第 30 卷第 6 期
2013 年 6 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 30 No. 6
Jun. 2013

DOI: 10.7641/CTA.2013.12039

一类状态时滞广义串级控制系统的H_∞镇定

杜昭平[1†], 岳 东[1], 胡松林[1], 张美生[2]

(1. 华中科技大学 控制科学与工程系, 湖北 武汉 430074; 2. 山东建筑大学 校长办公室, 山东 济南 250101)

摘要: 针对一类具有状态时滞的广义串级控制系统, 研究其H_∞控制问题. 考虑到当非零扰动作用于系统时采用静态反馈控制对于被控对象的输出具有不足性, 从而将串级控制应用于具有状态时滞的广义系统中, 并且建立了闭环广义串级控制系统新的模型. 提出的方法着重在于串级控制在广义系统中的应用. 利用线性矩阵不等式方法得到了相应系统正则、无脉冲和稳定的条件, 并进一步利用得到的条件给出了相应的H_∞稳定控制器设计方法. 最后通过仿真实例说明了所提方法的有效性.

关键词: H_∞控制; 广义系统; 串级控制; 状态时滞
中图分类号: TP273 **文献标识码**: A

H-infinity stabilization for a class of singular cascade control systems with time-delay in state

DU Zhao-ping[1†], YUE Dong[1], HU Song-lin[1], ZHANG Mei-sheng[2]

(1. Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan Hubei 430074, China;
2. Headmaster's Office, Shandong Jianzhu University, Jinan Shandong 250101, China)

Abstract: This paper investigates the H-infinity control for a class of singular cascade control systems with time-delay in state. Because of the undesirable output performance in the feedback control when non-zero disturbances exist the systems, the cascade control is applied for the singular systems with time-delay in state; and a new model is derived for the closed-loop singular cascade control system. The proposed method emphasizes the employment of cascade control to the singular systems. The resulting regularity, impulse-free and stability conditions are derived in terms of linear matrix inequalities; and the corresponding H-infinity stabilizing controller design technique is developed based on the above conditions. A simulation and experimental example is given to illustrate the effectiveness of the obtained results.

Key words: H-infinity control; singular systems; cascade control; time-delay in state

1 Introduction

Singular systems have been extensively studied for many years, because singular systems are more general and natural in describing the practical dynamical systems than standard state-space systems[1]. A great number of results on these systems have been reported in the literature. However, singular systems are much more complicated than standard state-space systems because they are required to be regular and impulse-free(for continuous singular systems) or causal (for discrete singular systems) simultaneously[2], while the two characteristics don't appear in the standard state-space systems.

Over the past decades, more and more attention has been focused to the stability analysis and system synthesis for singular systems with time-delay. Many fundamental system theories developed for standard state-space systems have been successively generalized to the singular systems with time-delay; for example, robustness and filtering ability[3–5], H-infinity control problem[6–7], stability analysis[8–9], etc. Although Lyapunov function candidate is an effective form for stability analysis, and most results can be expressed by linear matrix inequality (LMI); however, for singular systems with time-delay, the selection of Lyapunov function candidate and the calculation of its derivative along the motion locus of the systems are more difficult than those for state-space systems becausemore issues are needed to be considered simultaneously.

Cascade control is first proposed by Franks and Worley[10], and has been considered very effective to improve control system performances, especially for the systems with disturbances. Nowadays, cascade control has become one of the most important control forms and are applied in many industrial process systems,

Received 3 March 2012; revised 11 November 2012.
†Corresponding author: E-mail: duzhaoping98@163.com.
This work is supported by National Natural Science Foundation of China (Nos. 61074025, 61203024), the China Postdoctoral Science Foundation (No. 20110491148) and the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (No. 11KJD120001).

such as power plants[11–12], chemical plants[13], and so on. Because of the inner loop, a cascade control system quickly attenuates the disturbances in the inner loop, and leads to the improvement of the entire system performance[14].

Recently, much efforts have been paid to the study for the cascade control for different systems from theoretical point of view, such as networked control systems[14–15], generalized nonlinear systems[16–17], neural network systems[18] and so on. A great number of results have been reported in the literatures. However, most of those results are not concerned with the singular systems, especially the singular systems with time-delay. To the best of the authors' knowledge, the stability analysis and synthesis problems of singular systems with time-delay under cascade control, still remain challenging to scholars from a theoretical point of view; especially, the advantages of singular systems under cascade control which motivates the present study.

Over the past decades, the problem of H-infinity control has been studied for singular systems with time-delay, such as networked control systems[19–20]. As is known, taking into account of the external disturbances, H-infinity control is effective in analysis and synthesis of practical systems with disturbance. However,not all the methods are feasible for the singular cascade control systems with time-delay.

To investigate the problem of H-infinity control for a class of singular cascade control systems with time-delay. A new model of singular cascade control systems is proposed where the parameters have clear physical meanings and can be easily determined. The stability conditions are derived in terms of LMI and the corresponding H-infinity controller design technique is also developed.

Our investigation is the first attempt to consider the problems of cascade control for singular systems from the theoretical point of view, and a new model of singular cascade control system with time-delay and disturbances is constructed. Based on this model, and taking into account of the characteristics of the systems with time-delay, sufficient condition of regularity, impulse-free and stability is proposed by using LMI technique and the result implies that the systems are admissible under certain compatible initial conditions. According to the obtained result, we also derive the sufficient condition of controller existence for the singular cascade control systems with time-delay, and the corresponding primary controller and the secondary controller can be obtained simultaneously. Furthermore, we present the method of H-infinity controller design for the singular cascade control systems with time-delay and external disturbances. In this research, some inequalities are employed to simplify the calculation procedure by solving the corresponding LMI instead. Numerical example is given to illustrate the effectiveness and applications of the proposed method.

Most of the notations used here are standard in most respects. We use R to denote the set of real numbers. $\mathbb{R}^n$ and $\mathbb{R}^{n_1\times n_2}$ are sets of vectors and matrices of the specified dimensions, the symmetrical term in a symmetric matrix is denoted by $*$, M^{T} stands for the transpose of M.

2 Modeling of singular cascade control systems with time-delay and disturbances

To address the modeling problem, we introduce the following configuration diagram for the singular cascade control systems with time-delay and disturbances.

In Fig.1, P_1 is the primary plant, P_2 is the secondary plant, C_1 is the primary controller, C_2 is the secondary controller, and A is the actuator between the controller C_2 and the plant P_2, S_1 and S_2 are the sensors from the plants P_1 and P_2, respectively.

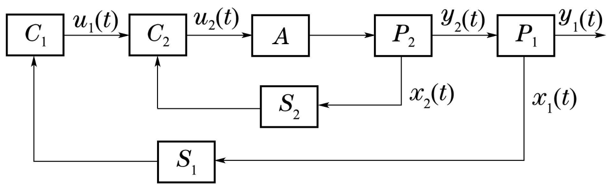

Fig. 1 Configuration diagram of singular cascade control systems

It can be seen from Fig.1, that the output of P_2 is as the input of P_1, the disturbance $w(t)$ is mainly existing in the inner loop of the systems, the states of P_1 and P_2 are sent to C_1 and C_2 by S_1 and S_2, respectively. The primary plant P_1 can be described by the standard state-space system form, and the secondary plant P_2 is modeled by a class of singular system with time-delay.

Consider the singular cascade control system shown in Fig.1, the primary plant is a continuous linear time system, which can be described by the following form

$$\begin{cases} \dot{x}_1(t) = A_1x_1(t) + B_1y_2(t), \\ y_1(t) = C_1x_1(t) + C_3w(t), \end{cases} \tag{1}$$

where $x_1(t)$ is the state vector of the primary plant, $y_1(t)$ and $y_2(t)$ are the output of the primary plant P_1 and the secondary plant P_2, respectively. A_1, B_1, C_1 and C_3 are the constant matrices with appropriate dimensions.

The secondary plant is modeled by a class of singular systems with time-delay and external disturbances which can be written as

$$\begin{cases} E\dot{x}_2(t) = A_2x_2(t) + A_3x_2(t-\tau)+ \\ \qquad\qquad B_2u_2(t) + B_3w(t), \\ y_2(t) = C_2x_2(t) + C_4w(t), \end{cases} \tag{2}$$

where $x_2(t)$ is the state vector of the secondary plant, $u_2(t)$ is the control input received by the actuator, $w(t) \in L_2[0,\infty)$ is the disturbance with limited energy, $y_2(t)$ is the output of the secondary plant, τ is the time-delay of the secondary plant. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular and $\mathrm{rank}(E) = r \leqslant n$, A_2, A_3,

B_2, B_3, C_2 and C_4 are the constant matrices with appropriate dimensions.

In this paper, we use a static state feedback controller as the primary controller, which can be described as

$$u_1(t) = K_1 x_1(t), \tag{3}$$

where $x_1(t)$ is the state vector of the primary plant, K_1 and $u_1(t)$ are the state feedback gain matrix and the control output of the primary controller, respectively.

A static state feedback controller is also adopted as the secondary controller, that is

$$u_2(t) = u_1(t) + K_2 x_2(t). \tag{4}$$

Combining (1)−(4), the closed-loop model of the singular cascade control systems can be described as

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 C_2 x_2(t) + B_1 C_4 w(t), \\ E\dot{x}_2(t) = \\ A_2 x_2(t) + A_3 x_2(t-\tau) + \\ B_2 K_1 x_1(t) + B_2 K_2 x_2(t) + B_3 w(t), \\ y_1(t) = C_1 x_1(t) + C_3 w(t). \end{cases} \tag{5}$$

The systems can be modeled as (5), it is worthy noting that this research is the first attempt to adopt cascade control for the singular systems with time-delay and external disturbances from theoretical point of view. With the zero reference, the H-infinity control problem for the singular cascade control systems will be considered in the following.

3 Stability analysis of singular cascade control systems with time-delay

In order to derive the sufficient condition of H-infinity control for the singular cascade control systems (5) with time-delay and external disturbances, the following definition and lemmas will be essential for the proof of the theorems.

Definition 1[1,21]

1) The pair (E, A_2) is said to be regular if $\det(sE - A_2)$ is not identically zero.

2) The pair (E, A_2) is said to be impulse-free if $\deg(\det(sE - A_2)) = \mathrm{rank}(E)$.

3) The unforced singular system of (2) with $w(t) = 0$ and time-delay is said to be regular and impulse-free, if the pair (E, A_2) is regular and impulse-free.

4) The unforced singular system of (2) with $w(t) = 0$ and time-delay is said to be admissible if it is regular, impulse-free and stable.

Lemma 1[22] Assume that $a(\cdot) \in \mathbb{R}^{n_a}$, $b(\cdot) \in \mathbb{R}^{n_b}$ and $N(\cdot) \in \mathbb{R}^{n_a \times n_b}$ are defined on the interval Ω. Then, for any matrices $X(\cdot) \in \mathbb{R}^{n_a \times n_a}$, $Y(\cdot) \in \mathbb{R}^{n_a \times n_b}$ and $Z(\cdot) \in \mathbb{R}^{n_b \times n_b}$, the following holds

$$-2\int_{\Omega} a^{\mathrm{T}}(\alpha) N b(\alpha) \mathrm{d}\alpha \leqslant$$
$$\int_{\Omega} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y - N \\ Y^{\mathrm{T}} - N^{\mathrm{T}} & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} \mathrm{d}\alpha,$$

where $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geqslant 0$.

Lemma 2[23] Given any real matrices Ω_1, Ω_2, and Ω_3, where $\Omega_1^{\mathrm{T}} = \Omega_1$, and $\Omega_2 = \Omega_2^{\mathrm{T}} > 0$, then $\Omega_1 + \Omega_3^{\mathrm{T}} \Omega_2^{-1} \Omega_3 < 0$ if and only if $\begin{bmatrix} \Omega_1 & \Omega_3^{\mathrm{T}} \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0$.

Lemma 3[24] For symmetric positive-definite matrix Q, and matrices P and R with appropriate dimensions, matrix inequality $P^{\mathrm{T}}R + R^{\mathrm{T}}P \leqslant R^{\mathrm{T}}QR + P^{\mathrm{T}}Q^{-1}P$ holds.

In the following, sufficient conditions of admissible for systems (5) with $w(t) = 0$ and time-delay are presented by using LMI technique.

Theorem 1 For given state feedback gain K_1 and K_2, if there exist a non-singular matrix P, symmetric positive-definite matrices R, Q, Z, and matrices X, Y, such that

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \geqslant 0, \tag{6}$$

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & E^{\mathrm{T}}ZE \end{bmatrix} \geqslant 0, \tag{7}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & P^{\mathrm{T}}A_3 - Y & \tau(A_2^{\mathrm{T}} + K_2^{\mathrm{T}}B_2^{\mathrm{T}})Z \\ * & \Omega_{22} & 0 & \tau K_1^{\mathrm{T}}B_2^{\mathrm{T}}Z \\ * & * & -Q & \tau A_3^{\mathrm{T}}Z \\ * & * & * & -\tau Z \end{bmatrix} < 0, \tag{8}$$

where

$$\Omega_{11} = P^{\mathrm{T}}(A_2 + B_2K_2) + (A_2 + B_2K_2)^{\mathrm{T}}P + \tau X + Y + Y^{\mathrm{T}} + Q,$$
$$\Omega_{12} = P^{\mathrm{T}}B_2K_1 + C_2^{\mathrm{T}}B_1^{\mathrm{T}}R,\ \Omega_{22} = R^{\mathrm{T}}A_1 + A_1^{\mathrm{T}}R,$$

then, the singular cascade control system (5) with $w(t) = 0$ is admissible for time-delay τ.

Proof To show the singular cascade control system (5) with $w(t) = 0$ and time-delay is admissible, first of all, we show the unforced singular system of (5) with $w(t) = 0$ is regular and impulse-free. Without loss of generality, we can assume that the matrices E, A_2 in (5) have the following forms:

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

From (6)−(8), it can be seen that

$$Y = \begin{bmatrix} Y_{11} & 0 \\ Y_{21} & 0 \end{bmatrix},\ P = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix},\ Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^{\mathrm{T}} & Z_{22} \end{bmatrix}.$$

From (7)−(8), we can obtain

$$P^{\mathrm{T}}A_2 + A_2^{\mathrm{T}}P + Y + Y^{\mathrm{T}} < 0,$$

which implies that A_{22} is nonsingular, and thus the pair (E, A_2) is regular and impulse-free.

Next, we show the system (5) with $w(t) = 0$ is stable. Choose a Lyapunov functional as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{9}$$

where

$$V_1(t) = x_2^{\mathrm{T}}(t)E^{\mathrm{T}}Px_2(t),$$
$$V_2(t) = \int_{-\tau}^{0}\int_{t+\beta}^{t}\dot{x}_2^{\mathrm{T}}(\alpha)E^{\mathrm{T}}ZE\dot{x}_2(\alpha)\mathrm{d}\alpha\mathrm{d}\beta,$$
$$V_3(t) = \int_{t-\tau}^{t}x_2^{\mathrm{T}}(\alpha)Qx_2(\alpha)\mathrm{d}\alpha,$$
$$V_4(t) = x_1^{\mathrm{T}}(t)Rx_1(t),$$

and P is defined in Theorem 1, Z, Q, R are symmetric positive-definite matrices to be determined.

Using Lemma 1, we can obtain the time derivative of $V_1(t)$ as

$$\dot{V}_1(t) \leqslant 2x_2^{\mathrm{T}}(t)P^{\mathrm{T}}[(A_2+A_3)x_2(t)+B_2K_1x_1(t)+B_2K_2x_2(t)]+\tau x_2^{\mathrm{T}}(t)(X+Y+Y^{\mathrm{T}})x_2(t)+2x_2^{\mathrm{T}}(t)(P^{\mathrm{T}}A_3-Y)x_2(t-\tau))-2x_2^{\mathrm{T}}(t)P^{\mathrm{T}}A_3\times x_2(t)-\int_{t-\tau}^{t}\dot{x}_2^{\mathrm{T}}(\alpha)E^{\mathrm{T}}ZE\dot{x}_2(\alpha)\mathrm{d}\alpha,$$

where X, Y, Z satisfy (7).

Since $V_2(t)$, $V_3(t)$ and $V_4(t)$ yield the relation

$$\dot{V}_2(t) = \tau\dot{x}_2^{\mathrm{T}}(t)E^{\mathrm{T}}ZE\dot{x}_2(t)-\int_{t-\tau}^{t}\dot{x}_2^{\mathrm{T}}(\alpha)E^{\mathrm{T}}ZE\dot{x}_2(\alpha)\mathrm{d}\alpha,$$
$$\dot{V}_3(t) = x_2^{\mathrm{T}}(t)Qx_2(t)-x_2^{\mathrm{T}}(t-\tau)Qx_2(t-\tau),$$
$$\dot{V}_4(t) = 2x_1^{\mathrm{T}}(t)R(A_1x_1(t)+B_1C_2x_2(t)).$$

Therefore, we have the time derivative of $V(t)$ along the trajectory of the system as

$$\dot{V}(t) \leqslant \xi^{\mathrm{T}}(t)\varOmega\xi(t), \tag{10}$$

where $\xi(t) = \begin{bmatrix} x_2^{\mathrm{T}}(t) & x_1^{\mathrm{T}}(t) & x_2^{\mathrm{T}}(t-\tau)\end{bmatrix}^{\mathrm{T}}$ and

$$\varOmega = \begin{bmatrix} \varOmega_{11} & \varOmega_{12} & P^{\mathrm{T}}A_3-Y+\tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})ZA_3 \\ * & \varOmega_{22} & 0 \\ * & * & \tau A_3^{\mathrm{T}}ZA_3-Q \end{bmatrix},$$

where

$$\varOmega_{11} = P^{\mathrm{T}}(A_2+B_2K_2)+\tau X+Y+Y^{\mathrm{T}}+Q+\tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})Z(A_2+B_2K_2)+(A_2+B_2K_2)^{\mathrm{T}}P,$$
$$\varOmega_{12} = P^{\mathrm{T}}B_2K_1+C_2^{\mathrm{T}}B_1^{\mathrm{T}}R+\tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})ZB_2K_1,$$
$$\varOmega_{22} = R^{\mathrm{T}}A_1+A_1^{\mathrm{T}}R+\tau K_1^{\mathrm{T}}B_2^{\mathrm{T}}ZB_2K_1.$$

It follows that the inequality $\varOmega < 0$ guarantees $\dot{V}(t) < 0$ for all non-zero $\xi(t)$. Hence, $\varOmega < 0$ guarantees the system stable for time-delay τ. Using Lemma 2, $\varOmega < 0$ is equivalent to the following LMI:

$$\begin{bmatrix} \varOmega_{11} & \varOmega_{12} & P^{\mathrm{T}}A_3-Y & \tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})Z \\ * & \varOmega_{22} & 0 & \tau K_1^{\mathrm{T}}B_2^{\mathrm{T}}Z \\ * & * & -Q & \tau A_3^{\mathrm{T}}Z \\ * & * & * & -\tau Z \end{bmatrix} < 0,$$

where

$$\varOmega_{11} = P^{\mathrm{T}}(A_2+B_2K_2)+(A_2+B_2K_2)^{\mathrm{T}}P+\tau X+Y+Y^{\mathrm{T}}+Q,$$
$$\varOmega_{12} = P^{\mathrm{T}}B_2K_1+C_2^{\mathrm{T}}B_1^{\mathrm{T}}R,\ \varOmega_{22} = R^{\mathrm{T}}A_1+A_1^{\mathrm{T}}R.$$

From this, we can conclude that the system (5) with $w(t) = 0$ and time-delay is regular, impulse-free and stable. That is, the system (5) with $w(t) = 0$ is admissible. This completes the proof.

Based on Theorem 1, we develop the method for the primary and secondary controller design in system (5) with $w(t) = 0$ and time-delay.

Theorem 2 If there exist a non-singular matrix $\hat{P}$, symmetric positive-definite matrices $\hat{R}, \hat{Q}, \hat{Z}$, and matrices $\hat{X}, \hat{Y}, W_1, W_2$, such that the following inequalities hold

$$\hat{P}^{\mathrm{T}}E^{\mathrm{T}} = E\hat{P} \geqslant 0, \tag{11}$$

$$\begin{bmatrix} \hat{X} & \hat{Y} \\ \hat{Y}^{\mathrm{T}} & \hat{P}^{\mathrm{T}}E^{\mathrm{T}}+E\hat{P}-E^{\mathrm{T}}\hat{Z}E \end{bmatrix} \geqslant 0, \tag{12}$$

$$\begin{bmatrix} \hat{\varOmega}_{11} & \hat{\varOmega}_{12} & A_3\hat{P}-Y & \tau\hat{P}^{\mathrm{T}}A_2^{\mathrm{T}}+\tau W_2^{\mathrm{T}}B_2^{\mathrm{T}} \\ * & \hat{\varOmega}_{22} & 0 & \tau W_1^{\mathrm{T}}B_2^{\mathrm{T}} \\ * & * & -\hat{Q} & \tau\hat{P}^{\mathrm{T}}A_3^{\mathrm{T}} \\ * & * & * & -\tau\hat{Z} \end{bmatrix} < 0, \tag{13}$$

where

$$\hat{\varOmega}_{11} = A_2\hat{P}+\hat{P}^{\mathrm{T}}A_2^{\mathrm{T}}+B_2W_2+W_2^{\mathrm{T}}B_2^{\mathrm{T}}+\tau\hat{X}+\hat{Y}+\hat{Y}^{\mathrm{T}}+\hat{Q},$$
$$\hat{\varOmega}_{12} = B_2W_1+\hat{P}^{\mathrm{T}}C_2^{\mathrm{T}}B_1^{\mathrm{T}},\ \hat{\varOmega}_{22} = A_1\hat{R}+\hat{R}^{\mathrm{T}}A_1^{\mathrm{T}},$$

then, the singular cascade control system (5) with $w(t) = 0$ and time-delay is admissible, and the desired primary controller gain can be obtained as $K_1 = W_1\hat{R}^{-1}$, the corresponding secondary controller gain is $K_2 = W_2\hat{P}^{-1}$.

Proof According to Theorem 1, inequality (8) is equivalent to

$$\begin{bmatrix} \varOmega_{11} & \varOmega_{12} & P^{\mathrm{T}}A_3-Y & \tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}}) \\ * & \varOmega_{22} & 0 & \tau K_1^{\mathrm{T}}B_2^{\mathrm{T}} \\ * & * & -Q & \tau A_3^{\mathrm{T}} \\ * & * & * & -\tau Z^{-1} \end{bmatrix} < 0, \tag{14}$$

where

$$\varOmega_{11} = P^{\mathrm{T}}(A_2+B_2K_2)+(A_2+B_2K_2)^{\mathrm{T}}P+\tau X+Y+Y^{\mathrm{T}}+Q,$$
$$\varOmega_{12} = P^{\mathrm{T}}B_2K_1+C_2^{\mathrm{T}}B_1^{\mathrm{T}}R,\ \varOmega_{22} = R^{\mathrm{T}}A_1+A_1^{\mathrm{T}}R.$$

Then, if there exist a non-singular matrix P, symmetric positive-definite matrices R, Q, Z, matrices X, Y satisfying (6), (7) and (8), the singular cascade control system (5) with $w(t) = 0$ is admissible.

Pre- and post-multiplying (14) by $\mathrm{diag}\{P^{-\mathrm{T}}, R^{-\mathrm{T}}, P^{-\mathrm{T}}, I\}$ and its transpose, respectively, we obtain

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \tau P^{-\mathrm{T}}(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}}) \\ * & \Omega_{22} & 0 & \tau R^{-\mathrm{T}}K_1^{\mathrm{T}}B_2^{\mathrm{T}} \\ * & * & -P^{-\mathrm{T}}QP^{-1} & \tau P^{-\mathrm{T}}A_3 \\ * & * & * & -\tau Z^{-1} \end{bmatrix} < 0,$$

where

$$\Omega_{11} = (A_2 + B_2K_2)P^{-1} + P^{-\mathrm{T}}(A_2 + B_2K_2)^{\mathrm{T}} + \tau P^{-\mathrm{T}}XP^{-1} + P^{-\mathrm{T}}YP^{-1} + P^{-\mathrm{T}}Y^{\mathrm{T}}P^{-1} + P^{-\mathrm{T}}QP^{-1},$$
$$\Omega_{12} = B_2K_1R^{-1} + P^{-\mathrm{T}}C_2^{\mathrm{T}}B_1^{\mathrm{T}},$$
$$\Omega_{22} = A_1R^{-1} + R^{-\mathrm{T}}A_1^{\mathrm{T}},$$
$$\Omega_{13} = A_3P^{-1} - P^{-\mathrm{T}}YP^{-1}.$$

Then, defining $\hat{P}=P^{-1}$, $\hat{R}=R^{-1}$, $\hat{X}=P^{-\mathrm{T}}XP^{-1}$, $\hat{Y} = P^{-\mathrm{T}}YP^{-1}$, $\hat{Q}=P^{-\mathrm{T}}QP^{-1}$, $\hat{Z}=Z^{-1}$, $W_1 = K_1R^{-1}$, $W_2 = K_2P^{-1}$, etc, we can obtain (13) easily.

Pre- and post-multiplying both sides of inequality (7) by $\mathrm{diag}\{P^{-\mathrm{T}}, P^{-\mathrm{T}}\}$ and its transpose, we obtain

$$\begin{bmatrix} \hat{X} & \hat{Y} \\ \hat{Y}^{\mathrm{T}} & P^{-\mathrm{T}}E^{\mathrm{T}}\hat{Z}^{-1}EP^{-1} \end{bmatrix} \geqslant 0. \tag{15}$$

From the proof of Theorem 1, we have

$$P^{-\mathrm{T}}E^{\mathrm{T}}\hat{Z}^{-1}EP^{-1} = \begin{bmatrix} P_1^{-\mathrm{T}}Z_{11}P_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0.$$

Using Lemma 3, we obtain

$$\hat{P}^{\mathrm{T}}E^{\mathrm{T}}+E\hat{P}-E^{\mathrm{T}}\hat{Z}E \leqslant P^{-\mathrm{T}}E^{\mathrm{T}}\hat{Z}^{-1}EP^{-1}. \tag{16}$$

Then, if (12) holds, we obtain the inequality (15).

According to Theorem 1, we can obtain that the closed-loop singular cascade control system (5) with $w(t) = 0$ and time-delay τ is regular, impulse-free and stable, and the desired primary controller gain is $K_1 = W_1\hat{R}^{-1}$, the corresponding secondary controller gain is $K_2 = W_2\hat{P}^{-1}$. This completes the proof.

Remark 1 In Theorem 2, (13) is a linear matrix inequality whose feasibility can be checked by using MATLAB LMI toolbox, (11) and (12) can also be calculated by MATLAB LMI toolbox. Then, for singular cascade control systems (5) with $w(t) = 0$ and time-delay, the controller parameters can be obtained and the desired primary controller and the corresponding secondary controller can be designed directly. However, the existing results in the literature about singular systems can not obtain the method of controller design for the singular cascade control systems, especially for the systems with time-delay.

4 H-infinity control of singular cascade control systems with time-delay and disturbances

In this section, based on Theorem 1 and Theorem 2, let us now concentrate to the controller design of the form (3) and (4), which guarantees system (5) regular, impulse-free and stable with H-infinity performance γ.

The following definition will be given for the proof of Theorem 3 and Theorem 4.

Definition 2 Given a certain constant $\gamma > 0$, if there exist state feedback control laws (3) and (4), which make the singular cascade control systems (5) regular, impulse-free and stable, and the primary plant output $y_1(t)$ and the disturbance $w(t)$ are subject to the H-infinity norm bounded constraint $\|y_1(t)\|_2 \leqslant \gamma\|w(t)\|_2$ under zero initial conditions, then there exists the H-infinity stabilization control law for systems (5), and the disturbance attenuation degree of the system is γ.

To facilitate the analysis of the problem, let's consider the closed-loop singular cascade control systems

$$\begin{cases} \dot{x}_1(t)=A_1x_1(t)+B_1C_2x_2(t)+B_1C_4w(t), \\ E\dot{x}_2(t) = \\ A_2x_2(t) + A_3x_2(t-\tau)+ \\ B_2K_1x_1(t) + B_2K_2x_2(t) + B_3w(t), \\ y_1(t) = C_1x_1(t) + C_3w(t), \end{cases} \tag{17}$$

where $y_1(t)$ is the output, $w(t)$ is the disturbance input that belongs to $L_2(0,\infty]$. $A_1, B_1, A_2, B_2, A_3, B_3, C_1, C_2, C_3$ and C_4 are known real constant matrices with appropriate dimensions.

Theorem 3 For a given state feedback gain K and constant $\gamma > 0$, if there exist a non-singular matrix P, symmetric positive-definite matrices R, Q, Z, and matrices X, Y, such that (6), (7) and the following LMI hold

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & P^{\mathrm{T}}B_3 & \Omega_{15} & 0 \\ * & \Omega_{22} & 0 & R^{\mathrm{T}}B_1C_4 & \tau K_1^{\mathrm{T}}B_2^{\mathrm{T}}Z & C_1^{\mathrm{T}} \\ * & * & -Q & 0 & \tau A_3^{\mathrm{T}}Z & 0 \\ * & * & * & -\gamma^2 I & \tau B_3^{\mathrm{T}} & C_3^{\mathrm{T}} \\ * & * & * & * & -\tau Z & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{18}$$

where

$$\Omega_{11} = P^{\mathrm{T}}(A_2 + B_2K_2) + (A_2 + B_2K_2)^{\mathrm{T}}P + \tau X + Y + Y^{\mathrm{T}} + Q,$$
$$\Omega_{12}=P^{\mathrm{T}}B_2K_1 + C_2^{\mathrm{T}}B_1^{\mathrm{T}}R,\ \Omega_{22}=R^{\mathrm{T}}A_1 + A_1^{\mathrm{T}}R,$$
$$\Omega_{13} = P^{\mathrm{T}}A_3 - Y,\ \Omega_{15} = \tau(A_2^{\mathrm{T}} + K_2^{\mathrm{T}}B_2^{\mathrm{T}})Z,$$

then, the system (17) is regular, impulse-free, and stable with H-infinity performance γ for time-delay τ.

Proof By choosing a Lyapunov functional as (9) and according to Theorem 1, the closed-loop singular cascade control system (17) with $w(t) = 0$ and time-delay is admissible if (6)−(8) hold.

Noting that

$$y_1^{\mathrm{T}}(t)y_1(t) - \gamma^2w^{\mathrm{T}}(t)w(t) = [C_1x_1(t) + C_3w(t)]^{\mathrm{T}}[C_1x_1(t) + C_3w(t)] - \gamma^2w^{\mathrm{T}}(t)w(t),$$

We obtain

$$\dot{V}(t) + y_1^{\mathrm{T}}(t)y_1(t) - \gamma^2w^{\mathrm{T}}(t)w(t) \leqslant \hat{\xi}^{\mathrm{T}}(t)\hat{\Omega}\hat{\xi}(t),$$

where

$$\hat{\xi}(t) = \begin{bmatrix} x_2^{\mathrm{T}}(t) & x_1^{\mathrm{T}}(t) & x_2^{\mathrm{T}}(t-\tau) & w^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}},$$

$$\hat{\Omega}=\begin{bmatrix}\hat{\Omega}_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14}\\ * & \Omega_{22} & 0 & \Omega_{24}\\ * & * & \tau A_3^{\mathrm{T}}ZA_3-Q & \Omega_{34}\\ * & * & * & \Omega_{44}\end{bmatrix},$$

and

$$\begin{aligned}\hat{\Omega}_{11}=&P^{\mathrm{T}}(A_2+B_2K_2)+\tau X+Y+Y^{\mathrm{T}}+\\&Q+\tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})Z(A_2+B_2K_2)+\\&(A_2+B_2K_2)^{\mathrm{T}}P,\\ \Omega_{12}=&P^{\mathrm{T}}B_2K_1+C_2^{\mathrm{T}}B_1^{\mathrm{T}}R+\\&\tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})ZB_2K_1,\\ \Omega_{13}=&P^{\mathrm{T}}A_3-Y+\tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})ZA_3,\\ \Omega_{22}=&R^{\mathrm{T}}A_1+A_1^{\mathrm{T}}R+\tau K_1^{\mathrm{T}}B_2^{\mathrm{T}}ZB_2K_1+C_1^{\mathrm{T}}C_1,\\ \Omega_{14}=&P^{\mathrm{T}}B_3+\tau(A_2^{\mathrm{T}}+K_2^{\mathrm{T}}B_2^{\mathrm{T}})ZB_3,\\ \Omega_{24}=&R^{\mathrm{T}}B_1C_4+\tau K_1^{\mathrm{T}}B_2^{\mathrm{T}}ZB_3+C_1^{\mathrm{T}}C_3,\\ \Omega_{34}=&\tau A_3^{\mathrm{T}}ZB_3,\ \Omega_{44}=C_3^{\mathrm{T}}C_3+\tau B_3^{\mathrm{T}}ZB_3-\gamma^2I.\end{aligned}$$

Using Lemma 2, $\hat{\Omega}<0$ is equivalent to LMI (18).

To show the singular cascade control systems to have H-infinity performance γ, we consider the following index

$$J_{y_1w}=\int_0^{\infty}[y_1^{\mathrm{T}}(t)y_1(t)-\gamma^2w^{\mathrm{T}}(t)w(t)]\mathrm{d}t.$$

Noting that, we have

$$\begin{aligned}&J_{y_1w}=\\&\int_0^{\infty}[y_1^{\mathrm{T}}(t)y_1(t)-\gamma^2w^{\mathrm{T}}(t)w(t)+\dot{V}(t)]\mathrm{d}t-\\&\lim_{t\to\infty}V(t)+V(0)=\\&\int_0^{\infty}\hat{\xi}^{\mathrm{T}}(t)\hat{\Omega}\hat{\xi}(t)\mathrm{d}t-\lim_{t\to\infty}V(t)+V(0).\end{aligned}$$

Since $\hat{\Omega}<0$, $V(0)=0$ under zero initial condition and $\lim\limits_{t\to\infty}V(t)\geqslant 0$, then the following LMI holds $J_{zw}<0$, this give the desired result $\|y_1(t)\|_2\leqslant\|w(t)\|_2$, which implies that the closed-loop singular cascade control system (17) has H-infinity performance γ.

Moreover, Theorem 3 implies that the singular cascade control system (17) with $w(t)=0$ is regular, impulse-free and stable. This completes the proof.

Now, based on Theorem 3, we consider the problem of H-infinity controller design for system (17).

Theorem 4 For a given constant $\gamma>0$, if there exist a non-singular matrix $\hat{P}$, symmetric positive-definite matrices $\hat{R},\hat{Q},\hat{Z}$, and matrices $\hat{X},\hat{Y},W_1,W_2$, such that (11)−(12) and the following LMI hold:

$$\begin{bmatrix}\hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & B_3 & \hat{\Omega}_{15} & 0\\ * & \hat{\Omega}_{22} & 0 & B_1C_4 & \tau W_1^{\mathrm{T}}B_2^{\mathrm{T}} & \hat{R}^{\mathrm{T}}C_1^{\mathrm{T}}\\ * & * & -\hat{Q} & 0 & \tau\hat{P}^{\mathrm{T}}A_3^{\mathrm{T}} & 0\\ * & * & * & -\gamma^2I & \tau B_3^{\mathrm{T}} & C_3^{\mathrm{T}}\\ * & * & * & * & -\tau\hat{Z} & 0\\ * & * & * & * & * & -I\end{bmatrix}<0, \tag{19}$$

where

$$\begin{aligned}\hat{\Omega}_{11}=&A_2\hat{P}+\hat{P}^{\mathrm{T}}A_2^{\mathrm{T}}+B_2W_2+W_2^{\mathrm{T}}B_2^{\mathrm{T}}+\\&\tau\hat{X}+\hat{Y}+\hat{Y}^{\mathrm{T}}+\hat{Q},\\ \hat{\Omega}_{12}=&B_2W_1+\hat{P}^{\mathrm{T}}C_2^{\mathrm{T}}B_1^{\mathrm{T}},\\ \hat{\Omega}_{13}=&A_3\hat{P}-Y,\ \hat{\Omega}_{15}=\tau\hat{P}^{\mathrm{T}}A_2^{\mathrm{T}}+\tau W_2^{\mathrm{T}}B_2^{\mathrm{T}},\\ \hat{\Omega}_{22}=&A_1\hat{R}+\hat{R}^{\mathrm{T}}A_1^{\mathrm{T}},\end{aligned}$$

then, the system (17) is regular, impulse-free, and stable with H-infinity performance γ for all time-delay τ and external disturbance $w(t)$, and the desired primary controller gain is $K_1=W_1\hat{R}^{-1}$, the corresponding secondary controller gain is $K_2=W_2\hat{P}^{-1}$.

Proof According to Theorem 3, and similar to the proof of Theorem 2, we can obtain (11), (12) and (19) easily. This completes the proof.

5 Simulation example

A simulation example is given to illustrate the effectiveness and applicability of the proposed method.

Example 1 Consider the following singular cascade control system with time-delay and disturbances

$$\begin{cases}\begin{bmatrix}1&0\\0&0\end{bmatrix}\dot{x}_2(t)=\\ \begin{bmatrix}0.3&1\\0.2&0\end{bmatrix}x_2(t)+\begin{bmatrix}0.2&0.1\\-0.3&1\end{bmatrix}\times\\ x_2(t-\tau)+\begin{bmatrix}0.5\\1\end{bmatrix}u_2(t)+\begin{bmatrix}-0.4\\0.1\end{bmatrix}w(t),\\ y_2(t)=[0\ \ 0.1]x_2(t)+0.1w(t).\end{cases}$$

The state-space representation of the primary plant is

$$\begin{cases}\dot{x}_1(t)=\begin{bmatrix}0.5&3\\-1&-2\end{bmatrix}x_1(t)+\begin{bmatrix}0\\-0.1\end{bmatrix}y_2(t),\\ y_1(t)=[-0.3\ \ 0.2]x_1(t)+0.2w(t),\end{cases}$$

where $\tau=1$, $\gamma=2$. The controller is adopted in the form (4). By using the MATLAB LMI toolbox and according to Theorem 4, we obtain the corresponding primary controller gain $K_1=[0.0021\quad 0.0038]$, and the desired secondary controller gain $K_2=[-2.8032\ -2.6799]$.

However, all the methods of controller design for singular systems in the literature are not feasible for this type of systems.

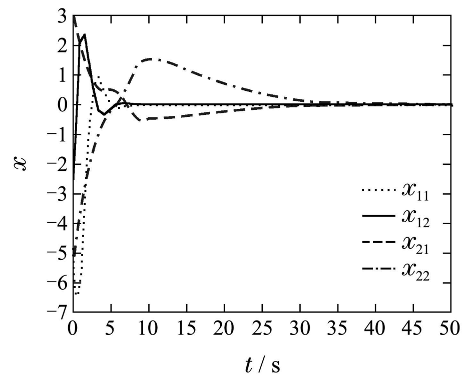

Fig. 2 State response for example 1

From Fig.2, it can be seen that the system is regular, impulse-free and stable with H-infinity performance $\gamma = 2$ when the initial condition $x(t_0) = [-4.45, -2.5, 3, -5.12]$, $t_0 \in [-1, 0]$, and the disturbance input $w(t) = \begin{cases} \sin t, & 0\,\mathrm{s} < t \leqslant 10\,\mathrm{s}, \\ 0, & \text{otherwise}. \end{cases}$

6 Conclusions

This paper is concerned with the problems of stability analysis and H-infinity control for a class of singular cascade control systems with time-delay and disturbances. It should be pointed out that it is the first attempt to investigate the cascade control theoretically and develop a novel computational approach for singular systems, especially for the systems with time-delay. Finally, a simulation example is performed to illustrate the applicability and usefulness of the developed theoretical results.

References:

[1] DAI L. *Singular Control Systems* [M]. Berlin: Springer-Verlag, 1989.

[2] XU S Y, DOOREN P V, STEFAN R, et al. Robust stability and stabilization for singular systems with time delay and parameter uncertainty [J]. *IEEE Transactions on Automatic Control*, 2002, 47(7): 1122 – 1128.

[3] LU R, SU H, CHU J, et al. Reduced-order H-infinity filtering for discrete-time singular systems with lossy measurements [J]. *IET Control Theory & Applications*, 2010, 4(1): 151 – 163.

[4] KIM J H. Delay-dependent robust H-infinity filtering for uncertain discrete-time singular systems with interval time-varying delay [J]. *Automatica*, 2010, 46(3): 591 – 597.

[5] YUE D, HAN Q L. Robust H-infinity filter design of uncertain descriptor systems with discrete and distributed delays [J]. *IEEE Transactions on Signal Processing*, 2004, 52(11): 3200 – 3212.

[6] 沃松林, 史国栋, 邹云. 不确定广义大系统分散鲁棒H-infinity保性能控制 [J]. 控制理论与应用, 2009, 26(9): 1035 – 1040.
(WO Songlin, SHI Guodong, ZOU Yun. Decentralized robust H-infinity and cost-guaranteed control for uncertain singular large-scale systems [J]. *Control Theory & Applications*, 2009, 26(9): 1035 – 1040.)

[7] MA S P, ZHANG C H, CHENG Z L. Delay-dependent robust H-infinity control for uncertain discrete-time singular systems with time-delays [J]. *Journal of Computational and Applied Mathematics*, 2008, 217(1): 194 – 211.

[8] DU Z P, ZHANG Q L, LIU L L. Delay-dependent stabilization of uncertain singular systems with multiple time-delays [J]. *International Journal of Innovative Computing, Information and Control*, 2009, 5(6): 1655 – 1664.

[9] WU Z G, SHI P, SU H Y, et al. Delay-dependent stability analysis for discrete-time singular markovian jump systems with time-varying delay [J]. *International Journal of Systems Science*, 2012, 43(11): 2095 – 2106.

[10] FRANKS R G, WORLEY C W. Quantitative analysis of cascade control [J]. *Industrial & Engineering Chemistry*, 1956, 48(6): 1074 – 1079.

[11] XU M, LI S, CAI W. Cascade generalized predictive control strategy for boiler drum level [J]. *ISA Transactions*, 2005, 44(3): 399 – 411.

[12] HAGIWARA M, MAEDA R, AKAGI H. Control and analysis of the modular multilevel cascade converter based on double-star chopper-cells (mmcc-dscc) [J]. *IEEE Transactions on Power Electronics*, 2011, 26(6): 1649 – 1658.

[13] TEO T, LAKSHMINARAYANAN S, RANGAIAH G. Performance assessment of cascade control systems [J]. *Journal of the Institution of Engineers*, 2005, 45(6): 27 – 38.

[14] HUANG C, BAI Y, LIU X. H-infinity state feedback control for a class of networked cascade control systems with uncertain delay [J]. *IEEE Transactions on Industrial Informatics*, 2010, 6(1): 62 – 72.

[15] HUANG C, BAI Y, LI X. Fundamental issues in networked cascade control systems [C] //*Proceedings of IEEE International Conference on Automation and Logistics*. Qingdao: IEEE, 2008: 3014 – 3018.

[16] KOLMANOVSKY I, MCCLAMROCH N. Hybrid feedback laws for a class of cascade nonlinear control systems [J]. *IEEE Transactions on Automatic Control*, 1996, 41(9): 1271 – 1282.

[17] WAI R J, KUO M A, LEE J D. Design of cascade adaptive fuzzy sliding-mode control for nonlinear two-axis inverted-pendulum servomechanism [J]. *IEEE Transactions on Fuzzy Systems*, 2008, 16(5): 1232 – 1244.

[18] GUO C Y, SONG Q, CAI W J. A neural network assisted cascade control system for air handling unit [J]. *IEEE Transactions on Industrial Electronics*, 2007, 54(1): 620 – 628.

[19] CHEN W H, ZHENG W X, SHEN Y J. Delay-dependent stochastic stability and H-infinity control of uncertain neutral stochastic systems with time delay [J]. *IEEE Transactions on Automatic Control*, 2009, 54(7): 1660 – 1667.

[20] BOUKAS E K. Singular linear systems with delay: H-infinity stabilization [J]. *Optimal Control Applications and Methods*, 2007, 28(4): 259 – 274.

[21] SHU Z, LAM J. Exponential estimates and stabilization of uncertain singular systems with discrete and distributed delays [J]. *International Journal of Control*, 2008, 81(6): 865 – 882.

[22] MOON Y, PARK P, KWON W, et al. Delay-dependent robust stabilization of uncertain state-delayed systems [J]. *International Journal of Control*, 2001, 74(14): 1447 – 1455.

[23] PETERSEN I. A stabilization algorithm for a class of uncertain linear systems [J]. *Systems & Control Letters*, 1987, 8(4): 351 – 357.

[24] XIE L, DE SOUZA C E. Robust H-infinity control for linear systems with norm-bounded time-varying uncertainty [J]. *IEEE Transactions on Automatic Control*, 1992, 37(8): 1188 – 1191.

作者简介:

杜昭平 (1978–), 男, 博士, 副教授, 目前研究方向为复杂网络控制系统、时滞广义系统, E-mail: duzhaoping98@163.com;

岳　东 (1964–), 男, 教授, 博士生导师, 目前研究方向为网络控制系统分析与综合、多智能体, E-mail: medongy@vip.163.com;

胡松林 (1982–), 男, 博士, 讲师, 目前研究方向为网络控制系统、触发机制系统, E-mail: songlin621@126.com;

张美生 (1979–), 男, 硕士, 讲师, 目前研究方向为网络控制系统, E-mail: meisheng@sdjzu.edu.cn.