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随机系统中能容忍连续丢包和测量时延的卡尔曼滤波

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摘要: 通过转换原线性系统到能容忍连续丢包和测量时延的随机参数系统, 推导了一个最优线性滤波器. 给出一个仿真例子, 比较已存在的结果, 仿真结果表明被提出的线性滤波器有优越的性能. 然而, 该滤波器不能应用于非线性系统. 从应用角度, 为非线性系统提出了一个增强型的滤波器. 而且, 该增强型的滤波器能成功地应用于不可靠的无线传感器网络场景来跟踪移动目标. 这些滤波器只依靠测量值的达到概率, 而不需要知道某一时刻测量是否接收. 仿真说明了被提出的增强型滤波器不仅能改善实时目标跟踪的鲁棒性, 而且比标准的扩展卡尔曼滤波器能够提供更精确的估计.

关键词: 滤波器设计; 连续丢包; 无线传感器网络; 测量时延

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Kalman filtering for stochastic systems with consecutive packet losses and measurement time delays

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Abstract: An optimal linear filter is derived through transferring the original linear systems to stochastic parameter systems with consecutive packet losses and time delays. A numerical simulation example is performed with results showing that this linear filter has superior performance to other existing approaches. However, the proposed filter cannot be applied to nonlinear systems. From the practical perspective, an enhanced filter is proposed and is extended to nonlinear systems. This enhanced filter has been applied successfully to an unreliable wireless sensor network (WSNs) scenario to track a moving target. The proposed filters depend only on the measurement arrival probability at all time but do not require knowing whether a measurement is received at a specific time instant. Simulations show that the proposed enhanced filter not only improves the robustness for real-time target tracking in WSNs, but also provides more accurate estimations than the standard extended Kalman filter.

Key words: filter design; consecutive packet losses; wireless sensor networks; measurement time delays

1 Introduction

Packet losses, time delays and constrained bandwidth are general problems across unreliable wireless communication links. Especially, in wireless sensor networks (WSNs), sensor nodes are limited in power, computational capacities, and memory[1]. So the research on these problems is more important in both theory and application.

The research on missing measurements can be traced back to Nahi[2] and Hadidi[3]. An optimal recursive filter with missing observations is first developed in [2]. Recently, many results are reported for systems with packet losses, time delays and uncertain observations. Two main methods are popular for modeling these uncertainties. The first one is called Markovian jump linear system approach, where packet losses, time delays, and missing measurements are all modeled as a Markov chain[4–7]. The other is to model the uncertainty by a stochastic Bernoulli binary sequence taking on 1 or 0[8–20]. In Markov chain approach the packet losses have been studied using jump linear systems, which are hybrid systems with model transitions modeled as Markov chains that switch among several discrete models. The filter design with packet losses consists of choosing the switching logic, determining the size of this finite set and assigning the filter gains[5]. Markov chain approach restricts their formulation to the steady-state case and the Kalman gain is constant. Furthermore, the transition probability and state error covariance matrices need to be computed exactly[6–7].

The general case of time-varying Kalman gain is considered and how packet dropouts can affect state estimation

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is discussed in [8–9]. It is illustrated that there exists a certain threshold of the packet dropout rate[8]. The estimation of multiple-input multiple-output dynamic system using Kalman filter is considered over a mobile fading communication channel[10]. The optimal filtering is considered for systems where multiple packets are dropped[11–13,21] in an unreliable network , where the original systems are transferred to the stochastic parameter systems by augmentation of the state and measurement based on a recursive received model[11–12]. Considering multiple packet dropouts in both sensor-to-controller and controller-to-actuator channels, the linear minimum variance filter is proposed using the orthogonality principle[14]. The problem of optimal filtering with random sensor delay, multiple packet dropout and uncertain observation is early investigated by transforming to a stochastic parameter in the system representation[15]. For online computation of filter gains, a Riccati equation approach is applied to develop an adaptive Kalman filtering with random sensor delays, multiple packet dropouts and missing measurements[16]. An innovation analysis approach is used to develop the optimal linear estimators via developing a unified model to describe random sensor delays, multiple packet dropouts and uncertain observations[17]. But the filtering methods in [16–17] only consider one-step time delay. Kalman filtering for linear continuous-time systems with multiple delayed measurements is proposed based on the reorganized innovation analysis approach[18], where packet losses, however, is not considered.

The above existing works dealing with packet losses and measurement time delays have mainly considered in linear systems. Different from the existing works, we develop a class of real-time filters with packet losses and time delays, considering both linear systems and nonlinear systems. We have derived some preliminary results with packet losses[19] in WSNs. In this paper, our main contributions are shown as follows: firstly, filter design for time-invariant systems in our previous work[20] is further extended to time-variant systems in this paper; then, the optimal linear filter is derived in linear systems and further extended to the time-varying nonlinear systems; at last, the designed nonlinear filters are successfully applied to track a moving target in WSNs. The simulation results show the effectiveness and robustness of the proposed methods.

2 Problem formulation

Consider linear time-varying stochastic systems as follows:

$$x(k+1) = A(k)x(k) + w(k), \tag{1a}$$

$$z(k) = C(k)x(k) + \nu(k), \tag{1b}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $z(k) \in \mathbb{R}^m$ is the sensor measurement, $A(k)$ and $C(k)$ are time-varying matrices with appropriate dimensions, $w(k)$ is state noise, $\nu(k)$ is measurement noise.

In WSNs and networked systems, sensor nodes measure the output of the systems at every time step and transmit the measurements to the processing center (the filter) over digital communication network (DCN). We model DCN as a module between the plant and the filter. Random time delays and packet losses are unavoidable across unreliable DCN. To reduce the effect of packet losses without overloading the network traffic too much, each sensor measurement is retransmitted several times. Assume that the largest time delays and packet losses are no more than $N+1$, and there is a packet arriving at the filter at each time step. Unlike our previous work[20], the following packet loss model[21] of the received measurement is adopted:

$$\begin{aligned} y(k) = {} & \gamma_0(k)z(k) + (1-\gamma_0(k))\gamma_1(k)z(k-1) + \\ & \cdots + (1-\gamma_0(k))(1-\gamma_1(k))\cdots \\ & (1-\gamma_{N-1}(k))z(k-N), \end{aligned} \tag{2}$$

where $k \geqslant N$ and $N \geqslant 1$, otherwise, when $k < N$, define $z(-1) = z(-2) = \cdots = z(-N) = 0$; $y(k)$ is the measurement received by the filter; N denotes the number of consecutive packets dropout, and $\gamma_i(k)$ is an independent binary stochastic variable taking 0 or 1 with the identity distribution. Probabilities of $\gamma_i(k)$ are represented by $\mathrm{P}(\gamma_i(k) = 1) = p$, and $\mathrm{P}(\gamma_i(k) = 0) = 1 - p$. We can derive that

$$\begin{cases} \mathrm{E}(\gamma_i(k)) = p, \ \mathrm{E}(\gamma_i^2(k)) = p, \\ \mathrm{E}(\gamma_i(k)(1-\gamma_i(k))) = 0. \end{cases} \tag{3}$$

Interestingly,

$$\gamma_0(k) + (1-\gamma_0(k))\gamma_1(k) + \cdots +$$
$$(1-\gamma_0(k))(1-\gamma_1(k))\cdots(1-\gamma_{N-1}(k)) = 1.$$

It implies that one packet is received at each time step even through there exists time delays or consecutive packet losses. We explain (2) when $N = 2$ in the following. At time step k, we see that $z(k)$ is received if $\gamma_0(k) = 1$, i.e., $y(k) = z(k)$ with the probability p, $z(k-1)$ is received if $\gamma_0(k) = 0$ and $\gamma_1(k) = 1$, i.e., $y(k) = z(k-1)$ with the probability $p(1-p)$, and $z(k-2)$ is received if $\gamma_0(k) = 0$ and $\gamma_1(k) = 0$, i.e., $y(k) = z(k-2)$ with the probability $(1-p)^2$. It is worth noting that measurement $z(k)$ is received probably on time by the filter, delayed or lost over DCN. The following Table 1 describes these three cases.

Table 1 Data transmission in DCN

k	1	2	3	4	5	6	7	8	9	10	11	12
$\gamma_0(k)$	1	1	0	0	1	1	1	0	0	0	1	1
$\gamma_1(k)$	$\star$	$\star$	1	1	$\star$	$\star$	$\star$	1	0	0	$\star$	$\star$
$y(k)$	z_1	z_2	z_2	z_3	z_5	z_6	z_7	z_7	z_7	z_8	z_{11}	z_{12}

From Table 1 ("$\star$" denotes arbitrary value taking 0 or 1), we see that $z(1)$, $z(2)$, $z(5)$, $z(6)$, $z(7)$, $z(11)$, and $z(12)$ are received on time, $z(3)$ and $z(8)$ are delayed, i.e., $z(3)$ and $z(8)$ are delayed one and two time steps respectively, $z(4)$, $z(9)$ and $z(10)$ are lost. Furthermore, $z(9)$ and $z(10)$ are lost consecutively. In addition, $z(2)$ and $z(7)$ are re-received, i.e., $z(2)$ and $z(7)$ are re-retransmitted one and two times respectively. The largest number of packet losses and time delays is $N = 2$ from Table 1. So model (2) describes effectively the consecutive packet losses and time delays.

Substituting (1) into (2), we have that

$$y(k) = \sum_{i=0}^{N} \alpha_i(k)[C(k-i)x(k-i) + \nu(k-i)], \tag{4}$$

where

$$\alpha_0(k) = \gamma_0(k), \tag{5a}$$

$$\alpha_i(k) = \gamma_i(k) \prod_{j=0}^{i-1} (1-\gamma_j(k)),\ 1 \leqslant i \leqslant N-1, \tag{5b}$$

$$\alpha_N(k) = \prod_{j=0}^{N-1} (1 - \gamma_j(k)). \tag{5c}$$

Our objective is to design an optimal minimum variance filter (OMVF) $\hat{x}(k+1|k+1)$ of state $x(k+1)$ based on the received measurements $y(0), y(1), \cdots, y(k+1)$ with the following form:

$$\hat{x}(k+1|k+1) = F(k+1)\hat{x}(k|k) + K(k+1)y(k+1), \tag{6}$$

by minimizing error covariance matrix $\mathrm{P}(k+1|k+1)$, where $\mathrm{P}(k+1|k+1)$ is defined as

$$\mathrm{P}(k+1|k+1) := \mathrm{E_x E_\gamma}[x(k+1) - \hat{x}(k+1|k+1)] \times [x(k+1) - \hat{x}(k+1|k+1)]^{\mathrm{T}}, \tag{7}$$

$F(k+1)$ and $K(k+1)$ are the unknown variables to be designed, $\mathrm{E_x}$ is the expectation based on $x(k), w(k)$ and $\nu(k)$, and $\mathrm{E_\gamma}$ is the expectation with respect to $\gamma(k)$.

We first show some assumptions and Lemmas before deriving main results.

Assumption 1 State noise $w(k)$ and measurement noise $\nu(k)$ in systems (1) are uncorrelated white noise with zero mean and covariance $Q(k)$ and $R(k)$ respectively.

Assumption 2 Initial state $x(0)$ is independent of $w(k)$ and $\nu(k)$, and satisfying $\mathrm{E}[x(0)] = m_0$,

$$\mathrm{E}[x(0)x^{\mathrm{T}}(0)] = S_0,\ \mathrm{E}[x(0) - m_0][x(0) - m_0]^{\mathrm{T}} = P_0.$$

Lemma 1 For (5), stochastic variable $\alpha_i(k) (i = 0, 1, \cdots, N)$ has properties as follows:

$$\alpha_0 := \mathrm{E}[\alpha_0(k)] = p, \tag{8a}$$

$$\alpha_i := \mathrm{E}[\alpha_i(k)] = p(1-p)^i,\ 1 \leqslant i \leqslant N, \tag{8b}$$

$$\alpha_N := \mathrm{E}[\alpha_N(k)] = (1-p)^N, \tag{8c}$$

$$\mathrm{E}[\alpha_i(k)\alpha_j(k)] = \begin{cases} \alpha_i, & i = j, \\ 0, & i \neq j. \end{cases} \tag{8d}$$

Proof It is easy to obtain (8a) according to (3) and (5). For (8b),

$$\alpha_i = \mathrm{E}[\alpha_i(k)] = \mathrm{E}[\gamma_i(k) \prod_{l=0}^{i-1} (1 - \gamma_l(k))] = p(1-p)^i.$$

If i=j, $\mathrm{E}[\alpha_i(k)\alpha_j(k)] = \mathrm{E}[\alpha_i^2(k)]$=$\mathrm{E}[\alpha_i(k)] = \alpha_i$; if $i < j$,

$$\mathrm{E}[\alpha_i(k)\alpha_j(k)] = \mathrm{E}[\gamma_i(k) \prod_{l=0}^{i-1} (1 - \gamma_l(k))][\gamma_j(k) \prod_{l=0}^{j-1} (1 - \gamma_l(k))] = \mathrm{E}[\prod_{l=0}^{i-1} (1 - \gamma_l(k))^2]\mathrm{E}[\gamma_i(k)(1 - \gamma_i(k))] \times \mathrm{E}[\gamma_j(k) \prod_{l=i+1}^{j-1} (1 - (\gamma_l(k))].$$

We recall $\mathrm{E}[\gamma_i(k)(1 - \gamma_i(k))] = 0$ in equation (3). So $\mathrm{E}[\alpha_i(k)\alpha_j(k)] = 0$. If $i > j$, we can derive similarly $\mathrm{E}[\alpha_i(k)\alpha_j(k)] = 0$. This proof is completed.

Lemma 2 For stochastic systems (1) and packet loss model (2), we have the following equations:

$$\mathrm{E_x E_\gamma}[(x(k+1) - \hat{x}(k+1|k+1))y^{\mathrm{T}}(t)] = 0, \tag{9}$$

$$\mathrm{E_x E_\gamma}[x(k)y^{\mathrm{T}}(t)] = \mathrm{E_x E_\gamma}[\hat{x}(k|k))y^{\mathrm{T}}(t)], \tag{10}$$

and

$$\mathrm{P}(k+1|k+1) = \mathrm{E_x E_\gamma}[x(k+1) - \hat{x}(k+1|k+1)]x^{\mathrm{T}}(k+1), \tag{11}$$

where $t = 0, 1, \cdots, k, \cdots$.

Proof Define estimation error $\tilde{x}(k+1) := x(k+1) - \hat{x}(k+1|k+1)$. Because $\tilde{x}(k+1)$ and $y(t)$ are orthogonal, $\mathrm{E_x E_\gamma}\tilde{x}(k+1)y^{\mathrm{T}}(t) = 0$, where $t = 0, 1, \cdots, k, \cdots$. We can obtain (9). It can derive (10) by (9). Because $\hat{x}(k+1|k+1)$ is linear function of $y(0), y(1), \cdots, y(k+1)$, it is derived that

$$\mathrm{E_x E_\gamma}[x(k+1) - \hat{x}(k+1|k+1)]\hat{x}^{\mathrm{T}}(k+1|k+1) = 0.$$

Thus, we have

$$\mathrm{P}(k+1|k+1) = \mathrm{E_x E_\gamma}[x(k+1) - \hat{x}(k+1|k+1)] \times [x(k+1) - \hat{x}(k+1|k+1)]^{\mathrm{T}} = \mathrm{E_x E_\gamma}[x(k+1) - \hat{x}(k+1|k+1)]x^{\mathrm{T}}(k+1). \tag{12}$$

This proof is completed.

3 Filter design with consecutive packet losses and random time delays

Our main results are derived based on the minimum error covariance matrix for linear stochastic systems in this section. Then the design method is extended to the nonlinear case for practical applications in WSNs.

3.1 Filter design for linear systems

Theorem 1 For time-varying stochastic systems (1) satisfying Assumptions 1–2, the parameters of OMVF are shown as follows:

$$F(k+1) = A(k) - K(k+1)J, \tag{13}$$

$$K(k+1) = [A(k)\mathrm{P}(k|k)J^{\mathrm{T}} + \alpha_0 Q(k)C^{\mathrm{T}}(k+1)] \times [J\mathrm{P}(k|k)J^{\mathrm{T}} - JS(k)J^{\mathrm{T}} + \bar{S} + \bar{Q} + \bar{R}]^{-1}, \tag{14}$$

with

$$\bar{S} = \alpha_0 C(k+1)A(k)S(k)A^{\mathrm{T}}(k)C^{\mathrm{T}}(k+1) + \alpha_1 C(k)S(k)C^{\mathrm{T}}(k) + \sum_{i=2}^{N} \alpha_i C(k+1-i)(\prod_{j=2}^{i} A(k+1-j))^{-1}S(k) \times (\prod_{j=2}^{i} A(k+1-j))^{-\mathrm{T}}C^{\mathrm{T}}(k+1-i),$$

$$\bar{Q} = \alpha_0 C(k+1)Q(k)C^{\mathrm{T}}(k+1) + \sum_{i=2}^{N} \alpha_i C(k+1-i)[\sum_{j=2}^{i} (\prod_{l=j}^{i} A(k+1-l))^{-1} \times Q(k+1-j)(\prod_{l=j}^{i} A(k+1-l))^{-\mathrm{T}}] \times$$

$C^{\mathrm{T}}(k+1-i),$

and

$$\bar{R}=\sum_{i=0}^{N}\alpha_i R(k+1-i),$$
$$\mathrm{P}(k+1|k+1)=$$
$$F(k+1)\mathrm{P}(k|k)A^{\mathrm{T}}(k)+$$
$$[I_n-\alpha_0 K(k+1)C(k+1)]Q(k), \tag{15}$$
$$J=\alpha_0 C(k+1)A(k)+\alpha_1 C(k)+$$
$$\sum_{i=2}^{N}\alpha_i C(k+1-i)(\prod_{j=2}^{i}A(k+1-j))^{-1}, \tag{16}$$
$$S(k+1)=A(k)S(k)A^{\mathrm{T}}(k)+Q(k), \tag{17}$$

where $\hat{x}(0|0)=m_0,\ \mathrm{P}(0|0)=P_0,\ S(0)=S_0$.

Proof The key derivations of Theorem 1 are to seek appropriate $F(k+1)$ and $K(k+1)$ such that the error covariance matrix $\mathrm{P}(k+1|k+1)$ is minimal. By iterations in (1a), we obtain

$$x(k+1-i)=$$
$$(\prod_{j=2}^{i}A(k+1-j))^{-1}x(k)-$$
$$\sum_{j=2}^{i}(\prod_{l=j}^{i}A(k+1-l))^{-1}w(k+1-j), \tag{18}$$

where assume that $A(k)$ is invertible. Substituting (1a) (6) and (4) into (9), and applying to Lemma 2, it is derived that

$$\mathrm{E_x E_\gamma}[x(k+1)-\hat{x}(k+1|k+1)]y^{\mathrm{T}}(t)=0.$$

Taking $t=k-N$ and applying Assumptions 1–2, we have

$$\mathrm{E}_\gamma\{A(k)-F(k+1)-$$
$$K(k+1)[\alpha_0(k+1)C(k+1)A(k)+$$
$$\alpha_1(k+1)C(k)+\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)\times$$
$$(\prod_{j=2}^{i}A(k+1-j))^{-1}]\mathrm{E_x}x(k)\times$$
$$[\sum_{i=0}^{N}\alpha_i(k-N)C(k-N-i)x(k-N-i)]^{\mathrm{T}}\}=0.$$

Obviously, $\mathrm{E_x}[x(k)[\sum_{i=0}^{N}\alpha_i(k-N)C(k-N-i)x(k-N-i)]^{\mathrm{T}}]\neq 0$, it follows that $\mathrm{E}_\gamma[A(k)-F(k+1)-K(k+1)[\alpha_0(k+1)C(k+1)A(k)+\alpha_1(k+1)C(k)+\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)(\prod_{j=2}^{i}A(k+1-j))^{-1}]]=0$. Using Lemma 1, equations (13) and (16) are derived from the above equation.

Taking $t=k+1$ for (9), and using (18), we obtain that

$$\mathrm{E_x E_\gamma}[x(k+1)-\hat{x}(k+1|k+1)]y^{\mathrm{T}}(k+1)=$$
$$\mathrm{E_x E_\gamma}\{A(k)x(k)+w(k)-F(k+1)\hat{x}(k|k)-$$
$$K(k+1)[\alpha_0(k+1)C(k+1)(A(k)x(k)+w(k))+$$
$$\alpha_1(k+1)C(k)x(k)+\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)\times$$
$$[(\prod_{j=2}^{i}A(k+1-j))^{-1}x(k)-$$
$$\sum_{j=2}^{i}(\prod_{l=j}^{i}A(k+1-l))^{-1}w(k+1-j)]+$$
$$\sum_{i=0}^{N}\alpha_i(k+1)\nu(k+1-i)]\}\{\alpha_0(k+1)C(k+1)\times$$
$$(A(k)x(k)+w(k))+\alpha_1(k+1)C(k)x(k)+$$
$$\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)[(\prod_{j=2}^{i}A(k+1-j))^{-1}\times$$
$$x(k)-\sum_{j=2}^{i}(\prod_{l=j}^{i}A(k+1-l))^{-1}w(k+1-j)]+$$
$$\sum_{i=0}^{N}\alpha_i(k+1)\nu(k+1-i)\}^{\mathrm{T}}=$$
$$A(k)\mathrm{P}(k|k)J^{\mathrm{T}}+K(k+1)[JS(k)J^{\mathrm{T}}-$$
$$J\mathrm{P}(k|k)J^{\mathrm{T}}-\bar{S}-\bar{Q}-\bar{R}]+$$
$$\alpha_0 Q(k)C^{\mathrm{T}}(k+1)=0.$$

Defining $S(k):=\mathrm{E_x}[x(k)\ \ x^{\mathrm{T}}(k)]$ in the above equation, it follows that

$$\bar{S}:=$$
$$\mathrm{E_x E_\gamma}\{\alpha_0(k+1)C(k+1)A(k)+$$
$$\alpha_1(k+1)C(k)+\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)\times$$
$$(\prod_{j=2}^{i}A(k+1-j))^{-1}\}x(k)x^{\mathrm{T}}(k)\{\alpha_0(k+1)\times$$
$$C(k+1)A(k)+\alpha_1(k+1)C(k)+$$
$$\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)(\prod_{j=2}^{i}A(k+1-j))^{-1}\}^{\mathrm{T}}=$$
$$\alpha_0 C(k+1)A(k)S(k)A^{\mathrm{T}}(k)C^{\mathrm{T}}(k+1)+$$
$$\alpha_1 C(k)S(k)C^{\mathrm{T}}(k)+\sum_{i=2}^{N}\alpha_i C(k+1-i)\times$$
$$(\prod_{j=2}^{i}A(k+1-j))^{-1}S(k)\times$$
$$(\prod_{j=2}^{i}A(k+1-j))^{-\mathrm{T}}C^{\mathrm{T}}(k+1-i),$$
$$\bar{Q}:=$$
$$\mathrm{E}_\gamma\{\alpha_0(k+1)C(k+1)w(k)-$$
$$\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)[\sum_{j=2}^{i}(\prod_{l=j}^{i}A(k+1-$$
$$l))^{-1}w(k+1-j)]\}\{\alpha_0(k+1)C(k+1)w(k)-$$
$$\sum_{i=2}^{N}\alpha_i(k+1)C(k+1-i)\times$$
$$[\sum_{j=2}^{i}(\prod_{l=j}^{i}A(k+1-l))^{-1}w(k+1-j)]\}^{\mathrm{T}}=$$
$$\alpha_0 C(k+1)Q(k)C^{\mathrm{T}}(k+1)+$$
$$\sum_{i=2}^{N}\alpha_i C(k+1-i)[\sum_{j=2}^{i}(\prod_{l=j}^{i}A(k+1-l))^{-1}\times$$
$$Q(k+1-j)(\prod_{l=j}^{i}A(k+1-l))^{-\mathrm{T}}]C^{\mathrm{T}}(k+1-i),$$

$$\bar{R} := \mathrm{E}_\gamma[\sum_{i=0}^{N} \alpha_i(k+1)\nu(k+1-i)] \times [\sum_{i=0}^{N} \alpha_i(k+1)\nu(k+1-i)]^{\mathrm{T}} = \sum_{i=0}^{N} \alpha_i R(k+1-i),$$

where (8d) is used.

The equation (14) is derived from the above equations. Next equations (15) and (17) will be obtained. In terms of Lemma 2 we obtain the error covariance matrix and the state variance matrix as follows:

$$\begin{aligned}&\mathrm{P}(k+1|k+1) = \\ &\mathrm{E_x E_\gamma}[x(k+1) - \hat{x}(k+1|k+1)]x^{\mathrm{T}}(k+1) = \\ &\mathrm{E_x E_\gamma}[A(k)x(k) + w(k) - F(k+1)\hat{x}(k|k) - \\ &K(k+1)[\alpha_0(k+1)C(k+1)x(k+1) + \\ &\alpha_1(k+1)C(k)x(k) + \sum_{i=2}^{N} \alpha_i(k+1) \times \\ &C(k+1-i)x(k+1-i) + \\ &\sum_{i=0}^{N} \alpha_i(k+1)\nu(k+1-i)]][A(k)x(k)+w(k)]^{\mathrm{T}} = \\ &A(k)\mathrm{P}(k|k)A^{\mathrm{T}}(k) - K(k+1)J\mathrm{P}(k|k)A^{\mathrm{T}}(k) + \\ &Q(k) - \alpha_0 K(k+1)C(k+1)Q(k), \\ &S(k+1) = \\ &\mathrm{E_x}[x(k+1)x^{\mathrm{T}}(k+1)] = \\ &\mathrm{E_x}[A(k)x(k) + w(k)][A(k)x(k) + w(k)]^{\mathrm{T}} = \\ &A(k)S(k)A^{\mathrm{T}}(k) + Q(k).\end{aligned}$$

This proof is completed.

Remark 1 OMVF is only suitable for linear time-varying stochastic systems. However, many practice applications are nonlinear cases. For example, state equations and measurement equations usually are nonlinear in target tracking of WSNs. In the following subsection we will derive an enhanced minimum variance filter in order to augment the proposed method to nonlinear case.

3.2 Filter design for nonlinear systems

Consider nonlinear time-varying systems:

$$x(k+1) = f(x(k)) + w(k), \tag{19a}$$

$$z(k) = h(x(k)) + \nu(k), \tag{19b}$$

where $f(\cdot)$ and $h(\cdot)$ are the nonlinear functions with respect to the state $x(k)$.

To obtain state estimation $\hat{x}(k+1|k+1)$, nonlinear function (19a) at estimation $\hat{x}(k|k)$ is expanded by the first-order Taylor series as

$$x(k+1) = f(\hat{x}(k|k)) + f'_{\mathrm{x}}(k)(k)(x(k) - \hat{x}(k|k)) + w(k), \tag{20}$$

where

$$f'_{\mathrm{x}}(k) = \frac{\mathrm{d}f(x(k))}{\mathrm{d}x(k)}|_{x(k)=\hat{x}(k|k)}$$

is Jacobian matrix of the function $f(x(k))$.

Similar to (20), we make linearization of the nonlinear measurement function (19b) by the first-order Taylor series at $\hat{x}(k|k-1)$ and substitute it into (2) yielding

$$y(k+1) = \sum_{i=0}^{N} \alpha_i(k)[h(\hat{x}(k-i|k-i-1)) + H(k-i)(x(k-i) - \hat{x}(k-i|k-i-1)) + \nu(k-i)], \tag{21}$$

where $H(k) = \frac{\mathrm{d}h(x(k))}{\mathrm{d}x(k)}|_{x(k)=\hat{x}(k|k-1)}$ is a Jacobian matrix of $h(x(k))$. Based on the alternative forms (20) and (21) of the nonlinear stochastic systems (19), we obtain the following theorem.

Theorem 2 For time-varying stochastic systems (20)–(21) satisfying Assumptions 1–2, the enhanced minimum variance filter (EMVF) is shown as follows:

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)[y(k+1) - \sum_{i=0}^{N} \alpha_i(k)h(\hat{x}(k+1-i|k-i))], \tag{22}$$

where

$$\hat{x}(k+1|k) = f(\hat{x}(k|k)), \tag{23}$$

$$\begin{aligned}&K(k+1) = \\ &[f'_{\mathrm{x}}(k)\mathrm{P}(k|k)J^{\mathrm{T}} + \alpha_0 Q(k)H^{\mathrm{T}}(k+1)] \times \\ &[J\mathrm{P}(k|k)J^{\mathrm{T}} - JS(k)J^{\mathrm{T}} + \bar{S}(k+1) + \\ &\bar{Q}(k+1) + \bar{R}(k+1)]^{-1},\end{aligned} \tag{24}$$

$$\begin{aligned}&\bar{S}(k+1) = \\ &\alpha_0 H(k+1)f'_{\mathrm{x}}(k)S(k)f'^{\mathrm{T}}_{\mathrm{x}}(k)H^{\mathrm{T}}(k+1) + \\ &\alpha_1 H(k)S(k)H^{\mathrm{T}}(k) + \sum_{i=2}^{N} \alpha_i H(k+1-i) \times \\ &(\prod_{j=2}^{i} f'_{\mathrm{x}}(k+1-j))^{-1}S(k) \times \\ &(\prod_{j=2}^{i} f'_{\mathrm{x}}(k+1-j))^{-\mathrm{T}}H^{\mathrm{T}}(k+1-i), \\ &\bar{Q}(k+1) = \\ &\alpha_0 H(k+1)Q(k)H^{\mathrm{T}}(k+1) + \sum_{i=2}^{N} \alpha_i H(k+ \\ &1-i)[\sum_{j=2}^{i}(\prod_{l=j}^{i} f'_{\mathrm{x}}(k+1-l))^{-1}Q(k+1-j) \times \\ &(\prod_{l=j}^{i} f'_{\mathrm{x}}(k+1-l))^{-\mathrm{T}}]H^{\mathrm{T}}(k+1-i), \\ &\bar{R}(k+1) = \sum_{i=0}^{N} \alpha_i R(k+1-i),\end{aligned}$$

$$\begin{aligned}&\mathrm{P}(k+1|k+1) = \\ &(f'_{\mathrm{x}}(k) - K(k+1)J)\mathrm{P}(k|k)f'^{\mathrm{T}}_{\mathrm{x}}(k) + \\ &[I_n - \alpha_0 K(k+1)H(k+1)]Q(k),\end{aligned} \tag{25}$$

$$J = \alpha_0 H(k+1)f'_{\mathrm{x}}(k) + \alpha_1 H(k) + \sum_{i=2}^{N} \alpha_i H(k+1-i)(\prod_{j=2}^{i} f'_{\mathrm{x}}(k+1-j))^{-1}, \tag{26}$$

$$S(k+1) = f'_{\mathrm{x}}(k)S(k)f'^{\mathrm{T}}_{\mathrm{x}}(k) + Q(k), \tag{27}$$

where $\hat{x}(0|0) = m_0,\ \mathrm{P}(0|0) = P_0,\ S(0) = S_0$.

Proof The proof of Theorem 2 is similar to that of

Theorem 1, so it is omitted.

Remark 2 OMVF and EMVF look like be complex, and in fact, they are a set of iterative formulas and very suitable for computer operation. Moreover, OMVF and EMVF are implemented only depending on the packet arrival probability p or α of stochastic variable $\gamma(k)$ and do not need to know whether the measurement is received at a particular time instant. It is different from the existing works[8–9], where the estimator is computed depending on whether the current measurement is received. That is, OMVF and EMVF are independent on binary stochastic variable $\gamma(k)$ and related only to the probability p.

Remark 3 Specially, when packet loss probability $p \to 1$, we have $\alpha_0 \to 1$ and $\alpha_i \to 0$. So OMVF (or EMVF) approximately reduces to the standard KF (or EKF)[22] without time delays and packet losses. That is, KF (or EKF) is the special case of OMVF (or EMVF); while $\alpha_i = 1 (i = 1, 2, \cdots, N)$, it means that the information from $(k-i)th$ time step is received. OMVF (or EMVF) updates the estimation by the measurement from $(k-i)th$ time step. That is, the filters have constant i time-step delays.

Remark 4 From the packet dropout model (2) and the analysis in Table 1, it is known that N in (2) denotes the largest time-delay steps or the number of the consecutive packet dropout. When N tends to infinite, N is replaced by ∞ and the computation is rather complex.

3.3 Filter design for nonlinear systems

Consider nonlinear time-varying systems

$$x(k+1) = f(x(k)) + w(k), \tag{28a}$$

$$z(k) = h(x(k)) + \nu(k), \tag{28b}$$

where $f(\cdot)$ and $h(\cdot)$ are the nonlinear functions with respect to the state $x(k)$.

To obtain state estimation $\hat{x}(k+1|k+1)$, nonlinear function (19a) at estimation $\hat{x}(k|k)$ is expanded by the first-order Taylor series as

$$x(k+1) = f(\hat{x}(k|k)) + f'_{\mathrm{x}}(k)(k)(x(k) - \hat{x}(k|k)) + w(k), \tag{29}$$

where

$$f'_{\mathrm{x}}(k) = \frac{\mathrm{d}f(x(k))}{\mathrm{d}x(k)}|_{x(k)=\hat{x}(k|k)}$$

is Jacobian matrix of the function $f(x(k))$.

Similar to (20), we make linearization of the nonlinear measurement function (19b) by the first-order Taylor series at $\hat{x}(k|k-1)$ and substitute it into (2) yielding

$$y(k+1) = \sum_{i=0}^{N} \alpha_i(k)[h(\hat{x}(k-i|k-i-1)) + H(k-i)(x(k-i) - \hat{x}(k-i|k-i-1)) + \nu(k-i)], \tag{30}$$

where

$$H(k) = \frac{\mathrm{d}h(x(k))}{\mathrm{d}x(k)}|_{x(k)=\hat{x}(k|k-1)}$$

is Jacobian matrix of $h(x(k))$. Based on the alternative forms (20) and (21) of the nonlinear stochastic systems (19), we obtain the following theorem.

Theorem 2 For time-varying stochastic systems (20)–(21) satisfying Assumption 1 and Assumption 2, the enhanced minimum variance filter (EMVF) is shown as follows:

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)[y(k+1) - \sum_{i=0}^{N} \alpha_i(k) h(\hat{x}(k+1-i|k-i))], \tag{31}$$

where

$$\hat{x}(k+1|k) = f(\hat{x}(k|k)), \tag{32}$$

$$K(k+1) = [f'_{\mathrm{x}}(k)\mathrm{P}(k|k)J^{\mathrm{T}} + \alpha_0 Q(k) H^{\mathrm{T}}(k+1)] \times [J\mathrm{P}(k|k)J^{\mathrm{T}} - JS(k)J^{\mathrm{T}} + \bar{S}(k+1) + \bar{Q}(k+1) + \bar{R}(k+1)]^{-1}, \tag{33}$$

$$\bar{S}(k+1) = \alpha_0 H(k+1) f'_{\mathrm{x}}(k) S(k) f'^{\mathrm{T}}_{\mathrm{x}}(k) H^{\mathrm{T}}(k+1) + \alpha_1 H(k) S(k) H^{\mathrm{T}}(k) + \sum_{i=2}^{N} \alpha_i H(k+1-i) \times (\prod_{j=2}^{i} f'_{\mathrm{x}}(k+1-j))^{-1} S(k) \times (\prod_{j=2}^{i} f'_{\mathrm{x}}(k+1-j))^{-\mathrm{T}} H^{\mathrm{T}}(k+1-i),$$

$$\bar{Q}(k+1) = \alpha_0 H(k+1) Q(k) H^{\mathrm{T}}(k+1) + \sum_{i=2}^{N} \alpha_i H(k+1-i)[\sum_{j=2}^{i} (\prod_{l=j}^{i} f'_{\mathrm{x}}(k+1-l))^{-1} Q(k+1-j) \times (\prod_{l=j}^{i} f'_{\mathrm{x}}(k+1-l))^{-\mathrm{T}}] H^{\mathrm{T}}(k+1-i),$$

$$\bar{R}(k+1) = \sum_{i=0}^{N} \alpha_i R(k+1-i),$$

$$\mathrm{P}(k+1|k+1) = (f'_{\mathrm{x}}(k) - K(k+1)J)\mathrm{P}(k|k) f'^{\mathrm{T}}_{\mathrm{x}}(k) + [I_n - \alpha_0 K(k+1) H(k+1)] Q(k), \tag{34}$$

$$J = \alpha_0 H(k+1) f'_{\mathrm{x}}(k) + \alpha_1 H(k) + \sum_{i=2}^{N} \alpha_i H(k+1-i) (\prod_{j=2}^{i} f'_{\mathrm{x}}(k+1-j))^{-1}, \tag{35}$$

$$S(k+1) = f'_{\mathrm{x}}(k) S(k) f'^{\mathrm{T}}_{\mathrm{x}}(k) + Q(k), \tag{36}$$

where $\hat{x}(0|0) = m_0,\ \mathrm{P}(0|0) = P_0,\ S(0) = S_0$.

Proof The proof of Theorem 2 is similar to that of Theorem 1, so it is omitted.

Remark 5 OMVF and EMVF look like be complex, and in fact, they are a set of iterative formulas and very suitable for computer operation. Moreover, OMVF and EMVF are implemented only depending on the packet arrival probability p or α of stochastic variable $\gamma(k)$ and do not need to know whether the measurement is received at a particular time instant. It is different from the existing works[8–9], where the estimator is computed depending on whether the current measurement is received. That is, OMVF and EMVF are independent on binary stochastic variable $\gamma(k)$ and related only to the probability p.

Remark 6 Specially, when packet loss probability $p \to$

1, we have $\alpha_0 \to 1$ and $\alpha_i \to 0$. So OMVF (or EMVF) approximately reduces to the standard KF (or EKF)[22] without time delays and packet losses. That is, KF (or EKF) is the special case of OMVF (or EMVF); while $\alpha_i = 1 (i = 1, 2, \cdots, N)$, it means that the information from $(k-i)\text{th}$ time step is received. OMVF (or EMVF) updates the estimation by the measurement from $(k-i)\text{th}$ time step. That is, the filters have constant i time-step delays. However, from practice perspective, when time-delay steps are more than a given bound N, it is considered that the data are invalid even though data are received by the filters in the application of WSNs, because these data can not improve estimation accuracy.

Remark 7 Similar to [21], when

$$\alpha_i(t) = \frac{1}{N+1-i},$$

it is able to derive

$$\alpha_i = \frac{1}{N+1-i},\ 0 \leqslant i \leqslant N,$$

which means the average random delay rate is $\frac{1}{N+1}$ for Theorem 1 and 2 in systems (1) and (2).

Remark 8 To decrease packet loss rate and increase estimation accuracy, the other sensor nodes in wireless networks are scheduled as tasking sensor nodes (See [22–25] in details) to cooperatively process information. The reason that the great deals of packets are lost or delayed comes possibly from unsuccessful operations of the current tasking sensor node.

3.4 Distributed sensor scheduling strategy

There are many sensor scheduling schemes for WSNs, such as the nearest sensor scheduling strategy (NSSS), the dynamic-group scheduling scheme[23], the adaptive sensor scheduling[24], and the distributed adaptive multisensor scheduling[25]. For simplification, we use NSSS to select next tasking sensor node. NSSS selects the nearest sensor node to the mobile target as the current tasking node at each time step[23].

The main operations of each tasking sensor node are summarized as the following Algorithm 1, where we assume neighboring position of each sensor node is known.

Algorithm 1 For time step k+1.

Requirements: state vector $\hat{x}(k|k)$, state variance matrix $S(k)$ and error covariance matrix $\mathrm{P}(k|k)$ from previous task node at time step k.

1) Compute parameter J by (26);

2) Compute prediction state $\hat{x}(k+1|k)$ by (23);

3) Compute the gain matrix $K(k+1)$ by (24);

4) Receive measurement distance $y(k+1)$ from DCN;

5) Update state estimation $\hat{x}(k+1|k+1)$ by (22);

6) Update error covariance matrix $\mathrm{P}(k+1|k+1)$ by (25);

7) Update state variance matrix $S(k+1)$ by (27);

8) Predict state $\hat{x}(k+2|k+1)$ by (23) for next time step;

9) Compute the distance between each neighboring node and moving target to be estimated by (30);

10) Select the sensor node that is the nearest distance to position of moving target to be estimated as next tasking node.

4 Computer simulation

4.1 Simulation for Theorem 1

For Theorem 1, consider a numerical example[21] as follows:

$$x(k+1) = \begin{bmatrix} 0.8 & 0 \\ 0.2\sin\dfrac{2k\pi}{N} & 0.95 \end{bmatrix} x(k) + \begin{bmatrix} 0.6 \\ 0.5 \end{bmatrix} w(k), \tag{37}$$

$$z(k) = [1\ \ 1]x(k) + v(k), \tag{38}$$

$$\begin{aligned} y(k) = {} & \gamma_0(k)z(k) + (1-\gamma_0(k))\gamma_1(k)z(k-1) + \\ & (1-\gamma_0(k))(1-\gamma_1(k))z(k-2), \end{aligned} \tag{39}$$

where $w(k)$ and $v(k)$ with zero mean and variance

$$Q(k) = \begin{bmatrix} 0.72 & 0.60 \\ 0.60 & 0.50 \end{bmatrix},$$

$R(k) = 10$ respectively. The other parameters are set as:

$$x(0) = [0.1\ \ 0]^{\mathrm{T}},\ \hat{x}(0|0) = [0\ \ 0]^{\mathrm{T}},$$
$$\mathrm{P}(0|0) = 0.1 \times \mathrm{diag}\{[1\ \ 1]\},$$
$$S(0) = \mathrm{P}(0|0),\ k = 0, 1, \cdots, 300.$$

We have 100 times Monte Carlo test for estimation error of two state components under the different packet arrival probability p.

Figures 1–2 show the estimation error comparison of two components of the state $x(k)$ for proposed filters and references [2, 8, 21] under $0.1 \leqslant p \leqslant 1$. From Figs.1–2 we know that the proposed OMVF has the best estimation accuracy than the existing other filters since it is the optimal filter with the small packet loss rate. The filter in [2] has the worst estimation accuracy since it updates estimation by the received noise when packet loss happens. The filters in [8, 21] have better estimation accuracy than the filter in [2]. However, the filter in [8] contains a random variable in the filtering equations and needs to know whether each measurement is received, so it is inconvenient to implement in real application. The proposed OMVF and the filter in [21] need only to know the packet arrival probability p, and not require knowing if each packet is received. Furthermore, the proposed OMVF is easily and conveniently extended to nonlinear systems.

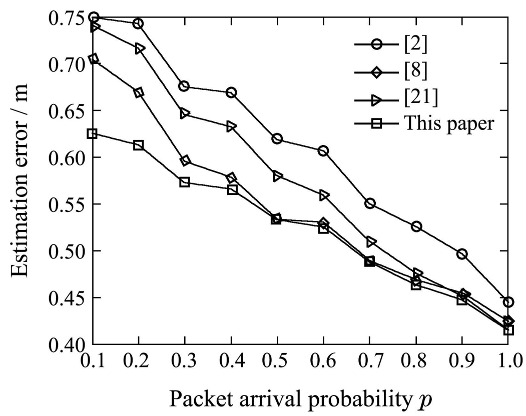

Fig. 1 The first component $x_1(k)$ comparison of estimation error for OMVF in this paper and [2, 8, 21]

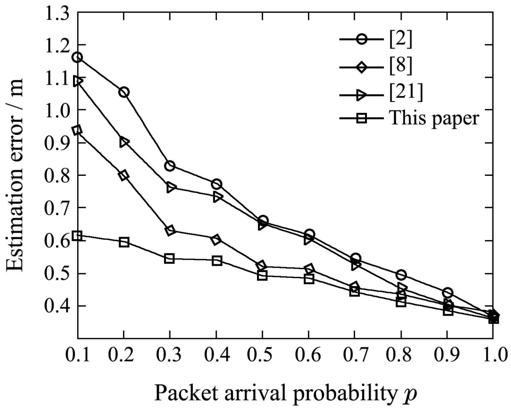

Fig. 2 The second component $x_2(k)$ comparison of estimation error for OMVF in this paper and [2, 8, 21]

4.2 Simulation for Theorem 2

4.2.1 State model and measurement model

The state of moving target usually is described by its position, speed and turn rate. In our experiments, a coordinated turn (CT) model is adopted, which is similar to [19, 25].

The CT is shown as follows:

$$x(k+1) = f(k)x(k) + w(k), \tag{40}$$

where

$$x(k) = (x_{\rm t}(k), y_{\rm t}(k), \dot{x}_{\rm t}(k), \dot{y}_{\rm t}(k), \omega(k))^{\rm T}, \tag{41}$$

$(x_{\rm t}(k), y_{\rm t}(k))$ denotes the position of moving target, $(\dot{x}_{\rm t}(k), \dot{y}_{\rm t}(k))$ and $\omega(k)$ represent velocity and angular velocity of moving target respectively. $w(k)$ is Gaussian white noise with zero mean and covariance $Q(k)$. The nonlinear state transfer matrix $f(k)$ and its derivation $f'_{\rm x}(k)$ are shown respectively as follows:

$$f(k) = \begin{bmatrix} 1 & 0 & \dfrac{\sin(k\omega(k))}{\omega(k)} & \dfrac{\cos(k\omega(k))-1}{\omega(k)} & 0 \\ 0 & 1 & -\dfrac{\cos(k\omega(k))-1}{\omega(k)} & \dfrac{\sin(k\omega(k))}{\omega(k)} & 0 \\ 0 & 0 & \cos(k\omega(k)) & -\sin(k\omega(k)) & 0 \\ 0 & 0 & \sin(k\omega(k)) & \cos(k\omega(k)) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$f'_{\rm x}(k) = \begin{bmatrix} 1 & 0 & \dfrac{\sin(k\omega(k))}{\omega(k)} & \dfrac{\cos(k\omega(k))-1}{\omega(k)} & \dfrac{\partial x_{\rm t}(k+1)}{\partial\omega(k)} \\ 0 & 1 & -\dfrac{\cos(k\omega(k))-1}{\omega(k)} & \dfrac{\sin(k\omega(k))}{\omega(k)} & \dfrac{\partial y_{\rm t}(k+1)}{\partial\omega(k)} \\ 0 & 0 & \cos(k\omega(k)) & -\sin(k\omega(k)) & \dfrac{\partial \dot{x}_{\rm t}(k+1)}{\partial\omega(k)} \\ 0 & 0 & \sin(k\omega(k)) & \cos(k\omega(k)) & \dfrac{\partial \dot{y}_{\rm t}(k+1)}{\partial\omega(k)} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where

$$\frac{\partial x_{\rm t}(k+1)}{\partial\omega(k)} = \frac{k\omega(k)\cos(k\omega(k)) - \sin(k\omega(k))}{\omega^2(k)}\dot{x}(k) - \frac{k\omega(k)\sin(k\omega(k)) + \cos(k\omega(k)) - 1}{\omega^2(k)}\dot{y}(k),$$

$$\frac{\partial y_{\rm t}(k+1)}{\partial\omega(k)} = \frac{k\omega(k)\sin(k\omega(k)) + \cos(k\omega(k)) - 1}{\omega^2(k)}\dot{x}(k) + \frac{k\omega(k)\cos(k\omega(k)) - \sin(k\omega(k))}{\omega^2(k)}\dot{y}(k),$$

$$\frac{\partial \dot{x}_{\rm t}(k+1)}{\partial\omega(k)} = -k\sin(k\omega(k))\dot{x}(k) - k\cos(k\omega(k))\dot{y}(k),$$

$$\frac{\partial \dot{y}_{\rm t}(k+1)}{\partial\omega(k)} = k\cos(k\omega(k))\dot{x}(k) - k\sin(k\omega(k))\dot{y}(k).$$

The measurement is based on the distance from the position of a sensor to one of a moving target to be estimated. The measurement model of the sensor node s is shown as

$$z_s(k) = h_s(x(k)) + \nu_s(k) = \mathscr{A} + \nu_s(k), \tag{42}$$

where

$$\mathscr{A} = \sqrt{(\hat{x}_{\rm t}(k|k-1) - x_s(k))^2 + (\hat{y}_{\rm t}(k|k-1) - y_s(k))^2},$$

$(\hat{x}_{\rm t}(k|k-1), \hat{y}_{\rm t}(k|k-1))$ is the predicted position of the moving target position $(x_{\rm t}(k), y_{\rm t}(k))$, $(x_s(k), y_s(k))$ is the known position of the sensor node s, and $\nu_s(k)$ is Gaussian white noise of sensor node s with zero mean and covariance $R_s(k)$ at time step k. Jacobian matrix of $h(x(k))$ is shown as

$$H(k) = \begin{pmatrix} \dfrac{\hat{x}_{\rm t}(k|k-1) - x_s(k)}{\mathscr{A}} \\ \dfrac{\hat{y}_{\rm t}(k|k-1) - y_s(k)}{\mathscr{A}} \\ 0 \\ 0 \end{pmatrix}^{\rm T}.$$

4.2.2 Simulation results

The monitored field is $100\,{\rm m} \times 100\,{\rm m}$ and covered by 20 sensors randomly deployed in Figs. 3, 5, 7 and 9, where NSSS is adopted in target tracking, and the little circles represent random sensor nodes deployed in the monitored WSNs area, and the little circles with symbol '*' represent sensor nodes scheduled in the process of the target tracking, and the associated tasking sensor of each estimated target position is indicated by the blue line between them. In the monitored field the target moves along the big circle trajectory.

For Theorem 2, initial state

$$\hat{x}(0|0) = [30\ 70\ 20\ 20\ -1],$$

initial covariance matrix

$${\rm P}(0|0) = 0.2 \times {\rm diag}\{0.1, 0.1, 0.1, 0.1, 0.1\},$$

and initial variance $S(0) = {\rm P}(0|0)$. Total time step

$$n = 65,\ q = 0.2,\ R(k) = 0.0001,\ Q(k) = B \times q \times B^{\rm T},$$

where $B = [0\ \ 0\ \ 0\ \ 0\ \ 1]^{\rm T}$. Define the estimation error at time step k as

$$Er(k) := \sqrt{\sum_{i=1}^{2}(x_i(k) - \hat{x}_i(k|k))^2},$$

where $x_i(k)$ and $\hat{x}_i(k|k)$ are the ith component of vector $x(k)$ and $\hat{x}(k|k-1)$, respectively, $i = 1, 2$. The estimation error of the EMVF under different packet arrival probability is shown in Figs.4, 6, 8 and 10 respectively, where Gamma (γ) denotes random variable taking value 0 or 1.

Figures 3, 5, 7 and 9 show tracking trajectory under different packet arrival probability for EKF and EMVF when N=1 and N=2, respectively. Fig. 4, 6, 8 and 10 show estimation error of EMVF under different packet arrival probability respectively.

We see that tracking trajectories of EKF and EMVF under $N = 1$ and $N = 2$ are approximately identical when $p = 1$ in Fig.3. Estimation error of the three cases is much low (less than 0.3) in Fig.4. Moreover, estimation error is the same for EMVF under $N = 1$ and $N = 2$.

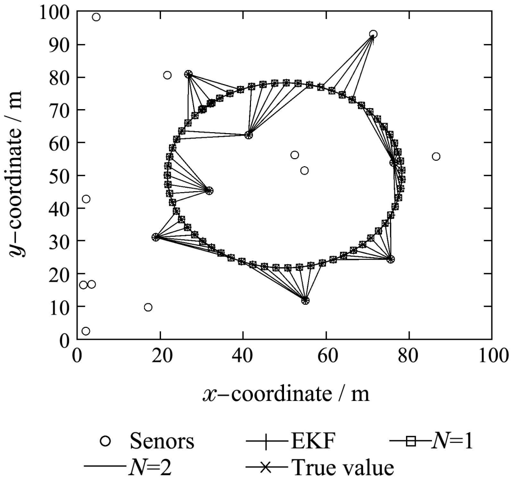

Fig. 3 Trajectory of EKF and EMVF with NSSS when $p = 1$

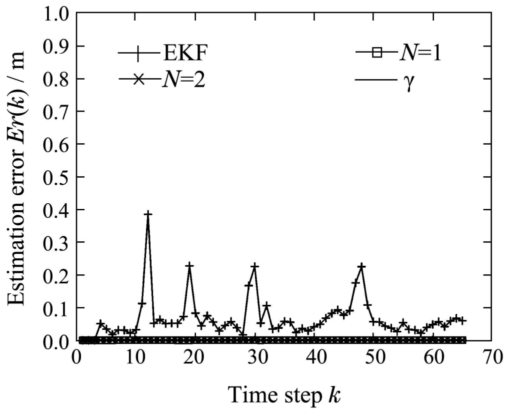

Fig. 4 Estimation error $Er(k)$ of EKF and EMVF when $p = 1$

EKF obviously deviates from real trajectory too much when $p = 0.98$ in Fig.5; while EMVF is still able to track real trajectory well. We know that estimation error of EKF is much high once packet loss happens, e.g., at time step $k = 46$ in Fig.6. However, estimation error of EMVF is rather low under $N = 1$ and $N = 2$. Therefore, compared with EKF, tracking performance of EMVF is significantly improved.

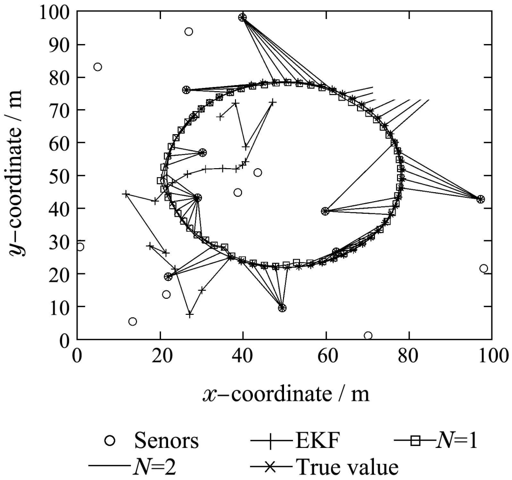

Fig. 5 Trajectory of EKF and EMVF with NSSS when $p = 0.98$

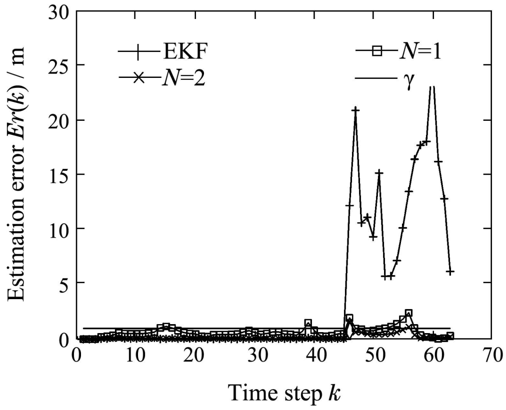

Fig. 6 Estimation error $Er(k)$ of EKF and EMVF when $p = 0.98$

EKF losses tracking function as packet arrival probability further decreases. Thus we only compare EMVF under $N = 1$ with $N = 2$ in Figs.7–10. Tracking performance under $N = 2$ is better than that under $N = 1$ when $p = 0.95$ and $p = 0.90$ respectively. For example, seven packets are dropped out in Fig.10, maximal estimation error reaches 17.5 under $N = 1$ while maximal estimation error approaches only 7–8 under $N = 2$.

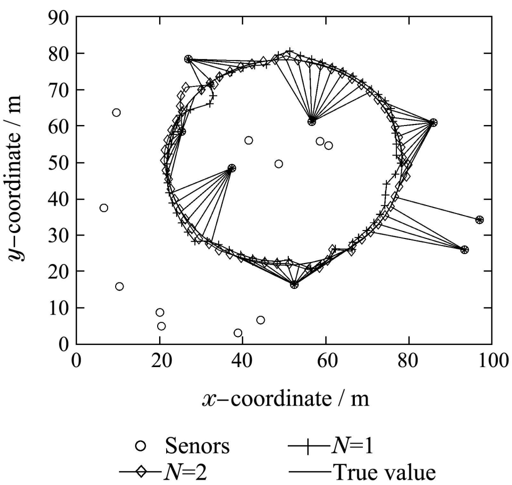

Fig. 7 Trajectory of EMVF with NSSS when $p = 0.95$

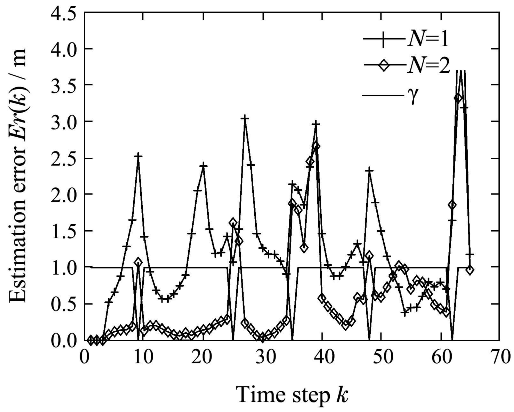

Fig. 8 Estimation error $Er(k)$ of EMVF when $p = 0.95$

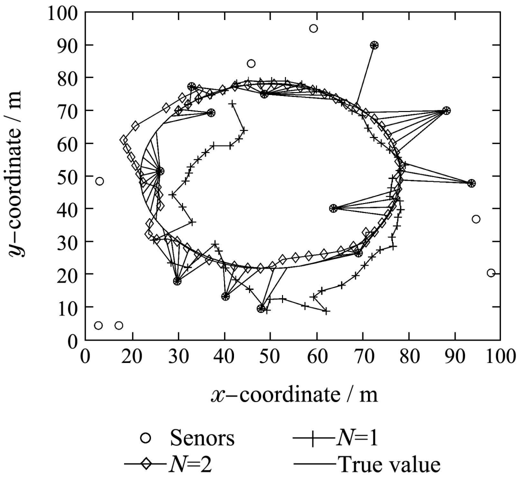

Fig. 9 Trajectory of EMVF with NSSS when $p = 0.90$

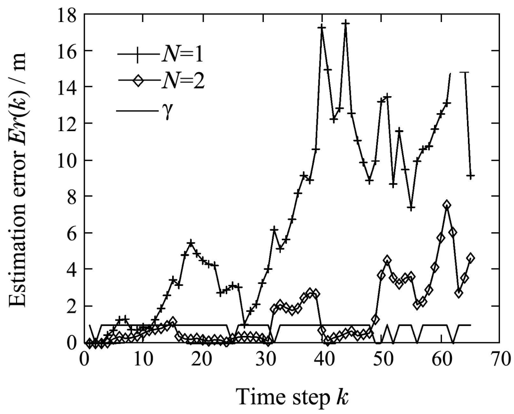

Fig. 10 Estimation error $Er(k)$ of EMVF when $p = 0.90$

To further illustrate the effectiveness of the EMVF, 300 ($m = 300$) times Monte Carlo simulations are conducted under different p when the number of consecutive packet losses N is taken different value. Define statistical average estimation error (SAEE) of m times Monte Carlo simulations

$$\text{SAEE} := \frac{1}{mn}\sum_{k=1}^{m}\sum_{k=1}^{n} Er(k).$$

For (2), define $y(k) := \nu(k)$ when N=0, which means that the current tasking node only receives measurement noise.

Table 2 illustrates that statistical average estimation error SAEE gradually decreases as packet arrival probability p or N increases.

Table 2 Comparison with SAEE (m) under different N, p

N	p				
	0.80	0.85	0.90	0.95	0.99
$N = 0$	15.11	13.47	12.60	10.78	8.62
$N = 1$	13.98	11.31	7.78	5.50	1.23
$N = 2$	4.86	4.80	3.54	2.47	0.59

The above simulation results show further the proposed EMVF is effective and robust to track moving target in WSNs.

5 Conclusions

We derive the optimal minimum variance filter (OMVF) for linear time-varying systems and extend to the enhanced minimum variance filter (EMVF) for nonlinear systems with consecutive packet losses and measurement time delays. The proposed filters with packet losses and time delays are independent on binary stochastic variable $\gamma(k)$ and related only to the probability p of $\gamma(k)$. Therefore, these filters have wide-range application prospects in many engineering fields, such as WSNs and networked control systems. For the linear OMVF, the estimation accuracy of the proposed OMVF is superior compared to the existing works. Furthermore, combined with NSSS, we develop a real-time algorithm for moving target tracking in WSNs. Simulations illustrate effectiveness and robustness of the proposed EMVF.

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