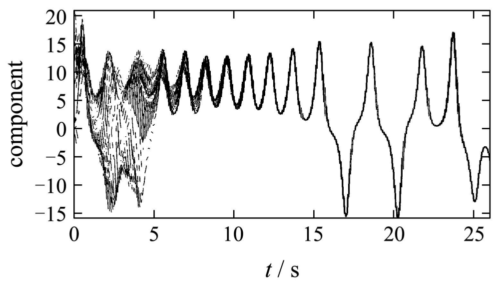

Fig. 5(a) Impulsive synchronization of one-component of each sub-system $(n = 9)$

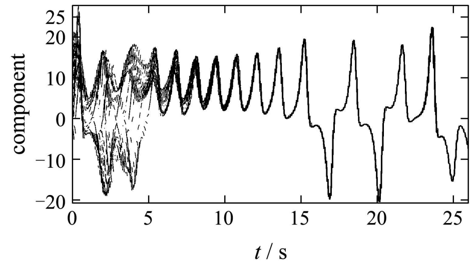

Fig. 5(b) Impulsive synchronization of two-component of each sub-system $(n = 9)$

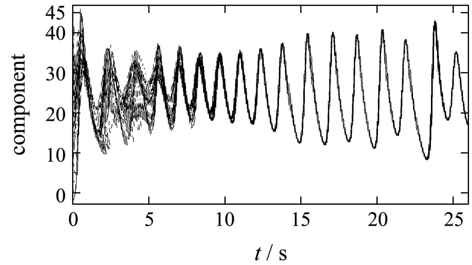

Fig. 5(c) Impulsive synchronization of three-component of each sub-system $(n = 9)$

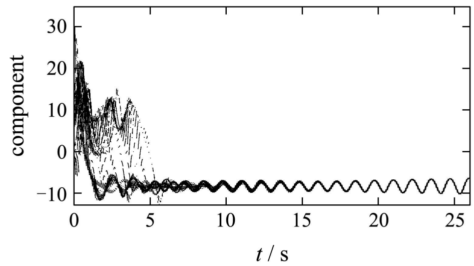

Fig. 6(a) Impulsive synchronization of one-component of each sub-system $(n = 16)$

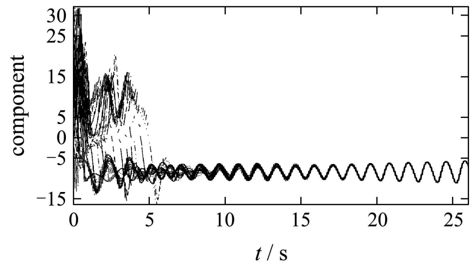

Fig. 6(b) Impulsive synchronization of two-component of each sub-system $(n = 16)$

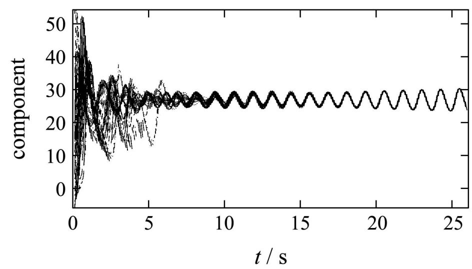

Fig. 6(c) Impulsive synchronization of three-component of each sub-system $(n = 16)$

4 Conclusion

Approaches to impulsive synchronization of multi-chaotic systems have been presented in this paper. Strong properties of global and asymptotic impulsive synchronization have been achieved in a finite number of steps. The techniques have been successfully applied to a typical delayed Hopfield neural networks and Lorenz system. Numerical simulations have verified the effectiveness of the method.

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