

# 采用惯性测量单元的移动机器人轨迹跟踪方法研究

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**摘要:** 对于非完整移动机器人的轨迹跟踪控制已有很多方法提出, 但是这些方法或者是基于动力学模型或者是采用复杂的运动学模型, 对于缺少强大计算设备且需要实时控制的工程应用是不适合的. 本文针对非完整移动机器人提出了一种基于比例微分(proportional-differential, PD)控制器的实时轨迹跟踪控制方法. 该方法运行在40 MHz的嵌入式控制器上的控制周期只有1~2 ms. 通过将一个用于直流电机控制的非线性PID速度控制器与提出的轨迹控制器进行集成, 实现了一个轮式移动机器人的运动控制. 机器人轨迹跟踪实验系统中采用微机电系统(micro electro-mechanical system, MEMS)惯性测量单元检测轮式移动机器人的偏航角, 实验结果验证了提出方法的有效性.

**关键词:** 轨迹跟踪; PD控制器; 轮式移动机器人  
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## Path tracking of a mobile robot using inertial measurement unit

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**Abstract:** Many approaches for trajectory tracking of nonholonomic mobile robots have been proposed. Most of them are designed based on the control of robot dynamics or adopt the complicated kinematics control, which are not suitable for real-time engineering applications without powerful computing devices. A real-time tracking controller based on PD controller is presented for nonholonomic mobile robot. This algorithm holds a time cost of only 1~2 ms for each control loop in a 40 MHz embedded controller. A nonlinear-PID based velocity controller for DC driving motors is integrated with the proposed tracking controller to fulfill the path tracking control of a wheeled mobile robot (WMR). To estimate the yaw angle of WMR, a low-cost micro electro-mechanical system (MEMS) measurement unit is employed in the experimental setup. The experimental results show the effectiveness of the proposed method.

**Key words:** trajectory tracking; PD controller; wheeled mobile robot

### 1 Introduction

Because of the complexity brought by nonholonomic constraints, the control of mobile robots has attracted considerable attention in recent years. Several controllers were proposed for mobile robots with nonholonomic constraints, and posture stabilization and trajectory tracking were mainly discussed. The purpose of posture stabilization is to stabilize the robot to a reference point, while the aim of trajectory tracking is to let the robot follow a reference trajectory. With the stabilization problem, trajectory tracking is a more practical issue for mobile robots. Many researchers have studied the controllers for trajectory tracking of nonholonomic mobile robots, which can be divided into two types: one utilizes the kinematic trajectory tracking controller<sup>[1-10]</sup> to handle only tracking issue, while the other integrates the kinematic trajectory tracking controller with the dynamic controller of the wheeled mobile robot (WMR)<sup>[11]</sup>.

For the kinematic control, there are trajectory tracking methods such as linearization kinematic models<sup>[1]</sup>, input-output linearization<sup>[2]</sup>, feedback control<sup>[3]</sup>, dynamic linearization<sup>[4-5]</sup>, recursive technique<sup>[6]</sup>, sliding-mode<sup>[7]</sup>, and backstepping approach<sup>[8-9]</sup>, fuzzy control<sup>[12]</sup>. An interesting experimental comparison between control laws proposed in literature has been presented in<sup>[11]</sup>. Other researchers dealt with the backstepping control architecture<sup>[13]</sup>, the hybrid of the kinematic control and the dynamic controller, and put forward trajectory tracking methods. The backstepping control architecture is frequently applied to practical applications recently<sup>[13-23]</sup>. To deal with the un-modeled disturbance and parameter uncertainty of the dynamics, neural networks<sup>[14-16]</sup>, adaptive control<sup>[17-19]</sup>, and sliding-mode control (SMC)<sup>[20-23]</sup> are widely adopted. In literature, a few experimental setups and results can be found; most of them<sup>[11, 16, 23-24]</sup> consist of a vision system for pose measurement, a host PC for the calculation of the con-

trol law, and a WMR with a low level controller for executing the control actions.

Although the kinematic controllers are easy to be implemented, most of them are only validated by simulations. The “perfect velocity tracking” assumption<sup>[1]</sup> of the kinematics controllers may not hold in practice. Other controllers that have considered the dynamics can achieve better motion performance, and the effects of disturbance and parameter uncertainty can be eliminated. However, the complexity of such controllers requires a high computation cost.

This paper focuses its attention to solve the problem of path tracking, guaranteeing the effectiveness and the real-time characteristic simultaneously. An important difference between path following and path tracking lies in the time dependence: path following is to pursue waypoints lying on the desired path, while path tracking is to track a time-parameterized reference path. Among the recent papers addressing path tracking control, [25] extended the preliminary work<sup>[26]</sup> by concerning the next bend ahead of the vehicle and combining a velocity controller, while [27] presented a higher level waypoints by tracking controller taking position, heading, and current velocity of the robot into the calculation module. The technique proposed in [28] tracks piece-wise linear paths which are an approximation of the feasible smooth reference path. Both [26] and [27] presented their experimental results.

Being different from those above controllers, the proposed tracking controller in this paper considers a human-like tracker-guide mode and implements it in a PD controller. Thus, it is more suitable for engineering applications. Besides, unlike those existing experimental setups, we adopt a self-designed WMR with low performance motors, and the control algorithm is implemented in a 40 MHz ARM chip-based control board instead of a powerful PC.

The organization of the remaining part of the paper is as follows. Section II is a concise description of the kinematical modeling of a differentially steered WMR. In Section III, the design of the proposed tracking controller and the low-level velocity controller are introduced. Section IV gives the detail of yaw angle

estimation of WMR using a low-cost inertial measurement unit (IMU). In Section V, real time experimental results of the proposed controller are presented for several trajectories. Section VI draws the conclusions.

## 2 Modeling

Consider a typical model of the differential-drive WMR consisting of a vehicle with two driving wheels mounted on the same axis and a free front wheel, while two DC motors provide necessary torques for the motion and orientation. With a symmetry structure, the barycenter of the WMR is assumed to be located in the middle of two driving wheels. The following is the kinematic model for the wheeled mobile robot with nonholonomic constraints of pure rolling without slipping phenomenon.

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} v \\ \omega \end{pmatrix} = \mathbf{J}(\theta) \cdot \mathbf{V}, \quad (1)$$

where  $\mathbf{J}(\theta) \in \mathbb{R}^{3 \times 2}$  and  $\mathbf{V} \in \mathbb{R}^2$  are the full rank velocity transformation matrix and velocity vector, respectively.  $v$  is the linear velocity and  $\omega$  is the angular velocity.  $(x, y)$  and  $(\dot{x}, \dot{y})$  are the actual position and translational velocity of the centroid of WMR in the inertial coordinate system, respectively.  $\theta$  is the angle between  $X$  axis and  $X_c$  axis and represents the heading direction, and  $\dot{\theta}$  is the angular velocity.

The angular velocities of the right actuated wheel and the left actuated wheel are denoted by  $\omega_r$  and  $\omega_l$ , respectively. Velocity conversion matrix between  $(\omega_r, \omega_l)$  and  $(v, \omega)$  is given by

$$\begin{pmatrix} \omega_r \\ \omega_l \end{pmatrix} = \begin{pmatrix} 1/r & d/r \\ 1/r & -d/r \end{pmatrix} \cdot \begin{pmatrix} v \\ \omega \end{pmatrix}, \quad (2)$$

where  $d$  is the half of the width of mobile robot, and  $r$  denotes the radius of the wheel.

## 3 Path tracking controller

In Fig.1, we present a path tracking strategy based on PD control method. This kinematics controller consists of a tracking controller and a low-level velocity controller. The tracking controller is designed based on PD control strategy while the low-level velocity controller is implemented based on a nonlinear PID controller.

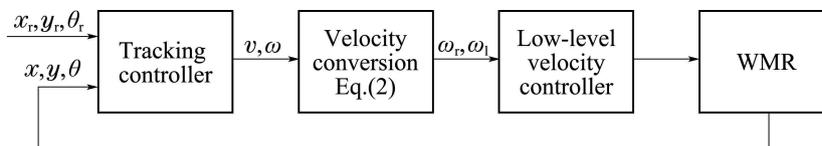


Fig. 1 Kinematics controller for the path tracking of WMR

As shown in Fig.2, a reference robot denoted by the configurations  $(x_r, y_r, \theta_r)$  moves along the desired path with a specified velocity  $v_r$ ; the real robot denoted by the configurations  $(x, y, \theta)$  aims to catch up with the reference one. There is an important concept called ‘sight-line-angle’ ( $e_o$  in Fig.2) which is used in this path tracking strategy for replacing the heading angle error  $\theta_r - \theta$  in most known strategies. To reduce the distance error  $e_p$  and the ‘sight-line-angle’  $e_o$ , the PD control law is designed as follows:

$$\begin{cases} v = k_{pp}e_p + k_{dp}\dot{e}_p, \\ \omega = k_{po}e_o + k_{do}\dot{e}_o \end{cases} \quad (3)$$

with

$$\begin{cases} e_p = \sqrt{(x_r - x)^2 + (y_r - y)^2}, \\ e_o = \theta - \text{angle}[(x_r - x) + i(y_r - y)], \end{cases} \quad (4)$$

where the function  $\text{angle}[\cdot]$  denotes the argument of the complex number,  $v$  and  $\omega$  represent the velocity input and angular velocity input respectively;  $k_{pp}, k_{dp}, k_{po}$  and  $k_{do} > 0$  are the control gains which need to be determined by using the trial and error method.

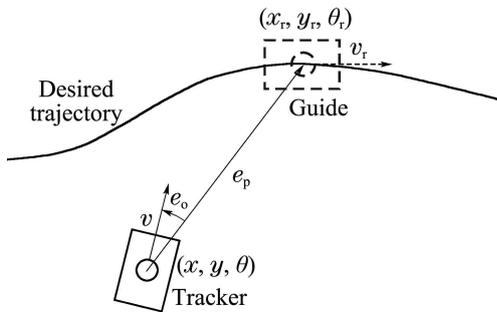


Fig. 2 Diagram of path tracking scheme

In equation (3), we decouple the control problem into two parts: one is to reduce the distance between the real robot and the reference robot, the other is to reduce the ‘sight-line-angle’. That strategy is similar to the human tracking mode according to our life experience. Once the distance and the ‘sight-line-angle’ are eliminated simultaneously, the tracker robot can catch up with the guide one.

From Fig.1, it can be seen that the tracking controller requires angular velocities of two actuated wheels as control inputs of the low-level velocity controller. A nonlinear PID control strategy is implemented to design the velocity controller. Several known amelioration technologies, including incomplete differential, variable-integrator and nonlinear proportion are applied. Combing with those technologies, we design the nonlinear PID controller as follows:

$$\begin{cases} K_P = k_p + b_p \left[ 1 - \frac{1}{\cosh(a_p \cdot \text{err}(k))} \right], \\ K_I = k_i \left[ 1 - \tanh \frac{a_i |\text{err}(k)|}{y_{\max}} \right], \\ u_D(k) = \alpha u_D(k-1) + k_d (1 - \alpha) (\text{err}(k) - \text{err}(k-1)), \\ u(k) = K_P \cdot \text{err}(k) + K_I \cdot \sum \text{err}(k) + u_D(k), \end{cases} \quad (5)$$

where  $k_p, k_i$  and  $k_d$  denote the parameters of a traditional PID controller,  $b_p, a_p, a_i > 0$  are parameters to be adjusted via the trial and error method,  $y_{\max}$  is the maximum output of the system,  $0 < \alpha < 1$  should be tuned in applications,  $\text{err}(k)$  is the feedback error at the  $k$ th sample time,  $u(k)$  represents the current control output.

For the tuning of control parameters in the nonlinear PID controller, we adopt the trial and error method. First, we choose values of  $k_p$  and  $k_i$  based on our experience; If the control effect is not satisfied, try to change those two values until the dynamic response of system meet the performance requirement. In case of the derivation action, the tuned  $k_p$  and  $k_i$  should be decreased a bit and then repeat the trial and error until the optimum values of  $k_p, k_i$  and  $k_d$  are found. Moreover, the other carefully adjusted control parameters,  $a_p, b_p, a_i, \alpha$  can guarantee better control effects but would not have a great impact on the control effectiveness. Then in tuning, those parameters can be chosen in a wide range.

In the velocity controller, the velocity estimation is a critical issue when using a low-quality position measurement device, i.e., in our developed WMR, the photoelectric encoder of the driving motor has only 16 lines. The differential signal obtained by the backward difference (BD) method is very noisy which limits the overall performance<sup>[29–30]</sup>. In view of that case, a tracking differentiator (TD) method<sup>[31]</sup> is implemented to provide a high-quality differential signal for more effective and robust performance in the presence of measurement noise. The viability for practical applications has been demonstrated via simulations and experiments<sup>[29–30, 32–33]</sup>.

For a boundary integrable function  $r(t)$ , system (6) can be used as a high-performance TD to provide two signals  $r_1(t)$  and  $r_2(t)$ :  $r_1(t) \rightarrow r(t)$  and  $r_2(t) \rightarrow \dot{r}(t)$ , respectively<sup>[31]</sup>:

$$\begin{cases} r_1(k+1) = r_1(k) + h \cdot r_2(k), \\ r_2(k+1) = \\ r_2(k) + h \cdot \text{fst}(r_1(k) - r(k), r_2(k), \delta, h), \end{cases} \quad (6)$$

where  $h$  is the sampling step, and  $\delta$  is a velocity



the desired forward velocity is specified to be 0.1 m/s. The initial posture of the WMR is  $\mathbf{q}(0) = [x(0) \ y(0) \ \theta(0)]^T = [0 \ 0 \ 0]^T$ .

Table 2 Control gains of the controllers

	$k_p, k_i, k_d$	1.2, 0.25, 1.0
Nonlinear PID controller	$a_p, b_p$	0.8, 1.0
	$a_i$	1.6
	$\alpha$	0.8
	$h$	0.02
Tracking differentiator	$\delta$	5500

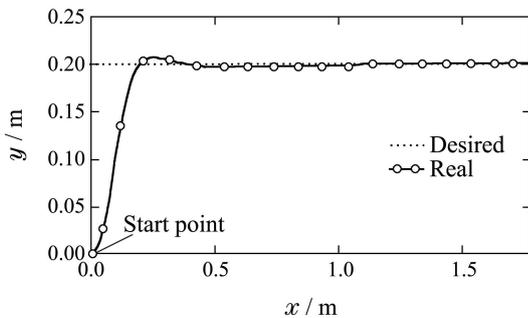


Fig. 5 Experimental result for the straight line tracking

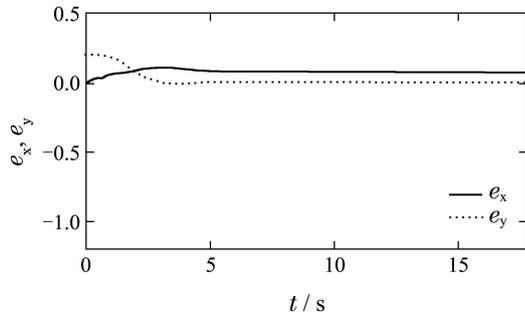


Fig. 6 Position errors for the straight line tracking

1) Results for the straight line path. In this experiment, the WMR follows the trajectory as denoted by the dashed line in Fig.5. Although a big initial position error exists, the robot quickly reduces this error under the control with a very small overshoot less than 0.01 m. As the robot catches up with the desired straight line trajectory, the orientation of the robot also approximates the desired value  $\theta_r = 0$ . As it can be seen in Fig.6, the position error along  $X$  axis,  $e_x = x_r - x$ , converges to a steady value less than 0.08 m, while the position error along  $Y$  axis,  $e_y = y_r - y$ , converges to 0. In practical applications, a tracking error less than 0.08 m is acceptable in real exploration applications.

2) Results for the circle path: The desired circle trajectory can be seen from Fig.7, the radius of which is 0.5 m. With the proposed controller (3), the robot can quickly catch up the desired motion in 4 s without overshoot even though an initial position error exists.

As can be seen in Fig.8, the steady position error is less than 0.04 m.

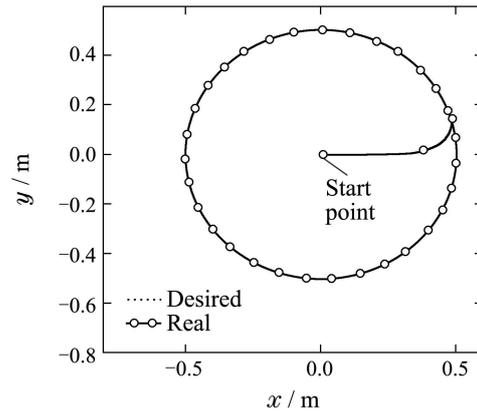


Fig. 7 Experimental result for the circle tracking

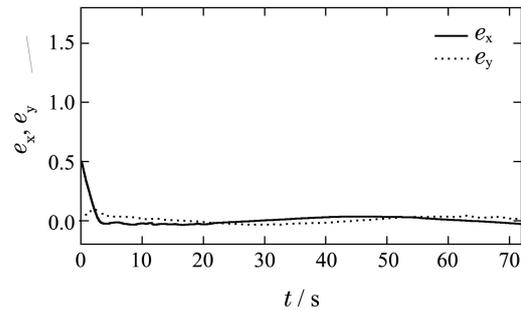


Fig. 8 Position errors for the circle tracking

## 6 Conclusions

A path tracking strategy for the nonholonomic mobile robot based on a PD controller is proposed. It mimics the humanlike 'tracker and guide' mode. Unlike known tracking methods, the proposed strategy takes the distance and the 'sight-line-angle' as the control inputs. Besides, to achieve the low-level velocity control, we also implement a nonlinear PID controller. For the estimation of the yaw angle of robot, a low-cost IMU is employed in the experimental setup. Two basic Path tracking experiments verify the effectiveness of the proposed tracking strategy and the low-level velocity controller. Of course, more engineering application experiments are required to confirm the practical value of the proposed approach.

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