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随机非完整链式系统的自适应状态反馈镇定

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摘要: 基于反步技术, 对带有不确定非线性项和不确定非线性系数的随机非完整链式系统, 设计了自适应状态反馈镇定器. 给出了能够保证系统依概率几乎渐近稳定到平衡点的切换控制. 最后用仿真验证了控制器的有效性.

关键词: 随机非完整系统; 状态反馈; 反步; 切换控制

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Adaptive state-feedback stabilization for stochastic nonholonomic chained systems

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Abstract: Based on the backstepping approach, an adaptive state-feedback backstepping controller is designed for stochastic nonholonomic systems with uncertain nonlinear terms and uncertain nonlinear coefficients. A switching control strategy for the original system is developed which can guarantee that the closed-loop system is almost asymptotically stable at the zero equilibrium in probability. A simulation example is provided to illustrate the effectiveness of the controller.

Key words: stochastic nonholonomic systems; state-feedback; backstepping; switching control strategy

1 Introduction

Let us consider stochastic nonholonomic chained systems described by

$$\mathrm{d}x_0 = d_0(t)u_0\mathrm{d}t, \tag{1a}$$

$$\begin{cases} \mathrm{d}x_i = d_i(t)u_0x_{i+1}\mathrm{d}t + f_i(x_0,\bar{x}_i,\theta)\mathrm{d}t+ \\ \qquad g_i^{\mathrm{T}}(x_0,\bar{x}_i)\varSigma(t)\mathrm{d}\omega, \\ \qquad i=1,\cdots,n-1, \\ \mathrm{d}x_n = d_n(t)u\mathrm{d}t + f_n(x_0,x,\theta)\mathrm{d}t+ \\ \qquad g_n^{\mathrm{T}}(x_0,x)\varSigma(t)\mathrm{d}\omega, \end{cases} \tag{1b}$$

where u_0 and u are control inputs, $x_0 \in \mathbb{R}$ and $x = (x_1,\cdots,x_n)^{\mathrm{T}} \in \mathbb{R}^n$ are system states, $\bar{x}_i = (x_1,\cdots, x_i)^{\mathrm{T}}$, $\bar{x}_n = x$, $\theta \in \mathbb{R}^m$ is an unknown constant vector, $f_i(x_0,\bar{x}_i,\theta) : \mathbb{R}^{i+1}\times\mathbb{R}^m \to \mathbb{R}(1 \leqslant i \leqslant n)$ are smooth functions, which can be also named uncertain parameter based nonlinear drifts, with $f_i(0,0,\theta)=0$, $g_i(x_0,\bar{x}_i) : \mathbb{R}^{i+1} \to \mathbb{R}^r(1 \leqslant i \leqslant n)$ are smooth functions with $g_i(0,0)=0$, $d_i(t) : \mathbb{R}^+ \to \mathbb{R}(0 \leqslant i \leqslant n)$ are unknown uncertain time-varying control coefficients with known sign, $\varSigma(t) : \mathbb{R}^+ \to \mathbb{R}^{r\times r}$ is a bounded Borel measurable function which is nonnegative definite for each t, time-varying coefficient, also, and $\omega \in \mathbb{R}^r$ is an r-dimensional independent standard Wiener process defined on a complete probability space $(\varOmega,\mathcal{F},P)$ with $\varOmega$ being a sample space, $\mathcal{F}$ being a filtration, and P being a probability measure.

During the past decades, many results have been reported on the stabilization problem of nonholonomic control systems. In the existing literature, three methods are adopted for stabilization of nonholonomic systems. The first is discontinuous time-invariant stabilization[1]. The second is smooth time-varying stabilization[2–3]. The third is hybrid stabilization[4]. It is known that a nonholonomic system could be transformed into a chained form system by using state and input trasformations in [5]. There has been increasing attention devoted to the stability problem of the chained form systems[6–10].

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It's well known that stochastic signals are very prevalent in practical engineering and much progress has been made in stabilization of stochastic differential equations (SDE). Especially, when backstepping designs were firstly introduced, stochastic nonlinear control had experienced a breakthrough[11–12]. Based on quartic Lyapunov functions, the asymptotical stabilization control in the large of the open-loop system was discussed in [13]. Further research was developed by the recent work[14–16].

The almost global adaptive asymptotical controllers of stochastic nonholonomic systems with unknown time-varying coefficients before $\mathrm{d}\omega$ were discussed by using discontinuous control, but the systems didn't contain nonlinear drifts and unknown time-varying coefficients before $\mathrm{d}t$[17]. When the subsystem (1a) is given by the system of ordinary differential equations, the problem of state-feedback stabilization control for a class of high order stochastic nonholonomic systems with nonlinear drifts and uncertain time-varying coefficients was studied by the backstepping approach, but nonlinear drifts in the systems didn't contain uncertain parameters[18]. So, there exists a problem which is how to design an adaptive state-feedback stabilizing controller for stochastic nonholonomic systems with unknown parameters based nonlinear drifts and uncertain time-varying coefficients simultaneously. The main idea of this paper is highlighted as follows:

i) A stabilization controller is designed for stochastic nonholonomic systems with uncertain parameters based nonlinear drifts and uncertain time-varying coefficients simultaneously by adaptive state-feedback backstepping technique.

ii) A switching control strategy for the original system is presented. It guarantees the closed-loop system is almost asymptotically stabilized at the zero equilibrium point in probability. The states are globally asymptotically stabilized to zero in probability.

The paper is organized as follows: Section 2 begins with the mathematical preliminaries. In Section 3, the adaptive state-feedback backstepping controller is designed. In Section 4, a switching control strategy for the original system is discussed. Finally, a simulation example is given to show the effectiveness of the controller in Section 5.

2 Preliminaries

The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all nonnegative real numbers, $\mathbb{R}^n$ denotes the real n-dimensional space. For a given vector or matrix X, X^{T} denotes its transpose, $\mathrm{tr}\{X\}$ denotes its trace, when X is square, $|X|$ denotes the Euclidean norm, $|X|_\infty = \sup\limits_{t\in\mathbb{R}_+} |X|$.

Consider the following stochastic nonlinear system

$$\mathrm{d}x = f(x)\mathrm{d}t + g^{\mathrm{T}}(x)\mathrm{d}\omega,\ x(0) = x_0 \in \mathbb{R}^n, \quad (2)$$

where $x \in \mathbb{R}^n$ is the state, the Borel measurable functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are locally Lipschitz in x, and $\omega \in \mathbb{R}^r$ is an r-dimensional independent standard Wiener process defined on the complete probability space $(\varOmega, \mathcal{F}, P)$. The following definitions and lemmas will be used in the paper.

Definition 1[11] For any given $V(x) \in \mathcal{C}^2$, associated with stochastic system (2), the differential operator $\mathcal{L}$ is defined as follows:

$$\mathcal{L}V(x) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2}\mathrm{tr}\{g(x)\frac{\partial^2 V}{\partial x^2} g^{\mathrm{T}}(x)\}. \quad (3)$$

Definition 2[13] The equilibrium $x = 0$ of Eq.(2) is

• globally stable in probability if for $\forall\varepsilon > 0$, there exists a class $\mathcal{K}$ function $\gamma(\cdot)$ such that

$$\mathrm{P}\{|x(t)| < \gamma(|x_0|)\} \geqslant 1 - \varepsilon,\ \forall t \geqslant 0,\ x_0 \in \mathbb{R}^n \setminus \{0\}.$$

• globally asymptotically stable in probability if it is globally stable in probability and

$$\mathrm{P}\{\lim_{t\to\infty} |x(t)| = 0\} = 1,\ \forall x_0 \in \mathbb{R}^n.$$

Lemma 1[13] Considering the stochastic system (2), if there exists a $\mathcal{C}^2$ function $V(x)$, class $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$, constants $c_1 > 0$, $c_2 \geqslant 0$, and a nonnegative function $W(x)$ such that

$$\begin{cases} \alpha_1(|x|) \leqslant V(x) \leqslant \alpha_2(|x|), \\ \mathcal{L}V(x) \leqslant -c_1 W(x) + c_2, \end{cases} \quad (4)$$

then

i) for Eq.(2), there exists an almost surely unique solution on $[0, \infty)$ for each $x_0 \in \mathbb{R}^n$;

ii) when $c_2 = 0$, $f(0) = 0$, $g(0) = 0$ and $W(x)$ is continuous, then the equilibrium $x = 0$ is globally stable in probability and $\mathrm{P}\{\lim\limits_{t\to\infty} W(x(t)) = 0\} = 1$ for $\forall x_0 \in \mathbb{R}^n$.

Lemma 2[19] Let x and y be real variables. Then, for any positive integers m, n and any real number $\varepsilon > 0$, the following inequality holds:

$$|x|^m|y|^n \leqslant \frac{m}{m+n}\varepsilon|x|^{m+n} + \frac{n}{m+n}\varepsilon^{-\frac{m}{n}}|y|^{m+n}. \quad (5)$$

Lemma 3[20] For any vector-valued continuous function $f(x, y)$, where $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, there are smooth scalar functions $a(x) \geqslant 1$ and $b(x) \geqslant 1$ such that

$$|f(x, y)| \leqslant a(x)b(y). \quad (6)$$

3 State feedback control

For system (1), the following assumptions and remarks are needed.

Assumption 1 For smooth functions $f_i(\cdot)$ and $g_i(\cdot)$, $i = 1, \cdots, n$, there exist known non-negative smooth functions $\bar{\gamma}_i : \mathbb{R}^{i+1} \times \mathbb{R}^m \to \mathbb{R}^+$ and $\bar{\xi}_i : \mathbb{R}^{i+1} \to \mathbb{R}^+$ such that for any $x_0, \bar{x}_i$ and θ:

$$|f_i(x_0,\bar{x}_i,\theta)| \leqslant (|x_1|+\cdots+|x_i|)\bar{\gamma}_i(x_0,\bar{x}_i,\theta),$$
$$|g_i(x_0,\bar{x}_i)| \leqslant (|x_1|+\cdots+|x_i|)\bar{\xi}_i(x_0,\bar{x}_i).$$

Remark 1 There exist positive smooth functions $c_i(\theta)$ and $\gamma_i(x_0,\bar{x}_i)$, $i=1,\cdots,n$, such that

$$|f_i(x_0,\bar{x}_i,\theta)| \leqslant (|x_1|+\cdots+|x_i|)\gamma_i(x_0,\bar{x}_i)c_i(\theta).$$

Assumption 2 For any $t>0$, there exist known positive constants λ and μ such that

$$\lambda \leqslant d_i(t) \leqslant \mu,\ i=0,1,\cdots,n.$$

Remark 2 Assumption 1 is similar to the Assumption 1 in [15], (H1) and (H2) in [20]. The Assumption 2 is same to the Assumption 3 in [18].

In the following two subsections, we will consider system (1) under the condition of $x_0(t_0)\neq 0$ and the case of $x_0(t_0)=0$ will be discussed in the Section 4.

3.1 The first state stabilization

Let us consider the subsystem (1a). The control u_0 is designed to guarantee that x_0 converges to zero but never crosses zero. So one can take u_0 as follows:

$$u_0=-\eta_0 x_0, \tag{7}$$

where η_0 is a positive gain. We employ a Lyapunov function of the form

$$V_0(x_0)=\frac{1}{4}x_0^4.$$

Obviously, for any nonzero initial condition $(t_0,x_0(t_0))$ with $t_0\geqslant 0$, solution of the subsystem (1a) is asymptotically stable and will not reach zero.

In subsection 3.2, other states will be regulated to the origin in probability by the design of the control input u.

3.2 Other states stabilization

Let us consider the subsystem (1b). In order to design a smooth adaptive state-feedback controller, the following state-input scaling discontinuous transformation defined by Eq.(8) is needed:

$$z_i=\frac{x_i}{u_0^{n-i}},\ 1\leqslant i\leqslant n, \tag{8}$$

under the new z-coordinate (8), the subsystem (1b) is transformed into

$$\begin{cases}\mathrm{d}z_i=d_i(t)z_{i+1}\mathrm{d}t+(\dfrac{f_i}{u_0^{n-i}}+\eta_0(n-i)d_0(t)z_i)\mathrm{d}t+\\ \qquad \dfrac{g_i^{\mathrm{T}}}{u_0^{n-i}}\varSigma(t)\mathrm{d}\omega,\ i=1,\cdots,n-1,\\ \mathrm{d}z_n=d_n(t)u\mathrm{d}t+f_n\mathrm{d}t+g_n^{\mathrm{T}}\varSigma(t)\mathrm{d}\omega.\end{cases} \tag{9}$$

Remark 3 For the initial state $x_0(t_0)\neq 0$, from the subsection 3.1, one can obtain that the transformation (8) is meaningful.

Remark 4 From the subsection 3.1 and the state-input scaling discontinuous transformation (8), we know that x_0, i.e., u_0 asymptotically converges to zero, which means $x_i(t)$ converge to zero in probability with $z_i(t)$ converge to zero in probability as t goes to infinity.

To deal with the uncertain nonlinear drifts and uncertain time-varying coefficients simultaneously, define the estimate parameter

$$\varTheta=\max_{1\leqslant i\leqslant n}\{c_i(\theta),\bar{\theta}\}, \tag{10}$$

and the error variables ε_i are given by

$$\varepsilon_1=z_1,\ \varepsilon_i=z_i-z_i^*(x_0,\bar{z}_{i-1},\hat{\varTheta}),\ i=2,\cdots,n, \tag{11}$$

where $\bar{\theta}=|\varSigma(t)\varSigma^{\mathrm{T}}(t)|_\infty$, $\bar{z}_i=(z_1,\cdots,z_i)^{\mathrm{T}}$, $z=\bar{z}_n$, $z_i^*(i=2,\cdots,n)$ are virtual smooth controllers and z_i^* will be designed later. Then, we have

$$\begin{cases}\mathrm{d}\varepsilon_i=d_i(t)z_{i+1}\mathrm{d}t+F_i\mathrm{d}t+G_i^{\mathrm{T}}\varSigma(t)\mathrm{d}\omega,\\ \qquad i=1,\cdots,n-1,\\ \mathrm{d}\varepsilon_n=d_n(t)u\mathrm{d}t+F_n\mathrm{d}t+G_n^{\mathrm{T}}\varSigma(t)\mathrm{d}\omega,\end{cases} \tag{12}$$

where

$$F_i(x_0,\bar{z}_i,\hat{\varTheta})=\frac{f_i}{u_0^{n-i}}+\eta_0(n-i)d_0(t)z_i-\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\{d_k(t)z_{k+1}+\frac{f_k}{u_0^{n-k}}+\eta_0(n-k)d_0(t)z_k\}-\eta_0 d_0(t)\frac{\partial z_i^*}{\partial x_0}x_0-\frac{\partial z_i^*}{\partial\hat{\varTheta}}\dot{\hat{\varTheta}}-\frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2 z_i^*}{\partial z_j\partial z_k}\frac{g_j^{\mathrm{T}}}{u_0^{n-j}}\varSigma(t)\varSigma^{\mathrm{T}}(t)\frac{g_k}{u_0^{n-k}},$$

$$G_i^{\mathrm{T}}(x_0,\bar{z}_i,\hat{\varTheta})=\frac{g_i^{\mathrm{T}}}{u_0^{n-i}}-\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\frac{g_k^{\mathrm{T}}}{u_0^{n-k}},\quad i=1,\cdots,n-1.$$

By Assumption 1, Remark 1, Eqs.(8) and (11), we have the following proposition, whose proof is given in Appendix.

Proposition 1 For smooth functions $f_i(\cdot)$ and $g_i(\cdot)$, $i=1,\cdots,n$, there exist known non-negative smooth functions $\gamma_{ij}(x_0,\bar{z}_i):\mathbb{R}^{i+1}\rightarrow\mathbb{R}^+$, $c_i(\theta):\mathbb{R}^m\rightarrow\mathbb{R}^+$, $j=1,2,3,4$, such that for any x_0, $\bar{x}_i$ and θ:

$$|\frac{f_i}{u_0^{n-i}}|\leqslant(\sum_{k=1}^{i}|\varepsilon_k|)\gamma_{i1}c_i(\theta), \tag{13a}$$

$$|\frac{g_i^{\mathrm{T}}}{u_0^{n-i}}|\leqslant(\sum_{k=1}^{i}|\varepsilon_k|)\gamma_{i2}, \tag{13b}$$

$$|\frac{g_i^{\mathrm{T}}}{u_0^{n-i}}-\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\frac{g_k}{u_0^{n-k}}|\leqslant(\sum_{k=1}^{i}|\varepsilon_k|)\gamma_{i3}, \tag{13c}$$

$$|-\frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2 z_i^*}{\partial z_j\partial z_k}\frac{g_j^{\mathrm{T}}}{u_0^{n-j}}\frac{g_k}{u_0^{n-k}}|\leqslant(\sum_{k=1}^{i-1}|\varepsilon_k|)\gamma_{i4}. \tag{13d}$$

Now we design the adaptive backstepping controller of the subsystem (1b).

Step 1 Define the 1st Lyapunov candidate function

$$V_1(x_0,z_1,\hat{\varTheta})=\frac{1}{4}x_0^4+\frac{1}{4}\varepsilon_1^4+\frac{1}{2}\tilde{\varTheta}^2, \tag{14}$$

where $\tilde{\Theta} = \Theta - \hat{\Theta}$ is the parameter estimation error.

$$\mathcal{L}V_1 \leqslant -\lambda\eta_0 x_0^4 + \varepsilon_1^3 d_1(t) z_2 + \varepsilon_1^3 \eta_0 (n-1) d_0(t) z_1 + |\varepsilon_1|^3 |\frac{f_1}{u_0^{n-1}}| + \frac{3}{2}\varepsilon_1^2 |\frac{g_1^{\mathrm{T}}}{u_0^{n-1}}|^2 |\Sigma(t)\Sigma^{\mathrm{T}}(t)|_\infty - \tilde{\Theta}\dot{\hat{\Theta}}. \tag{15}$$

By Proposition 1, there exist nonnegative smooth functions $\gamma_{11}(x_0, z_1)$, $c_1(\theta)$ and $\gamma_{12}(x_0, z_1)$, such that

$$|\frac{f_1}{u_0^{n-1}}| \leqslant |\varepsilon_1| \gamma_{11}(x_0, z_1) c_1(\theta), \tag{16}$$

$$|\frac{g_1}{u_0^{n-1}}| \leqslant |\varepsilon_1| \gamma_{12}(x_0, z_1). \tag{17}$$

Substituting Eqs.(16) and (17) into (15), we have

$$\mathcal{L}V_1 \leqslant -\lambda\eta_0 x_0^4 + \varepsilon_1^3 d_1(t)(z_2 - z_2^*) + \varepsilon_1^3 d_1(t) z_2^* + \eta_0 \mu (n-1) \varepsilon_1^4 + \Theta\{\gamma_{11} + \frac{3}{2}\gamma_{12}^2\}\varepsilon_1^4 - \tilde{\Theta}\dot{\hat{\Theta}}. \tag{18}$$

Suppose that

$$z_2^*(x_0, z_1, \hat{\Theta}) = -\alpha_1(x_0, z_1, \hat{\Theta})\varepsilon_1,$$

where $\alpha_1(\cdot) \geqslant 0$ is a smooth function to be chosen. Thus, by Assumption 2, we have

$$d_1 \varepsilon_1^3 z_2^* = -d_1 \varepsilon_1^4 \alpha_1 \leqslant -\lambda \varepsilon_1^4 \alpha_1 = \lambda \varepsilon_1^3 z_2^*. \tag{19}$$

Then, adding and subtracting the term $\bar{c}_1 \Theta \varepsilon_1^4 + \bar{\bar{c}}_1 \varepsilon_1^4$ on the right-hand side of Eq.(18), and using Eq.(19), one gets

$$\mathcal{L}V_1 \leqslant -\lambda\eta_0 x_0^4 - \bar{c}_1 \Theta \varepsilon_1^4 - \bar{\bar{c}}_1 \varepsilon_1^4 + d_1(t)\varepsilon_1^3 (z_2 - z_2^*) + \lambda \varepsilon_1^3 \{z_2^* + \frac{1}{\lambda}(\sqrt{1+\hat{\Theta}^2} H_{11} + H_{12})\varepsilon_1\} + \tilde{\Theta}\{\tau_1 - \dot{\hat{\Theta}}\}, \tag{20}$$

where

$$\begin{cases} H_{11} = \bar{c}_1 + \gamma_{11} + \frac{3}{2}\gamma_{12}^2, \ \tau_1 = H_{11}\varepsilon_1^4, \\ H_{12} = \bar{\bar{c}}_1 + \lambda\mu(n-1). \end{cases} \tag{21}$$

Choosing the virtual smooth control z_2^* as follows:

$$\begin{cases} z_2^*(x_0, z_1, \hat{\Theta}) = -\alpha_1(x_0, z_1, \hat{\Theta})\varepsilon_1, \\ \alpha_1(x_0, z_1, \hat{\Theta}) = \frac{1}{\lambda}(\sqrt{1+\hat{\Theta}^2} H_{11} + H_{12}), \end{cases} \tag{22}$$

and substituting Eq.(22) into Eq.(20), one can obtain

$$\mathcal{L}V_1 \leqslant -\lambda\eta_0 x_0^4 - \bar{c}_1 \Theta \varepsilon_1^4 - \bar{\bar{c}}_1 \varepsilon_1^4 + d_1(t)\varepsilon_1^3 (z_2 - z_2^*) + \tilde{\Theta}\{\tau_1 - \dot{\hat{\Theta}}\}. \tag{23}$$

Step i $(2 \leqslant i \leqslant n)$ Suppose that the design steps from 1 to $i-1$ have been finished, the smooth virtual control z_j^*, the updating law for $\hat{\Theta}_{1,j-1}$ and the tuning function τ_{j-1} for Step $j-1 (j = 2, \cdots, i)$ have been chosen as follows:

$$\begin{cases} z_j^*(x_0, \bar{z}_{j-1}, \hat{\Theta}) = -\alpha_{j-1}(x_0, \bar{z}_{j-1}, \hat{\Theta})\varepsilon_{j-1}, \\ \tau_{j-1} = \tau_{j-2} + H_{j-1,1}\varepsilon_{j-1}^4, \end{cases} \tag{24}$$

where α_{j-1} and τ_{j-1} are smooth functions, and the $(i-1)$th Lyapunov candidate function

$$V_{i-1}(x_0, \bar{\varepsilon}_{i-1}, \hat{\Theta}) = V_{i-2}(x_0, \bar{\varepsilon}_{i-2}, \hat{\Theta}) + \frac{1}{4}\varepsilon_{i-1}^4, \tag{25}$$

where $\bar{\varepsilon}_i = (\varepsilon_1, \cdots, \varepsilon_i)^{\mathrm{T}}$, for Step $i-1$ satisfies

$$\mathcal{L}V_{i-1} \leqslant -(\lambda\eta_0 - \sum_{j=2}^{i-1}\beta_j)x_0^4 - \Theta\sum_{j=1}^{i-1}(\bar{c}_j - \sum_{k=j+1}^{i-1}\bar{\rho}_{kj})\varepsilon_j^4 - \sum_{j=1}^{i-1}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i-1}\bar{\bar{\rho}}_{kj})\varepsilon_j^4 + d_{i-1}(t)\varepsilon_{i-1}^3(z_i - z_i^*) + (\tilde{\Theta} + \sum_{k=2}^{i-1}\varepsilon_k^3 \frac{\partial z_k^*}{\partial\hat{\Theta}})\{\tau_{i-1} - \dot{\hat{\Theta}}\}. \tag{26}$$

In the following, we will prove that Eq.(26) also holds for i.

Define the ith Lyapunov candidate function

$$V_i(x_0, \bar{\varepsilon}_i, \hat{\Theta}) = V_{i-1}(x_0, \bar{\varepsilon}_{i-1}, \hat{\Theta}) + \frac{1}{4}\varepsilon_i^4. \tag{27}$$

From Eqs.(12)(27) and It$\hat{o}$ formula, one has

$$\begin{aligned} \mathcal{L}V_i \leqslant & -(\lambda\eta_0 - \sum_{j=2}^{i-1}\beta_j)x_0^4 - \Theta\sum_{j=1}^{i-1}(\bar{c}_j - \sum_{k=j+1}^{i-1}\bar{\rho}_{kj})\varepsilon_j^4 - \\ & \sum_{j=1}^{i-1}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i-1}\bar{\bar{\rho}}_{kj})\varepsilon_j^4 + d_{i-1}(t)\varepsilon_{i-1}^3(z_i - z_i^*) + \\ & (\tilde{\Theta} + \sum_{k=2}^{i-1}\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}})\{\tau_{i-1} - \dot{\hat{\Theta}}\} + d_i(t)\varepsilon_i^3 z_{i+1}^* + \\ & d_i(t)\varepsilon_i^3(z_{i+1} - z_{i+1}^*) + \varepsilon_i^3\{\frac{f_i}{u_0^{n-i}} + \\ & \lambda(n-i)d_0(t)z_i - \sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}(d_k(t)z_{k+1} + \frac{f_k}{u_0^{n-k}} + \\ & \lambda(n-k)d_0(t)z_k) - \lambda d_0(t)\frac{\partial z_i^*}{\partial x_0}x_0 - \varepsilon_i^3\frac{\partial z_i^*}{\partial\hat{\Theta}}\dot{\hat{\Theta}} - \\ & \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2 z_i^*}{\partial z_j\partial z_k}\frac{g_j^{\mathrm{T}}}{u_0^{n-j}}\Sigma(t)\Sigma^{\mathrm{T}}(t)\frac{g_k}{u_0^{n-k}}\} + \\ & \frac{3}{2}\varepsilon_i^2|\frac{g_i^{\mathrm{T}}}{u_0^{n-i}} - \sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\frac{g_k^{\mathrm{T}}}{u_0^{n-k}}|^2|\Sigma\Sigma^{\mathrm{T}}|_\infty. \end{aligned} \tag{28}$$

By Lemma 2, Assumptions 1–2, Proposition 1, Eqs.(8) and (11), one can obtain the following inequalities, which are proved in Appendix:

$$d_{i-1}(t)\varepsilon_{i-1}^3(z_i - z_i^*) \leqslant \frac{3}{4}\mu\varepsilon_{i,i-1,1}\varepsilon_{i-1}^4 + \frac{1}{4}\mu\varepsilon_{i,i-1,1}^{-3}\varepsilon_i^4, \tag{29}$$

$$\varepsilon_i^3\frac{f_i}{u_0^{n-i}} \leqslant \Theta\sum_{k=1}^{i-1}\frac{1}{4}\varepsilon_{i,k,2}\varepsilon_k^4 + \Theta\{\sum_{k=1}^{i-1}\frac{3}{4}\varepsilon_{i,k,2}^{-\frac{1}{3}}\gamma_{i1}^{\frac{4}{3}} + \gamma_{i1}\}\varepsilon_i^4, \tag{30}$$

$$\varepsilon_i^3\eta_0(n-i)d_0(t)z_i \leqslant \frac{1}{4}\mu\eta_0(n-i)\varepsilon_{i,i-1,3}\varepsilon_{i-1}^4 + \frac{3}{4}\mu\eta_0(n-i)\varepsilon_{i,i-1,3}^{-\frac{1}{3}}\alpha_{i-1}^{\frac{4}{3}}\varepsilon_i^4, \tag{31}$$

$$-\varepsilon_i^3\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}d_k(t)z_{k+1}\leqslant$$
$$\mu\sum_{k=1}^{i-2}\frac{1}{4}\varepsilon_{i,k+1,4}\varepsilon_{k+1}^4+\mu\sum_{k=1}^{i-1}\frac{1}{4}\varepsilon_{i,k,5}\varepsilon_k^4+$$
$$\mu\{\sum_{k=1}^{i-2}\frac{3}{4}\varepsilon_{i,k+1,4}^{-\frac{1}{3}}(\sqrt{1+(\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}+\sqrt{1+(\frac{\partial z_i^*}{\partial z_{i-1}})^2}+$$
$$\sum_{k=1}^{i-1}\frac{3}{4}\varepsilon_{i,k,5}^{-\frac{1}{3}}(\sqrt{1+(\alpha_k\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}\}\varepsilon_i^4,\tag{32}$$
$$-\varepsilon_i^3\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\frac{f_k}{u_0^{n-k}}\leqslant$$
$$\Theta\sum_{k=1}^{i-1}\sum_{j=1}^{k}\frac{1}{4}\varepsilon_{i,j,6}\varepsilon_j^4+$$
$$\Theta\sum_{k=1}^{i-1}\sum_{j=1}^{k}\frac{3}{4}\varepsilon_{i,j,6}^{-\frac{1}{3}}(\sqrt{1+(\gamma_{k1}\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}\varepsilon_i^4,\tag{33}$$
$$-\varepsilon_i^3\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\eta_0(n-k)d_0(t)z_k\leqslant$$
$$\mu\eta_0\sum_{k=1}^{i-1}(n-k)\frac{1}{4}\{\varepsilon_{i,k-1,8}\varepsilon_{k-1}^4+\varepsilon_{i,k,7}\varepsilon_k^4\}+$$
$$\mu\eta_0\sum_{k=1}^{i-1}(n-k)\{\frac{3}{4}\varepsilon_{i,k,7}^{-\frac{1}{3}}(\sqrt{1+(\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}+$$
$$\frac{3}{4}\varepsilon_{i,k-1,8}^{-\frac{1}{3}}(\sqrt{1+(\alpha_{k-1}\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}\}\varepsilon_i^4,\tag{34}$$
$$-\varepsilon_i^3\eta_0d_0(t)\frac{\partial z_i^*}{\partial x_0}x_0\leqslant$$
$$\mu\eta_0\{\frac{1}{4}\varepsilon_{i,0,9}x_0^4+\frac{3}{4}\varepsilon_{i,0,9}^{-\frac{1}{3}}(\sqrt{1+(\frac{\partial z_i^*}{\partial x_0})^2})^{\frac{4}{3}}\varepsilon_i^4\},\tag{35}$$
$$-\frac{1}{2}\varepsilon_i^3\sum_{j,k=1}^{i-1}\frac{\partial^2 z_i^*}{\partial z_j\partial z_k}(\frac{g_j}{u_0^{n-j}})^{\mathrm{T}}\Sigma(t)\Sigma^{\mathrm{T}}(t)(\frac{g_k}{u_0^{n-k}})\leqslant$$
$$\Theta\sum_{k=1}^{i-1}\frac{1}{4}\varepsilon_{i,k,10}\varepsilon_k^4+\Theta\sum_{k=1}^{i-1}\frac{3}{4}\varepsilon_{i,k,10}^{-\frac{1}{3}}\gamma_{i4}^{\frac{4}{3}}\varepsilon_i^4,\tag{36}$$
$$\frac{3}{2}\varepsilon_i^2|\frac{g_i}{u_0^{n-i}}-\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\frac{g_k}{u_0^{n-k}}|^2\Sigma(t)\Sigma^{\mathrm{T}}(t)\leqslant$$
$$\Theta\{\frac{3}{4}i\sum_{k=1}^{i-1}\gamma_{i3}^4\varepsilon_{i,k,11}^{-1}+\frac{3}{2}\gamma_{i3}^2\}\varepsilon_i^4+$$
$$\Theta\frac{3}{4}i\sum_{k=1}^{i-1}\varepsilon_{i,k,11}\varepsilon_k^4,\tag{37}$$

and substituting Eqs.(29)–(37) into Eq.(28), one gets

$$\mathcal{L}V_i\leqslant$$
$$-(\lambda\eta_0-\sum_{j=2}^{i-1}\beta_j)x_0^4-\Theta\sum_{j=1}^{i-1}(\bar{c}_j-\sum_{k=j+1}^{i-1}\bar{\rho}_{kj})\varepsilon_j^4-$$
$$\sum_{j=1}^{i-1}(\bar{\bar{c}}_j-\sum_{k=j+1}^{i-1}\bar{\bar{\rho}}_{kj})\varepsilon_j^4+d_i(t)\varepsilon_i^3(z_{i+1}-z_{i+1}^*)+$$
$$(\tilde{\Theta}+\sum_{k=2}^{i-1}\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}})\{\tau_{i-1}-\dot{\hat{\Theta}}\}+d_i(t)\varepsilon_i^3z_{i+1}^*+$$
$$\{\frac{1}{4}\mu\eta_0\varepsilon_{i,0,9}\}x_0^4+\Theta\bar{H}_{i1}\varepsilon_{i-1}^4+\bar{H}'_{i2}\varepsilon_i^4+$$
$$\Theta\sum_{k=1}^{i-1}\{\frac{1}{4}\varepsilon_{i,k,2}+\frac{1}{4}\varepsilon_{i,k,10}+\frac{3}{4}i\varepsilon_{i,k,11}\}\varepsilon_k^4+$$
$$\{\frac{3}{4}\mu\varepsilon_{i,i-1,1}+\mu\eta_0(n-i)\frac{1}{4}\varepsilon_{i,i-1,3}\}\varepsilon_{i-1}^4+$$
$$\Theta\sum_{k=1}^{i-1}\sum_{j=1}^{k}\frac{1}{4}\varepsilon_{i,j,6}\varepsilon_j^4+\mu\sum_{k=1}^{i-2}\frac{1}{4}\varepsilon_{i,k+1,4}\varepsilon_{k+1}^4+$$
$$\{\mu\sum_{k=1}^{i-1}\frac{1}{4}\varepsilon_{i,k,5}+\mu\eta_0\sum_{k=1}^{i-1}(n-k)\frac{1}{4}\varepsilon_{i,k,7}\}\varepsilon_k^4+$$
$$\mu\eta_0\sum_{k=1}^{i-1}(n-k)\frac{1}{4}\varepsilon_{i,k-1,8}\varepsilon_{k-1}^4-\varepsilon_i^3\frac{\partial z_i^*}{\partial\hat{\Theta}}\dot{\hat{\Theta}},\tag{38}$$

where

$$\bar{H}_{i1}=\sum_{k=1}^{i-1}\frac{3}{4}\{\varepsilon_{i,k,2}^{-\frac{1}{3}}\gamma_{i1}^{\frac{4}{3}}+\varepsilon_{i,k,10}^{-\frac{1}{3}}\gamma_{i4}^{\frac{4}{3}}+i\gamma_{i3}^4\varepsilon_{i,k,11}^{-1}\}+$$
$$\gamma_{i1}+\sum_{k=1}^{i-1}\sum_{j=1}^{k}\frac{3}{4}\varepsilon_{i,j,6}^{-\frac{1}{3}}(\sqrt{1+(\gamma_{k1}\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}+\frac{3}{2}\gamma_{i3}^2,$$
$$\bar{H}'_{i2}=\frac{1}{4}\mu\varepsilon_{i,1,1}^{-3}+\mu\eta_0(n-i)(1+\frac{3}{4}\varepsilon_{i,i-1,3}^{-\frac{1}{3}}\alpha_{i-1}^{\frac{4}{3}})+$$
$$\sum_{k=1}^{i-2}\frac{3}{4}\mu\varepsilon_{i,k+1,4}^{-\frac{1}{3}}(\sqrt{1+(\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}+$$
$$\mu\sqrt{1+(\frac{\partial z_i^*}{\partial z_{i-1}})^2}+$$
$$\frac{3}{4}\mu\eta_0\varepsilon_{i,0,9}^{-\frac{1}{3}}(\sqrt{1+(\frac{\partial z_i^*}{\partial x_0})^2})^{\frac{4}{3}}+$$
$$\sum_{k=1}^{i-1}\frac{3}{4}\mu\varepsilon_{i,k,5}^{-\frac{1}{3}}(\sqrt{1+(\alpha_k\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}+$$
$$\frac{3}{4}\mu\eta_0\sum_{k=1}^{i-1}(n-k)\varepsilon_{i,k,7}^{-\frac{1}{3}}(\sqrt{1+(\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}+$$
$$\frac{3}{4}\mu\eta_0\varepsilon_{i,k-1,8}^{-\frac{1}{3}}(\sqrt{1+(\alpha_{k-1}\frac{\partial z_i^*}{\partial z_k})^2})^{\frac{4}{3}}.$$

From simple operation, one can obtain

$$\mu\sum_{k=1}^{i-2}\frac{1}{4}\varepsilon_{i,k+1,4}\varepsilon_{k+1}^4=\mu\sum_{k=2}^{i-1}\frac{1}{4}\varepsilon_{i,k,4}\varepsilon_k^4,\tag{39}$$
$$\Theta\sum_{k=1}^{i-1}\sum_{j=1}^{k}\frac{1}{4}\varepsilon_{i,j,6}\varepsilon_j^4=\Theta\sum_{k=1}^{i-1}\frac{1}{4}(i-k)\varepsilon_{i,k,6}\varepsilon_k^4,\tag{40}$$
$$\mu\eta_0\sum_{k=1}^{i-1}(n-k)\frac{1}{4}\varepsilon_{i,k-1,8}\varepsilon_{k-1}^4=$$
$$\mu\eta_0\sum_{k=1}^{i-2}(n-k-1)\frac{1}{4}\varepsilon_{i,k,8}\varepsilon_k^4.\tag{41}$$

Suppose that

$$z_{i+1}^*(x_0,\bar{z}_i,\hat{\Theta})=-\alpha_i(x_0,\bar{z}_i,\hat{\Theta})\varepsilon_i,$$

where $\alpha_i(\cdot)\geqslant 0$ is a smooth function to be chosen. Thus, by Assumption 2, we have

$$d_i\varepsilon_i^3z_{i+1}^*\leqslant\lambda\varepsilon_i^3z_{i+1}^*.\tag{42}$$

Substituting Eqs.(39)–(42) into Eq.(38), adding and subtracting the term $\Theta\bar{c}_i\varepsilon_i^4+\bar{\bar{c}}_i\varepsilon_i^4$ on the right-hand side of Eq.(38), we have

$$\mathcal{L}V_i \leqslant -(\lambda\eta_0 - \sum_{j=2}^{i}\beta_j)x_0^4 - \Theta\sum_{j=1}^{i}(\bar{c}_j - \sum_{k=j+1}^{i}\bar{\rho}_{kj})\varepsilon_j^4 - \sum_{j=1}^{i-1}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i-1}\bar{\bar{\rho}}_{kj})\varepsilon_j^4 + \lambda\varepsilon_i^3 z_{i+1}^* - \bar{\bar{c}}_i\varepsilon_i^4 + d_i(t)\varepsilon_i^3(z_{i+1} - z_{i+1}^*) + H_{i2}'\varepsilon_i^4 + \bar{\bar{\rho}}_{i1}'\varepsilon_1^4 + \hat{\Theta}H_{i1}\varepsilon_i^4 + \sum_{j=2}^{i-2}\bar{\bar{\rho}}_{ij}'\varepsilon_j^4 + \bar{\bar{\rho}}_{i,i-1}'\varepsilon_{i-1}^4 - \sum_{k=2}^{i-1}\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}}H_{i1}\varepsilon_i^4 + (\tilde{\Theta} - \varepsilon_i^3\tau_i\frac{\partial z_i^*}{\partial\hat{\Theta}} + \sum_{k=2}^{i}\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}})\{\tau_i - \dot{\hat{\Theta}}\}, \tag{43}$$

where

$$\beta_i = \frac{1}{4}\mu\eta_0\varepsilon_{i,0,9},\ \tau_i = \tau_{i-1} + H_{i1}\varepsilon_i^4, \tag{44}$$

$$\bar{\rho}_{ik} = \frac{1}{4}\varepsilon_{i,k,2} + \frac{1}{4}(i-k)\varepsilon_{i,k,6} + \frac{1}{4}\varepsilon_{i,k,10} + \frac{3}{4}i\varepsilon_{i,k,11},\ k = 1,\cdots,i-2, \tag{45}$$

$$\bar{\rho}_{i,i-1} = \frac{3}{4}\varepsilon_{i,i-1,1} + \frac{1}{4}\varepsilon_{i,i-1,2} + \frac{1}{4}\varepsilon_{i,i-1,6} + \frac{1}{4}\varepsilon_{i,i-1,10} + \frac{3}{4}i\varepsilon_{i,i-1,11}, \tag{46}$$

$$\begin{aligned}
&\bar{\bar{\rho}}_{i1}' = \frac{1}{4}\mu\{\varepsilon_{i,1,5} + \eta_0(n-1)\varepsilon_{i,1,7} + \eta_0(n-2)\varepsilon_{i,1,8}\},\\
&\bar{\bar{\rho}}_{ik}' = \frac{1}{4}\mu\varepsilon_{i,k,4} + \frac{1}{4}\mu\varepsilon_{i,k,5} + \frac{1}{4}\mu\eta_0(n-k)\varepsilon_{i,k,7} + \frac{1}{4}\mu\eta_0(n-k-1)\varepsilon_{i,k,8},\ k = 2,\cdots,i-2,\\
&\bar{\bar{\rho}}_{i,i-1}' = \frac{3}{4}\mu\varepsilon_{i,i-1,1} + \frac{1}{4}\mu\eta_0(n-i)\varepsilon_{i,i-1,3} + \frac{1}{4}\mu\varepsilon_{i,i-1,4} + \frac{1}{4}\mu\varepsilon_{i,i-1,5} + \frac{1}{4}\mu\eta_0(n-i+1)\varepsilon_{i,k,7},\\
&H_{i1} = \bar{c}_i + \bar{H}_{i1},\ H_{i2}' = \bar{\bar{c}}_i + \bar{H}_{i2}'.
\end{aligned} \tag{47}$$

By Eq.(43) and Lemma 2, the following inequalities hold:

$$-\varepsilon_i^3\tau_i\frac{\partial z_i^*}{\partial\hat{\Theta}} \leqslant \sum_{k=1}^{i-1}\frac{1}{4}\varepsilon_{i,k,12}\varepsilon_k^4 + \sqrt{1 + (\varepsilon_i^3 H_{i1}\frac{\partial z_i^*}{\partial\hat{\Theta}})^2}\varepsilon_i^4 + \sum_{k=1}^{i-1}\frac{3}{4}\varepsilon_{i,k,12}^{-\frac{1}{3}}(\sqrt{1 + (\varepsilon_k^3 H_{k1}\frac{\partial z_i^*}{\partial\hat{\Theta}})^2})^{\frac{4}{3}}\varepsilon_i^4, \tag{48}$$

$$-\sum_{k=2}^{i-1}\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}}H_{i1}\varepsilon_i^4 \leqslant \sum_{k=2}^{i-1}\sqrt{1 + (\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}}H_{i1})^2}\varepsilon_i^4. \tag{49}$$

Substituting Eqs.(48) and (49) into Eq.(43) results in

$$\mathcal{L}V_i \leqslant -(\lambda\eta_0 - \sum_{j=2}^{i}\beta_j)x_0^4 - \Theta\sum_{j=1}^{i}(\bar{c}_j - \sum_{k=j+1}^{i}\bar{\rho}_{kj})\varepsilon_j^4 - \sum_{j=1}^{i}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i}\bar{\bar{\rho}}_{kj})\varepsilon_j^4 + d_i(t)\varepsilon_i^3(z_{i+1} - z_{i+1}^*) + \lambda\varepsilon_i^3 z_{i+1}^* + \lambda\{\frac{1}{\lambda}(\sqrt{1+\hat{\Theta}^2}H_{i1} + H_{i2})\}\varepsilon_i^4 + (\tilde{\Theta} + \sum_{k=2}^{i}\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}})\{\tau_i - \dot{\hat{\Theta}}\}, \tag{50}$$

where

$$\bar{\bar{\rho}}_{ik} = \bar{\bar{\rho}}_{ik}' + \frac{1}{4}\varepsilon_{i,k,12},\ k = 1,\cdots,i-1, \tag{51}$$

$$H_{i2} = \sqrt{1 + (\varepsilon_i^3 H_{i1}\frac{\partial z_i^*}{\partial\hat{\Theta}})^2} + \sum_{k=2}^{i-1}\sqrt{1 + (\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}}H_{i1})^2} + \sum_{k=1}^{i-1}\frac{3}{4}\varepsilon_{i,k,12}^{-\frac{1}{3}}(\sqrt{1 + (\varepsilon_k^3 H_{k1}\frac{\partial z_i^*}{\partial\hat{\Theta}})^2})^{\frac{4}{3}} + H_{i2}'. \tag{52}$$

Choosing the virtual smooth control z_{i+1}^* as follows:

$$\begin{cases} z_{i+1}^*(x_0, \bar{z}_i, \hat{\Theta}) = -\alpha_i(x_0, \bar{z}_i, \hat{\Theta})\varepsilon_i, \\ \alpha_i(x_0, \bar{z}_i, \hat{\Theta}) = \dfrac{1}{\lambda}(\sqrt{1+\hat{\Theta}^2}H_{i1} + H_{i2}), \end{cases} \tag{53}$$

and substituting Eq.(53) into Eq.(50), one can obtain

$$\mathcal{L}V_i \leqslant -(\lambda\eta_0 - \sum_{j=2}^{i}\beta_j)x_0^4 - \Theta\sum_{j=1}^{i}(\bar{c}_j - \sum_{k=j+1}^{i}\bar{\rho}_{kj})\varepsilon_j^4 - \sum_{j=1}^{i}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i}\bar{\bar{\rho}}_{kj})\varepsilon_j^4 + d_i(t)\varepsilon_i^3(z_{i+1} - z_{i+1}^*) + (\tilde{\Theta} + \sum_{k=2}^{i}\varepsilon_k^3\frac{\partial z_k^*}{\partial\hat{\Theta}})\{\tau_i - \dot{\hat{\Theta}}\}. \tag{54}$$

In the end, when $i = n$, $z_{n+1} = z_{n+1}^* = u$ is the actual control. By Choosing the actual control law and the adaptive laws for $\hat{\Theta}$

$$\begin{cases} u(x_0, \bar{z}_n, \hat{\Theta}) = -\alpha_n(x_0, \bar{z}_n, \hat{\Theta})\varepsilon_n, \\ \dot{\hat{\Theta}} = \tau_n = \sum_{i=1}^{n}H_{i1}\varepsilon_i^4, \end{cases} \tag{55}$$

where α_n and $H_{i1}(i = 1,\cdots,n)$ are smooth functions. We choose the nth Lyapunov candidate function

$$V_n(x_0, \bar{\varepsilon}_n, \hat{\Theta}) = \frac{1}{4}x_0^4 + \sum_{k=1}^{n}\frac{1}{4}\varepsilon_k^4 + \frac{1}{2}\hat{\Theta}^2, \tag{56}$$

where $\varepsilon = \varepsilon_n = (\varepsilon_1,\cdots,\varepsilon_n)^{\mathrm{T}}$. One can easily get

$$\mathcal{L}V_n \leqslant -(\lambda\eta_0 - \sum_{j=2}^{n}\beta_j)x_0^4 - \sum_{j=1}^{n}(\bar{\bar{c}}_j - \sum_{k=j+1}^{n}\bar{\bar{\rho}}_{kj})\varepsilon_j^4 - \Theta\sum_{j=1}^{n}(\bar{c}_j - \sum_{k=j+1}^{n}\bar{\rho}_{kj})\varepsilon_j^4. \tag{57}$$

4 Switching control stability

In Section 3, we have considered the case of $x_0(t_0) \neq 0$. The controllers Eqs.(7) and (55) for system (1) are given. Now we turn to the case of $x_0(t_0) = 0$. If the initial is zero, one can choose an open loop control $u_0 = u_0^* \neq 0$ to drive the state x_0 away from zero. So there exists $t_{\mathrm{s}}^* > 0$ such that $x_0(t_{\mathrm{s}}^*) \neq 0$. After that, controllers u_0 and u given in Eqs.(7) and (55) can be used. Based on the above analysis, we give the main results of this paper.

Theorem 1 Suppose that Assumptions 1 and 2 hold. If the following switching control procedure is applied to system (1):

i) When the initial state belongs to

$$\{(x_0(t_0), x(t_0)) \in \mathbb{R}^{n+1} | x_0(t_0) \neq 0\},$$

we design control inputs u_0 and u in form Eqs.(7) and (55), respectively;

ii) When the initial state belongs to

$$\{(x_0(t_0), x(t_0)) \in \mathbb{R}^{n+1} | x_0(t_0) = 0\}.$$

If $t \in [t_0, t_s^*)$, one can choose the control law $u_0 = u_0^*$ and $u = u^*$; If $t \in [t_s^*, +\infty)$, at the time $t = t_s^*$, we switch the control inputs u_0 and u into Eqs.(7) and (55), respectively.

Then, for any initial conditions in the state space, system (1) will be almost asymptotically stabilized in probability at the equilibrium and specifically, the states are globally asymptotically regulated to zero in probability.

Proof Firstly, we consider the case that the initial state belongs to $\{(x_0(t_0), x(t_0)) \in \mathbb{R}^{n+1} | x_0(t_0) \neq 0\}$. One can obtain that x_0 is asymptotically stable and will not be zero. One can choose $\bar{c}_j$ and $\bar{\bar{c}}_j$ such that $\mathcal{L}V_n < 0$. From Lemma 1 and Eq.(57), one gets signals $\varepsilon_1, \cdots, \varepsilon_n$ are globally asymptotically regulated to zero in probability and bounded in probability, signal $\hat{\Theta}$ is bounded in probability also. From $\hat{\Theta}$ are bounded in probability and Eq.(11), it is easy to see that z_1 and z_2^* are bounded and globally stable in probability. By $z_2 = \varepsilon_2 + z_2^*$, we have z_2 is bounded in probability and converges to zero in probability. With the similar method, the same results also hold for $z_3, \cdots, z_n$. So $z_1, \cdots, z_n$ globally asymptotically regulated to zero in probability and bounded in probability. As a result of Eq.(8), one gets $x_0, x_1, \cdots, x_n$ are globally asymptotically converge to zero in probability and all bounded in probability.

Secondly, when the initial state belongs to

$$\{(x_0(t_0), x(t_0)) \in \mathbb{R}^{n+1} | x_0(t_0) = 0\},$$

we use the constant control $u_0 = u_0^* \neq 0$ in order to drive x_0 far away from the origin. Meanwhile, by application of the design procedure proposed in Section 3, we construct a controller $u = u^*$, which guarantees that all the signals are bounded in probability during $[t_0, t_s^*)$. Then, in view of $x_0(t_s^*) \neq 0$, the switching control strategy is applied to system (1) at the time instant $t_s^* > 0$.

5 Simulation example

Consider the following system:

$$\begin{cases} \mathrm{d}x_0 = d_0(t)u_0\mathrm{d}t, \\ \mathrm{d}x_1 = d_1(t)x_2u_0\mathrm{d}t + x_1\theta\mathrm{d}t + x_1\Sigma(t)\mathrm{d}\omega, \\ \mathrm{d}x_2 = d_2(t)u\mathrm{d}t + x_2\Sigma(t)\mathrm{d}\omega, \end{cases} \tag{58}$$

where $d_0(t) = 1.5$, $d_1(t) = 1 + 0.1\sin t$, $d_2(t) = 0.9 + 0.2\sin t$, $\Sigma(t) = 1 + 0.125\sin t$.

One can easily obtain that Proposition 1 is satisfied with $n = 2$, $\gamma_{11} = \gamma_{12} = 1$, $c_1(\theta) = \sqrt{1+\theta^2}$, $\gamma_{21} = 0$, $\gamma_{22} = \gamma_{23} = 1$. In simulation, choose $\theta = 1$. Obviously, there exist positive constants $\lambda = 0.7$ and $\mu = 1.6$ to satisfy $\lambda \leqslant d_i(t) \leqslant \mu (i = 0, 1, 2)$, $\eta_1 = \eta_2 = 1.6$ which satisfy Assumption 2. According to Eq.(7), one gets the control

$$u_0 = -\eta_0 x_0. \tag{59}$$

Defining $\bar{\theta} = |\Sigma(t)\Sigma^{\mathrm{T}}(t)|_\infty$ and $\Theta = \max\{c_1(\theta), \bar{\theta}\}$. According to Eq.(21), it is easy to obtain $H_{11} = \bar{c}_1 + \gamma_{11} + \dfrac{3}{2}\gamma_{12}^2$, $\tau_1 = H_{11}\varepsilon_1^4$, $H_{12} = \bar{\bar{c}}_1 + \lambda\mu$, where $\bar{c}_1 > 0$ and $\bar{\bar{c}}_1 > 0$ are design parameters which will be chosen later. Thus, by Eq.(22), one gets the virtual smooth control z_2^*,

$$\begin{cases} z_2^*(x_0, z_1, \hat{\Theta}) = -\alpha_1(x_0, z_1, \hat{\Theta})\varepsilon_1, \\ \alpha_1(x_0, z_1, \hat{\Theta}) = \dfrac{1}{\lambda}(\sqrt{1+\hat{\Theta}^2}H_{11} + H_{12}). \end{cases} \tag{60}$$

From Eq.(59), we have $\dfrac{\partial z_2^*}{\partial z_1} = -\alpha_1$, $\dfrac{\partial^2 z_2^*}{\partial z_1^2} = 0$, $\dfrac{\partial z_2^*}{\partial x_0} = 0$, $\dfrac{\partial z_2^*}{\partial \hat{\Theta}} = -\dfrac{\hat{\Theta}H_{11}}{\lambda\sqrt{1+\hat{\Theta}^2}}\varepsilon_1$. Next, let $i = 2$ in Section 3 and $\varepsilon_{2,i,j} = 1 (i = 0, 1;\ j = 1, \cdots, 10)$, one can obtain

$$\beta_2 = \frac{1}{4}\eta_0\mu,\ \bar{\rho}_{21} = \frac{1}{2},\ \bar{\bar{\rho}}_{21} = \mu + \frac{1}{4}\eta_0\mu + 2,$$

$$H_{21} = \bar{c}_2 + \frac{3}{4}\gamma_{21}^{\frac{4}{3}} + \gamma_{21} + \frac{3}{4}(\sqrt{1+(\gamma_{11}\frac{\partial z_2^*}{\partial z_1})^2})^{\frac{4}{3}},$$

$$\begin{aligned} H_{22} = {} & \bar{\bar{c}}_2 + \frac{1}{4}\mu + \frac{3}{4}\mu(\sqrt{1+(\alpha_1\frac{\partial z_2^*}{\partial z_1})^2})^{\frac{4}{3}} + \frac{3}{2}\gamma_{23}^4 + \\ & 3\gamma_{23}^2 + \frac{3}{4}\eta_0\mu(\sqrt{1+(\frac{\partial z_2^*}{\partial z_1})^2})^{\frac{4}{3}} + \\ & \frac{3}{4}\eta_0(\sqrt{1+(\frac{\partial z_2^*}{\partial x_0})^2})^{\frac{4}{3}} + \frac{3}{4}\sqrt{1+(\gamma_{24}\frac{\partial^2 z_2^*}{\partial z_1^2})^2} + \\ & \sqrt{1+(\varepsilon_2^3 H_{21}\frac{\partial z_2^*}{\partial \hat{\Theta}})^2}\varepsilon_2^4 + \mu\sqrt{1+(\frac{\partial z_2^*}{\partial z_1})^2} + \\ & \frac{3}{4}(\sqrt{1+(\varepsilon_1^3 H_{11}\frac{\partial z_2^*}{\partial \hat{\Theta}})^2})^{\frac{4}{3}}\varepsilon_2^4, \end{aligned}$$

$$\tau_2 = \tau_1 + H_{21}\varepsilon_2^4,$$

and the updating law for Θ and the smooth control u as follows:

$$\begin{cases} \dot{\hat{\Theta}} = H_{11}\varepsilon_1^4 + H_{21}\varepsilon_2^4,\ u = -\alpha_2(x_0, z_1, \hat{\Theta})\varepsilon_2, \\ \alpha_2(x_0, z_1, \hat{\Theta}) = \dfrac{1}{\lambda}(\sqrt{1+\hat{\Theta}^2}H_{21} + H_{22}), \end{cases} \tag{61}$$

where $\bar{c}_2 > 0$ and $\bar{\bar{c}}_2 > 0$ are design parameters.

In order to satisfy $\mathcal{L}V_2 \leqslant 0$, we choose $\eta_0 = 3$, $\bar{c}_1 = 0.6$, $\bar{\bar{c}}_1 = 4.2$, $\bar{c}_2 = \bar{\bar{c}}_2 = 0.1$, and the initial values $x_0(0) = 0.3$, $x_1(0) = 0.09$, $x_2(0) = -1.3$, $\hat{\Theta}(0) = 0.8$. Fig.1 gives the response of the closed-loop system consisting of Eqs.(58)– (60), from which, the effectiveness of the controller is demonstrated.

Remark 5 From Fig.1–3, the effectiveness of the controller is demonstrated. However, by Eq.(61), one can obtain that α_2 has great effect on the controller u, that is, the value of u is determined by that of $\hat{\Theta}$, H_{21}. In simulation, one can choose the parameters $0 < \varepsilon_{2,i,j} \neq 1 (i = 0, 1;\ j = 1, \cdots, 10)$ and the stochastic non-holonomic system (1) is globally asymptotically stable, also.

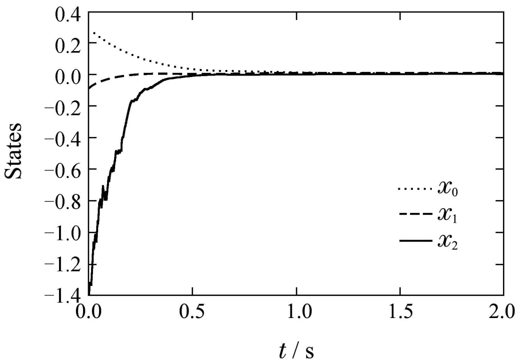

Fig. 1 The responses of states x_0, x_1 and x_2 with respect to time

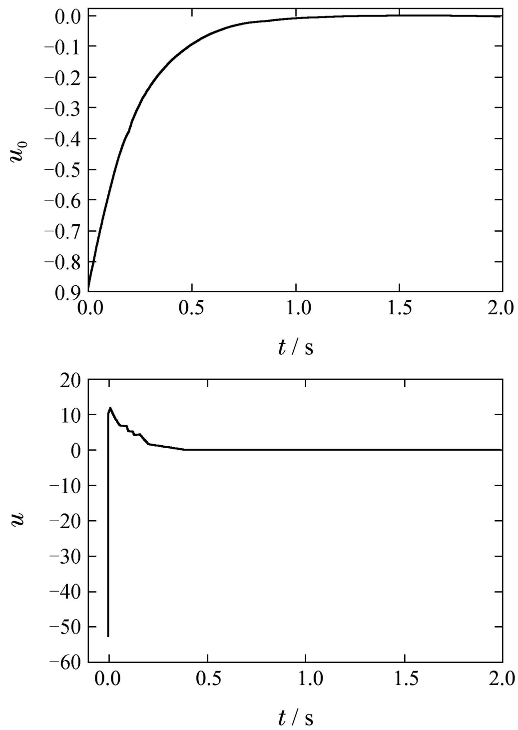

Fig. 2 The responses of controllers u_0 and u with respect to time

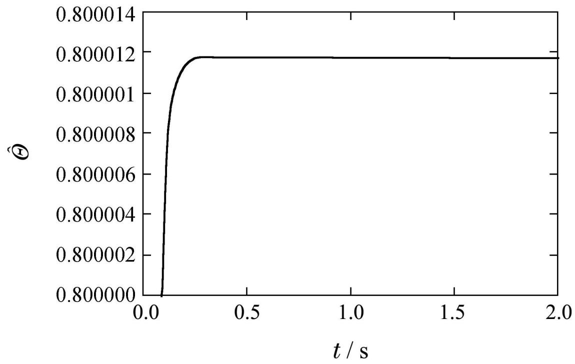

Fig. 3 The responses of estimate parameter $\hat{\Theta}$ with respect to time

6 Conclusions

This paper studies the adaptive state-feedbacks stabilization of stochastic nonholonomic systems with unknown parameters. A recursive adaptive state-feedback backstepping controllers is designed. A switching control strategy for the original system is given which can guarantee the closed-loop system is almost asymptotically stabilized at the origin in probability.

There are some remaining problems to be discussed. For example, how to design the controller for the stochastic nonholonomic systems when the first subsystem is stochastic differential equation with uncertain parameters, especially, in the visual serving feedback control of nonholonomic moving mobile robots.

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Appendix

i) We only prove Eq.(13a). The proofs of Eqs.(13b)–(13d) are similar to that of Eq.(13a). From Assumption 1, Remark 1, Eqs.(8) and (11), it is easy to see

$$|\frac{f_i}{u_0^{n-i}}(x_0,\bar{x}_i,\theta)| \leqslant \frac{\sum\limits_{k=1}^{i}|x_k|}{|u_0^{n-i}|}\bar{\gamma}_i(x_0,\bar{x}_i,\theta) \leqslant \sum_{k=1}^{i}|z_k|\tilde{\gamma}_i(x_0,\bar{x}_i)c_i(\theta) \leqslant \sum_{k=1}^{i}|\varepsilon_k|(1+\sum_{j=1}^{i-1}\alpha_j)\tilde{\gamma}_i c_i(\theta) = (|\varepsilon_1|+\cdots+|\varepsilon_i|)\gamma_{i1}(x_0,\bar{x}_i)c_i(\theta).$$

ii) In the following, we will prove the inequality Eq.(30). The proofs of Eqs.(29)(31)–(37) and (48)–(49) are similar to that of Eq.(30). By Lemma 2, Assumption1, 2, Proposition 1, Eqs.(8) and (11), one can obtain

$$\varepsilon_i^3\frac{f_i}{u_0^{n-i}} \leqslant |\varepsilon_i|^3\sum_{k=1}^{i-1}|\varepsilon_k|\gamma_{i1}c_i(\theta)+\varepsilon_i^4\gamma_{i1}c_i(\theta) \leqslant \sum_{k=1}^{i-1}(\frac{1}{4}\varepsilon_{i,k,2}\varepsilon_k^4+\frac{3}{4}\varepsilon_{i,k,2}^{-\frac{1}{3}}\gamma_{i1}^{\frac{4}{3}}\varepsilon_i^4)c_i(\theta)+\gamma_{i1}\varepsilon_i^4c_i(\theta) \leqslant \varTheta\sum_{k=1}^{i-1}\frac{1}{4}\varepsilon_{i,k,2}\varepsilon_k^4+\varTheta\{\sum_{k=1}^{i-1}\frac{3}{4}\varepsilon_{i,k,2}^{-\frac{1}{3}}\gamma_{i1}^{\frac{4}{3}}+\gamma_{i1}\}\varepsilon_i^4.$$

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