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带有内动态的离散不确定非线性系统的滑模预测跟踪控制

张利军[1,2], 赵杰梅[1], 齐　雪[3]

(1. 哈尔滨工程大学 自动化学院, 黑龙江 哈尔滨 150001;

2. 西北工业大学 航海学院, 陕西 西安 710072; 3. 安徽科技学院 理学院, 安徽 蚌埠 233100)

摘要: 对于一类带有内动态的单输入-单输出不确定离散非线性系统, 基于滑模预测控制技术设计了一个控制器. 通过反馈校正和滚动优化技术, 可以及时补偿不确定性的影响, 提高了匹配和不匹配不确定项的鲁棒性. 然后, 通过滚动优化技术得到期望的滑模控制律. 特别地, 通过预测控制, 滑模控制的抖振现象可以消除. 最后, 在不确定项的界未知的情况下, 得到闭环系统的所有信号都是有界的, 并且跟踪误差是鲁棒稳定的. 仿真例子说明所提出控制方法的有效性.

关键词: 离散非线性系统; 预测控制; 滑模控制; 内动态

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Sliding-mode prediction tracking control for discrete-time uncertain nonlinear systems with internal dynamics

ZHANG Li-jun[1,2], ZHAO Jie-mei[1], QI Xue[3]

(1. College of Automation, Harbin Engineering University, Harbin Heilongjiang 150001, China;
2. School of Marine, Northwestern Polytechnical University, Xi’an Shaanxi 710072, China;
3. School of Science, Anhui Science and Technology University, Bengbu Anhui 233100, China)

Abstract: Based on the sliding-mode predictive control (SMPC) technique, a controller for a class of discrete-time nonlinear uncertain single-input-single-output (SISO) systems with internal dynamics is proposed. Due to the feedback correction and receding horizon optimization, the effect of uncertainty can be compensated in time, and the robustness to matched or unmatched uncertainties has been improved. The desired sliding-mode control law is obtained by using the receding horizon optimization subsequently. Especially, the chattering of sliding-mode control (SMC) can be eliminated by predictive control techniques. It is shown that all the signals of the closed-loop systems are bounded and the tracking error is robustly stable, without requiring the boundary knowledge of the uncertainty. Simulation result is given to demonstrate the advantages of the proposed method.

Key words: discrete-time nonlinear systems; predictive control; sliding mode control; internal dynamic

1 Introduction

Sliding-mode control (SMC) is a popular control approach for systems containing uncertainties or unknown disturbances. It is derived from variable structure control (VSC) which was studied originally by [1]. For a broad class of systems, this kind of control is particularly appealing due to its ability to deal with nonlinearities, time-variance, as well as uncertainties and disturbances, in a direct manner in the face of modeling imprecisions (see [2−4]). The first step in SMC is to define a sliding surface. The second step is to synthesize a suitable control law to globally drive the trajectory onto the predefined sliding surface in finite time and maintain them there for all subsequent time. The experimental results show that this control strategy provides good performance even in the presence of unknown nonlinear parameters.

Model predictive control (MPC) has become one of the most popular control methodologies in both industry and academia. It has been successfully implemented in many industrial control, showing good performance (see [5-16]). The basic idea of MPC is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The performance index to be optimized is the expectation of a quadratic function measuring the distance between the predictive system output and a predictive reference sequence over the horizon, and a quadratic function representing the control effort.

However, it is well known that chattering is a

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defect for SMC, which can be reduced by some approaches, such as the integral SMC and range-bounded SMC technique. However, the reduction of chattering will often decrease the robustness of the closed-loop systems. The sliding mode predictive control (SMPC) strategy is proposed to tackle this problem in some literature. The characteristics of prediction and receding horizon can help to improve the performance of the reaching mode in SMC, which is one of the goals that the SMPC strategy achieves. Another achievement is the capability of controlling the processes with large time delays and high controllability ratio. Moreover, the implementation problem of a SMC when the state is not accessible can be solved with the predictive strategy.

Some work based on the SMPC technique can be found. For example, a dual mode control scheme characterized by non-linear SMPC was presented in [9]; Ref.[10] applied SMPC to a solar air conditioning plant. But all of the above are specified by continuous-time systems. In fact, due to the widespread application of digital implementations, discrete-time systems are very common in real plants. What is more, a stable continuous-time system may become unstable after being discretized. Therefore, it is necessary to design controllers for discrete-time systems directly. Ref. [11] investigated a SMPC algorithm for a class of discrete-time n-joint rigid robotic manipulator systems, where authors claim SMPC can remove the chattering. Stability analysis for a triangular discrete-time nonlinear systems is studied in [12] via the SMPC approach. Explicit/multi-parametric MPC of linear discrete-time systems is investigated in [13]. Explicit/multi-parametric MPC of linear discrete-time systems is investigated in [14]. But none of these studies has ever dealt with the discrete-time systems with nonlinear internal dynamics which will seriously increase the complexity of the considered system.

In this paper, a controller for a class of discrete-time nonlinear uncertain SISO systems with nonlinear internal dynamics is presented. The main contributions of this paper are: i) a predictive value of the sliding mode is estimated based on a constructed prediction model, which tracks the expected sliding mode reference value, thus a chattering issue can be avoided; ii) through feedback correction and receding horizon optimization, a controller is obtained which guarantees the tracking error is robustly stable while all the signals of the closed-system are bounded; iii) for the internal dynamics, an input-to-state stability(ISS) theory is applied, which ensures the overall system for stability. Simulation result is provided to illustrate the effectiveness of the proposed method.

2 Problem formulation

Consider an SISO discrete-time system described by the following normal form:

$$\begin{cases} x_i(k+1) = x_{i+1}(k),\ i = 1, \cdots, r-1, \\ x_{\mathrm{r}}(k+1) = \\ f(\xi(k), \eta(k)) + g(\xi(k), \eta(k))u(k) + d(k), \\ \eta(k+1) = p(\xi(k), \eta(k)), \\ y(k) = x_1(k), \end{cases} \tag{1}$$

where $\xi(k) = [x_1(k)\ \ x_2(k)\ \ \cdots\ \ x_{\mathrm{r}}(k)]^{\mathrm{T}} \in \mathbb{R}^r$, $\eta(k) \in \mathbb{R}^{n-r}$, $u(k) \in \mathbb{R}$ and $y(k) \in \mathbb{R}$ are the state, the internal dynamic, the input and the output, respectively. $f(\xi(k), \eta(k))$, $g(\xi(k), \eta(k))$ and $p(\xi(k), \eta(k))$ are the known functions and $g(\xi(k), \eta(k)) \neq 0$, $d(k)$ is the uncertainty and external disturbance.

Based on the development of differential geometry, many systems can be transformed into the Brunowsky-like canonical form by differential homeomorphism map (see [17]).

The control objective is to design a control law $u(k)$ via SMPC so that the output $y(k)$ of system with external disturbance tracks the desired trajectory $y_{\mathrm{d}}(k)$ while all the signals in the closed-loop system are bounded.

By system (1), it is easy to see $y(k+i) = x_{i+1}(k)$, $i = 0, 1, \cdots, r-1$. So the vector $\xi(k)$ can be rewritten as

$$\xi(k) = [y(k)\ \ y(k+1)\ \ \cdots\ \ y(k+r-1)]^{\mathrm{T}}.$$

Similarly,

$$\xi_{\mathrm{d}}(k) = [y_{\mathrm{d}}(k)\quad y_{\mathrm{d}}(k+1) \cdots y_{\mathrm{d}}(k+r-1)]^{\mathrm{T}}.$$

Then the original system (1) is converted to

$$\begin{cases} y(k+r) = \\ f(\xi(k), \eta(k)) + g(\xi(k), \eta(k))u(k) + d(k), \\ \eta(k+1) = p(\xi(k), \eta(k)). \end{cases} \tag{2}$$

Now let $\tilde{y}(k) = y(k) - y_{\mathrm{d}}(k)$. Define a discrete linear filter as follows:

$$\begin{aligned} &e(k) = \\ &\tilde{y}(k+r-1) + \lambda_{r-1}\tilde{y}(k+r-2) + \cdots + \\ &\lambda_1 \tilde{y}(k) = [\varLambda^{\mathrm{T}}\ 1]\tilde{\xi}(k), \end{aligned} \tag{3}$$

where $\tilde{\xi}(k) = \xi(k) - \xi_{\mathrm{d}}(k)$, $\varLambda = [\lambda_1\ \ \lambda_2\ \ \cdots\ \ \lambda_{r-1}]^{\mathrm{T}}$, λ_i, $i = 1, 2, \cdots, r-1$ are chosen such that the polynomial $H(z) := z^{r-1} + \lambda_{r-1}z^{r-2} + \cdots + \lambda_1$ is Schur.

Based on Eqs. (2) and (3), an error dynamic system can be expressed as follows:

$$\begin{cases} e(k+1) = \\ [0 \ \ \Lambda^{\mathrm{T}}]\tilde{\xi}(k) - y_{\mathrm{d}}(k+r) + f(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k)) + \\ g(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k))u(k) + d(k), \\ \eta(k+1) = p(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k)), \\ \tilde{y}(k) = y(k) - y_{\mathrm{d}}(k). \end{cases} \tag{4}$$

We will design controller and analyze the stability of system (4). To this end, we will apply the SMPC technique to system (4).

3 Main results

Predictive control has three steps, prediction model, feedback correction and receding horizon optimization. During the design process, because of feedback correction and receding horizon optimization, the effect of uncertainties can be compensated in time, and strong robustness to matched or unmatched uncertainties is ensured. Simultaneously, the control signal can be optimized continually and online. Therefore, the combination of predictive control with SMC can do great help to controller design.

3.1 Design of sliding mode prediction model

The prediction model of system (1) is given as follows:

$$\begin{cases} x_{\mathrm{m},i}(k+1) = x_{\mathrm{m},i+1}(k), \ i = 1, \cdots, r-1, \\ x_{\mathrm{m},r}(k+1) = \\ f(\xi(k), \eta(k)) + g(\xi(k), \eta(k))u(k), \\ \eta_{\mathrm{m}}(k+1) = p(\xi(k), \eta(k)), \\ y_{\mathrm{m}}(k) = x_{\mathrm{m},1}(k). \end{cases} \tag{5}$$

Define the sliding mode surface as

$$s(k) = e(k) = [\Lambda^{\mathrm{T}} \ \ 1]\tilde{\xi}(k). \tag{6}$$

In order to use the predictive control strategy to improve the performance of SMC, a suitable sliding-mode prediction model should be created at first. According to recursive sliding mode approach which described in [11,12,15−16], we construct the following sliding mode prediction model:

$$\begin{aligned} &s_{\mathrm{m}}(k+1) = \\ &[0 \ \ \Lambda^{\mathrm{T}}]\tilde{\xi}(k) - y_{\mathrm{d}}(k+r) + \\ &f(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k)) + \\ &g(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k))u(k) + \gamma s(k), \end{aligned} \tag{7}$$

where $0 < \gamma < 1$ is a manipulative parameter.

3.2 Design of control law

In practice, there exist errors between the model output and the real output. So feedback correction is needed to solve this problem. Let $\tilde{s}_{\mathrm{m}}(k+1)$ be the feedback correction value of $s_{\mathrm{m}}(k+1)$. Then

$$\tilde{s}_{\mathrm{m}}(k+1) = s_{\mathrm{m}}(k+1) + \sigma(s(k) - s_{\mathrm{m}}(k)), \tag{8}$$

where σ is a weighted feedback correction rate. From the viewpoint of practice, $0 < \sigma < 1$ is the appropriate range of σ (see [11]).

Let $\bar{s}(k) = s(k) - s_{\mathrm{m}}(k)$ for the sake of clarification. Then (8) reduced to

$$\begin{aligned} &\tilde{s}_{\mathrm{m}}(k+1) = s_{\mathrm{m}}(k+1) + \sigma\bar{s}(k) = \\ &h(k) + g(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k))u(k), \end{aligned} \tag{9}$$

where $h(k) = [0 \ \ \Lambda^{\mathrm{T}}]\tilde{\xi}(k) - y_{\mathrm{d}}(k+r) + f(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k)) + \gamma s(k) + \sigma\bar{s}(k)$.

Now, performance index is given as

$$J = (\tilde{s}_{\mathrm{m}}(k+1) - s_{\mathrm{r}})^2 + \lambda u^2(k), \tag{10}$$

where s_{r} is a sliding mode reference value.

Since the SMC objective is to keep states on the sliding surface and maintain them there for all subsequent time, the desired sliding mode reference value should be $s_{\mathrm{r}} = 0$. In this case, when the sliding mode reference value is tracked exactly, the sliding surface can be reached precisely. Therefore, the performance index (10) can be reduced to

$$J = \tilde{s}^2_{\mathrm{m}}(k+1) + \lambda u^2(k), \tag{11}$$

where λ is a weight coefficient, which adjusts the relation between the closed-loop output of sliding mode predictive model and the control signal.

According to equation (9), performance index (11) can be rewritten as

$$J = [h(k) + g(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k))u(k)]^2 + \lambda u^2(k). \tag{12}$$

Minimizing (12) gives the control signal $u(k)$. By setting the partial derivative of J to zero, that is, $\dfrac{\partial J}{\partial u} = 0$, solving the resulting equation, the optimal solution to $u(k)$ is

$$u(k) = -\frac{h(k)g(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k))}{g^2(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k)) + \lambda}. \tag{13}$$

Up to now, the control input $u(k)$ for closed-loop system (4) has been obtained.

3.3 Robust stability analysis

From performance index (12), one can see that $u(k)$ affects J less when associating with the decreasing of λ. For the sake of simplicity, according to optimal control theory, suppose that $\lambda = 0$, i.e., the case when control signal is not optimal. Thus, control law (13) reduces to

$$u(k) = -\frac{h(k)}{g(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k))}. \tag{14}$$

Consider system (4) and sliding-mode function (6), it follows

$$s(k+1) =$$

$$[0\ \ \Lambda^{\mathrm{T}}]\tilde{\xi}(k) - y_{\mathrm{d}}(k+r) + f(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k)) + g(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k))u(k) + d(k). \tag{15}$$

Putting control law (14) into (15), we can get

$$s(k+1) = -\gamma s(k) - \sigma\bar{s}(k) + d(k). \tag{16}$$

According to (7) and (9), we have

$$s_{\mathrm{m}}(k) = h(k-1) - \sigma\bar{s}(k-1). \tag{17}$$

From system (4) and $e(k) = s(k)$, we obtain

$$s(k)-d(k-1)=h(k-1)-\sigma\bar{s}(k-1)-\gamma s(k-1).$$

Thus, (17) is described in another form

$$s_{\mathrm{m}}(k) = s(k) - d(k-1) + \gamma s(k-1).$$

Accordingly,

$$\bar{s}(k)=s(k)-s_{\mathrm{m}}(k)=d(k-1)-\gamma s(k-1). \tag{18}$$

Therefore, (16) turns to

$$\begin{aligned}&s(k+1) = -\gamma s(k) - \sigma\bar{s}(k) + d(k) = \\ &-\gamma s(k) - \sigma[d(k-1) - \gamma s(k-1)] + d(k) = \\ &\gamma M + N,\end{aligned} \tag{19}$$

where $M=-s(k)+\sigma s(k-1)$, $N=d(k)-\sigma d(k-1)$.

Theorem 1 If the change rate of disturbances is bounded, i.e., the following inequality holds,

$$|d(k) - \sigma d(k-1)| \leqslant \mu, \tag{20}$$

where μ is a positive constant, then the closed-loop system (4) which is constructed by (6) and (14) is robustly stable.

Proof Consider the characteristic polynomial of M,

$$-1 + z^{-1} = 0. \tag{21}$$

Obviously, the root of equation (21) is $z = \sigma$. Because $0 < \sigma < 1$, M is stable.

While $0 < \gamma < 1$, γM is stable. Namely, $\forall\varepsilon > 0$, $\exists k_0$ such that $|M| < \varepsilon/\gamma$ when $k > k_0$.

According to (20), $|N| \leqslant \mu$, then

$$|s(k+1)| = |\gamma M + N| \leqslant |\gamma M| + |N| \leqslant \varepsilon + \mu.$$

Consequently, the practical sliding mode motion of the closed-loop system will converge to a ε vicinity of sliding surface and stay on it subsequently. In addition, because the stability of sliding surface has been guaranteed by (6), $\tilde{y}(k)$ is robustly stable with the control law (13), that is, $y(k)$ tracks the desired trajectory $y_{\mathrm{d}}(k)$.

3.4 Internal dynamic

The stability of internal dynamics is very important[18–22] and it is affected by the control input; in other words, different control laws may yield either stable or unstable zero dynamics. Therefore, to avoid this problem, we strengthen the assumption on the internal dynamics. In this section, we will consider the stability of the second equation of system (4) while the variable $\tilde{\xi}(k)$ is stablizable. Here we will apply the ISS theory of discrete-time version.

Consider the following subsystem of (4):

$$\eta(k+1) = p(\tilde{\xi}(k), \xi_{\mathrm{d}}(k), \eta(k)). \tag{22}$$

Definition 1[23] A continuous function $V : \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$ is called an ISS-Lyapunov function for system (22) if the following hold:

1) There exist $\mathcal{K}_\infty$-functions α_1, α_2 such that

$$\alpha_1(\|\eta\|) \leqslant qV(\eta) \leqslant q\alpha_2(\|\eta\|).$$

2) There exist a $\mathcal{K}_\infty$-function α_3 and a $\mathcal{K}$-function α_4, such that

$$V(p(\tilde{\xi}, \xi_{\mathrm{d}}, \eta)) - V(\eta) \leqslant -\alpha_3(\|\eta\|) + \alpha_4(\|(\tilde{\xi}, \xi_{\mathrm{d}})\|).$$

Lemma 1[23] If system (22) admits a continuous ISS-Lyapunov function, then it is ISS.

Assumption 1 System (22) is globally exponentially stable at $\eta = 0$ informally with respect to ξ_{d} when $\tilde{\xi} = 0$. Hence, there exists a function $V_\eta(\eta)$ (see [24], Theorem 2) satisfying

$$\begin{aligned}&c_1\|\eta\|^2 \leqslant V_\eta(\eta) \leqslant c_2\|\eta\|^2, \\ &V_\eta[p(0, \xi_{\mathrm{d}}, \eta)] - V_\eta(\eta) \leqslant -c_3\|\eta\|^2\end{aligned} \tag{23}$$

with some positive constants c_i, $i = 1, 2, 3$.

Note that (23) implies that the zero dynamic of system (4) is globally informally exponentially stable regardless of $\tilde{\xi}_{\mathrm{d}}$.

Assumption 2 p is Lipschitz in $\tilde{\xi}$ uniformly in ξ_{d} and η, i.e., $\|p(\tilde{\xi}, \xi_{\mathrm{d}}, \eta) - p(0, \xi_{\mathrm{d}}, \eta)\| \leqslant L\|\tilde{\xi}\|$, where $L > 0$.

Lemma 2[24] Assume that system (22) is globally exponentially stable, then there exists a Lyapunov function V satisfying (23) such that for any given variables ζ_1, ζ_2 and some constant $c_4 > 0$, the following holds

$$\|V(\zeta_1) - V(\zeta_2)\| \leqslant qc_4(\|\zeta_1\| + \|\zeta_2\|)\|\zeta_1 - \zeta_2\|. \tag{24}$$

Based on Assumptions 1 and 2 and (24), for the internal dynamics in (4), $\Delta V_\eta(\eta)$ along the trajectories of system (4) is given by

$$\begin{aligned}&\Delta V_\eta(\eta) = V_\eta[p(\tilde{\xi}, \xi_{\mathrm{d}}, \eta)] - V_\eta(\eta) = \\ &V_\eta[p(0, \xi_{\mathrm{d}}, \eta)] - V_\eta(\eta) + \\ &V_\eta[p(\tilde{\xi}, \xi_{\mathrm{d}}, \eta)] - V_\eta[p(0, \xi_{\mathrm{d}}, \eta)] \leqslant \\ &-c_3\|\eta\|^2 + 2c_4P\|p(\tilde{\xi}, \xi_{\mathrm{d}}, \eta) - p(0, \xi_{\mathrm{d}}, \eta)\| \leqslant \\ &-c_3\|\eta\|^2 + 2c_4PL\|\tilde{\xi}\|,\end{aligned}$$

where P is the upper bound of $\|p(\tilde{\xi}, \xi_{\mathrm{d}}, \eta)\|$.

From Definition 1, we obtain $V_\eta(\eta)$ is ISS-

Lyapunov function. According to Lemma 1, the internal dynamics in (4) is ISS.

By Theorem 1, we get $\|s\| \leqslant \varepsilon+\mu$. Applying (6), we find $\|\tilde{\xi}\|$ is bounded, that is, there exists a $\delta > 0$ such that $\|\tilde{\xi}\| < \delta$. Thus, $\Delta V_\eta(\eta) \leqslant -c_3\|\eta\|^2 + 2c_4PL\delta$. If $\|\eta\| > \dfrac{\sqrt{2c_4PL\delta}}{\sqrt{c_3}}$, then $\Delta V_\eta(\eta) < 0$.

Therefore, from the above discussion, we get the following result.

Theorem 2 For system (4) regulated by the control law (14), under Assumptions 1 and 2, all the closed loop signals are bounded while the tracking error is attracted to a neighborhood of the origin.

Remark 1 If relative degree $r = 2$ and internal dynamics $\eta = 0$ in system (1), we have a typical discrete-time example of 1-joint rigid robotic manipulator (see [11]).

Remark 2 If a sliding surface is defined, the aim of SMC is to synthesize a suitable control law to globally drive the trajectory onto the predefined sliding surface in finite time (ideal quasi-sliding mode dynamics) or its neighbourhood (non-ideal quasi-sliding mode dynamics). Because of disturbances of model, sliding surface is inaccurate. Then control law is discontinuous across the sliding surface. Therefore, chattering is produced. In SMPC, by predicting the future value of sliding mode function, the control signal can be adjusted immediately to prevent system states from crossing the sliding surface, hence chattering will be avoided.

4 Simulation results

This example involves simulation results carried out from a model in [25].

Example 1 The model for course control of ship as follows:

$$T\ddot{\psi}(t) + \dot{\psi}(t) + \alpha\dot{\psi}^3(t) + d(t) = K\delta(t), \quad (25)$$

where α is the nonlinear parameter, K is the rudder angle gain, T is a constant, $\psi(t)$ is the course angle, $\delta(t)$ is the rudder angle, $d(t)$ is the external disturbance.

Suppose that zero order holder is applied in the system, discretizing system (25), i.e., $\dot{\psi}(t) = (\psi(k+1) - \psi(k))/t_\mathrm{s}$, $\ddot{\psi}(t) = (\psi(k+2) - 2\psi(k+1) + \psi(k))/t_\mathrm{s}^2$ and defining $\psi(k) = x_1(k)$, $\psi(k+1) = x_2(k)$ and $y(k) = x_1(k)$, then

$$\begin{cases} x_1(k+1) = x_2(k), \\ x_2(k+1) = f(\xi(k)) + \dfrac{t_\mathrm{s}^2K}{T}\delta(k), \\ y(k) = x_1(k), \end{cases} \quad (26)$$

where t_s is the sampling period and

$$f(\xi(k)) = -\frac{t_\mathrm{s}^2K}{T}d(k) + (\frac{t_\mathrm{s}}{T} - 1)x_1(k) + (2 - \frac{t_\mathrm{s}}{T})x_2(k) - \frac{\alpha}{Tt_\mathrm{s}}[x_2(k) - x_1(k)]^3.$$

The vessel data comes from a ship which has an overall length of $160.9\,\mathrm{m}$. The motion of the ship described by the model has the following dynamic parameters at a speed of $V = 4\,\mathrm{m/s}$, $T = 114.64\,\mathrm{s}$, $K = 0.063\,\mathrm{s}^{-1}$, $\alpha = 30\,\mathrm{s}^2$. Let $d(k) = 0.05$ and initial value $x_1(0) = -0.05$, $x_2(0) = 0$.

From (14), we can get controller $\delta(k)$. The simulation results are illustrated in Figs.1, 2 and 3. Thus, the designed controller has good performance and the excellent tracking of output is obtained by SMPC algorithm.

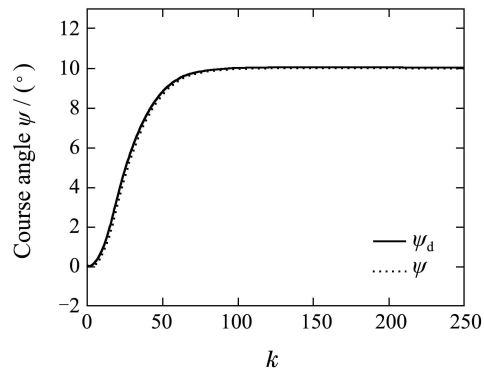

Fig. 1 Course angle $\psi(k)$ and reference course $\psi_\mathrm{d}(k)$.

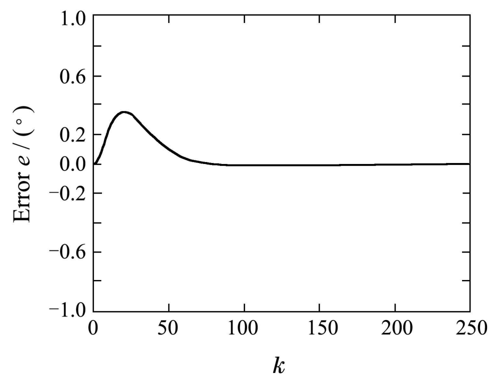

Fig. 2 Error $e(k)$

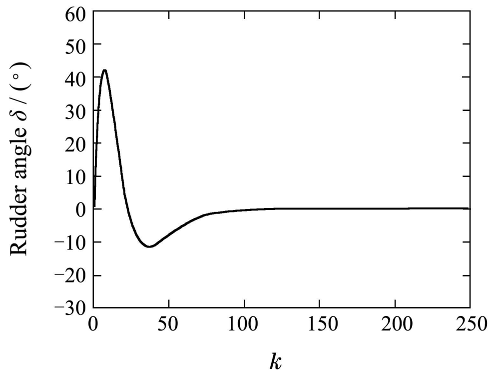

Fig. 3 Rudder angle $\delta(k)$

5 Conclusions

In this paper, the SMPC technique is considered for a class of discrete-time nonlinear uncertain systems with internal dynamic. The desired control law is constructed by using special sliding mode prediction model, feedback correction and receding horizon optimization. It is shown that all the signals of the closed-loop system are bounded and the tracking error is robustly stable. Robust stability analysis shows that the closed-loop system has strong robustness to uncertainty with unknown boundary. The stability of internal dynamic was investigated by ISS theory of discrete-time version. Simulation result verifies the efficacy of the proposed method.

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作者简介:

张利军 (1973–), 男, 博士, 教授, 目前研究方向为非线性控制理论与应用, E-mail: zhanglj@hrbeu.edu,cn;

赵杰梅 (1984–), 女, 博士研究生, 目前研究方向为非线性控制理论与应用, E-mail: jiemeizhao@163.com;

齐　雪 (1981–), 女, 博士, 讲师, 目前研究方向为水下机器人控制, E-mail: qixuesnow@163.com.