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高阶时滞多智能体系统一致性控制

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摘要: 本文讨论了多输入网络化时滞多智能体系统的一致性问题. 应用无向或有向图描述网络化系统的拓扑结构. 多智能体的一致性收敛问题可以转化为线性时滞系统的稳定性问题. 通过双线性矩阵不等式可以得到高阶多智能体系统时滞相依以及时滞独立的一致性准则. 最后, 举例一个无向拓扑结构的时滞多智能体来说明本文方法的有效性.

关键词: 一致性; 多智能体系统; 线性时不变系统; 李雅普诺夫函数; 双线性矩阵不等式

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Consensus control of higher-order multi-agent systems with delays

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Abstract: This paper discusses consensus problems for multi-input networked multi-agent systems (MAS) with communication delay. Directed or undirected graphs are used to represent the topology of a networked system. Consensus convergence problems of MAS are converted into stability of linear delayed systems. Consensus criteria of higher-order MAS for both delay-independent and delay-dependent asymptotical stabilities are derived in terms of bilinear matrix inequalities (BMI). Finally, a MAS with a delay and an undirected communication is used as an example to show the effectiveness of our strategies.

Key words: consensus; multi-agent systems (MAS); linear time-invariant systems (LTIs); Lyapunov function; bilinear matrix inequalities (BMI)

1 Introduction

A basic problem arising in distributed coordination and control is the so-called consensus problem, which plays a pivotal role in a group of spatially mobile agents with a common mission or task. The goal is to develop a distributed algorithm that can be used by a group of agents to seek to agree upon certain quantities of interest by exchanging information among them. Application of consensus algorithm can be found in cooperative control of vehicles, formation control, flocking, parallel computing, etc. Thus many researchers have focused on the consensus problems[1–12].

The most conventional method to describe the agents' interconnection structure is directed or undirected graphs and the graph Laplacian is often used as a state feedback gain. In most works[3–8], the proof of the states' convergence is usually given using LaSalle's Invariant Principle[13]. And such a graph Laplacian based algorithm is generally limited structure with low dimension and the control specification is also simple. However, in practice, the quantities of each agent are very complex and many aspects should be considered such as position, velocity, temperature, and so on. One-dimensional state consensus only needs to be investigated, if the quantities can be decoupled. But in fact, there exist many situations where these quantities are dependent on each other. Hence, it is necessary to extend the state space of each other from $\mathbb{R}$ to $\mathbb{R}^n$.

Therefore, in the last two years, research interest in consensus problem has been devoted to networks of high-order-integrator agents from a number of different perspectives and under various assumptions[14–24]. The consensus problem for agents modeled as integrator chains of length greater than two was reported in [14–15]. In addition, the consensus problem considering agents modeled by LTIs (linear time-invariant systems) was formulated in [17–18]. Further, references [19–20] studied output feed-back consensus problem for higher-order LTIs. A distributed observer-type protocol solving the consensus problem was also presented in [21–22]. Very recently, Zhai, Okuno, Imae, and Tomoaki[23] reduced the consensus problem on hand to solving a strict matrix inequality with respect to a Lyapunov matrix and a controller gain matrix. Semsar-Kazerooni and Khorasani[24] used a linear-matrix-inequality formulation to address an optimal control design

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strategy for guaranteeing consensus achievement in a network of multi-agent systems.

One-dimensional state consensus is often related to topology construction of communication network, for example a connected undirected graph[3] or a directed graph having a spanning tree[4]. However, just as the description in [18–24], the consensus of multi-input networked MAS is not only related to network topology but also related to state matrix and some gain matrices. As a whole, consensus analysis of multi-input networked MAS is more difficult, and some useful knowledge like graph theory, matrix theory and control theory can serve as effective tools. Communication delays are ubiquitous in networks owing to long distance or the confine of medium. However, except [21], there are few reports on consensus problem of higher-order MAS with communication delay up to now. Motivated by the above-mentioned reasons, this paper mainly focuses on the consensus problem for multi-input networked MAS with delay. In our network model, the dynamics of agents is also assumed to be LTIs. Then the consensus analyses are systematically performed under two cases that the communication topology is a directed or an undirected graph. Consensus analysis of MAS with directed graph is hard, for the corresponding Laplacian matrix often has complex eigenvalues. Therefore, a technique is used to solve the complex eigenvalues. Through some Jordan transformation, consensus question is converted to stability analysis. Next, consensus conditions for delay-independent and delay-dependent asymptotical stabilities are derived in terms of BMIs, and the control gain is solved by using of an effective algorithm.

The remainder of this paper is organized as follows. Section 2 reviews some preliminaries about graph theory, and introduces MAS model, control algorithm for consensus, some necessary definitions and lemmas. The main results on consensus analysis for MAS are presented under two cases in Section 3. In Section 4, numerical simulations are proposed to show validity of the results. Finally, Section 5 concludes the whole paper.

2 Notation and preliminaries

Throughout this paper, some standard notations are utilized. $\mathbb{R}^{n\times n}(\mathbb{R}^n)$ and $\mathbb{C}^{n\times n}(\mathbb{C}^n)$ denote real and complex matrices (vectors) set, respectively. The superscript T represents transpose for real matrices. I_N stands for the identity matrix with dimension N and I denotes the identity matrix of an appropriate dimension. Let $\mathbf{1}$ be the vector with all entries equal to one. For $\zeta \in \mathbb{C}$, Re(ζ) and Im(ζ) mean the real part and imaginary part of ζ, respectively. For matrix A, $\lambda_k(A)$ represents the kth eigenvalue of A. The Kronecker product of matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \vdots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}$$

2.1 Graph theory

A directed graph $\mathcal{G}$ consists of a node set $\mathcal{V}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, in which an edge is represented by a pair of distinct nodes of $\mathcal{G}$: $(i, j) \in \mathcal{E}$, where i is the parent node, j is the child node, and j is neighboring to i. A graph with the property that $(i, j) \in \mathcal{E}$ implies that $(j, i) \in \mathcal{E}$ is said to be undirected. A path on $\mathcal{G}$ from nodes i_1 to i_l is a sequence of ordered edges in the form of (i_k, i_{k+1}), $k = 1, \cdots, l-1$. A directed graph is said to be strongly connected if, for any pair of distinct nodes, there exists a path between them. A directed graph has or contains a directed spanning tree if there exists a node called root such that there exists a directed path from this node to every other node. For an undirected graph, if there exists a path from any node i to any other node j, then it is connected.

Suppose that there are N nodes in a graph. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is defined by $a_{ii} = 0$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and 0 otherwise. Thus, $i, j, \cdots$ represent both nodes and indices, which should not cause confusion from the context. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ is defined as

$$l_{ii} = \sum_{j\neq i} = a_{ij},\ l_{ij} = -a_{ij}$$

for $i \neq j$. For an undirected graph, the Laplacian matrix $\mathcal{L}$ is symmetric, furthermore, all the nonzero eigenvalues of $\mathcal{L}$ are positive, and zero is a simple eigenvalue of $\mathcal{L}$ with eigenvector $\mathbf{1}$ if the undirected graph is connected. For a directed graph, all of the nonzero eigenvalues of $\mathcal{L}$ have positive real parts.

Lemma 1[4] The Laplacian matrix $\mathcal{L}$ of directed graph $\mathcal{G}$ has a simple zero eigenvalue if and only if graph $\mathcal{G}$ contains a spanning tree.

2.2 System model

Consider a MAS which consists of N identical agents with general linear dynamics. The dynamics of ith agent is described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),\ i \in \mathcal{I}, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are state and input of agent i, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are constant matrices with appropriate dimensions, $\mathcal{I} = \{1, 2, \cdots, N\}$ is indices set of agents.

In a MAS, each agent can be considered as a node in a generalized graph, and the information flow between two agents can be regarded as a directed or an undirected path between the nodes. Thus, the interconnection topology in a MAS can be described by a directed or an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$.

Definition 1 For system (1), if there exists an appropriate state feedback $u_i(t)$ such that all agents' states converge to the same vector, i.e.

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \forall\, i, j \in \mathcal{I}.$$

Then it is said that system (1) achieves consensus, and the same vector is called group decision value.

Let $x(t) = [x_1^{\mathrm{T}}(t)\ \ x_2^{\mathrm{T}}(t)\ \ \cdots\ \ x_N^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $u(t) = [u_1^{\mathrm{T}}(t)\ \ u_2^{\mathrm{T}}(t)\ \ \cdots\ \ u_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, then the collective dynamics of system (1) can be written as

$$\dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)u(t). \tag{2}$$

The proposed control law for ith agent with communication delay is represented as

$$u_i(t) = K \sum_{j=1}^{N} (x_i(t-\tau) - x_j(t-\tau)), \tag{3}$$

where $K \in \mathbb{R}^{m\times n}$ is a control gain and τ is a constant delay.

With the control law (3), MAS (2) is expressed as

$$\dot{x}(t) = (I_N \otimes A)x(t) + (\mathcal{L} \otimes BK)x(t-\tau). \tag{4}$$

The following lemma is useful in deriving the criteria.

Lemma 2[25] For any constant matrix $M \in \mathbb{R}^{n\times n}$, $M = M^{\mathrm{T}} > 0$, scalar $\gamma > 0$, and vector function $\dot{x} : [-\gamma, 0] \to \mathbb{R}$ such that the following integration is well defined, then

$$-\gamma \int_{-\gamma}^{0} \dot{x}^{\mathrm{T}}(t+\xi) M \dot{x}(t+\xi)\mathrm{d}\xi \leqslant \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -M & M \\ M & -M \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}.$$

3 Main results

In this section, consensus convergence of MAS (4) is mainly considered under the conditions that communication topology is a directed or an undirected graph.

3.1 Consensus analysis for MAS with undirected graph

Property 1 Suppose communication topology $\mathcal{G}$ of MAS (4) is an undirected and connected graph, coefficient matrix A has no positive real eigenvalue. Then, under the algorithm (3), MAS (4) achieves consensus if the zero solutions of following $N-1$ linear delay systems are asymptotically stable:

$$\dot{z}_i(t) = Az_i(t) + \lambda_i BK z_i(t-\tau),\ i = 2, \cdots, N, \tag{5}$$

where $z_i(t) \in \mathbb{R}^n$, A, B are the coefficient matrices in system (1), K is the feedback gain, and λ_i, $i = 2, \cdots, N$ are nonzero eigenvalues of Laplacian matrix $\mathcal{L}$.

Proof Since the communication topology $\mathcal{G}$ is an undirected and connected graph, the Laplacian matrix $\mathcal{L}$ has N real eigenvalues denoted as

$$0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \leqslant \lambda_3(\mathcal{L}) \leqslant \cdots \leqslant \lambda_N(\mathcal{L}),$$

and zero is a simple eigenvalue for $\mathcal{L}$. Then there exists a matrix S such that $\mathcal{L}$ has the following Jordan form

$$S^{-1}\mathcal{L}S = \varLambda = \mathrm{diag}\{0, \varLambda_{\mathrm{e}}\}.$$

Do coordinate transformation for $x(t)$ as $z(t) = (S^{-1} \otimes I_n)x(t)$, system (4) is written as

$$\begin{aligned}\dot{z}_i(t) = {} & (S^{-1}S\otimes A)z(t) + (S^{-1}\mathcal{L}S \otimes BK)z(t-\tau) = \\ & (I_N \otimes A)z(t) + (\varLambda \otimes BK)z(t-\tau),\end{aligned} \tag{6}$$

where $z(t) = [z_1^{\mathrm{T}}(t)\ \ z_2^{\mathrm{T}}(t)\ \ \cdots\ \ z_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{N_n}$. Let $z_{\mathrm{e}}(t) = [z_2^{\mathrm{T}}(t)\ \ z_3^{\mathrm{T}}(t)\ \ \cdots\ \ z_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, and system (6) can be decomposed as two equations:

$$\dot{z}_1(t) = Az_1(t), \tag{7}$$

$$\dot{z}_{\mathrm{e}}(t) = (I_{N-1} \otimes A)z_{\mathrm{e}}(t) + (\varLambda_{\mathrm{e}} \otimes BK)z_{\mathrm{e}}(t-\tau). \tag{8}$$

Obviously, if $z_{\mathrm{e}}(t)$ converges to zero as $t \to \infty$, MAS (4) achieves consensus. In fact, for symmetric matrix $\mathcal{L}$, its Jordon form is

$$\varLambda = \begin{bmatrix} 0 & & & \\ & \lambda_2(\mathcal{L}) & & \\ & & \ddots & \\ & & & \lambda_N(\mathcal{L}) \end{bmatrix}.$$

Therefore, equation (8) is equal to

$$\begin{aligned} \dot{z}_2(t) &= Az_2(t) + \lambda_2(\mathcal{L})BKz_2(t-\tau), \\ \dot{z}_3(t) &= Az_3(t) + \lambda_3(\mathcal{L})BKz_3(t-\tau), \\ &\vdots \\ \dot{z}_N(t) &= Az_N(t) + \lambda_N(\mathcal{L})BKz_N(t-\tau). \end{aligned}$$

At last, it is concluded that if linear system (5) is asymptotically stable about zero for $i = 2, \cdots, N$, then MAS (4) achieves consensus. The proof is completed.

Remark 1 From [19], it stated that group decision value of MAS (4) is $N^{1/2}\exp(At)(l_l^{\mathrm{T}} \otimes I_n)$ which can be obtained from (7) easily, where l_l is left-eigenvector of $\mathcal{L}$ corresponding to 0.

Property 1 has converted consensus problem of MAS into stability analysis of linear delay systems. Therefore, using the Lyapunov-Krasovskii functional approach, some sufficient conditions for ensuring the consensus of MAS (4) are derived as follows.

Theorem 1 Suppose that communication topology $\mathcal{G}$ of MAS (4) is an undirected and connected graph. If there exist positive-definite matrices P, Q and matrix K, such that

$$\varSigma(P, Q, K) = \begin{bmatrix} A^{\mathrm{T}}P + PA + Q & \lambda_i PBK \\ \lambda_i K^{\mathrm{T}}B^{\mathrm{T}}P & -Q \end{bmatrix} < 0, \tag{9}$$

$i = 2, \cdots, N$, then MAS (4) achieves consensus asymptotically for any time delay $\tau > 0$.

Proof Choose a Lyapunov-Krasovskii functional as

$$V(z_i(t)) = z_i^{\mathrm{T}}(t)Pz_i(t) + \int_{t-\tau}^{t} z_i^{\mathrm{T}}(\alpha)Qz_i(\alpha)\mathrm{d}\alpha, \tag{10}$$

where P and Q are positive-definite matrices. Taking time derivative of $V(z_i(t))$ along any trajectory of system (5) is

$$\begin{aligned}\dot{V}(z_i(t)) = {} & \dot{z}_i^{\mathrm{T}}(t)Pz_i(t) + z_i^{\mathrm{T}}(t)P\dot{z}_i(t) + \\ & z_i^{\mathrm{T}}(t)Qz_i(t) - z_i^{\mathrm{T}}(t-\tau)Qz_i(t-\tau) = \\ & z_i^{\mathrm{T}}(t)[A^{\mathrm{T}}P + PA + Q]z_i(t) - \\ & z_i^{\mathrm{T}}(t-\tau)Qz_i(t-\tau) + \\ & 2\lambda_i z_i^{\mathrm{T}}(t-\tau)K^{\mathrm{T}}B^{\mathrm{T}}Pz_i(t) = \\ & Z_i^{\mathrm{T}}(t)\varSigma Z_i(t),\end{aligned}$$

where $Z_i(t) = [z_i^{\mathrm{T}}(t)\ \ z_i^{\mathrm{T}}(t-\tau)]^{\mathrm{T}}$,

$$\varSigma = \begin{bmatrix} A^{\mathrm{T}}P + PA + Q & \lambda_i PBK \\ \lambda_i K^{\mathrm{T}}B^{\mathrm{T}}P & -Q \end{bmatrix}.$$

When inequality (9) holds, we have $\dot{V}_i(z_i(t)) < 0$, for $i = 2, \cdots, N$. From Lyapunov-Krasovskii Stability Theorem, it is known that system (5) is asymptotically stable. Therefore, by Proposition 1, it is derived that MAS (4) achieves consensus asymptotically. This completes the proof.

Theorem 1 is a delay-independent stability criterion for MAS (4). As we know, delay-independent stability criteria are suitable for the system which is asymptotically stable for an arbitrary delay. However, such criteria are often very conservative if the delay is already known and small. In contrast with delay-independent stability, delay-dependent stability is concerned with the size of the delay and provides an upper bound of the delay such that the system is stable for any delay less than the upper bound. In addition, delay-dependent stability is less conservative than delay-independent one. In the following, a delay-dependent criterion for the asymptotical consensus of MAS (4) is provided.

Theorem 2 Suppose that communication topology $\mathcal{G}$ of MAS (4) is an undirected and connected graph, and the time-invariant delay $\tau \in [0, h]$ for some $h < \infty$. If there exist positive-definite matrices P, R and matrix K, such that

$$\varPi(P,R,K) = \begin{bmatrix} A^{\mathrm{T}}P+PA-R & PBK+R & hA^{\mathrm{T}}R \\ \lambda_i K^{\mathrm{T}}B^{\mathrm{T}}P+R & -R & \lambda_i h K^{\mathrm{T}}B^{\mathrm{T}}R \\ hRA & \lambda_i hRBK & -R \end{bmatrix} < 0, \tag{11}$$

$i = 2, \cdots, N$, then MAS (4) achieves consensus asymptotically for any time delay $\tau \in [0, h]$.

Proof Choose a Lyapunov-Krasovskii functional as

$$V(z_i(t)) = V_1(z_i(t)) + V_2(z_i(t)), \tag{12}$$

where

$$V_1(z_i(t)) = z_i^{\mathrm{T}}(t)Pz_i(t);$$
$$V_2(z_i(t)) = \int_{t-h}^{t}(h-t+\alpha)\dot{z}_i^{\mathrm{T}}(\alpha)hR\dot{z}_i(\alpha)\mathrm{d}\alpha,$$

where P and R are positive-definite matrices. Taking time derivative of $V(z_i(t))$ along any trajectory of system (5) is

$$\begin{aligned}&\dot{V}(z_i(t)) = \dot{V}_1(z_i(t)) + \dot{V}_2(z_i(t)) = \\ &z_i^{\mathrm{T}}(t)[A^{\mathrm{T}}P+PA]z_i(t) + 2\lambda_i z_i^{\mathrm{T}}PBKz_i(t-\tau) + \\ &\dot{z}_i^{\mathrm{T}}(t)h^2R\dot{z}_i(t) - \int_{t-h}^{t}\dot{z}_i^{\mathrm{T}}(\alpha)hR\dot{z}_i(\alpha)\mathrm{d}\alpha \leqslant \\ &z_i^{\mathrm{T}}(t)[A^{\mathrm{T}}P+PA]z_i(t) + 2\lambda_i z_i^{\mathrm{T}}PBKz_i(t-\tau) + \\ &[Az_i(t)+\lambda_i BKz_i(t-\tau)]^{\mathrm{T}}h^2R\cdot \\ &[Az_i(t)+\lambda_i BKz_i(t-\tau)] + \\ &\begin{bmatrix} z_i(t) \\ z_i(t-\tau)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -R & R \\ R & -R\end{bmatrix} \begin{bmatrix} z_i(t) \\ z_i(t-\tau)\end{bmatrix},\end{aligned}$$

where the inequality above is obtained from Lemma 2. Then, it follows that $\dot{V}_i(z_i(t)) \leqslant Z_i^{\mathrm{T}}(t)\varXi Z_i(t)$, where

$$Z_i(t) = [z_i^{\mathrm{T}}(t)\ \ z_i^{\mathrm{T}}(t-\tau)]^{\mathrm{T}},$$
$$\varXi = \begin{bmatrix} \varXi_{11} & \varXi_{12} \\ \varXi_{12}^{\mathrm{T}} & \varXi_{22}\end{bmatrix},$$

where

$$\varXi_{11} = A^{\mathrm{T}}P + PA + A^{\mathrm{T}}h^2RA - R,$$
$$\varXi_{12} = \lambda_i PBK + \lambda_i A^{\mathrm{T}}h^2RBK + R,$$
$$\varXi_{22} = \lambda_i^2 K^{\mathrm{T}}B^{\mathrm{T}}h^2RBK - R.$$

From Schur complements, it is verity that inequality (11) is equivalent to $\varXi < 0$. Therefore, we have $\dot{V}_i(z_i(t)) < 0$, for $i = 2, \cdots, N$. From Lyapunov-Krasovskii Stability Theorem, it is known that systems (5) is asymptotically stable. Using Proposition 1, it is concluded that MAS (4) achieves consensus asymptotically. This ends the proof.

Through Theorem 1 or Theorem 2, consensus problem of MAS (4) is formulated as a feasible problem with BMI constrains:

Problem 1 $\min t$, s.t. $\varSigma < tI(\varPi < tI)$.

It is intuitively obvious inequalities (9) and (11) are both bilinear matrix inequalities about matrices P, $Q(R)$ and K. Then, a alternate algorithm which is used in [26–27] is adopted in order to solve BMIs (9) and (11).

Algorithm 1

Step 1 Initialization. Let $k = 1$, give the initial value to $P = P^0$, $Q = Q^0 (R = R^0)$;

Step 2 Repeat. Let $k = k + 1$, solve problem

$$\min t,\ \text{s.t.}\ \varSigma(P^{k-1}, Q^{k-1}, K) < tI(\varPi(P^{k-1}, R^{k-1}, K) < tI)$$

to obtain the solution K, denote $K^k = K$, solve the problem

$$\min t,\ \text{s.t.}\ \varSigma(P, Q, K^k) < tI(\varPi(P, R, K^k) < tI)$$

to obtain the solution $P, Q(R)$, denote $P^k = P$, $Q^k = Q$ $(R^k = R)$;

Step 3 End. The solution P^k, $Q^k(R^k)$, K^k satisfy inequality

$$\varSigma(P^k, Q^k, K^k) < tI(\varPi(P^k, R^k, K^k) < tI),$$

or there is no feasible solution to Problem 1.

By applying Algorithm 1, the control gain K in control law (3) is obtained.

3.2 Consensus analysis for MAS with directed graph

As we know directed graphs are more complex than undirected graphs since direct graphs not only describe coupling relations between nodes but also specify coupling direction. MSA (4) with directed communication topology $\mathcal{G}$ is more general in reality and is well worth consensus analysis. The Laplacian matrix $\mathcal{L}$ of a directed graph $\mathcal{G}$ is often asymmetric and reducible and cannot be diagonalizable. Therefore, when the communication topology of MAS (4) is a directed graph $\mathcal{G}$ which contains a spanning tree, the properties of Jordan block matrix are adopted to analyze consensus of MAS (4).

Property 2 Suppose communication topology $\mathcal{G}$ of MAS (4) is directed and contains a spanning tree, and coefficient matrix A has no positive real eigenvalue. Then, under the algorithm (3), MAS (4) achieves consensus if the zero solutions of following $k-1$ linear delay systems are asymptotically stable:

$$\dot{z}_i(t) = Az_i(t) + \lambda_i BKz_i(t-\tau),\ i = 2, \cdots, k. \tag{13}$$

where $z_i(t) \in \mathbb{C}^n$, A, B are the coefficient matrices in system (1), K is the feedback gain, and λ_i, $i = 2, \cdots, k$ are nonzero eigenvalues of Laplacian matrix $\mathcal{L}$.

Proof Similar to the proof of Property 1, there exists a nonsingular matrix $\bar{S}$ such that $\bar{S}^{-1}\mathcal{L}\bar{S} = \bar{\Lambda}$. Let $\eta(t) = (\bar{S}^{-1} \otimes I_n)x(t)$, MAS (4) is rewritten as

$$\dot{\eta}(t) = (I_N \otimes A)\eta(t) + (\bar{\Lambda} \otimes BK)\eta(t-\tau). \quad (14)$$

When the graph $\mathcal{G}$ is directed and contains a spanning tree, its Laplacian matrix $\mathcal{L}$ is asymmetric and reducible, from Lemma 1, Jordan matrix of $\mathcal{L}$ has the following form:

$$\bar{\Lambda} = \begin{bmatrix} 0 & & & \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_k \end{bmatrix},$$

where J_i is the Jordan block corresponding to the n_i multiple eigenvalue λ_i of $\mathcal{L}$:

$$J_i = \begin{bmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{bmatrix}_{n_i \times n_i}, \ i = 2, \cdots, k.$$

From the expression of $\bar{\Lambda}$, equation (14) is decomposed as

$$\dot{\eta}_1(t) = A\eta_1(t), \quad (15)$$

$$\dot{\eta}_{\mathrm{e}}(t) = (I_{N-1} \otimes A)\eta_{\mathrm{e}}(t) + (\bar{\Lambda}_{\mathrm{e}} \otimes BK)\eta_{\mathrm{e}}(t-\tau), \quad (16)$$

where

$$\eta_{\mathrm{e}}(t) = [\eta_2^{\mathrm{T}}(t) \ \ \eta_3^{\mathrm{T}}(t) \ \ \cdots \ \ \eta_k^{\mathrm{T}}(t)]^{\mathrm{T}}$$

with

$$\eta_i(t) = [\eta_{i1}^{\mathrm{T}}(t) \ \ \eta_{i2}^{\mathrm{T}}(t) \ \ \cdots \ \ \eta_{in_i}^{\mathrm{T}}(t)]^{\mathrm{T}}$$

and $\eta_{ij}(t) \in \mathbb{C}^n$ for $1 \leqslant j \leqslant n_i (i = 2, \cdots, k)$. (15) and (16) show that if $\eta_{\mathrm{e}}(t)$ converges to zero as $t \to \infty$, MAS (4) achieves consensus. Next, it is only needed to prove equation (16) is asymptotically stable about zero solutions. When $2 \leqslant i \leqslant k$, $\eta_i(t)$ has the following forms:

$$\begin{cases} \dot{\eta}_{i1}(t) = A\eta_{i1}(t) + \lambda_i BK\eta_{i1}(t-\tau) + \\ \qquad BK\eta_{i2}(t-\tau), \\ \dot{\eta}_{i2}(t) = A\eta_{i2}(t) + \lambda_i BK\eta_{i2}(t-\tau) + \\ \qquad BK\eta_{i3}(t-\tau), \\ \vdots \\ \dot{\eta}_{in_i-1}(t) = A\eta_{in_i-1}(t) + \lambda_i BK\eta_{in_i-1} \\ \qquad (t-\tau) + BK\eta_{in_i}(t-\tau), \\ \dot{\eta}_{in_i}(t) = A\eta_{in_i}(t) + \lambda_i BK\eta_{in_i}(t-\tau). \end{cases} \quad (17)$$

Via above deduction, it can be concluded that if linear system (17) is asymptotically stable, zero solutions of following dynamical systems

$$\dot{\eta}_i(t) = (I_{n_i} \otimes A)\eta_i(t) + (J_i \otimes BK)\eta_i(t-\tau), \\ i = 2, \cdots, k,$$

are asymptotically stable. Therefore, if zero solutions of linear delay systems (13) are asymptotically stable, system (16) is asymptotically stable about zero solution. This ends the proof.

When communication graph $\mathcal{G}$ is directed and has a spanning tree, Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is not symmetric and irreducible and may has complex eigenvalues λ_i, $i = 2, \cdots, k$. As a result, in liner system (13), $z_i(t)$ is a complex vector. In that cases, let $z_i(t) = \mathrm{Re}(z_i(t)) + \mathrm{j}\mathrm{Im}(z_i(t))$ be the solution of system(13) and $\lambda_i = \alpha_i + \mathrm{j}\beta_i$, in which j is the imaginary unit, then

$$\begin{aligned} \mathrm{Re}(\dot{z}_i(t)) = & \ A\mathrm{Re}(z_i(t)) + \alpha_i BK\mathrm{Re}(z_i(t-\tau)) - \\ & \ \beta_i BK\mathrm{Im}(z_i(t-\tau)), \\ \mathrm{Im}(\dot{z}_i(t)) = & \ A\mathrm{Im}(z_i(t)) + \alpha_i BK\mathrm{Im}(z_i(t-\tau)) + \\ & \ \beta_i BK\mathrm{Re}(z_i(t-\tau)), \\ & \ i = 2, \cdots, k. \end{aligned} \quad (18)$$

Denote $\varphi_i(t) = [\mathrm{Re}(z_i(t)) \ \ \mathrm{Im}(z_i(t))]^{\mathrm{T}} \in \mathbb{R}^{2n}$, $\bar{A} = \mathrm{diag}\{A, A\}$ and

$$\Delta_i = \begin{bmatrix} \alpha_i BK & -\beta_i BK \\ \beta_i BK & \alpha_i BK \end{bmatrix},$$

it follows from (18) that

$$\dot{\varphi}_i(t) = \bar{A}\varphi_i(t) + \Delta_i\varphi_i(t-\tau), \ i = 2, \cdots, k. \quad (19)$$

Here, linear delay systems (19) are in real field. Choose Lyapunov-Krasovskii functionals (10) and (12) for (19), two corresponding consensus criteria are derived as follows: one is delay-independent and the other is delay-dependent.

Theorem 3 Suppose communication topology $\mathcal{G}$ of MAS (4) is directed and contains a spanning tree. If there exist positive-definite matrices P, Q and matrix K, such that

$$\bar{\Sigma}(P, Q, K) = \begin{bmatrix} \bar{A}^{\mathrm{T}}P + P\bar{A} + Q & P\Delta_i \\ \Delta_i^{\mathrm{T}}P & -Q \end{bmatrix} < 0, \quad (20)$$

$i = 2, \cdots, k$, where

$$\Delta_i = \begin{bmatrix} \alpha_i BK & -\beta_i BK \\ \beta_i BK & \alpha_i BK \end{bmatrix},$$

then MAS (4) achieves consensus asymptotically for any delay $\tau > 0$.

Theorem 4 Suppose communication topology $\mathcal{G}$ of MAS (4) is directed and contains a spanning tree, and the time-invariant delay $\tau \in [0, h]$ for some $h < \infty$. If there exist positive-definite matrices P, R and matrix K, such that

$$\bar{\Pi}(P, R, K) = \begin{bmatrix} \bar{A}^{\mathrm{T}}P + P\bar{A} + R & P\Delta_i + R & h\bar{A}^{\mathrm{T}}R \\ \Delta_i^{\mathrm{T}}P + R & -R & h\Delta_i^{\mathrm{T}}R \\ hR\bar{A} & hR\Delta_i & -R \end{bmatrix} < 0, \quad (21)$$

$i = 2, \cdots, k$, where

$$\Delta_i = \begin{bmatrix} \alpha_i BK & -\beta_i BK \\ \beta_i BK & \alpha_i BK \end{bmatrix},$$

then MAS (4) achieves consensus asymptotically for any time delay $\tau \in [0, h]$.

Applying Algorithm 1 to BMIs (20) and (21), control gain K can be solved easily. So, we don't repeat the steps here.

4 Examples and simulations

In this section, some simulations of a network with an undirected topology are provided to illustrate the effectiveness of our approach.

Example 1 Consider a network consisting of six agents, and the dynamics of agent i is two-dimensional linear system that is given by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),\ i = 1, 2, \cdots, 6, \tag{22}$$

where $x_i(t) \in \mathbb{R}^2$, $u_i(t) \in \mathbb{R}^2$, and for simplicity let the following numerical parameters for above system:

$$A = \begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix},\ B = I_2.$$

Suppose that the communication topology of multi-agent system (22) is an undirected graph $\mathcal{G}$ which is described by Fig. 1. From Fig. 1, the Laplacian matrix $\mathcal{L}_a$ of undirected graph $\mathcal{G}_a$ is shown as follows:

$$\mathcal{L}_a = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & -1 & -1 & 0 \\ 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix},$$

and the eigenvalues of $\mathcal{L}_a$ are $\lambda_1(\mathcal{L}_a) = 0$, $\lambda_2(\mathcal{L}_a) = 1$ (2 multiplicity), $\lambda_3(\mathcal{L}_a) = 3$ (2 multiplicity), $\lambda_4(\mathcal{L}_a) = 4$. Let initial value be $P^0 = Q^0 = \mathrm{diag}\{1, 1\}$, using Algorithm 1 for Theorem 1, the control gain is obtained as

$$K = \begin{bmatrix} 0.277 & -0.2064 \\ -0.1956 & 0.4780 \end{bmatrix}.$$

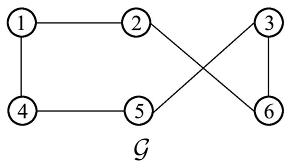

Fig. 1 Undirected and connect communication topology graph

Figure 2 shows that MAS (22) achieves consensus with delay $\tau = 3$, obtained control gain K and initial values

$$x_{10} = [3\ \ 1.2]^{\mathrm{T}},\ x_{20} = [3.5\ -1]^{\mathrm{T}},$$
$$x_{30} = [-2\ -1]^{\mathrm{T}},\ x_{40} = [-2\ \ 1.2]^{\mathrm{T}},$$
$$x_{50} = [4\ \ 1.5]^{\mathrm{T}},\ x_{60} = [-1.5\ \ 3]^{\mathrm{T}}.$$

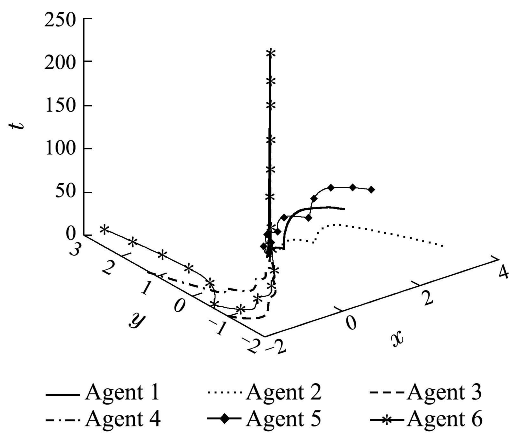

Fig. 2 States trajectory of MAS (22) with communication topology G.

For delay-dependent criterion, apply Algorithm 1 to Theorem 2 and choose initial value $P^0 = R^0 = I_2$, then a control gain

$$K = \begin{bmatrix} 0.0168 & -0.0413 \\ 0.0068 & 0.0229 \end{bmatrix}$$

is obtained with the upper bound of delay $h = 0.12$. Fig. 3 shows that MAS (22) achieves consensus with delay $\tau = 0.12$, obtained control gain K and same initial values $x_{10}, x_{20}, x_{30}, x_{40}, x_{50}, x_{60}$.

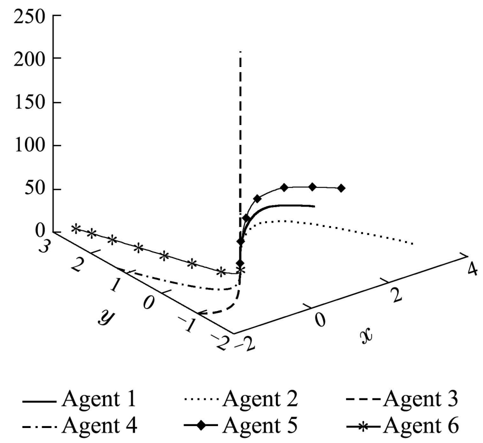

Fig. 3 States trajectory of MAS (22) with communication topology G

5 Conclusions

This paper has studied the consensus problems of higher-order MAS with communication delay and the dynamics of each agents is assumed to be general LITs. Our consensus algorithm is designed by a state feedback gain. Here, it is supposed that the communication topology is a directed or an undirected graph. By coordinate transformation, the consensus problems of MAS have been converted into stability problems of some LITs. Then, two kinds of consensus conditions which are expressed by BMIs have been derived: one is delay-independent and the other is delay-dependent. The control gain is solved from the BMIs by an effective algorithm. At last, a MAS with undirected graph and fixed delay is presented as a example to demonstrate the effectiveness of our theoretical results. In future, the consensus of MAS with switching network topologies and each agent having different dynamics will be analyzed.

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