第 30 卷第 7 期
2013 年 7 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 30 No. 7
Jul. 2013

DOI: 10.7641/CTA.2013.12054

产品质量与生产过程的集成控制: 双率采样数据方法

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摘要: 本文在集成控制的框架下研究了工业过程中产品质量和生产过程的实时一体控制问题. 首先, 基于双速率采样数据提出了产品质量和生产过程的集成控制方案. 然后, 把控制器设计问题归结为求解一组非线性矩阵不定式, 进而提出基于同伦算法的控制器设计方法. 最后, 通过数值仿真说明所提方法的有效.

关键词: 产品质量与生产过程的集成控制; 双采样率数据; 同伦算法

中图分类号: TP273　　**文献标识码**: A

Integrated control of product quality and production process for process industry: dual-rate sampled-data method

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Abstract: In an integrated framework, we investigate the real-time united control of the product quality and the production process for a process industry. On the basis of dual-rate sampled-data, an integrated product and process control (IPPC) scheme is proposed. The design problem is reduced to solving matrix inequalities which is solved by using a homotopy algorithm. Finally, an example is given to illustrate the proposed method.

Key words: integrated control of product quality and production process; dual-rate sampled-data; homotopy algorithm

1 Introduction

Product quality control (QC) plays a key role in manufacturing. Traditional QC is based on quality inspection and statistical analysis which usually leads to significant time delay. Thus it is not sufficient to meet the high product quality standard in modern manufacturing. To overcome the shortcoming of the traditional QC method, some real-time control methods are proposed to assure the product quality starting from the earliest stage of product manufacturing[1–5]. In [1], the so-called new generation quality control is presented based on reliable real-time process control. In [2], a practical approach based on on-line measurements and off-line analysis to the control of final product quality in semibatch reactors is established. [3] proposes an integrated process and product dynamic modeling approach, which facilitates simultaneous control of both process and product quality. [4] proposes a cascade control scheme called integrated product and process control (IPPC). As shown in Fig.1, the inner loop of the system is for process control, while the outer loop is for product control. The results of [4] are extended to multiple-input and multiple-output case in [5].

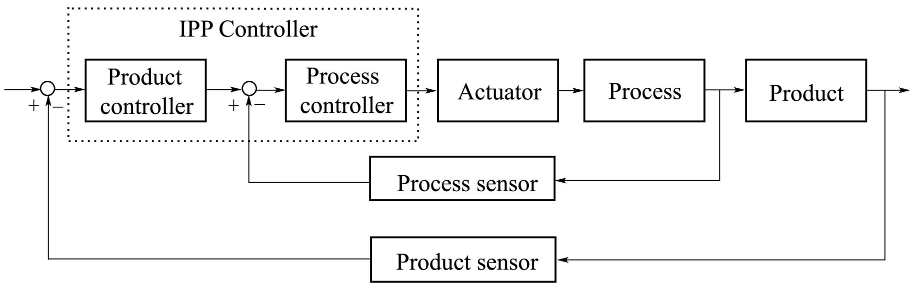

Fig. 1 The general IPP control scheme

Received 13 March 2012; revised 14 May 2012.
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This work was supported by the National Natural Science Foundation of China (Nos. 60904009, 61020106003) and the Fundamental Research Funds for the Central Universities (Nos. N100406010, 2013QNA50).

On the other hand, sampled-data control is a research frontier in the past decades because modern control systems usually employ digital technology for controller implementation. There are mainly three approaches for analysis and design of sampled-data systems. The first one is based on the lifting technique in which the problem is transformed to an equivalent finite-dimensional discrete control problem[6]. The second approach is based on the representation of the sampled-data system in the form of impulsive model[7]. The third one is the input delay approach, where the system is modeled as a continuous-time system with the delayed control input and the solution is established by the Lyapunov-Krasovskii functional method[8]. A common idea to deal with multi-rate sampled-data control problems is transforming the multi-rate problem into a single-rate one by the lifting technique[9], which usually requires the sampling rates to be fixed. Recently, the input delay approach is applied to stabilization problem of linear systems under uncertain sampling rate[10].

The IPPC methods proposed by [4–5] are based on the measurements on process and product and assume that the signals are available all the time. However, in many practical applications, quality measurements have slower sampling rate than the process variables. Thus this papers considers dual-rate sampled-data IPPC of process industry. First, a dual-rate sampled-data IPPC scheme is proposed. Then the design problem is reduced to solving matrix inequalities and a homotopy algorithm is given to solve the matrix inequalities. Finally, an example is given to show the effectiveness of proposed method.

2 Problem formulation

As shown in [5], a key component of IPPC design is the dynamic characterization of a product that is being manufactured. This requires product dynamic modeling beside the usual process dynamic modeling. Assume that the product quality dynamics model and the process dynamics model are as follows:

$$\dot{\gamma}(t) = A_{\gamma}\gamma(t) + B_{\mathrm{y}}y(t), \tag{1}$$

$$\dot{y}(t) = A_{\mathrm{y}}y(t) + B_{\mathrm{u}}u(t). \tag{2}$$

In the IPPC design scheme shown in Fig.1 given by [4–5], the inner loop of the system is for process control, while the outer loop is for product quality control. Based on this design scheme, we propose a dual-rate sampled-data IPPC scheme shown in Fig.2 (ZOH means zero-order holder).

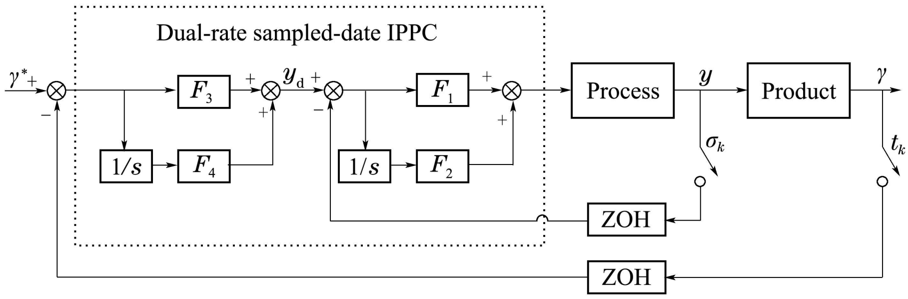


Fig. 2 Dual-rate sampled-data IPPC scheme

There have been many methods for process control (see [4–5] and the references therein). Thus we assume that the process controller is designed by the existing methods and given in this paper. We just need to design the product quality controller such that the dual-rate sampled-data system can tracking any given product quality setpoint γ^*.

Assume that the sampling instants for $\gamma(t)$ and $y(t)$ are t_k and σ_k, respectively. Let $\eta_1 = t - \sigma_k$, $\sigma_k \leqslant t \leqslant \sigma_{k+1}$, then $0 \leqslant \eta_1 \leqslant \sigma_{k+1} - \sigma_k \triangleq h_1$. Let $\eta_2 = t - t_k$, $t_k \leqslant t \leqslant t_{k+1}$, then $0 \leqslant \eta_2 \leqslant t_{k+1} - t_k \triangleq h_2$. Then the closed-loop system is as follows:

$$\begin{cases} \dot{y}(t) = A_{\mathrm{y}}y(t) + B_{\mathrm{u}}u(t),\ \dot{\xi}_1(t) = y_{\mathrm{d}}(t) - y(t-\eta_1), \\ \dot{\gamma}(t) = A_{\gamma}\gamma(t) + B_{\mathrm{y}}y(t),\ \dot{\xi}_2(t) = \gamma^* - \gamma(t-\eta_2), \\ y_{\mathrm{d}}(t) = F_3[\gamma^* - \gamma(t-\eta_2)] + F_4\xi_2(t), \\ u(t) = F_1[y_{\mathrm{d}}(t) - y(t-\eta_1)] + F_2\xi_1(t). \end{cases} \tag{3}$$

Let $e(t) = \gamma^* - \gamma(t)$, then the closed loop system is transformed into

$$\begin{cases} \dot{e}(t) = A_{\gamma}e(t) - B_{\mathrm{y}}y(t) - A_{\gamma}\gamma^*, \\ \dot{y}(t) = A_{\mathrm{y}}y(t) + B_{\mathrm{u}}[F_1(F_3e(t-\eta_2)+ \\ \qquad F_4\xi_2(t) - y(t-\eta_1)) + F_2\xi_1(t)], \\ \dot{\xi}_1(t) = F_3e(t-\eta_2) + F_4\xi_2(t) - y(t-\eta_1), \\ \dot{\xi}_2(t) = e(t-\eta_2). \end{cases} \tag{4}$$

Let $z(t) = [e(t)\ \ y(t)\ \ \xi_1(t)\ \ \xi_2(t)]$ and omit the constant term, we have

$$\begin{aligned} \dot{z}(t) = &\ (A + BK_1)z(t) + A_{\mathrm{d}1}z(t-\eta_1) + \\ &\ (A_{\mathrm{d}2} + BK_2)z(t-\eta_2) = \\ &\ A_1z(t) + A_2z(t-\eta_1) + A_3z(t-\eta_2), \end{aligned} \tag{5}$$

where

$$A = \begin{bmatrix} A_{\gamma} & -B_{\mathrm{y}} & 0 & 0 \\ 0 & A_{\mathrm{y}} & B_{\mathrm{u}}F_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A_{\mathrm{d}1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -B_{\mathrm{u}}F_1 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$K_1 = [0\ \ 0\ \ 0\ \ F_4],\ K_2 = [F_3\ \ 0\ \ 0\ \ 0],$$

$$A_1 = A + BK_1,\ A_2 = A_{\mathrm{d}1},\ A_3 = A_{\mathrm{d}2} + BK_2,$$

$$A_{\mathrm{d}2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ B_{\mathrm{u}}F_1 \\ I \\ 0 \end{bmatrix}.$$

Note the fact that system (4) has a unique equilibrium $[0\ \ y_{\mathrm{e}}\ \ \xi_{1\mathrm{e}}\ \ \xi_{2\mathrm{e}}]^{\mathrm{T}}$ for any given constat γ^* if it is asymptotically stable. As a result, $\lim\limits_{t\to\infty} e(t) = 0$ if system (5) is asymptotically stable. Thus the tracking control problem is converted into stabilization problem of system (5) for which F_1 and F_2 are given and F_3 and F_4 are to be determined.

3 Main results

3.1 Main theorem

Theorem 1 Given $F_i(i=1,2,3,4)$, system (5) is asymptotically stable if there exist matrices $P>0$, $M_i>0, U_i, V_i(i=1,2)$ satisfying

$$\begin{bmatrix} \varPhi_{11} & * & * & * & * \\ \varPhi_{21} & \varPhi_{22} & * & * & * \\ \varPhi_{31} & \varPhi_{32} & \varPhi_{33} & * & * \\ U_1^{\mathrm T} & V_1^{\mathrm T} & 0 & -h_1^{-1}M_1 & * \\ U_2^{\mathrm T} & 0 & V_2^{\mathrm T} & 0 & -h_2^{-1}M_2 \end{bmatrix} < 0, \tag{6}$$

where

$$\varPhi_{11}=A_1^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_1+PA_1+A_1^{\mathrm T}P+\sum_{i=1}^{2}(U_i+U_i^{\mathrm T}),$$

$$\varPhi_{21}=A_2^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_1+A_2^{\mathrm T}P+V_1-U_1^{\mathrm T},$$

$$\varPhi_{31}=A_3^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_1+A_3^{\mathrm T}P+V_2-U_2^{\mathrm T},$$

$$\varPhi_{22}=A_2^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_2-V_2-V_2^{\mathrm T},$$

$$\varPhi_{32}=A_3^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_2,$$

$$\varPhi_{33}=A_3^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_3-V_3-V_3^{\mathrm T}.$$

Proof By Schur complement lemma, inequality (6) is equivalent to

$$\begin{bmatrix} \varPhi_{11} & * & * \\ \varPhi_{21} & \varPhi_{22} & * \\ \varPhi_{31} & \varPhi_{32} & \varPhi_{33} \end{bmatrix} + h_1\begin{bmatrix} U_1M_1^{-1}U_1^{\mathrm T} & * & * \\ V_1M_1^{-1}U_1^{\mathrm T} & V_1M_1^{-1}V_1^{\mathrm T} & * \\ 0 & 0 & 0 \end{bmatrix} + h_2\begin{bmatrix} U_2M_2^{-1}U_2^{\mathrm T} & * & * \\ 0 & 0 & * \\ V_2M_2^{-1}U_2^{\mathrm T} & 0 & V_2M_2^{-1}V_2^{\mathrm T} \end{bmatrix} < 0. \tag{7}$$

Then there must exist matrices $P>0$, $M_i>0$, U_i, V_i,

$$\begin{bmatrix} X_i & Y_i \\ Y_i^{\mathrm T} & Z_i \end{bmatrix} \geqslant 0,\ i=1,2,$$

satisfying

$$\begin{bmatrix} U_iM_i^{-1}U_i^{\mathrm T} & * \\ V_iM_i^{-1}U_i^{\mathrm T} & V_iM_i^{-1}V_i^{\mathrm T} \end{bmatrix} \leqslant \begin{bmatrix} X_i & Y_i \\ Y_i^{\mathrm T} & Z_i \end{bmatrix},\ i=1,2, \tag{8}$$

$$\begin{bmatrix} \varPhi_{11} & * & * \\ \varPhi_{21} & \varPhi_{22} & * \\ \varPhi_{31} & \varPhi_{32} & \varPhi_{33} \end{bmatrix} + h_1\begin{bmatrix} X_1 & * & * \\ Y_1^{\mathrm T} & Z_1 & * \\ 0 & 0 & 0 \end{bmatrix} + h_2\begin{bmatrix} X_2 & * & * \\ 0 & 0 & * \\ Y_2^{\mathrm T} & 0 & Z_2 \end{bmatrix} < 0. \tag{9}$$

Choose the Lyapunov function as follows:

$$\begin{cases} V(t)=V_1(t)+V_2(t), \\ V_1(t)=z^{\mathrm T}(t)Pz(t), \\ V_2(t)=\displaystyle\int_{-h_1}^{0}\int_{t+\beta}^{t}\dot z^{\mathrm T}(\alpha)M_1\dot z(\alpha)\mathrm d\alpha\mathrm d\beta+ \\ \qquad\displaystyle\int_{-h_2}^{0}\int_{t+\beta}^{t}\dot z^{\mathrm T}(\alpha)M_2\dot z(\alpha)\mathrm d\alpha\mathrm d\beta. \end{cases} \tag{10}$$

Computing the derivative of the Lyapunov function $V(t)$ along the trajectories of system (5), we have

$$\begin{cases} \dot V_1(t)=2z^{\mathrm T}P\dot z= \\ 2z^{\mathrm T}P[A_1z(t)+A_2z(t-\eta_1)+A_3z(t-\eta_2)], \\ \dot V_2(t)= \\ h_1\dot z^{\mathrm T}M_1\dot z+h_2\dot z^{\mathrm T}M_2\dot z- \\ \displaystyle\int_{t-h_1}^{t}\dot z^{\mathrm T}(\alpha)M_1\dot z(\alpha)\mathrm d\alpha-\int_{t-\eta_2}^{t}\dot z^{\mathrm T}(\alpha)M_2\dot z(\alpha)\mathrm d\alpha. \end{cases} \tag{11}$$

By the Newton-Leibniz formula, we have

$$\int_{t_1}^{t_2}\dot z(\alpha)\mathrm d\alpha=z(t_2)-z(t_1).$$

Then, for any matrices U_i, V_i, $i=1,2$, it holds that

$$\varLambda_i=2[z(t)\ \ z(t-\eta_i)][U_i\ \ V_i]^{\mathrm T}\times[z(t)-z(t-\eta_i)-\int_{t-\eta_i}^{t}\dot z(\alpha)\mathrm d\alpha]=0. \tag{12}$$

In addition, for any matrices

$$\begin{bmatrix} X_i & Y_i \\ Y_i^{\mathrm T} & Z_i \end{bmatrix} \geqslant 0,\ i=1,2,$$

we have

$$\begin{cases} \varPsi_i=h_i\varPi_i-\displaystyle\int_{t-\eta_i}^{t}\varPi_i\mathrm d\alpha\geqslant 0, \\ \varPi_i=\begin{bmatrix} z(t) \\ z(t-\eta_i) \end{bmatrix}^{\mathrm T}\begin{bmatrix} X_i & Y_i \\ Y_i^{\mathrm T} & Z_i \end{bmatrix}\begin{bmatrix} z(t) \\ z(t-\eta_i) \end{bmatrix}. \end{cases} \tag{13}$$

From (11)–(14), we have

$$\dot V(t)\leqslant\dot V_1(t)+\dot V_2(t)+\sum_{i=1}^{2}(\varLambda_i+\varPsi_i)=$$

$$\psi^{\mathrm T}(t)\hat\varPhi\psi(t)+\sum_{i=1}^{2}\int_{t-\eta_i}^{t}\varphi_i^{\mathrm T}(t,\alpha)\varOmega_i\varphi_i(t,\alpha)\mathrm d\alpha, \tag{14}$$

where

$$\psi(t)=[z(t)\ \ z(t-\eta_1)\ \ z(t-\eta_2)]^{\mathrm T},$$

$$\varphi_i(t,\alpha)=[z(t)\ \ z(t-\eta_i)\ \ \dot z(\alpha)]^{\mathrm T},\ i=1,2,$$

$$\varOmega_i=\begin{bmatrix} -X_i & * & * \\ -Y_i^{\mathrm T} & -Z_i & * \\ -U_i^{\mathrm T} & -V_i^{\mathrm T} & -M_i \end{bmatrix},\ \hat\varPhi=\begin{bmatrix} \hat\varPhi_{11} & * & * \\ \hat\varPhi_{21} & \hat\varPhi_{22} & * \\ \hat\varPhi_{31} & \hat\varPhi_{32} & \hat\varPhi_{33} \end{bmatrix},$$

$$\hat\varPhi_{11}=\sum_{i=1}^{2}(h_iA_1^{\mathrm T}M_iA_1+U_i+U_i^{\mathrm T}+h_iX_i)+PA_1+A_1^{\mathrm T}P,$$

$$\hat\varPhi_{21}=A_2^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_1+A_2^{\mathrm T}P+V_1-U_1^{\mathrm T}+h_1Y_1^{\mathrm T},$$

$$\hat\varPhi_{31}=A_3^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_1+A_3^{\mathrm T}P+V_2-U_2^{\mathrm T}+h_2Y_2^{\mathrm T},$$

$$\hat\varPhi_{22}=A_2^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_2-V_2-V_2^{\mathrm T}+h_1Z_1,$$

$$\hat\varPhi_{32}=A_3^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_2,$$

$$\hat\varPhi_{33}=A_3^{\mathrm T}\sum_{i=1}^{2}(h_iM_i)A_3-V_3-V_3^{\mathrm T}+h_2Z_2.$$

It is noted that inequality (9) implies $\hat\varPhi<0$, and (8) is equivalent to $\varOmega_i<0,\ i=1,2$. Then, the closed-loop system (5) is asymptotically stable. This proof is completed.

Remark 1 Theorem 1 proposes a sufficient condition for the multiple time-delays system (5) to be asymptotically stable. By the dual-rate sampled-data IPPC scheme, the time delays $\eta_i(t),\ i=1,2$ are time-varying and satisfy $\dot{\eta}_i(t)=1, \forall t \neq t_k$ or σ_k. This information may be helpful to reduce the conservatism of the stability criterion, as will be considered in our future work.

Remark 2 Inequality (6) is nonlinear with respect to the controller gains F_3 and F_4. Thus it is significant to propose an efficient algorithm to solve it, as will be discussed in the next subsection.

3.2 Computational issue: Homotopy algorithm

By Schur complement lemma and inequality (6) is equivalent to

$$\begin{bmatrix} \Pi & * & * & * & * & * & * \\ A_2^{\mathrm{T}}P+V_1-U_1^{\mathrm{T}} & -V_1-V_1^{\mathrm{T}} & * & * & * & * & * \\ A_3^{\mathrm{T}}P+V_2-U_2^{\mathrm{T}} & 0 & -V_2-V_2^{\mathrm{T}} & * & * & * & * \\ U_1^{\mathrm{T}} & V_1^{\mathrm{T}} & 0 & -h_1^{-1}M_1 & * & * & * \\ U_2^{\mathrm{T}} & 0 & V_2^{\mathrm{T}} & 0 & -h_2^{-1}M_2 & * & * \\ A_1 & A_2 & A_3 & 0 & 0 & -h_1^{-1}M_1^{-1} & * \\ A_1 & A_2 & A_3 & 0 & 0 & 0 & -h_2^{-1}M_2^{-1} \end{bmatrix} < 0, \tag{15}$$

where $\Pi = PA_1 + A_1^{\mathrm{T}}P + \sum\limits_{i=1}^{2}(U_i+U_i^{\mathrm{T}})$. Substituting the system matrices in (15), we have

$$\begin{bmatrix} \Theta_1 & * & * & * & * & * & * \\ \Theta_2 & -V_1-V_1^{\mathrm{T}} & * & * & * & * & * \\ \Theta_3 & 0 & * & -V_2-V_2^{\mathrm{T}} & * & * & * \\ U_1^{\mathrm{T}} & V_1^{\mathrm{T}} & 0 & -h_1^{-1}M_1 & * & * & * \\ U_2^{\mathrm{T}} & 0 & V_2^{\mathrm{T}} & 0 & -h_2^{-1}M_2 & * & * \\ A+BK_1 & A_{\mathrm{d}1} & A_{\mathrm{d}2}+BK_2 & 0 & 0 & -h_1^{-1}M_1^{-1} & * \\ A+BK_1 & A_{\mathrm{d}1} & A_{\mathrm{d}2}+BK_2 & 0 & 0 & 0 & -h_2^{-1}M_2^{-1} \end{bmatrix} < 0, \tag{16}$$

where

$$\Theta_1 = P(A+BK_1)+(A+BK_1)^{\mathrm{T}}P+\sum_{i=1}^{2}(U_i+U_i^{\mathrm{T}}),$$

$$\Theta_2 = A_{\mathrm{d}1}^{\mathrm{T}}P+V_1-U_1^{\mathrm{T}},$$

$$\Theta_3 = (A_{\mathrm{d}2}+BK_2)^{\mathrm{T}}P+V_2-U_2^{\mathrm{T}}.$$

Pre- and post-multiplying (16) by

$$\mathrm{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I\};$$

and letting $\bar{P}=P^{-1}$, $\bar{V}_i=P^{-1}V_iP^{-1}$, $\bar{U}_i=P^{-1}U_iP^{-1}$, $\bar{M}_i=M_i^{-1}$, $i=1,2$, we have

$$\begin{bmatrix} \bar{\Theta}_1 & * & * & * & * & * & * \\ \bar{P}A_{\mathrm{d}1}^{\mathrm{T}}P+\bar{V}_1-\bar{U}_1^{\mathrm{T}} & -\bar{V}_1-\bar{V}_1^{\mathrm{T}} & * & * & * & * & * \\ \bar{\Theta}_2 & 0 & -\bar{V}_2-\bar{V}_2^{\mathrm{T}} & * & * & * & * \\ \bar{U}_1^{\mathrm{T}} & \bar{V}_1^{\mathrm{T}} & 0 & -h_1^{-1}P\bar{M}_1^{-1}\bar{P} & * & * & * \\ \bar{U}_2^{\mathrm{T}} & 0 & \bar{V}_2^{\mathrm{T}} & 0 & -h_2^{-1}P\bar{M}_2 & * & * \\ (A+BK_1)\bar{P} & A_{\mathrm{d}1}\bar{P} & (A_{\mathrm{d}2}+BK_2)\bar{P} & 0 & 0 & -h_1^{-1}\bar{M}_1^{-1} & * \\ (A+BK_1)\bar{P} & A_{\mathrm{d}1}\bar{P} & (A_{\mathrm{d}2}+BK_2)\bar{P} & 0 & 0 & 0 & -h_2^{-1}\bar{M}_2^{-1} \end{bmatrix} < 0, \tag{17}$$

where

$$\bar{\Theta}_1 = (A+BK_1)\bar{P}+\bar{P}(A+BK_1)^{\mathrm{T}}+\sum_{i=1}^{2}(\bar{U}_i+\bar{U}_i^{\mathrm{T}}),$$

$$\bar{\Theta}_2 = \bar{P}(A_{\mathrm{d}2}+BK_2)^{\mathrm{T}}+\bar{V}_2-\bar{U}_2^{\mathrm{T}}.$$

Note that $M_i > 0$, we have $(M_i-\bar{P})M^{-1}(M_i-\bar{P}) \geqslant 0$, which is equivalent to

$$-\bar{P}M^{-1}\bar{P} \leqslant M_i - 2\bar{P}, i=1,2. \tag{18}$$

Then (17) is guaranteed by

$$\begin{bmatrix} \bar{\Theta}_1 & * & * & * & * & * & * \\ \bar{P}A_{\mathrm{d}1}^{\mathrm{T}}+\bar{V}_1-\bar{U}_1^{\mathrm{T}} & -\bar{V}_1-\bar{V}_1^{\mathrm{T}} & * & * & * & * & * \\ \bar{\Theta}_2 & 0 & -\bar{V}_2-\bar{V}_2^{\mathrm{T}} & * & * & * & * \\ \bar{U}_1^{\mathrm{T}} & \bar{V}_1^{\mathrm{T}} & 0 & h_1^{-1}(M_1-2\bar{P}) & * & * & * \\ \bar{U}_2^{\mathrm{T}} & 0 & \bar{V}_2^{\mathrm{T}} & 0 & h_2^{-1}(M_2-2\bar{P}) & * & * \\ (A+BK_1)\bar{P} & A_{\mathrm{d}1}\bar{P} & (A_{\mathrm{d}2}+BK_2)\bar{P} & 0 & 0 & -h_1^{-1}\bar{M}_1^{-1} & * \\ (A+BK_1)\bar{P} & A_{\mathrm{d}1}\bar{P} & (A_{\mathrm{d}2}+BK_2)\bar{P} & 0 & 0 & 0 & -h_2^{-1}\bar{M}_2^{-1} \end{bmatrix} < 0. \tag{19}$$

which implies, by Theorem 1, that the closed-loop system (5) is asymptotically stable. Inequality (19) is a bilinear matrix inequality (BLMI). Neglecting the structure of K_i, and letting $L_i=K_i\bar{P},\ i=1,2$, we have

$$\begin{bmatrix} \check{\Theta}_1 & * & * & * & * & * & * \\ \bar{P}A_{\mathrm{d1}}^{\mathrm{T}}+\bar{V}_1-\bar{U}_1^{\mathrm{T}} & -\bar{V}_1-\bar{V}_1^{\mathrm{T}} & * & * & * & * & * \\ \check{\Theta}_2 & 0 & -\bar{V}_2-\bar{V}_2^{\mathrm{T}} & * & * & * & * \\ \bar{U}_1^{\mathrm{T}} & \bar{V}_1^{\mathrm{T}} & 0 & h_1^{-1}(\bar{M}_1-2\bar{P}) & * & * & * \\ \bar{U}_2^{\mathrm{T}} & 0 & \bar{V}_2^{\mathrm{T}} & 0 & h_2^{-1}(\bar{M}_2-2\bar{P}) & * & * \\ A\bar{P}+BL_1 & A_{\mathrm{d1}}\bar{P} & A_{2d}\bar{P}+BL_2 & 0 & 0 & -h_1^{-1}\bar{M}_1 & * \\ A\bar{P}+BL_1 & A_{\mathrm{d1}}\bar{P} & A_{2d}\bar{P}+BL_2 & 0 & 0 & 0 & -h_2^{-1}\bar{M}_2 \end{bmatrix} < 0, \quad (20)$$

where

$$\check{\Theta}_1 = A\bar{P} + BL_1 + \bar{P}A^{\mathrm{T}} + L_1^{\mathrm{T}}B^{\mathrm{T}} + \sum_{i=1}^{2}(\bar{U}_i + \bar{U}_i^{\mathrm{T}}),$$

$$\check{\Theta}_2 = \bar{P}A_{\mathrm{d2}}^{\mathrm{T}} + L_2^{\mathrm{T}}B^{\mathrm{T}} + \bar{V}_2 - \bar{U}_2^{\mathrm{T}},$$

which is a linear matrix inequality (LMI) and can be checked by LMI ToolBox of MATLAB.

Now, we solve the BMI (19) by adopting the idea of the homotopy method[11]. Let us introduce a real number λ varying from 0 to 1, and consider a matrix function

$$L(K_1, K_2, P, \lambda) = F((1-\lambda)K_1^0 + \lambda K_1, (1-\lambda)K_2^0 + \lambda K_2, P), \quad (21)$$

where $F(\cdot)$ denotes the left matrix of (19), $K_i^0, i = 1, 2$ are full-state-feedback gains which can be obtained from (20), and K_i are partial-state feedback gains with the structure. Thus, the term $(1-\lambda)K_i^0 + \lambda K_i$ in (21) defines a homotopy interpolating a full-state feedback controller and a desired partial-state-feedback controller, and our problem of finding a solution to (19) is embedded in the family of problems

$$L(K_1, K_2, P, \lambda),\ \lambda \in [0, 1]. \quad (22)$$

To carry out the homotopy method, we first need the solution K_i, P of (22) at $\lambda = 0$, which we denote by K_i^0, $i = 1, 2$ can be obtained from (20). Now, our problem is how to make a homotopy path to connect K_i^0, P at $\lambda = 0$ and K_i, P at $\lambda = 1$ in (22). Let N be a positive integer and consider $(N+1)$ points $\lambda_k = k/N$, $k = 0, 1, 2, \ldots, N$ in the interval [0, 1] to generate a family of problems

$$L(K_1, K_2, P, \lambda_k) < 0, \quad (23)$$

where $k = 0, 1, 2, \ldots, N$. If the problem at the kth point is feasible, we denote the obtained solution by solving it as LMIs with some variables are fixed as $K_i = K_{ik}$, or $P = P_k$. If the family of problems (23) are all feasible, a set of solution of the BMI (19) is obtained at $k = N$ (i.e. $\lambda = 1$). If it is not the case, we can consider more points in the interval [0, 1] by increasing N, and repeat the procedure.

We formulate this algorithm in the following procedure.

Algorithm 1

Step 1 Obtain the full-state-feedback gains $K_i^0 = L_iP^{-1}$ by solving (20).

Step 2 Set $k = 0$, $K_{ik} = 0$, and $P_k = P_0$;

Step 3 Set $k = k + 1$, $\lambda_k = k/N$. Compute a set solutions K_{1k}, K_{2k} of $L(K_1, K_2, P_{k-1}, \lambda_k) < 0$, if it is feasible, goto Step 4; if it is not feasible, compute a common solution P_k of $L(K_{1(k-1)}, K_{2(k-1)}, P, \lambda_k) < 0$, if it is feasible, goto Step 4, if it is not feasible, set $N = 2N$ and goto Step 4;

Step 4 If $N > N_{\max}$, where $N_{\max}$ is a prescribed upper bound, then the algorithm ends without feasible solution, else if $k < N_{\max}$, goto Step 3, and if $k = N$, the obtained K_{1N}, K_{2N}, P_N are the feasible solutions.

Remark 3 The homotopy algorithm is motivated by [11]. There is no convergence guarantees to an acceptable solution, the choice of initial value is important. The practice indicates that the P_0 with minimal trace will work well in practice.

Remark 4 In the design procedure, the sampling periods are not necessary fixed. The dual-rate sampled-data IPPC is valid as long as sampling periods are bounded by h_1 and h_2, respectively.

Remark 5 In practice, the product variables and process variables are usually constrained. Thus the design problem taking into account the constraints is significant for further consideration.

4 Example

In this section, the proposed dual-rate sampled-data IPPC design methodology is used to study an automotive coating curing problem which was considered in [3, 5]. In automotive manufacturing, it is a challenging problem to ensure coating quality. In production, vehicle bodies, when entering each baking oven, are covered by a thin layer of wet polymeric film. These vehicles loaded on a conveyor need to travel through the baking oven one by one at a constant speed for coating curing[5].

According to [3], the curing process dynamical model is as follows:

$$\begin{cases} \dot{T} = \dfrac{a}{\rho_{\mathrm{m}}C_{\mathrm{pm}}Z_{\mathrm{m}}}v_{\mathrm{in}}^{0.7}(T_{\mathrm{a}} - T), \\ \dot{P} = \dfrac{PA}{V}(v_{\mathrm{in}} - v_{\mathrm{out}}), \end{cases} \quad (24)$$

where T_{a} is the convection air temperature, v_{in} is the inlet convection air velocity, T is the panel temperature, a is the heat transfer coefficient, and $\rho_{\mathrm{m}}, C_{\mathrm{pm}}, Z_{\mathrm{m}}$ are the density, the heat capacity, and the thickness of the metal substrate, respectively. P is the air pressure within the oven, A is the area of the ventilation openings at the inlet and outlet of the oven, and v_{out} is the outlet convection air velocity.

The coating quality is governed by this model:

$$\begin{cases} \dot{x} = \varsigma\exp(-\dfrac{R_{\mathrm{r}}}{RT})(1 - x), \\ \dot{r} = \dfrac{\beta}{m_0}(\alpha P - \exp(17.22 - \dfrac{3137}{T - 94.43})0.2r^7), \end{cases} \quad (25)$$

where x is the cross linking conversion, R_{r} is the reaction activation energy, ς is the reaction frequency factor, and R is the gas constant. r is the percentage of the solvent residue in the coating film, α is the weight percentage of vapor phase within the drying air, β is the mass transfer coefficient, and m_0 is the initial amount of solvent within the coating film.

Linearizing the nonlinear equations (24)–(25) and using the parameter values given by [3, 5], we have the process dynamics model and the product quality dynamics model in the form of (1) and (2), respectively, with

$$A_{\mathrm{y}} = \begin{bmatrix} -0.0023 & 0 \\ 0 & -0.0025 \end{bmatrix}, B_{\mathrm{u}} = \begin{bmatrix} 0.023 & 0.04 \\ 0 & 1.667 \end{bmatrix},$$
$$A_\gamma = \mathrm{diag}\{-0.0042, -0.0126\},$$
$$B_{\mathrm{y}} = \begin{bmatrix} 8.9632\times 10^{-5} & 0 \\ -2.3966\times 10^{-4} & 1.892\times 10^{-3} \end{bmatrix}.$$

By [5], the inner loop process controller is chosen as

$$F_1 = \begin{bmatrix} 60.770 & 1.458 \\ 0 & 0.838 \end{bmatrix}, F_2 = \begin{bmatrix} 43.478 & -1.043 \\ 0 & 0.6 \end{bmatrix}.$$

Let $t_k = k,\ \sigma_k = 0.1k,\ k = 1, 2, \cdots$. Using Algorithm 1, we have

$$F_3 = \begin{bmatrix} 23.273 & -0.842 \\ 3.115 & 8.360 \end{bmatrix}, F_4 = \begin{bmatrix} 0.367 & -0.051 \\ 0.044 & 0.416 \end{bmatrix}.$$

To show the tracking performance of the closed-loop system, let the γ^* be square waves as shown in Fig.3 (the solid lines). It can be seen that the product quality variable can track the given performance quickly. Fig.4 shows that the inner system can track perfectly the dynamical setpoints produced by the outer loop system.

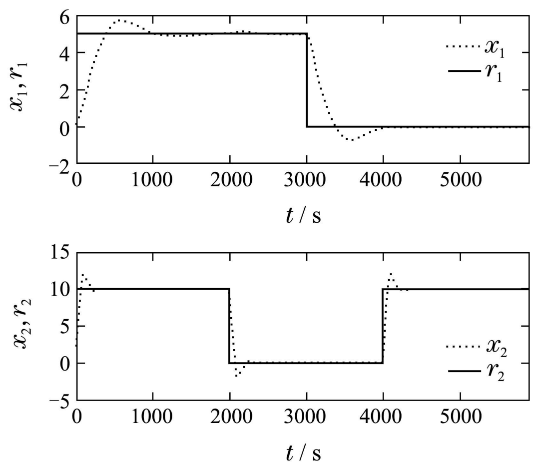
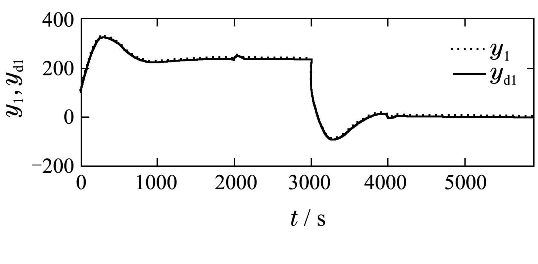


Fig. 3 Outer loop tracking performance for square waves



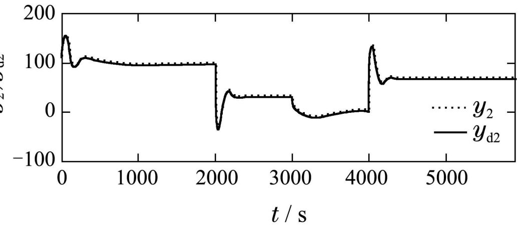


Fig. 4 Inner loop tracking performance for square waves

5 Conclusion

In this paper, we have considered the problem of realtime control for product quality of process industry in the framework of integrated product and process control (IPPC). A dual-rate sampled-data IPPC scheme has been proposed to achieve different sampling rate in the control. The design problem has been reduced to solving matrix inequalities and a homotopy algorithm has been given to solve the matrix inequalities. An example has been presented to show the effectiveness of the proposed dual-rate sampled-data IPPC method.

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